

How 'Heat' Moves

Review of past terms:

- Define "Energy":

The ability to do work or cause change.

- What is the basic unit of measure for energy?

Joules.

How 'Heat' Moves

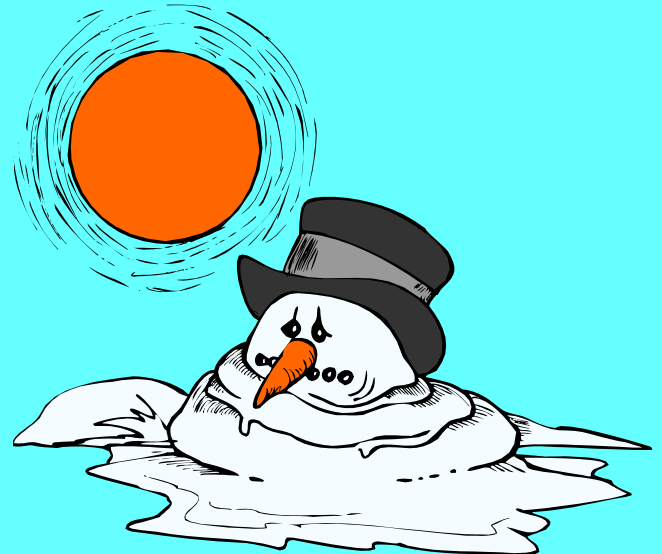
- Define "Heat":

Heat is the movement of thermal energy from a substance at a higher temperature to another substance at a lower temperature.

The Nature of Heat

Heat moves in only one direction:

- Under normal conditions and in nature, heat energy will **ALWAYS** flow the warmer object to the cooler object.
- Heat energy will flow from one substance to another until the two substances have the same temperature.



How 'Heat' Moves

Thermal energy in the form of heat can move in three ways.

Conduction

Convection

Radiation

Conduction

CONDUCTION:

- *The transfer of heat from one particle of matter to another by direct particle to particle contact.*
 - Conduction occurs primarily in solids because the particles are tightly packed together.
 - The particles themselves DO NOT change positions.

Conduction

Example: A metal spoon in a pot of water being heated on an electric stove.

- a. First, the electrical energy is converted to thermal energy by the stove.
- b. The rapidly vibrating particles of the hot electric coil collide with the particles of the cool pot.
- c. Heat energy is transferred, causing the particles in the pot to vibrate faster.

Conduction

- d. The rapidly vibrating particles of the pot now collide with the particles of the water at the bottom of the pot.
- e. The water particles absorb energy and vibrate and flow more rapidly and its temperature increases.
- f. Now, the energetic (hot) particles of water collide with the particles of the submerged end of the spoon.
- g. As the particles of the spoon absorb energy and vibrate more rapidly. The temperature of the spoon increases.

Conduction

- h. As the particles at this end of the spoon absorb energy and vibrate faster they collide with other particles in the spoon. As they collide, energy is transferred to the other particles (similar to momentum) and they begin to vibrate more rapidly.
- i. This process of conduction is repeated all along the metal spoon until the entire metal spoon becomes hot.

Conduction

Brainstorming: What are other examples of conduction?

Application: Describe the process of conduction when you place a hot spoon into a bowl of ice cream.

Convection

Convection: the transfer of thermal energy (heat) through the bulk movement of matter.

- Convection occurs in **FLUIDS** (liquids and gases).
- Convection produces **CURRENTS** in both gases and liquids.
- Thermal Energy heat is carried by the particles as they move from one location to another.

Convection

Example: Heating water:

- a. When the water at the bottom of the pot (nearest the burner) is heated, the particles absorb energy by conduction as they touch the hot pot.
- b. The water particles vibrate more rapidly.
- c. The particles also move farther apart and the hot water becomes less dense than the surrounding cool water.
- d. This causes the heated (hot) water to rise.

Convection

- e. The surrounding denser cooler water is forced downward near the burner by the rising hot water.
- f. This process continues to repeat.
- g. This **FLOW** creates a circular motion known as a **convection current** .

Application: How do convection currents form in a room when the heater is turned on?

Convection

- The warm air from the heater vent will rise. Why?
 - The warm air is less dense than the surrounding cooler air.
- The cool air is pushed down by the rising warm air.

What is the best location for a heat vent in a room and why? Near the ceiling or the floor?

Floor:

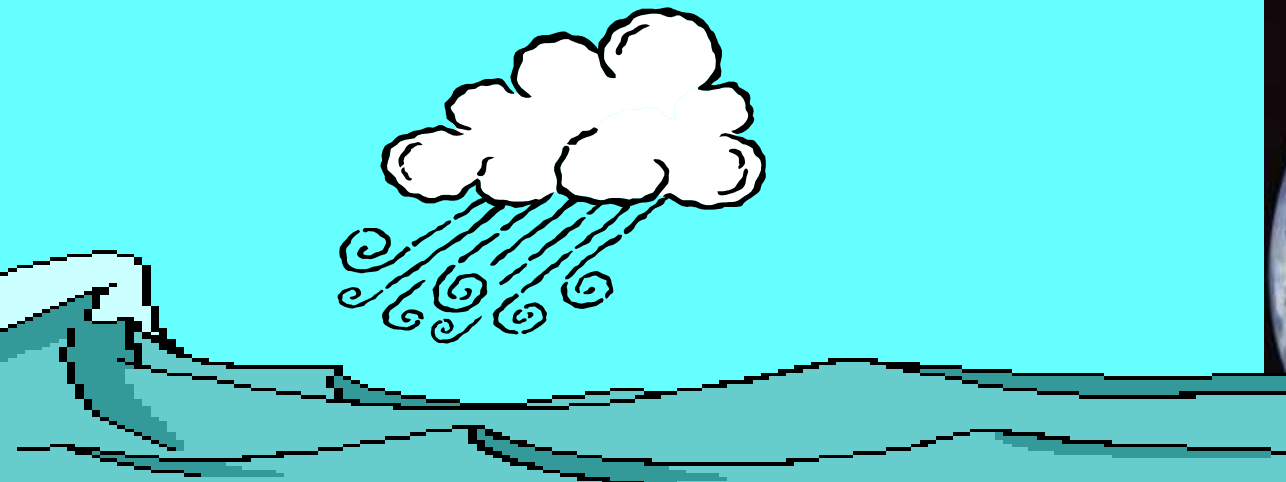
Because the warm air will rise to the ceiling.

How about the return vent?

Convection

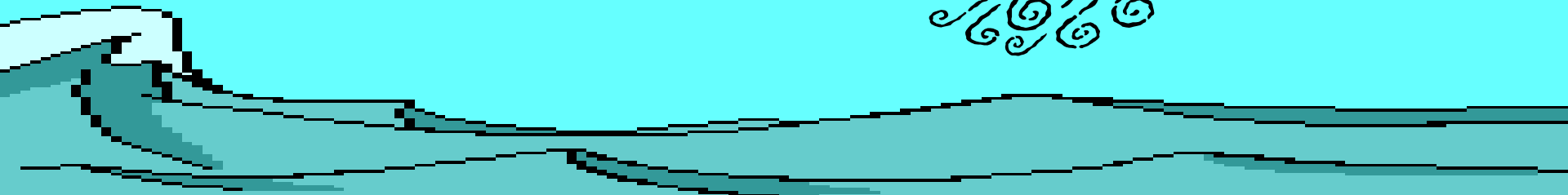
Convection currents occur in the environment as well. They produce:

- Global winds that contribute to Earth's weather.
- Ocean and lake currents



Convection

Brainstorming: On a hot summer day the breeze near the beach blows toward the water. However, later in the day the breeze reverses direction and blows toward land and will get increasingly stronger. Why?



Convection

Answer: In the morning the water may be warmer than the sand causing the air over the water to rise.

In the afternoon, the sand has become much hotter than the water and the air above it rises. The air over the water rushes in to fill its void causing a wind.

Radiation

- **Radiation:** the transfer of (thermal) energy by electromagnetic waves.
 - Radiation does not require matter to transfer thermal energy.
 - All the sun's energy that reaches Earth travels through millions of kilometers of empty space (a vacuum).
 - All matter can radiate energy.
 - You feel the radiation of thermal energy from a bonfire, a heat lamp and a light bulb.

Radiation

- Other examples of the transfer of heat by Radiation:
 - a. Charcoal grill.
 - b. Hot tin roof.
 - c. Burner on a stove top.
 - d. ?
 - e. ?

Radiation

Key Point: For radiation to be felt as heat it must first be absorbed by a material.

Example: Why do blue jeans feel hotter in the sun than a yellow shirt, even though they are both exposed to the same amount of sunlight?

- The blue jean fabric absorbs more radiant energy from the sun than the yellow shirt because of its dark color.

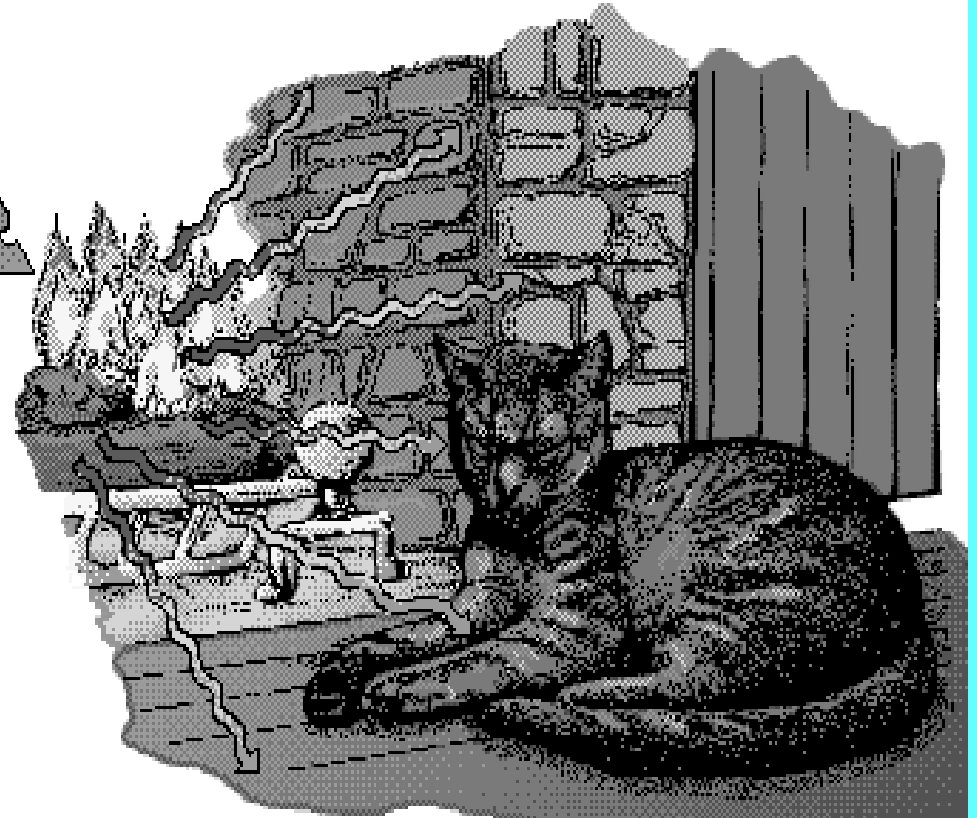
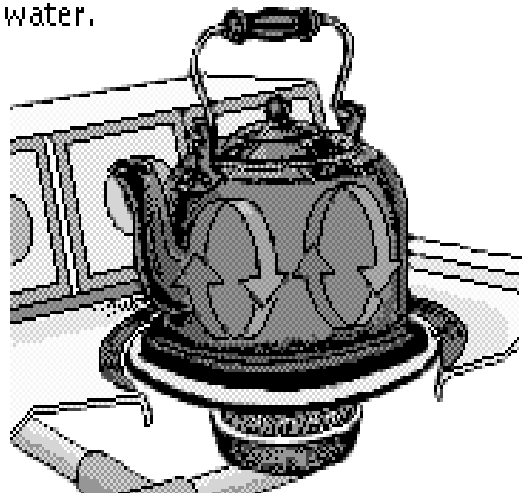
Conduction, Convection & Radiation



Conduction- Heat travels from the warmed end of the poker toward the cool end.

Radiation- Heat travels through space in the form of infrared radiation.

Convection- Heat from burner is transferred to the circulating water.



Energy from the Sun



Convection, Conduction & Radiation



The Nature of Heat

What happens when you put ice in a warm soft drink?

- The heat energy moves from the soft drink into the ice by conduction (particle to particle contact) causing the ice to melt.

Review

Describe the three kinds of heat transfer.

