MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu.

Department of Mathematics
Question Bank - Academic Year (2021-22)

Course Code & Course Name : 19BSS23 - Transforms and Partial Differential Equations

Name of the Faculty : M.Nazreen Banu

Year/Sem/Sec : II / III / AI &DS

Unit-I : Fourier Transform
Part-A (2 Marks)

1. State Fourier integral theorem.
2. Write down the Fourier transform pair.
3. If F(s) is the Fourier transform of f(x) show that $F[f(x-a)] = e^{ias}F(s)$.
4. State and prove the change of scale of property of Fourier transform.
5. Find the Fourier transform of $f(x) = \begin{cases} e^{ikx}, a < x < b \\ 0 \ , x \leq a \ and \ x > b \end{cases}$
6. State the convolution theorem for Fourier transform.
7. State the parseval's identity on Fourier transform
8. Find the Fourier sine transform of $f(x) = e^{-ax}$, a>0
9. Find the Fourier sine transform of $\frac{1}{x}$
10. Find the Fourier Cosine transform of $e^{-ax}, x \geq 0$

Part-B (16 Marks)

1. Find the Fourier transform of $f(x) = \begin{Bmatrix} a-|x| & in\ x < a \\ 0, & in\ \ x > a \end{Bmatrix}$ and hence evaluate (16)

   $(i)\ \int_0^\infty \left(\frac{sinx}{x}\right)^2 dx$ (ii) $\int_0^\infty \left(\frac{sinx}{x}\right)^4 dx$

2. Find the Fourier transform of $f(x) = \begin{Bmatrix} 1-|x| & in\ |x| < 1 \\ 0, & in\ |x| > 1 \end{Bmatrix}$ and hence (16)

   evaluate $(i) \int_0^\infty \left(\frac{sinx}{x}\right)^2 dx$ (ii) $\int_0^\infty \left(\frac{sinx}{x}\right)^4 dx$

3. Find the Fourier transform of $f(x) = \begin{Bmatrix} a^2 - x^2 & in\ |x| < a \\ 0, & in\ |x| > a \end{Bmatrix}$. Hence deduce that (16)

   (i) $\int_0^\infty \frac{sinx - xcos\ x}{x^3} dx = \frac{\pi}{4}$ (ii) $\int_0^\infty \frac{xcos\ x - sinx}{x^3} cos\left(\frac{x}{2}\right) dx = \frac{3\pi}{16}$ (iii) $\left(\frac{sinx - xcosx}{x^3}\right)^2 = \frac{\pi}{15}$

4. (i) Find the Fourier transform of $e^{-a^2x^2}$ for any a> 0 and hence show that $e^{-x^2/2}$ is self-Reciprocal under Fourier transform. (16)

   (ii) Find the Fourier Cosine transform of $e^{-a^2x^2}$ and hence, show that $e^{\frac{-x^2}{2}}$ is a self (16) reciprocal under Fourier Cosine transform.

5.(i). Using the Fourier Sine and Cosine transforms of $\mathbf{f(x) = e^{-ax}}$ evaluate: $(i)\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ (16)

$(ii)\int_0^\infty \frac{x^2 dx}{(x^2+a^2)^2}$

(8)

(ii) Using the Fourier Sine and Cosine transforms of $\mathbf{f(x) = e^{-ax}}$ and e^{-bx}, evaluate (i) $\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)}$ $(ii)\int_0^\infty \frac{x^2 dx}{(a^2+x^2)(b^2+x^2)}$.

Unit-II : Z - Transforms And Difference Equations

Part-A (2 Marks)

1. Find Z – Transform of a^n
2. Find the Z Transform of *n*.
3. Find Z –Transform of na^n .
4. Find Z – Transform of $cos\frac{n\pi}{2}$
5. Find Z – Transform of $cos\frac{n\pi}{2}$ and $sin\frac{n\pi}{2}$
6. Find Z – Transform of $\frac{1}{n}$.
7. Prove that $Z\left[\frac{1}{n+1}\right] = z.log\left(\frac{z}{z-1}\right)$
8. State Initial value theorem on Z Transform.
9. State Final value theorem on Z Transform.
10. Define Convolution theorem of Z Transform.