- a. **Conduction** - transfer of heat energy from one particle to another by direct contact. (Primarily in solids)
- b. **Convection** - transfer of heat energy in fluids-gases and liquids) through the bulk movement of matter from one place to another. (Produces currents)
- c. **Radiation** - transfer of energy through electromagnetic waves. (Matter is not required!) (Radiant & infrared radiation from the sun)

Conduction

- Direct contact of particles
- Solids/liquids /gases
- The handle of a cooking utensil

Radiation

- Transfer of energy by waves
- Only radiant energy that is absorbed becomes thermal energy
- Lightbulb
- Fireplace

Convection

- Transfer of energy by bulk movement of matter (fluids)
- Currents (wind, water)
- Hot air balloon

Contrast:
Conduction
Convection
Radiation

- Direct contact of particles
- Solids/liquids/gases
- Solids - good conductors
- Gases - poor conductors

Conduction

- Transfer of energy by waves
- Only radiant energy that is absorbed becomes thermal energy
- Shiny/light colors - reflect
- Dull/dark colors - absorb

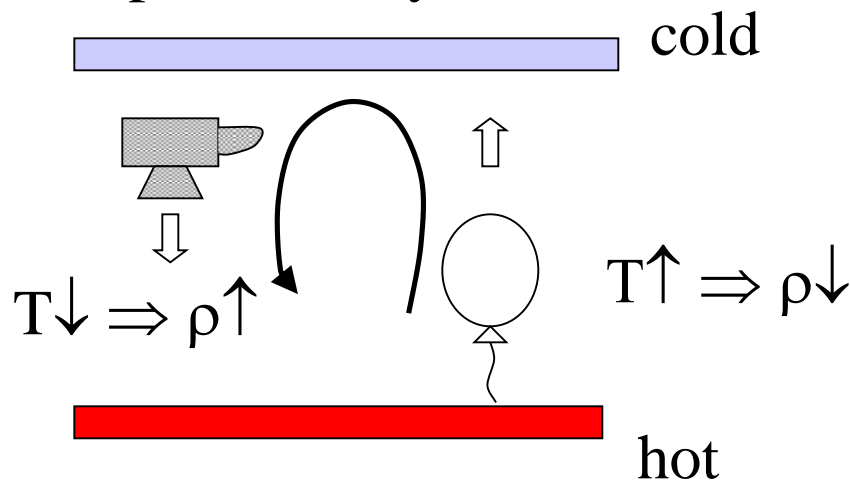
Radiation

- Transfer of energy by bulk movement of matter (fluids)
- Currents (wind, water)
- Hot air balloon

Convection

Free Convection

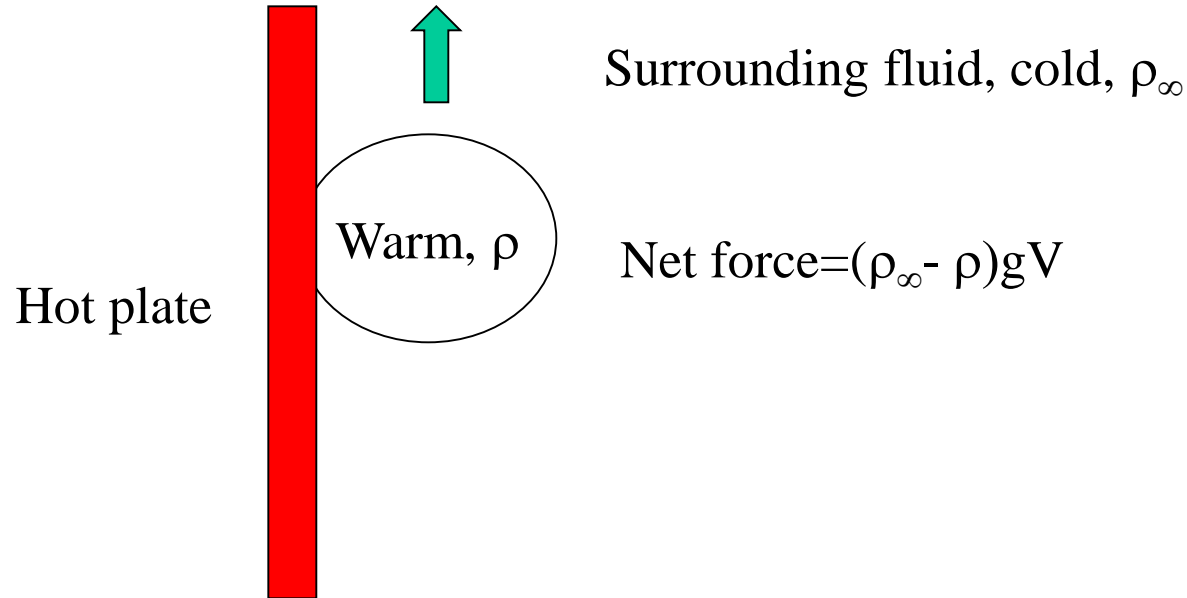
A free convection flow field is a self-sustained flow driven by the presence of a temperature gradient. (As opposed to a forced convection flow where external means are used to provide the flow.) As a result of the temperature difference, the density field is not uniform also. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer. It is therefore important only when there is no external flow exists.



Flow is unstable and a circulatory pattern will be induced.

Basic Definitions

Buoyancy effect:



The density difference is due to the temperature difference and it can be characterized by their volumetric thermal expansion coefficient, β :

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P \approx -\frac{1}{\rho} \frac{\rho_\infty - \rho}{T_\infty - T} = -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T}$$

$$\Delta \rho \approx \beta \Delta T$$

Grashof Number and Rayleigh Number

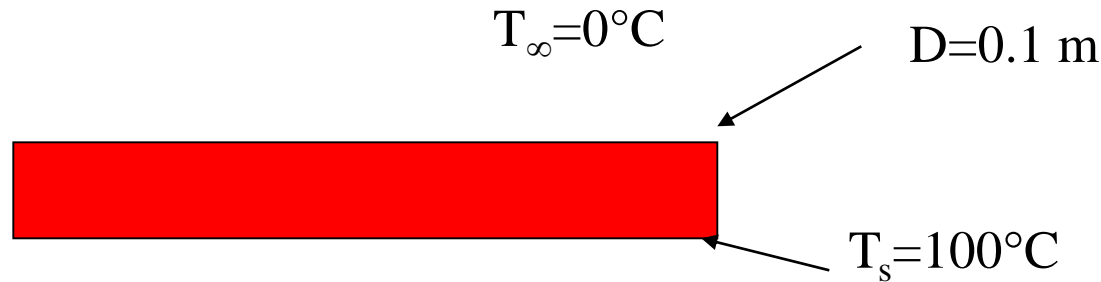
Define Grashof number, Gr , as the ratio between the buoyancy force and the viscous force:

$$Gr = \frac{g \beta \Delta T L^3}{\nu^2} = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2}$$

- Grashof number replaces the Reynolds number in the convection correlation equation. In free convection, buoyancy driven flow sometimes dominates the flow inertia, therefore, the Nusselt number is a function of the Grashof number and the Prandtl number alone. $Nu=f(Gr, Pr)$. Reynolds number will be important if there is an external flow. (see chapter 11.5, combined forced and free convection).
- In many instances, it is better to combine the Grashof number and the Prandtl number to define a new parameter, the Rayleigh number, $Ra=GrPr$. The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow. For example, when $Ra>10^9$, the vertical free convection boundary layer flow over a flat plate becomes turbulent.

Example

Determine the rate of heat loss from a heated pipe as a result of natural (free) convection.



Film temperature (T_f): averaged boundary layer temperature $T_f = 1/2(T_s + T_{\infty}) = 50^{\circ}\text{C}$.
 $k_f = 0.03 \text{ W/m.K}$, $\text{Pr} = 0.7$, $\nu = 2 \times 10^{-5} \text{ m}^2/\text{s}$, $\beta = 1/T_f = 1/(273 + 50) = 0.0031 \text{ (1/K)}$

$$Ra = \frac{g\beta(T_s - T_{\infty})L^3}{\nu^2} \text{Pr} = \frac{(9.8)(0.0031)(100 - 0)(0.1)^3}{(2 \times 10^{-5})^2} (0.7) = 7.6 \times 10^6.$$

$$Nu_D = \left\{ 0.6 + \frac{0.387 Ra^{1/6}}{[1 + (0.559 / \text{Pr})^{9/16}]^{8/27}} \right\}^2 = 26.0 \quad (\text{equation 11.15 in Table 11.1})$$

$$h = \frac{k_f}{D} Nu_D = \frac{0.03}{0.1} (26) = 7.8 \text{ (W / m}^2 \text{ K)}$$

$$q = hA(T_s - T_{\infty}) = (7.8)(\pi)(0.1)(1)(100 - 0) = 244.9 \text{ (W)}$$

Can be significant if the pipe are long.

Heat Transfer/Heat Exchanger

- How is the heat transfer?
- Mechanism of Convection
- Applications .
- Mean fluid Velocity and Boundary and their effect on the rate of heat transfer.
- Fundamental equation of heat transfer
- Logarithmic-mean temperature difference.
- Heat transfer Coefficients.
- Heat flux and Nusselt correlation
- Simulation program for Heat Exchanger

How is the heat transfer?

- Heat can transfer between the surface of a solid conductor and the surrounding medium whenever temperature gradient exists.

Conduction

Convection

Natural convection

Forced Convection

Natural and forced Convection

- Natural convection occurs whenever heat flows between a solid and fluid, or between fluid layers.
- As a result of heat exchange
Change in density of effective fluid layers taken place, which causes upward flow of heated fluid.

If this motion is associated with heat transfer mechanism only, then it is called **Natural Convection**

Forced Convection

- If this motion is associated by mechanical means such as pumps, gravity or fans, the movement of the fluid is enforced.
- And in this case, we then speak of **Forced convection**.

Heat Exchangers

- A device whose primary purpose is the transfer of energy between two fluids is named a **Heat Exchanger**.

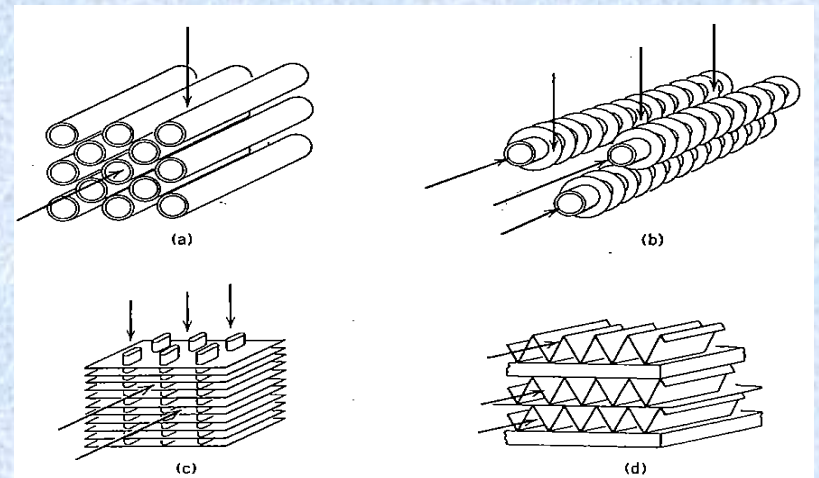
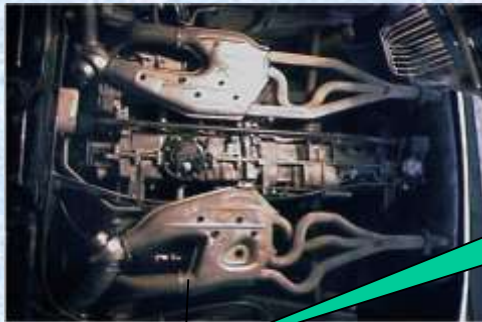


Figure 22.4 Compact heat-exchanger configurations.



Applications of Heat Exchangers



Heat Exchangers prevent car engine overheating and increase efficiency



Heat exchangers are used in Industry for heat transfer



Heat exchangers are used in AC and furnaces



- The closed-type exchanger is the most popular one.
- One example of this type is the Double pipe exchanger.

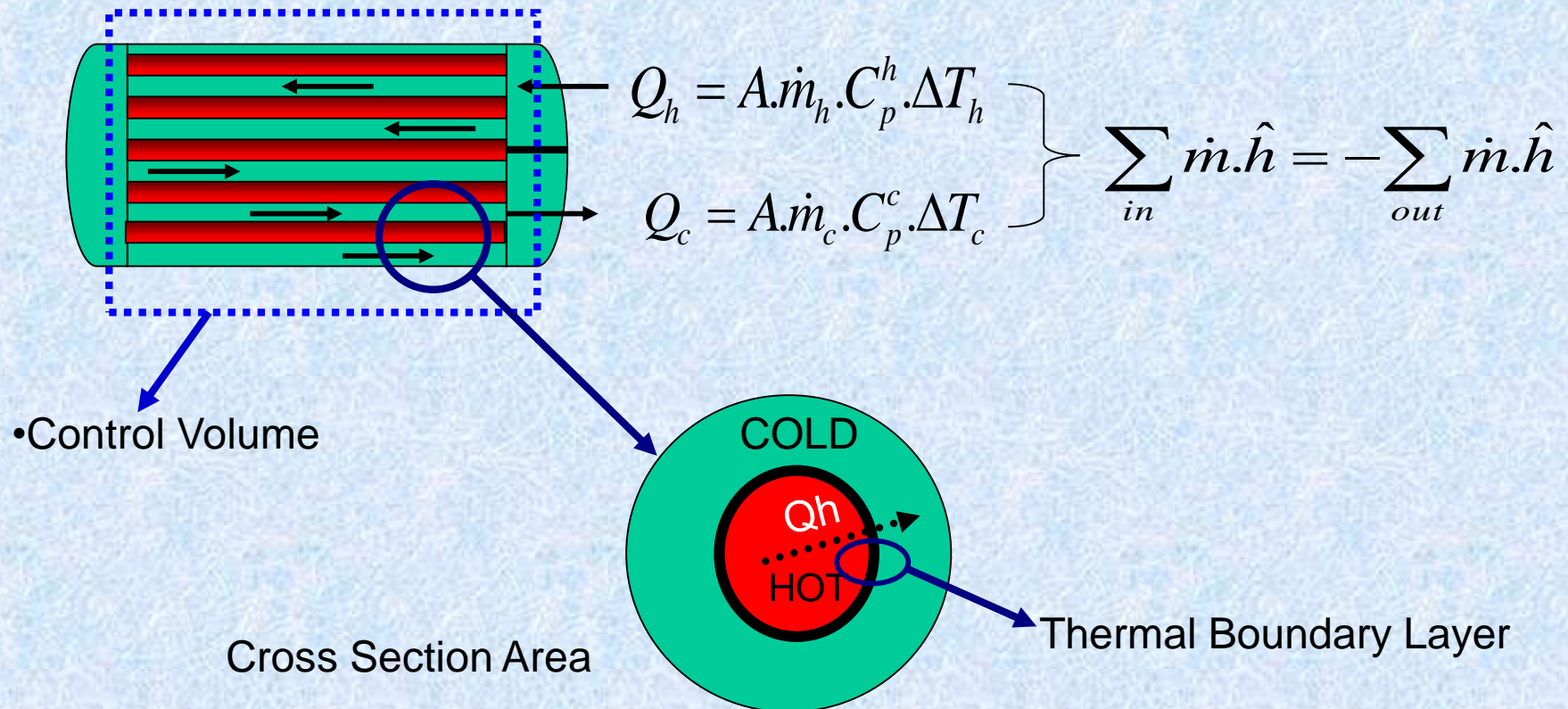


- In this type, the hot and cold fluid streams do not come into direct contact with each other. They are separated by a tube wall or flat plate.

Principle of Heat Exchanger

- First Law of Thermodynamic: “Energy is conserved.”

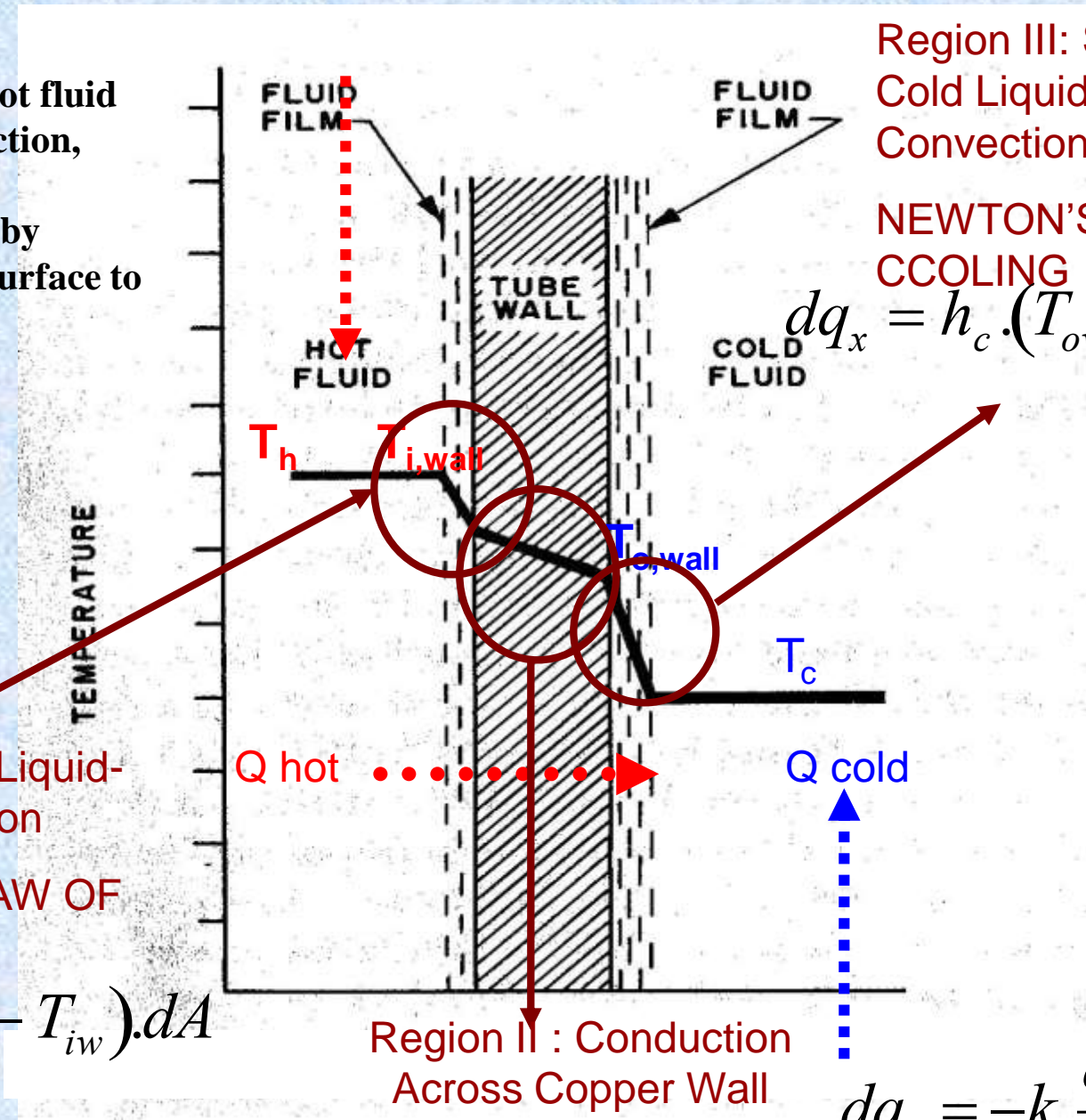
$$\frac{dE}{dt} = \left(\sum_{in} \dot{m} \cdot \hat{h}_{in} - \sum_{out} \dot{m} \cdot \hat{h}_{out} \right) + \dot{q} + \dot{w}_s + \dot{e}_{generated}$$



THERMAL

BOUNDARY LAYER

Energy moves from hot fluid to a surface by convection, through the wall by conduction, and then by convection from the surface to the cold fluid.



Region III: Solid – Cold Liquid Convection

NEWTON'S LAW OF COOLING

$$dq_x = h_c \cdot (T_{ow} - T_c) \cdot dA$$

Region I : Hot Liquid-Solid Convection

NEWTON'S LAW OF COOLING

$$dq_x = h_h \cdot (T_h - T_{iw}) \cdot dA$$

Region II : Conduction Across Copper Wall

FOURIER'S LAW

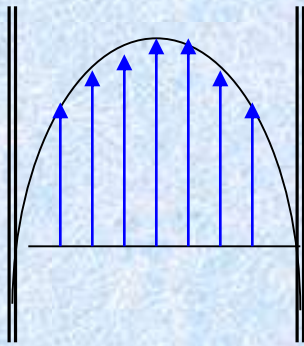
$$dq_x = -k \cdot \frac{dT}{dr}$$

- **Velocity distribution and boundary layer**

When fluid flow through a circular tube of uniform cross-section and fully developed,

The velocity distribution depend on the type of the flow.

In laminar flow the volumetric flowrate is a function of the radius.

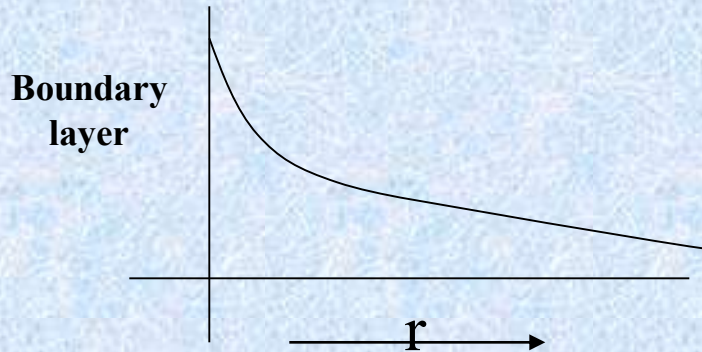


$$V = \int_{r=0}^{r=D/2} u 2\pi r dr$$

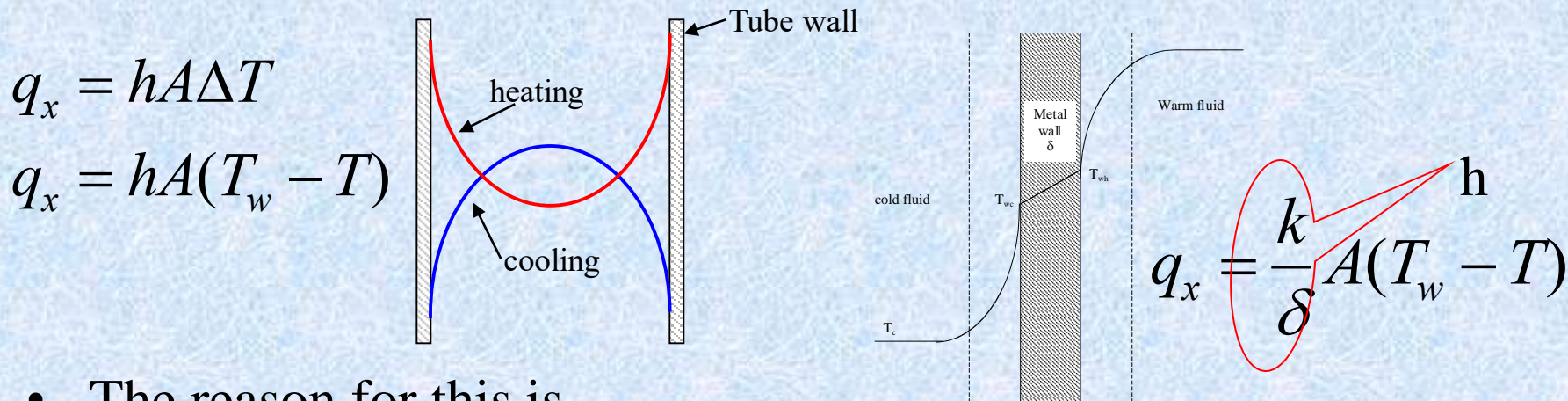
V = volumetric flowrate

u = average mean velocity

- In turbulent flow, there is no such distribution.
- The molecule of the flowing fluid which adjacent to the surface have zero velocity because of mass-attractive forces. Other fluid particles in the vicinity of this layer, when attempting to slid over it, are slow down by viscous forces.



- Accordingly the temperature gradient is larger at the wall and through the viscous sub-layer, and small in the turbulent core.



- The reason for this is
 - 1) Heat must transfer through the boundary layer by conduction.
 - 2) Most of the fluid have **a low thermal conductivity (k)**
 - 3) While in the turbulent core there are a rapid moving eddies, which they are equalizing the temperature.

U = The Overall Heat Transfer Coefficient [W/m.K]

Region I : Hot Liquid –
Solid Convection

$$q_x = h_{hot} \cdot (T_h - T_{iw}) \cdot A \quad \rightarrow \quad T_h - T_{iw} = \frac{q_x}{h_h \cdot A_i}$$

Region II : Conduction
Across Copper Wall

$$q_x = \frac{k_{copper} \cdot 2\pi L}{\ln \frac{r_o}{r_i}} \quad \rightarrow \quad T_{o,wall} - T_{i,wall} = \frac{q_x \cdot \ln \left(\frac{r_o}{r_i} \right)}{k_{copper} \cdot 2\pi L}$$

Region III : Solid –
Cold Liquid Convection

$$q_x = h_c (T_{o,wall} - T_c) A_o \quad \rightarrow \quad T_{o,wall} - T_c = \frac{q_x}{h_c \cdot A_o}$$

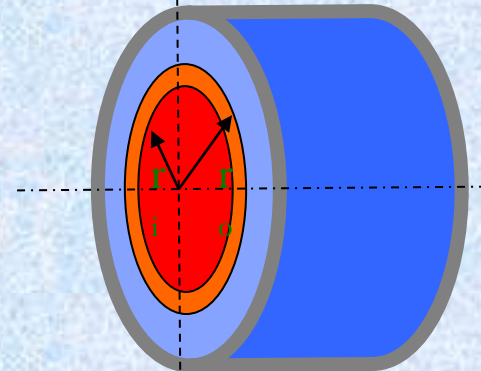
$$T_h - T_c = \frac{q_x}{R_1 + R_2 + R_3}$$

$$q_x = U \cdot A \cdot (T_h - T_c)$$

$$T_h - T_c = q_x \left[\frac{1}{h_h \cdot A_i} + \frac{\ln \left(\frac{r_o}{r_i} \right)}{k_{copper} \cdot 2\pi L} + \frac{1}{h_c \cdot A_o} \right] +$$

$$U = \frac{1}{A \cdot \Sigma R}$$

$$U = \left[\frac{r_o}{h_{hot} \cdot r_i} + \frac{r_o \cdot \ln \left(\frac{r_o}{r_i} \right)}{k_{copper} \cdot r_i} + \frac{1}{h_{cold}} \right]^{-1}$$



Calculating U using Log Mean Temperature

$$\begin{aligned} \text{Hot Stream : } dq_h &= \dot{m}_h \cdot C_p^h \cdot dT_h \\ \text{Cold Stream: } dq_c &= \dot{m}_c \cdot C_p^c \cdot dT_c \end{aligned} \left. \begin{array}{l} d(\Delta T) = dT_h - dT_c \\ \Delta T = T_h - T_c \end{array} \right\} \rightarrow d(\Delta T) = \left(\frac{dq_h}{\dot{m}_h \cdot C_p^h} - \frac{dq_c}{\dot{m}_c \cdot C_p^c} \right)$$

$$\left. \begin{array}{l} dq = -dq_{hot} = dq_{cold} \\ -dq = -U \cdot \Delta T \cdot dA \end{array} \right\} d(\Delta T) = -U \cdot \Delta T \cdot dA \cdot \left(\frac{1}{\dot{m}_h \cdot C_p^h} + \frac{1}{\dot{m}_c \cdot C_p^c} \right)$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U \cdot \left(\frac{\Delta T_h}{q_h} + \frac{\Delta T_c}{q_c} \right) \cdot \int_{A_1}^{A_2} dA$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U \cdot \left(\frac{1}{\dot{m}_h \cdot C_p^h} + \frac{1}{\dot{m}_c \cdot C_p^c} \right) \cdot \int_{A_1}^{A_2} dA$$