Part-B (16 Marks)

1. (i) Find the Z transform of $\left(\frac{1}{(n+1)(n+2)}\right)$. (8)

(ii) Find the Z transform of $a^n cosn\theta$ and $a^n sinn\theta$. (8)

2. (i) State and prove Initial & Final value theorem. (8)

(ii) State and prove Second shifting theorem. (8)

3. (i) Find by Partial fraction Method: (i) $Z^{-1}\left[\frac{z-4}{(z-1)(z-2)^2}\right]$, (ii) $Z^{-1}\left[\frac{z^2}{(z+2)(z^2+4)}\right]$ (8)

(ii) Find (i) $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ (ii) $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ (8) (8)

4. (i) Find $Z^{-1}\left[\frac{z^2}{(z+a)^2}\right]$ and (iv) $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ by Convolution theorem. (8)

(ii) Find by residue method (i) $Z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$, (ii) $Z^{-1}\left[\frac{z^2}{(z^2+4)}\right]$ and (8)

(iii) $Z^{-1}\left[\frac{z}{(z^2-2z+2)}\right]$.

5.(i). Solve the following difference equation by Z-Transform Technique: (8)

$y(n+2)+6y(n+1)+9y(n)=2n, y(0)=0, y(1)=0$

(ii) Solve the following difference equation by Z-Transform Technique: (8)

$y(n+2)+y(n)=2, y(0)=0, y(1)=0$

Unit-III: Fourier Series
Part-A (2 Marks)

1. Write the Dirichlet's conditions on the existence of Fourier series
2. Find the constant term in th expansion of $cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$
3. Give the expression for the Fourier Series co-efficient b_n for the function $f(x)$ defined in $(-2,2)$.
4. Obtain the first term of the Fourier series for the function $f(x)=x^2, (-\pi,\pi)$.
5. Given: $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^2}{n^2}\cos nx$ in $(-\pi,\pi)$, deduce that: $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\ldots\ldots=\frac{\pi^2}{6}$
6. The value of a_n in the Fourier series expansion of $(x)=x^3\ in -\pi<x<\pi$.
7. If $f(x)=2x$ in the interval $(0,4)$, find the value of a_2.
8. Find the root mean square value of $f(x)=x^2$ in $(0,\pi)$.
9. Without finding the value of a_0, a_n & b_n for the function $f(x)=x^2$ in $(0,\pi)$, find the value of $\frac{{a_0}^2}{2}+\sum_{n=1}^{\infty}({a_n}^2+{b_n}^2)$
10. Define harmonic and write the first two harmonic

Part-B (16 Marks)

1. (i) Expand $f(x)=\begin{cases} x & (0,\pi) \\ 2\pi-x & (\pi,2\pi)\end{cases}$ as Fourier series and hence deduce that (8)

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}\ldots\ldots\ldots=\frac{\pi^2}{8}$$

(ii) Find the Fourier series of $f(x)=\begin{cases} \pi+x & -\pi<x<0 \\ \pi-x & 0<x<\pi\end{cases}$ (8)

Hence, find the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots.$

2. (i) Find the Fourier series for $f(x)=x^2$ in $(-\pi.\pi)$ and hence show that (8)

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\ldots\ldots\ldots=\frac{\pi^2}{6}$ (b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\ldots\ldots\ldots=\frac{\pi^2}{12}$

(ii) Expand in Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)$ in $(0, 2\pi)$ (8)

3. (i) Expand $f(x) = |x|$ as a Fourier series in $(-\pi, \pi)$. (8)

(ii) Find the Fourier series for $f(x) = \begin{cases} 0 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$ (8)

4. (i) Find the Fourier series for f(x) = x^2 in $(-\pi, \pi)$ and also prove that (8)

$$1 + \frac{1}{1^4} + \frac{1}{3^4} \ldots\ldots\ldots = \frac{\pi^4}{90}$$

(ii) Find a Fourier series to represent $f(x) = 2x - x^2$ in the range $(0, 3)$ (8)