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -\frac{U \cdot A}{q} (\Delta T_h + \Delta T_c) = -\frac{U \cdot A}{q} \left[(T_h^{in} - T_h^{out}) - (T_c^{in} - T_c^{out}) \right]$$

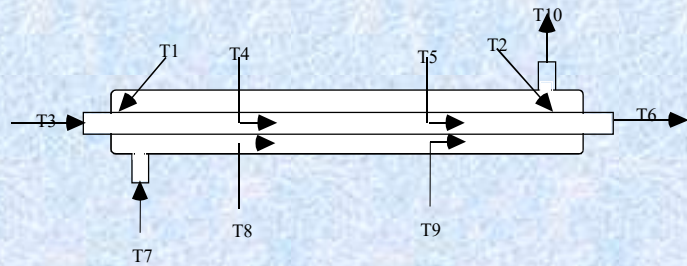
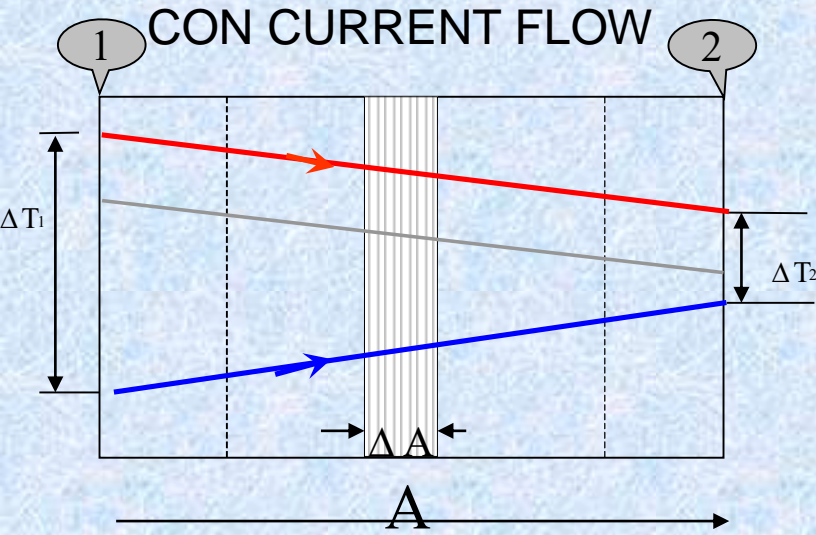
$$q = U \cdot A \cdot \frac{\Delta T_2 - \Delta T_1}{\ln \left(\frac{\Delta T_2}{\Delta T_1} \right)}$$

Log Mean Temperature

Log Mean Temperature evaluation

$$\Delta T_{Ln} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

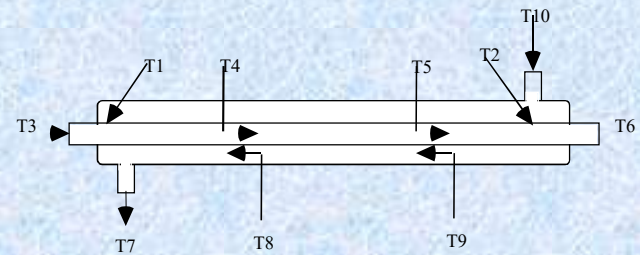
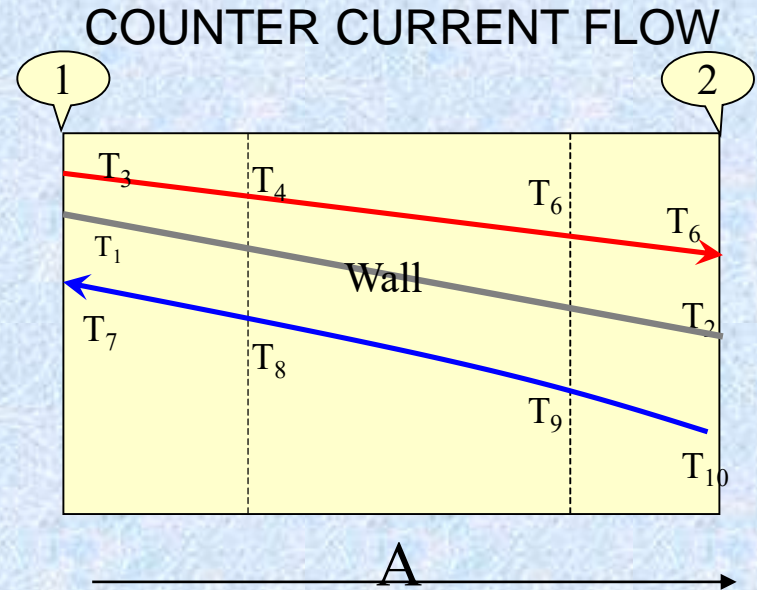
$$U = \frac{\dot{m}_h \cdot \dot{C}_p^h \cdot (T_3 - T_6)}{A \cdot \Delta T_{Ln}} = \frac{\dot{m}_c \cdot \dot{C}_p^c \cdot (T_7 - T_{10})}{A \cdot \Delta T_{Ln}}$$



Parallel Flow

$$\Delta T_1 = T_h^{in} - T_c^{in} = T_3 - T_7$$

$$\Delta T_2 = T_h^{out} - T_c^{out} = T_6 - T_{10}$$



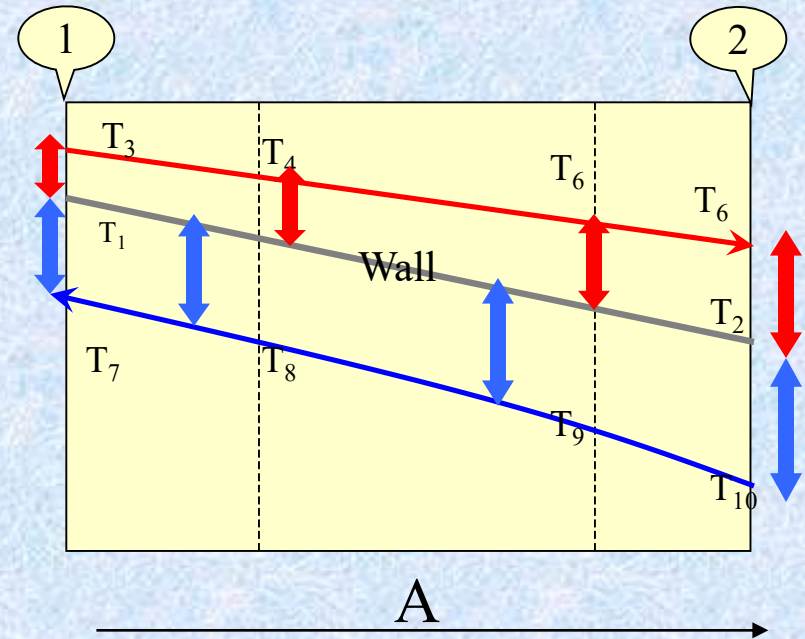
Counter - Current Flow

$$\Delta T_1 = T_h^{in} - T_c^{out} = T_3 - T_7$$

$$\Delta T_2 = T_h^{out} - T_c^{in} = T_6 - T_{10}$$

$$q = h_h A_i \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{(T_3 - T_1) - (T_6 - T_2)}{\ln \frac{(T_3 - T_1)}{(T_6 - T_2)}}$$



$$q = h_c A_o \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{(T_1 - T_7) - (T_2 - T_{10})}{\ln \frac{(T_1 - T_7)}{(T_2 - T_{10})}}$$

DIMENSIONLESS ANALYSIS TO CHARACTERIZE A HEAT EXCHANGER

$$Nu = f(Re, Pr, L/D, \mu_b / \mu_o)$$

$\frac{h.D}{k}$ $\frac{v.D.\rho}{\mu}$ $\frac{C_p.\mu}{k}$

• **Further Simplification:**

$$Nu = a.Re^b .Pr^c$$

$$Nu = \frac{D}{\delta}$$

Can Be Obtained from 2 set of experiments

One set, run for constant Pr

And second set, run for constant Re

$$q = \frac{k}{\delta} A(T_w - T)$$

•Empirical Correlation

- For laminar flow

$$Nu = 1.62 (Re \cdot Pr \cdot L/D)$$

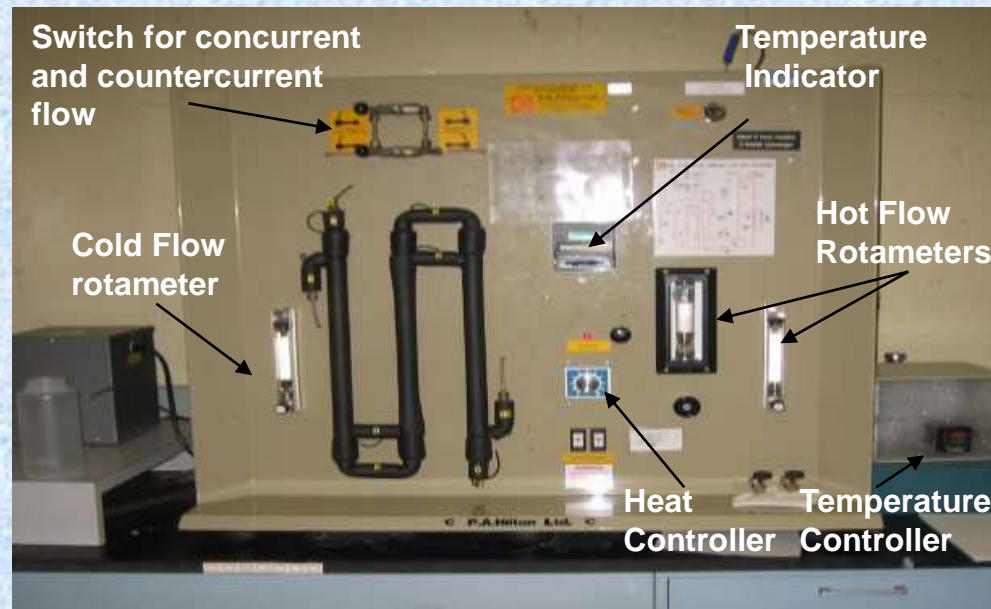
- For turbulent flow

$$Nu_{Ln} = 0.026 \cdot Re^{0.8} \cdot Pr^{1/3} \cdot \left(\frac{\mu_b}{\mu_o} \right)^{0.14}$$

- Good To Predict within 20%
- Conditions: $L/D > 10$
 $0.6 < Pr < 16,700$
 $Re > 20,000$

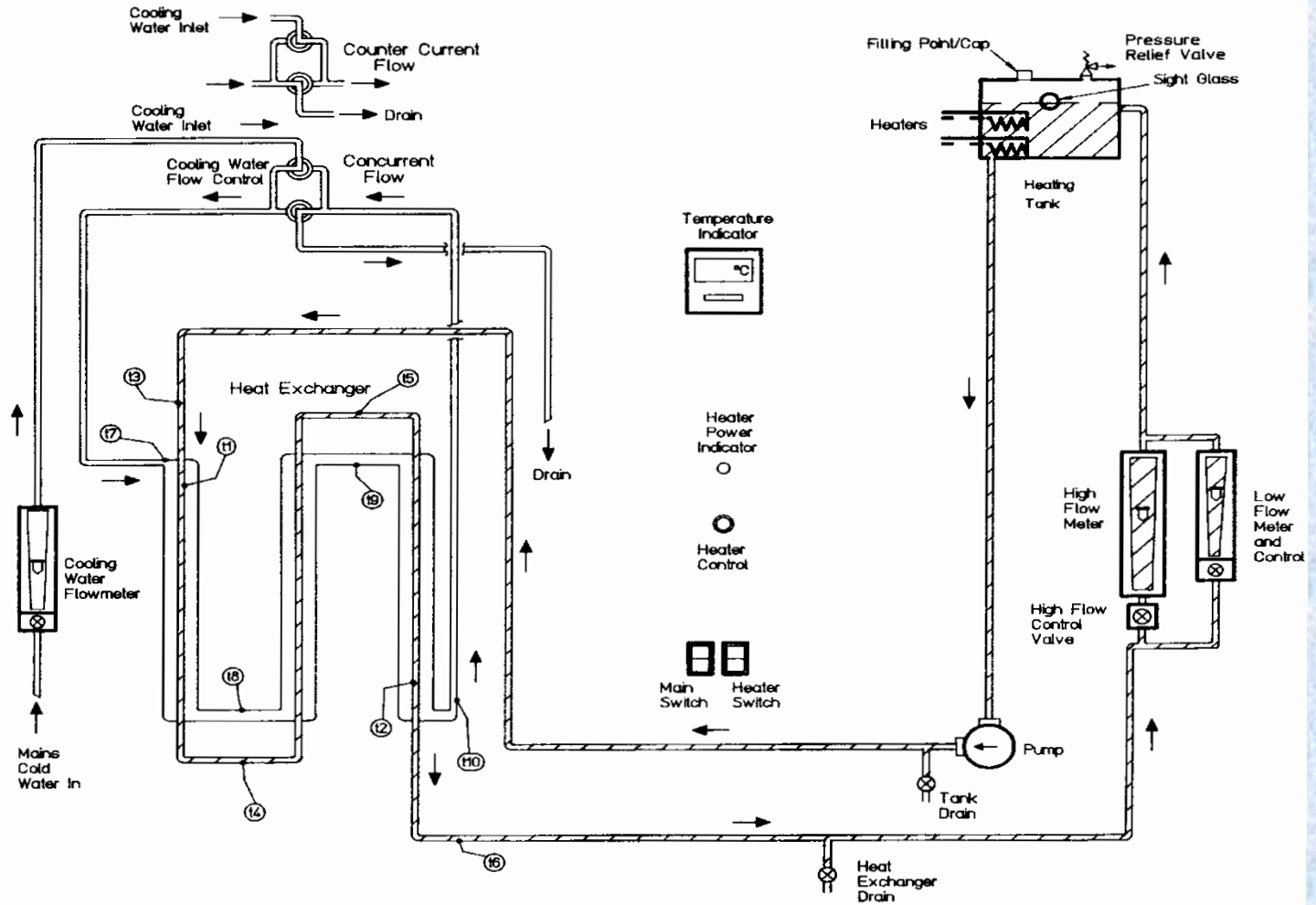
Experimental

Apparatus



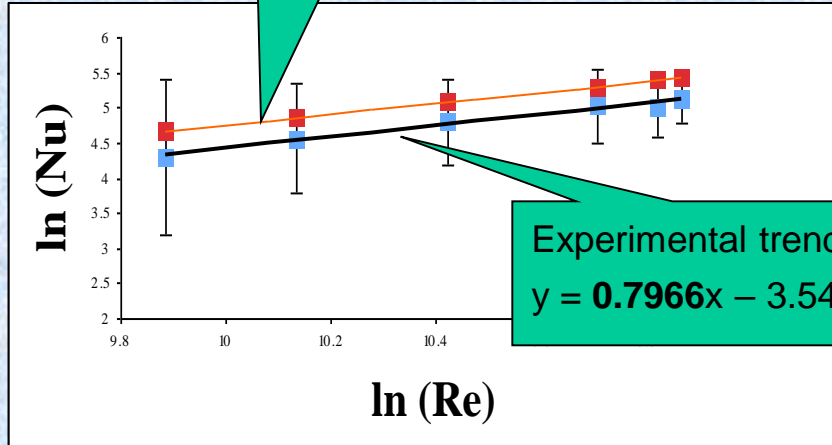
- Two copper concentric pipes
 - Inner pipe (ID = 7.9 mm, OD = 9.5 mm, L = 1.05 m)
 - Outer pipe (ID = 11.1 mm, OD = 12.7 mm)
- Thermocouples placed at 10 locations along exchanger, T1 through T10

WATER-WATER TURBULENT FLOW HEAT EXCHANGER



Examples of Exp. Results

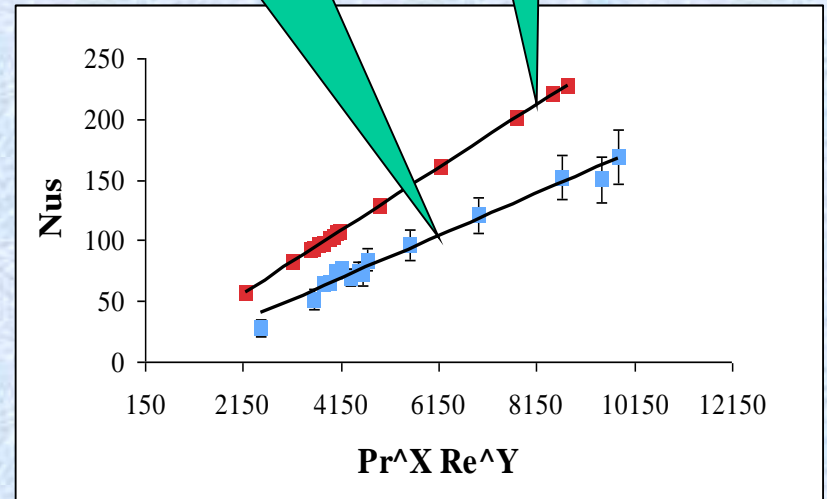
Theoretical trend
 $y = 0.8002x - 3.0841$



Experimental trend
 $y = 0.7966x - 3.5415$

Theoretical trend
 $y = 0.026x$

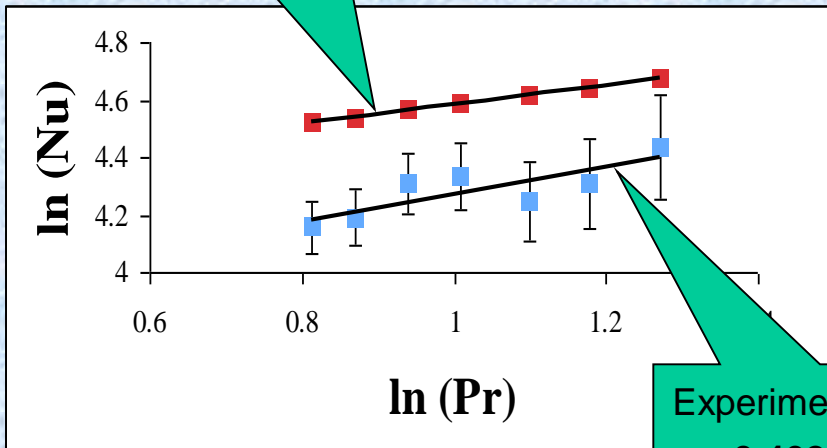
Experimental trend
 $y = 0.0175x - 4.049$



Experimental $Nu = 0.0175Re^{0.7966}Pr^{0.4622}$

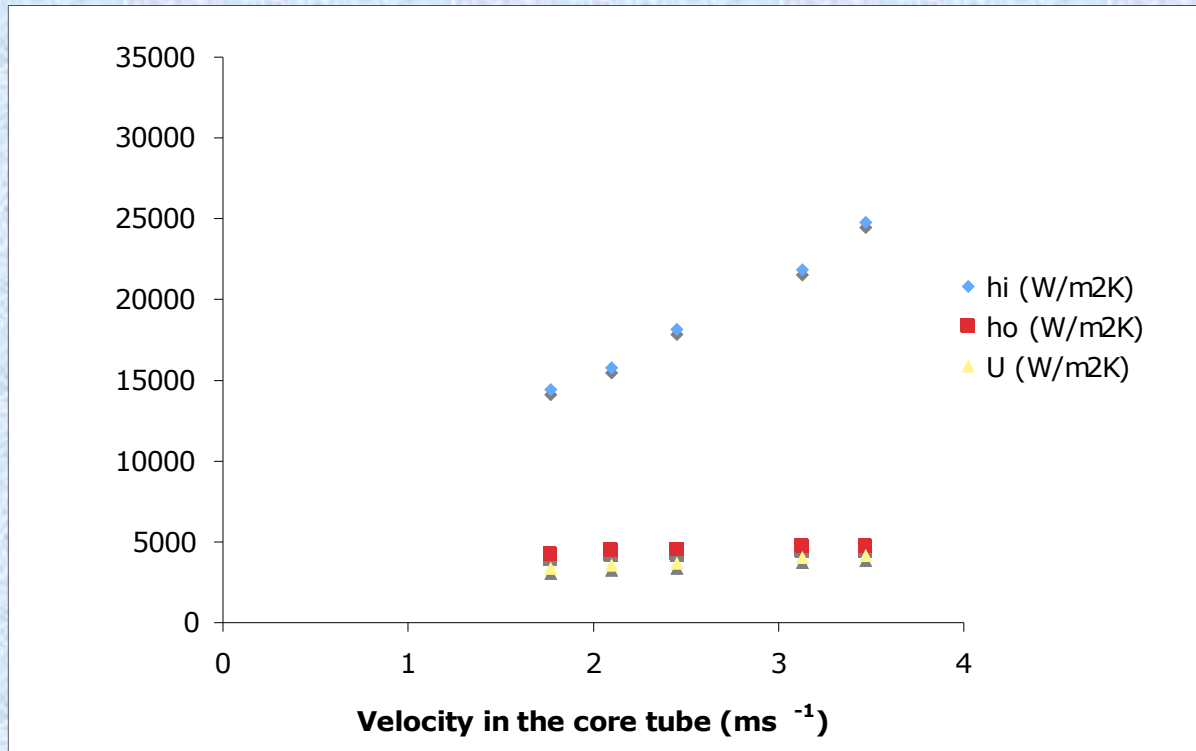
Theoretical $Nu = 0.026Re^{0.8}Pr^{0.33}$

Theoretical trend
 $y = 0.3317x + 4.2533$



Experimental trend
 $y = 0.4622x - 3.8097$

Effect of core tube velocity on the local and over all Heat Transfer coefficients



Radiation Exchange Between Surfaces: Enclosures with Nonparticipating Media

Basic Concepts

- **Enclosures** consist of two or more surfaces that envelop a region of space (typically gas-filled) and between which there is radiation transfer. **Virtual**, as well as real, **surfaces** may be introduced to form an enclosure.
- A **nonparticipating medium** within the enclosure neither emits, absorbs, nor scatters radiation and hence has no effect on radiation exchange between the surfaces.
- Each surface of the enclosure is assumed to be **isothermal**, **opaque**, **diffuse** and **gray**, and to be characterized by **uniform radiosity** and **irradiation**.

The View Factor (also Configuration or Shape Factor)

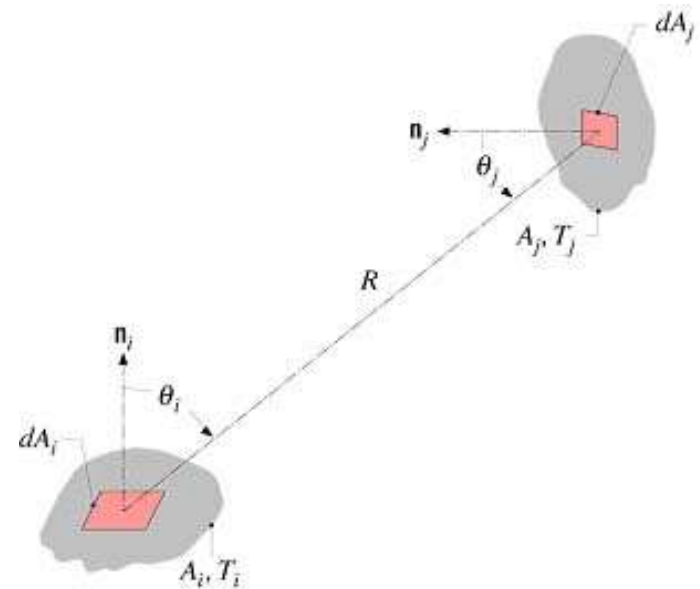
- The view factor, F_{ij} , is a geometrical quantity corresponding to the **fraction of the radiation leaving surface i that is intercepted by surface j** .

$$F_{ij} = \frac{q_{i \rightarrow j}}{A_i J_i}$$

- The **view factor integral** provides a general expression for F_{ij} . Consider exchange between differential areas dA_i and dA_j :

$$dq_{i \rightarrow j} = I_i \cos \theta_i dA_i d\omega_{j-i} = J_i \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$



View Factor Relations

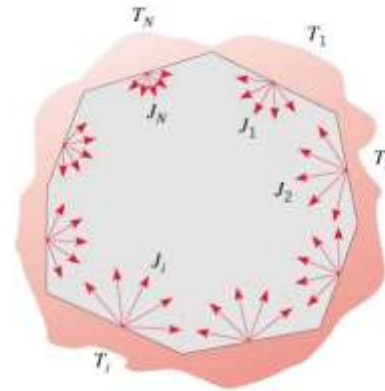
- **Reciprocity Relation.** With

$$F_{ji} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$

$$A_i F_{ij} = A_j F_{ji}$$

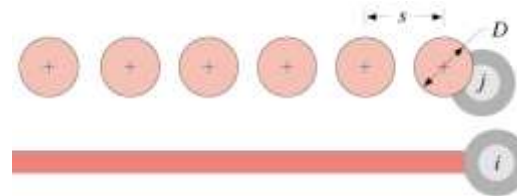
- **Summation Rule** for Enclosures.

$$\sum_{j=1}^N F_{ij} = 1$$



- **Two-Dimensional Geometries** (Table 13.1) For example,

An Infinite Plane and a Row of Cylinders



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s} \right)^2 \right]^{1/2} + \left(\frac{D}{s} \right) \tan^{-1} \left(\frac{s^2 - D^2}{D^2} \right)^{1/2}$$