5.(i). Obtain the half range cosine series $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 < x < 2 \end{cases}$ (8)

(ii) Find Cosine Series for $f(x)$ = x in $(0, \pi)$, hence or otherwise Show that: (8)

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} \ldots\ldots\ldots = \frac{\pi^4}{90}.$$

Unit-IV : Boundary Value Problems

Part-A (2 Marks)

1. Classify the PDE: $3u_{xx} + 4u_{xy} + 3u_y - 2u_x = 0$

2. Classify the PDE: $3u_{xx} + 4u_{xy} + 6u_{yy} - 2u_x + u_y - u = 0$

3. Classify the PDE $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

4. Write down all possible solutions of one dimensional wave equation.

5. In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, what does c^2 stands for?

6. Write down the possible solutions of one dimensional heat equation.

7. The ends A and B of a rod of length 10cm long have their temperature kept at 20°c and 70°c. Find the steady state temperature distribution of the rod.

8. A rod 40cm long with insulated sides has its ends A and B kept at 20°c and 60°c.Find the steady state temperature at a location 15cm from A.

9. A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest in this position, write the boundary conditions.

10. Write all three possible solutions of steady state two-dimensional heat equation.

Part-B (16 Marks)

1. A string is stretched and fastened to two points $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{l}$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t = 0. Find the displacement of any point of the string at a distance x from one end at any time t. (16)

2. A string is stretched and fastened to two points $x = 0$ *and* $x = 2l$ is initially at rest in its equilibrium position if the initial velocity is given by (16)

$$V = \begin{cases} \frac{c}{l}x, & 0 < x < l \\ \frac{c}{l}(2l - x) & l < x < 2l \end{cases}$$

. Find the displacement function $y(x,t)$.

3. A tightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$. The mid-point of the string is then taken to a height 'h' and then released from rest in that position. Find the displacement of at a distance x from one end of the rod and at any time t seconds. (16)

4. A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. Find an expression for the transverse displacement of the string at any time during the subsequent motion. (16)

5. A rod 30cm long has its end A and B kept at 20ºc and 80ºc respectively, until steady state conditions prevails. The temperature at each end is then suddenly reduced to 0ºc and kept so. Find the resulting temperature function u(x, t) at any point x from one end of the rod and at time t seconds. (16)

Unit- V : Partial Differential Equation

Part-A (2 Marks)

1. Form the PDE by eliminating the arbitrary constants a and b from $z = (x^2 + a^2)(y^2+b^2)$
2. Form the PDE by eliminating the arbitrary constants from $z = a^2 x + ay^2 + b$
3. Form the PDE by eliminating from the relation $z = f(x^2 + y^2) + x + y$
4. Form the PDE by eliminating the arbitrary function from $z^2 - xy = f(\frac{x}{z})$
5. Solve $p + q = pq$
6. Solve $(D^3 - 2D^2D')z = 0$
7. Solve $(D^2 - 2DD' + D'^2) z = 0$
8. Find the particular integral of $(D^2 - 2DD' + D'^2) z = e^{x-y}$)
9. Solve $(D^2 - 7DD' + 6D'^2)z = 0$

10. Solve $(D - D')^3 z = 0$

Part-B (16 Marks)

1. (i) Solve: $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ (8)

(ii) Solve: $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (8)

2. (i) Solve: $(mz - ny)p + (nx - lz)q = ly - mx$ (8)

(ii) Solve: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ (8)

3. (i) Solve: $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ (8)

(ii) Solve: $x(y - z)p + y(z - x)q = z(x - y)$ (8)

4. (i) Solve: $(D^3 + D^2D' - DD'^2 - D'^3)z = e^{2x+y} + \cos(x + y)$ (8)

(ii) Solve: $(D^3 - 7DD'^2 - 6D'^3)z = e^{3x+y} + \sin(x + 2y) + x^2y$ (8)

5.(i). Solve: $(D^2 + 2DD' + D'^2)z = \sinh(x + y) + e^{x+2y}$ (8)

(ii) Solve: $(D^2 + DD' - 6D'^2)z = y\cos x$ (8)

Course Faculty **HoD**