- **Three-Dimensional Geometries** (Table 13.2). For example,

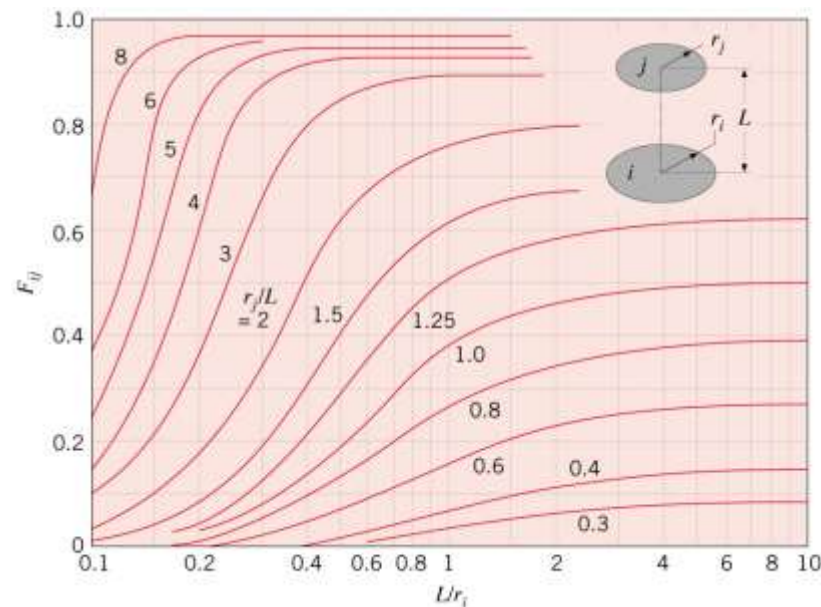
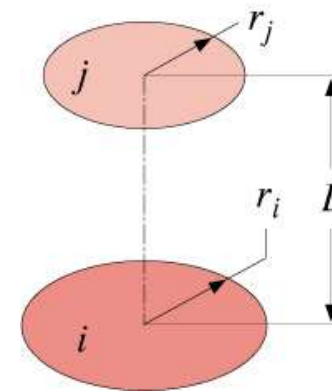
Coaxial Parallel Disks

$$F_{ij} = \frac{1}{2} \left\{ S - \left[S^2 - 4 \left(r_j / r_i \right)^2 \right]^{1/2} \right\}$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

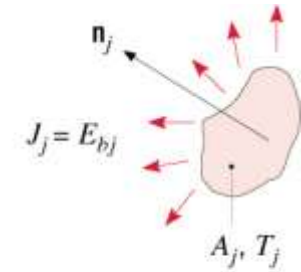
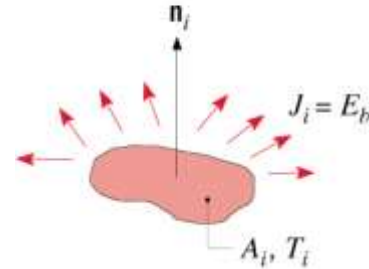
$$R_i = r_i / L$$

$$R_j = r_j / L$$



Blackbody Radiation Exchange

- For a blackbody, $J_i = E_{bi}$.
- Net radiative exchange between two surfaces that can be approximated as blackbodies \rightarrow **net rate at which radiation leaves surface i due to its interaction with j**



or **net rate at which surface j gains radiation due to its interaction with i**

$$q_{ij} = q_{i \rightarrow j} - q_{j \rightarrow i}$$

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

$$q_{ij} = A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

- Net radiation transfer from surface i due to exchange with all (N) surfaces of an enclosure:

$$q_i = \sum_{j=1}^N A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

General Radiation Analysis for Exchange between the N Opaque, Diffuse, Gray Surfaces of an Enclosure ($\varepsilon_i = \alpha_i = 1 - \rho_i$)

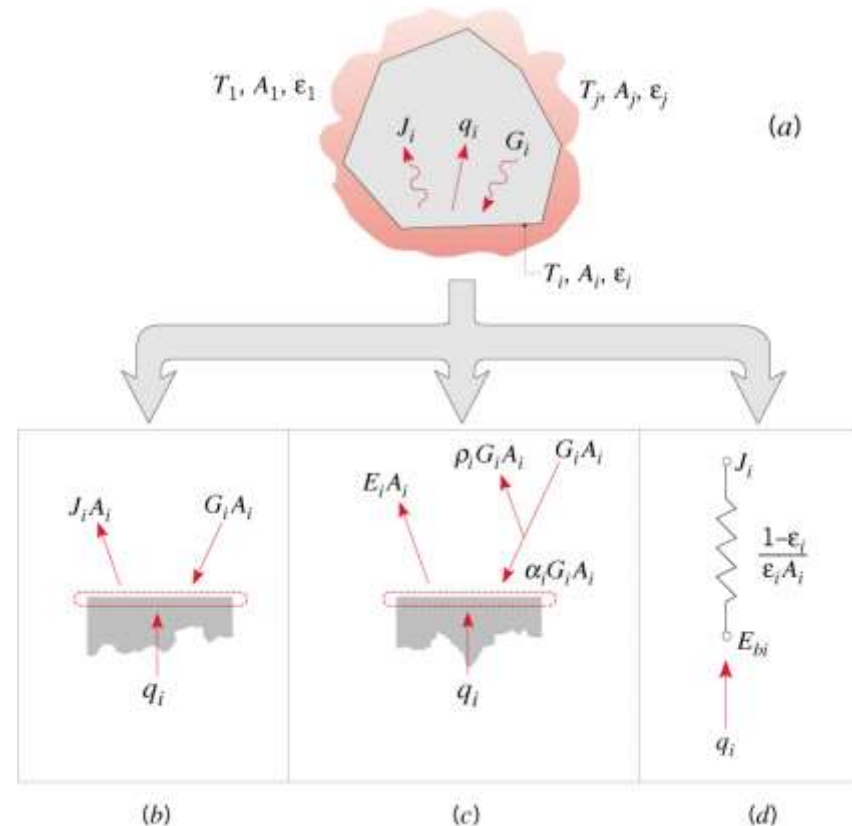
- Alternative expressions for **net radiative transfer from surface i** :

$$q_i = A_i (J_i - G_i) \rightarrow \text{Fig. (b)} \quad (1)$$

$$q_i = A_i (E_i - \alpha_i G_i) \rightarrow \text{Fig. (c)} \quad (2)$$

$$q_i = \frac{E_{bi} - J_i}{(1 - \varepsilon_i) / \varepsilon_i A_i} \rightarrow \text{Fig. (d)} \quad (3)$$

↳ Suggests a **surface radiative resistance** of the form: $(1 - \varepsilon_i) / \varepsilon_i A_i$



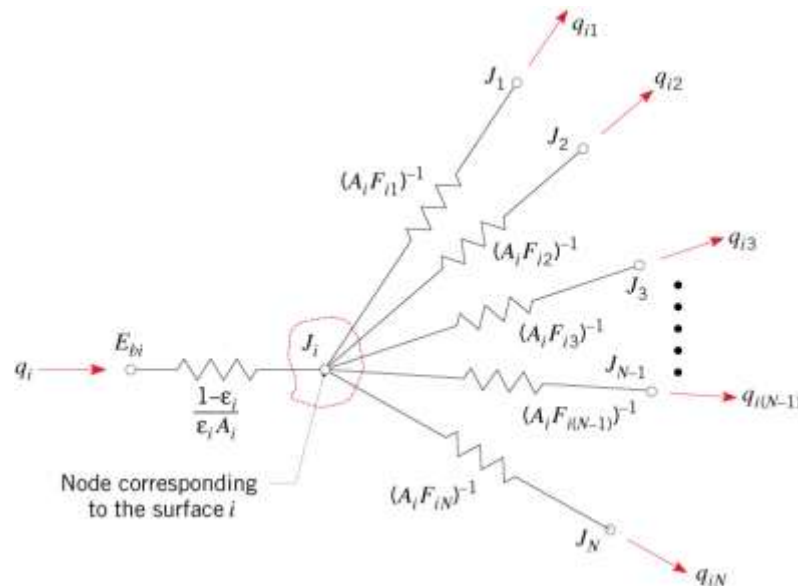
$$q_i = \sum_{j=1}^N A_i F_{ij} (J_i - J_j) = \sum_{j=1}^N \frac{J_i - J_j}{(A_i F_{ij})^{-1}} \quad (4)$$

↳ Suggests a **space or geometrical resistance** of the form: $(A_i F_{ij})^{-1}$

- Equating Eqs. (3) and (4) corresponds to a radiation balance on surface i :

$$\frac{E_{bi} - J_i}{(1 - \varepsilon_i) / \varepsilon_i A_i} = \sum_{j=1}^N \frac{J_i - J_j}{(A_i F_{ij})^{-1}} \quad (5)$$

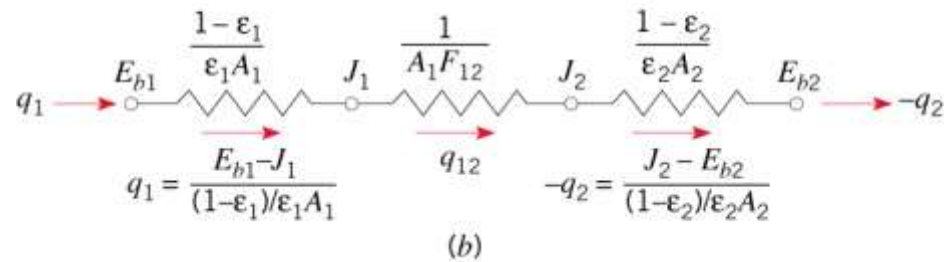
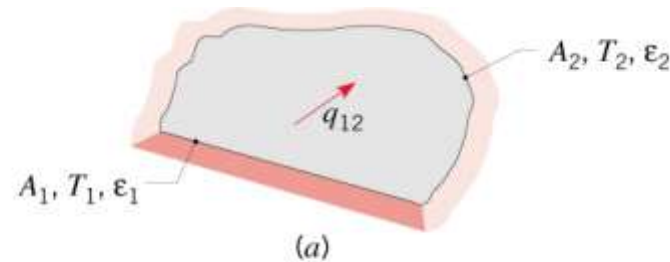
which may be represented by a **radiation network** of the form



- **Methodology of an Enclosure Analysis**
 - Apply Eq. (4) to each surface for which the net radiation heat rate q_i is known.
 - Apply Eq. (5) to each of the remaining surfaces for which the temperature T_i , and hence E_{bi} , is known.
 - Evaluate all of the view factors appearing in the resulting equations.
 - Solve the system of N equations for the unknown radiosities, J_1, J_2, \dots, J_N .
 - Use Eq. (3) to determine q_i for each surface of known T_i and T_i for each surface of known q_i .
- Treatment of the **virtual surface** corresponding to an **opening (aperture)** of area A_i , through which the interior surfaces of an enclosure exchange radiation with large surroundings at T_{sur} :
 - Approximate the opening as blackbody of area, A_i , temperature, $T_i = T_{sur}$, and properties, $\varepsilon_i = \alpha_i = 1$.

Two-Surface Enclosures

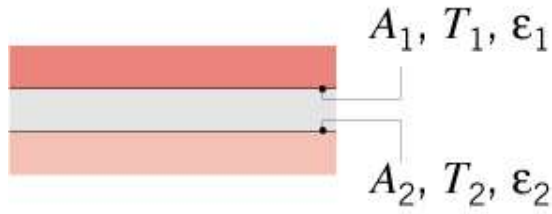
- Simplest enclosure for which radiation exchange is exclusively between two surfaces and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange.



$$q_1 = -q_2 = q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

- Special cases are presented in Table 13.3. For example,

- Large (Infinite) Parallel Plates



$$A_1 = A_2 \equiv A$$

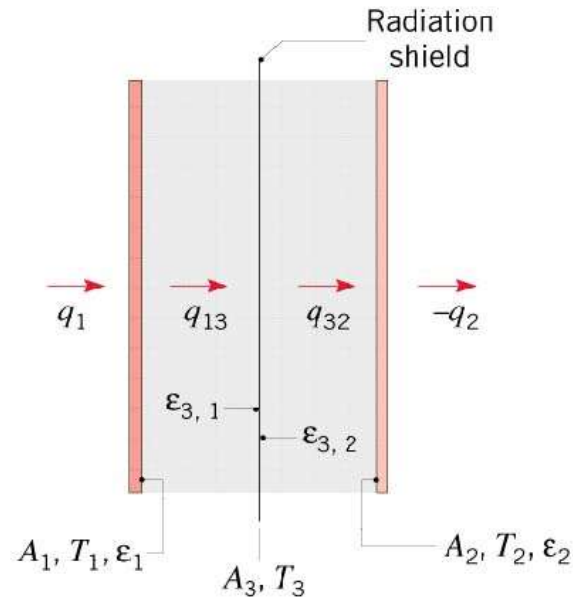
$$F_{12} = 1$$

$$q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

- Note result for *Small Convex Object in a Large Cavity*.

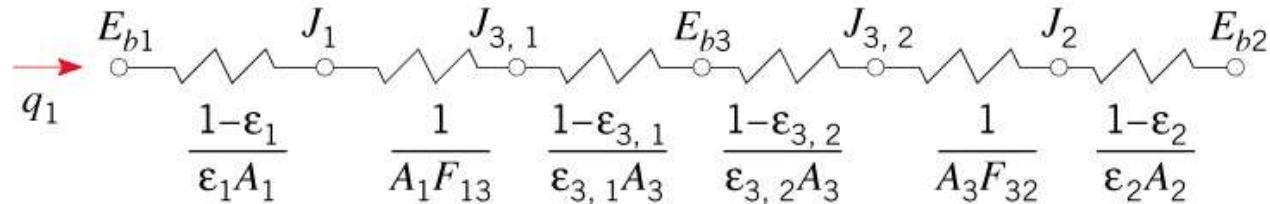
Radiation Shields

- High reflectivity (low $\alpha = \varepsilon$) surface(s) inserted between two surfaces for which a reduction in radiation exchange is desired.
- Consider use of a **single shield** in a two-surface enclosure, such as that associated with **large parallel plates**:



Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.

- Radiation Network:

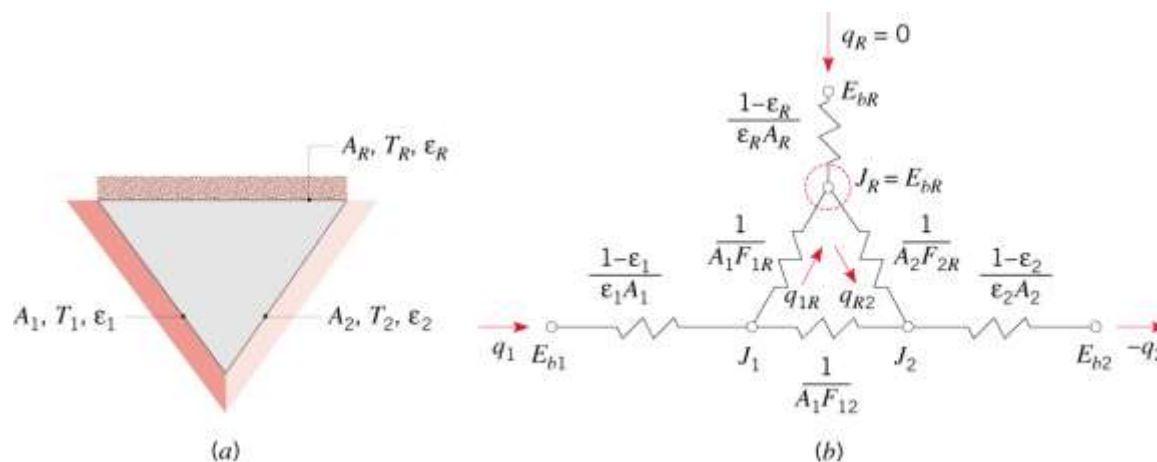


$$q_{12} = q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1} A_3} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2} A_3} + \frac{1}{A_3 F_{32}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

- The foregoing result may be readily extended to account for multiple shields and may be applied to long, concentric cylinders and concentric spheres, as well as large parallel plates.

The Reradiating Surface

- An idealization for which $G_R = J_R$. Hence, $q_R = 0$ and $J_R = E_{bR}$.
- Approximated by surfaces that are **well insulated on one side** and for which **convection is negligible on the opposite (radiating) side**.
- **Three-Surface Enclosure with a Reradiating Surface:**



$$q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left[\left(\frac{1}{A_1 F_{1R}} \right) + \left(\frac{1}{A_2 F_{2R}} \right) \right]^{-1}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

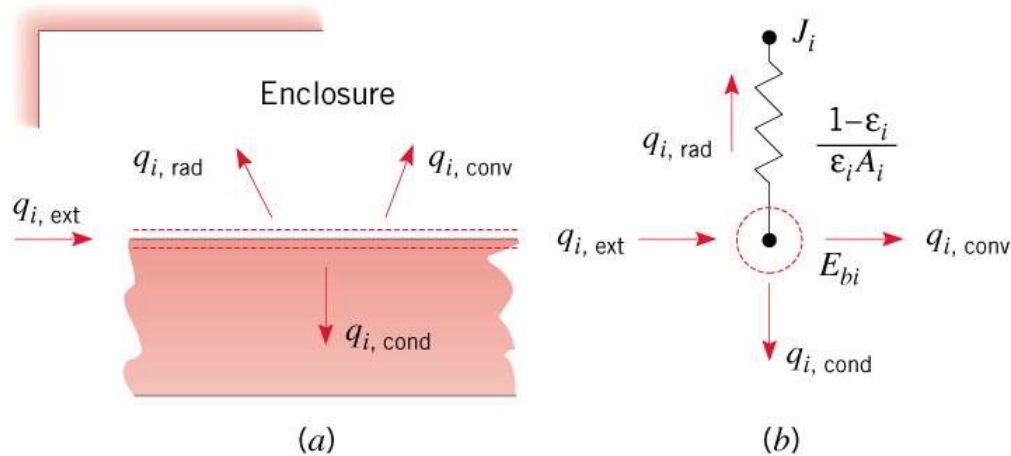
- Temperature of reradiating surface T_R may be determined from knowledge of its radiosity J_R . With $q_R = 0$, a radiation balance on the surface yields

$$\frac{J_1 - J_R}{(1/A_1 F_{1R})} = \frac{J_R - J_2}{(1/A_2 F_{2R})}$$

$$T_R = \left(\frac{J_R}{\sigma} \right)^{1/4}$$

Multimode Effects

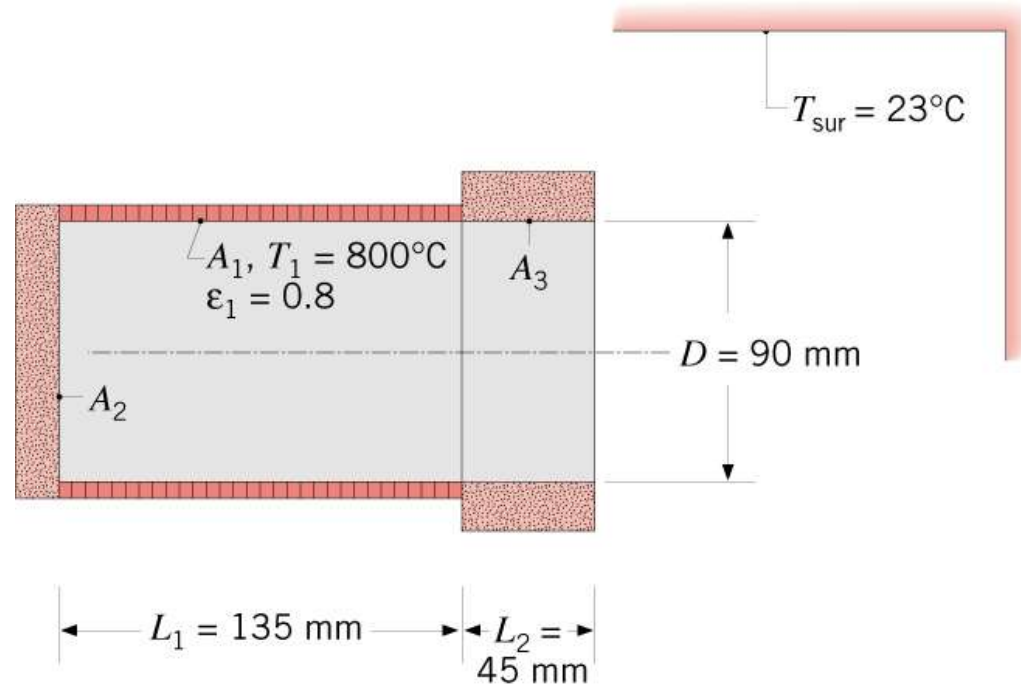
- In an enclosure with conduction and convection heat transfer to or from one or more surfaces, the foregoing treatments of radiation exchange may be combined with surface energy balances to determine thermal conditions.
- Consider a general surface condition for which there is external heat addition (e.g., electrically), as well as conduction, convection and radiation.



$$q_{i,ext} = q_{i,rad} + q_{i,conv} + q_{i,rad}$$

$q_{i,rad} \rightarrow$ Appropriate analysis for N -surface, two-surface, etc., enclosure.

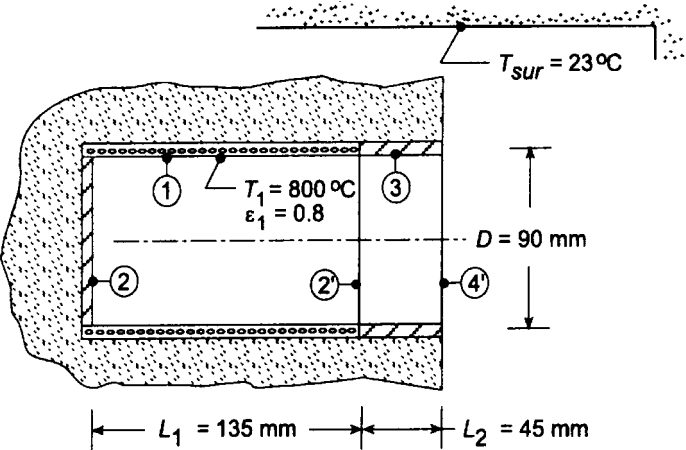
Problem 13.88: Power requirement for a cylindrical furnace with two reradiating surfaces and an opening to large surroundings.



KNOWN: Cylindrical furnace of diameter $D = 90 \text{ mm}$ and overall length $L = 180 \text{ mm}$. Heating elements maintain the refractory lining ($\epsilon = 0.8$) of section (1), $L_1 = 135 \text{ mm}$, at $T_1 = 800^\circ\text{C}$. The bottom (2) and upper (3) sections are refractory lined, but are insulated. Furnace operates in a spacecraft vacuum environment.

FIND: Power required to maintain the furnace operating conditions with the surroundings at 23°C .

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are diffuse gray, and (2) Uniform radiosity over the sections 1, 2, and 3.

ANALYSIS: By defining the furnace opening as the hypothetical area A_4 , the furnace can be represented as a four-surface enclosure.

The power required to maintain A_1 at T_1 is q_1 , the net radiation leaving A_1 .

To obtain q_1 , we must determine the radiosity at each surface by simultaneously solving radiation energy balance equations of the form

$$q_i = \frac{E_{bi} - J_i}{(1 - \epsilon_i) / \epsilon_i A_i} = \sum_{j=1}^N \frac{J_j - J_i}{1 / A_i F_{ij}} \tag{1,2}$$

However, since $\varepsilon_4 = 1$, $J_4 = E_{b4}$, and only three energy balances are needed for A_1 , A_2 , and A_3 .

$$A_1: \quad \frac{E_{b1} - J_1}{(1 - \varepsilon_1) / \varepsilon_1 A_1} = \frac{J_1 - J_2}{1 / A_1 F_{12}} + \frac{J_1 - J_3}{1 / A_1 F_{13}} + \frac{J_1 - J_4}{1 / A_1 F_{14}} \quad (3)$$

$$A_2: \quad 0 = \frac{J_2 - J_1}{1 / A_2 F_{21}} + \frac{J_2 - J_3}{1 / A_2 F_{23}} + \frac{J_2 - J_4}{1 / A_2 F_{24}} \quad (4)$$

$$A_3: \quad 0 = \frac{J_3 - J_1}{1 / A_3 F_{31}} + \frac{J_3 - J_2}{1 / A_3 F_{32}} + \frac{J_3 - J_4}{1 / A_3 F_{34}} \quad (5)$$

where $q_2 = q_3 = 0$ since the surfaces are insulated (adiabatic) and hence reradiating.

From knowledge of J_1 , q_1 can be determined using Eq. (1).

Of the $N^2 = 4^2 = 16$ view factors, $N(N - 1)/2 = 6$ must be independently evaluated, while the remaining can be determined by the summation rule and appropriate reciprocity relations. The six independently determined F_{ij} are:

By inspection: (1) $F_{22} = 0$ (2) $F_{44} = 0$

Problem: Furnace in Spacecraft Environment (cont)

Coaxial parallel disks: From Table 13.2,

$$(3) \quad F_{24} = 0.5 \left\{ S - \left[S^2 - 4(r_4 / r_2)^2 \right]^{1/2} \right\} = 0.05573$$

where

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + 0.250^2}{0.250^2} = 18.00 \quad R_2 = r_2 / L = 45 / 180 = 0.250 \quad R_4 = r_4 / L = 0.250$$

Enclosure 1-2-2' : From the summation rule for A_2 ,

$$(4) \quad F_{21} = 1 - F_{22'} = 1 - 0.09167 = 0.9083$$

where $F_{22'}$ can be evaluated from the coaxial parallel disk relation, Table 13.2, with $R_2 = r_2/L_1 = 45/135 = 0.333$, $R_{2'} = r_2/L_1 = 0.333$, and $S = 11.00$.

From the summation rule for A_1 ,

$$(5) \quad F_{11} = 1 - F_{12} - F_{12'} = 1 - 0.1514 - 0.1514 = 0.6972$$

From symmetry $F_{12} = F_{12'}$ and using reciprocity

$$F_{12} = A_2 F_{21} / A_1 = \left[\pi (0.090\text{m}) (2/4) \right] \times 0.9083 / \pi \times 0.090\text{m} \times 0.135\text{m} = 0.1514$$

Enclosure 2'-3-4: From the summation rule for A_4 ,

$$(6) \quad F_{43} = 1 - F_{42'} - F_{44} = 1 - 0.3820 - 0 = 0.6180$$

where $F_{44} = 0$ and using the coaxial parallel disk relation from Table 13.2, $F_{42'} = 0.3820$ with $R_4 = r_4/L_2 = 45/45 = 1$, $R_{2'} = r_2/L_2 = 1$, and $S = 3$.

The View Factors: Using summation rules and appropriate reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the F_{ij} are

$$\begin{array}{ccc} 0.6972* & 0.1514 & 0.097040.05438 \\ 0.9083* & 0* & 0.035970.05573* \\ 0.2911 & 0.017980.3819 & 0.3090 \\ 0.3262 & 0.055730.6180* & 0* \end{array}$$

The F_{ij} shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5), can be solved simultaneously to obtain the radiosities,

$$J_1 = 73,084 \text{ W/m}^2 \quad J_2 = 67,723 \text{ W/m}^2 \quad J_3 = 36,609 \text{ W/m}^2$$

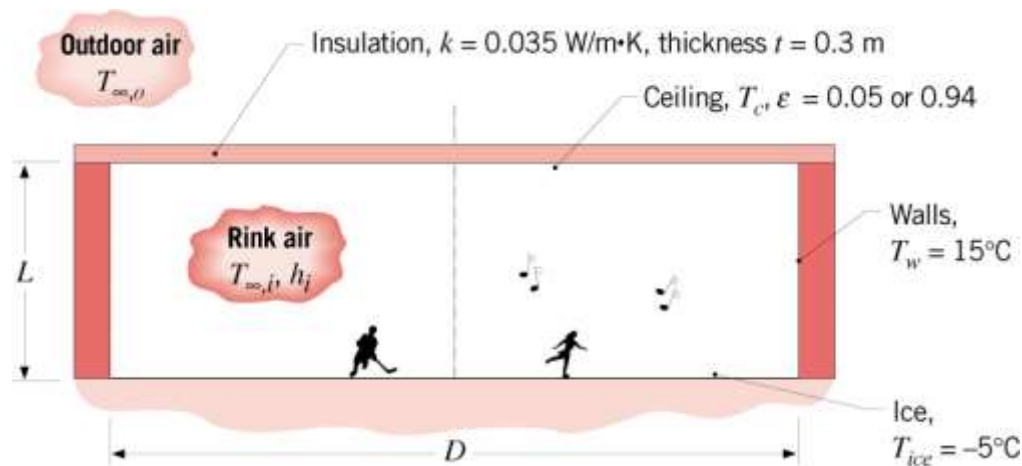
The net heat rate leaving A_1 can be evaluated using Eq. (1) written as

$$q_1 = \frac{E_{b1} - J_1}{(1 - \varepsilon_1) / \varepsilon_1 A_1} = \frac{(75,159 - 73,084) \text{ W/m}^2}{(1 - 0.8) / 0.8 \times 0.03817 \text{ m}^2} = 317 \text{ W} \quad <$$

where $E_{b1} = \sigma T_1^4 = \sigma(800 + 273\text{K})^4 = 75,159 \text{ W/m}^2$ and $A_1 = \pi DL_1 = \pi \times 0.090\text{m} \times 0.135\text{m} = 0.03817 \text{ m}^2$.

COMMENTS: Recognize the importance of defining the furnace opening as the hypothetical area A_4 which completes the four-surface enclosure representing the furnace. The temperature of A_4 is that of the surroundings and its emissivity is unity since it absorbs all radiation incident on it.

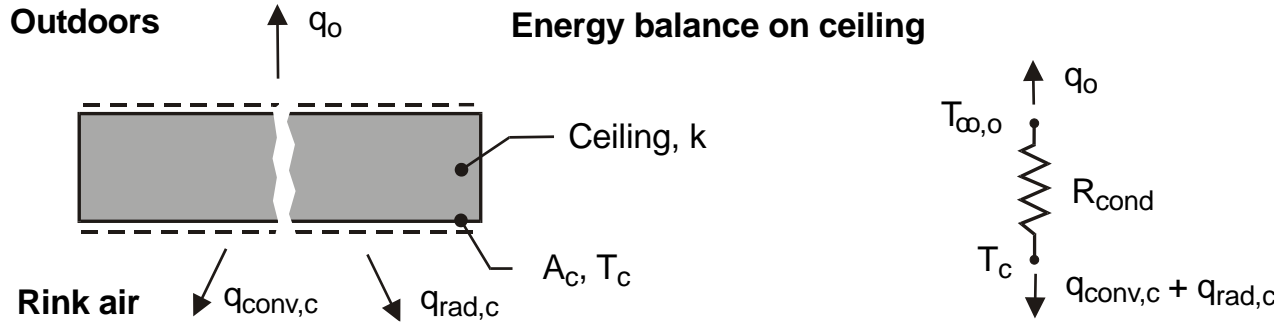
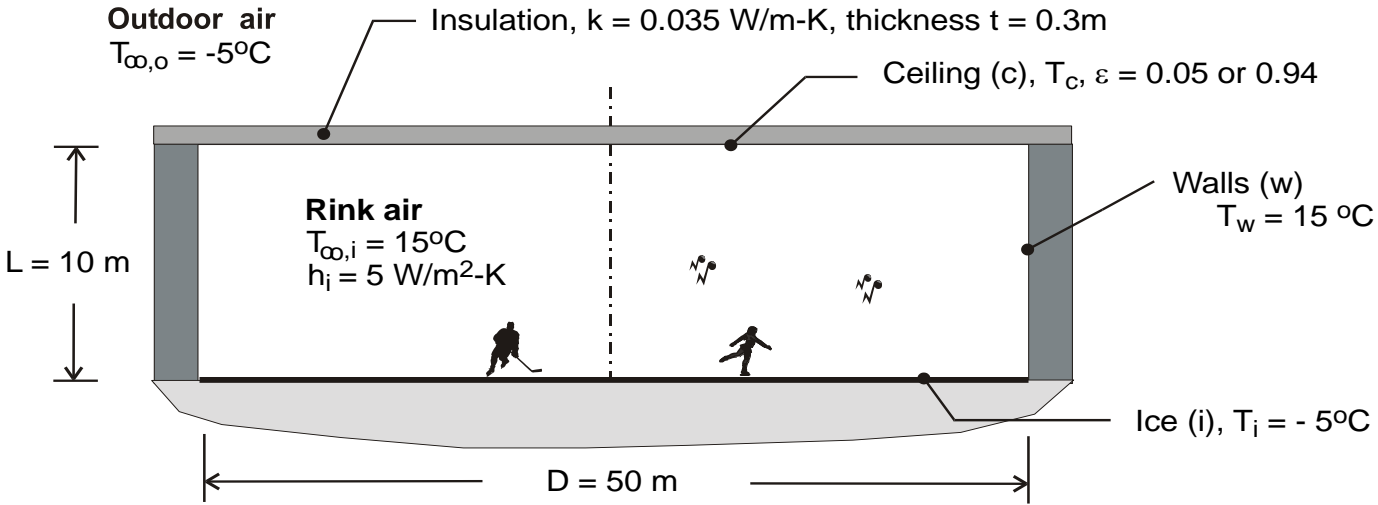
Problem 13.93: Assessment of ceiling radiative properties for an ice rink in terms of ability to maintain surface temperature above the dewpoint.



KNOWN: Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

FIND: (a) Temperature of the ceiling, T_c , for an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \leq t \leq 1 \text{ m}$; identify conditions for which condensation will occur on the ceiling.

SCHEMATIC:



ASSUMPTIONS: (1) Rink comprised of the ice, walls and ceiling approximates a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoor side of the ceiling is negligible compared to the conduction resistance of the ceiling insulation.

PROPERTIES: *Psychrometric chart* (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{\infty,i} = 15^\circ\text{C}$; relative humidity, $\text{RH} = 70\%$): Dew point temperature, $T_{dp} = 9.4^\circ\text{C}$.

ANALYSIS: Applying an energy balance to the inner surface of the ceiling and treating all heat rates as energy *outflows*,

$$\begin{aligned} \dot{E}_{\text{in}} - \dot{E}_{\text{out}} &= 0 \\ -q_o - q_{\text{conv},c} - q_{\text{rad},c} &= 0 \end{aligned} \quad (1)$$

where the rate equations for each process are

$$q_o = (T_c - T_{\infty,o}) / R_{\text{cond}} \quad R_{\text{cond}} = t / kA_c \quad (2,3)$$

$$q_{\text{conv},c} = h_i A_c (T_c - T_{\infty,i}) \quad (4)$$

$$q_{\text{rad},c} = \varepsilon E_b A_c - \alpha A_w F_{wc} E_b = \alpha A_i F_{ic} E_b \quad (5)$$

Since the ceiling panels are diffuse-gray, $\alpha = \varepsilon$.

From Table 13.2 for parallel, coaxial disks

$$F_{ic} = 0.672$$

From the summation rule applied to the ice (i) and the reciprocity rule,

$$F_{ic} + F_{iw} = 1 \quad F_{iw} = F_{cw} \text{ (symmetry)}$$

$$F_{cw} = 1 - F_{ic}$$

$$F_{wc} = A_c / A_w (F_{cw} = A_c / A_w (1 - F_{ic})) = 0.410$$

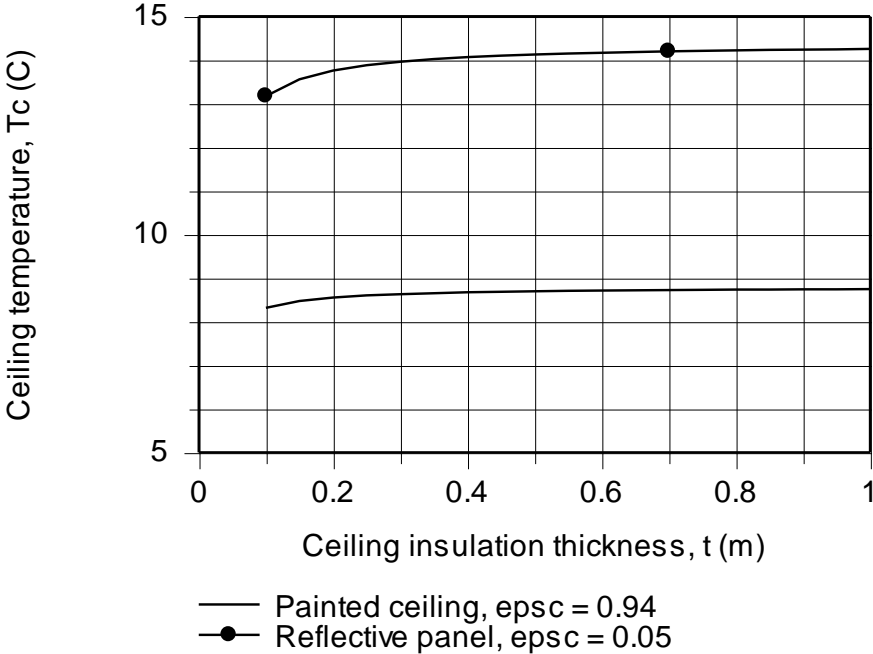
where $A_c = \pi D^2/4$ and $A_w = \pi DL$.

Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

<i>Ceiling panel</i>	ϵ	T_c (°C)	
Reflective	0.05	14.0	
Paint	0.94	8.6	$T_c < T_{dp}$ <

. Condensation will occur on the painted panel since $T_c < T_{dp}$.

(b) Applying the foregoing model for $0.1 \leq t \leq 1.0$ m, the following result is obtained



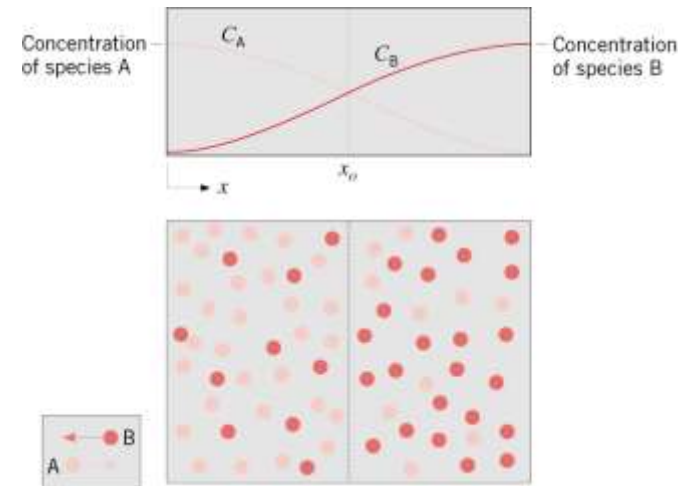
For the reflective panel ($\varepsilon = 0.05$), the ceiling surface temperature is considerably above the dew point. Therefore, condensation will not occur for the range of insulation thicknesses. For the painted panel ($\varepsilon = 0.94$), the ceiling surface temperature is always below the dew point, and condensation occurs for the range of insulation thicknesses.

COMMENTS: From the analysis, recognize that radiative exchange between the ice and the ceiling has the dominant effect on the ceiling temperature. With the reflective panel, the rate is reduced nearly 20-fold relative to that for the painted panel. With the painted panel ceiling, condensation will occur for most of the conditions likely to exist in the rink.

Diffusion Mass Transfer

General Considerations

- **Mass transfer refers to mass in transit due to a species concentration gradient in a mixture.**
 - Must have a mixture of two or more species for mass transfer to occur.
 - The species concentration gradient is the driving potential for transfer.
 - Mass transfer by diffusion is analogous to heat transfer by conduction.
- Physical Origins of Diffusion:
 - Transfer is due to random molecular motion.
 - Consider two species A and B at the same T and p , but initially separated by a partition.
 - Diffusion in the direction of decreasing concentration dictates net transport of A molecules to the right and B molecules to the left.
 - In time, uniform concentrations of A and B are achieved.



Definitions

C_i : **Molar concentration** (kmol/m^3) of species i.

ρ_i : **Mass density** (kg/m^3) of species i.

\mathcal{M}_i : **Molecular weight** (kg/kmol) of species i.

$$\rho_i = \mathcal{M}_i C_i$$

J_i^* : **Molar flux** ($\text{kmol}/\text{s} \cdot \text{m}^2$) of species i **due to diffusion**.

➤ Transport of i relative to molar average velocity (v^*) of mixture.

N_i'' : **Absolute molar flux** ($\text{kmol}/\text{s} \cdot \text{m}^2$) of species i.

➤ Transport of i relative to a fixed reference frame.

j_i : **Mass flux** ($\text{kg}/\text{s} \cdot \text{m}^2$) of species i **due to diffusion**.

➤ Transport of i relative to mass-average velocity (v) of mixture.

n_i'' : **Absolute mass flux** ($\text{kg}/\text{s} \cdot \text{m}^2$) of species i.

➤ Transport of i relative to a fixed reference frame.

x_i : **Mole fraction** of species i ($x_i = C_i / C$).

m_i : **Mass fraction** of species i ($m_i = \rho_i / \rho$).

Property Relations

- **Mixture Concentration:**

$$C = \sum_i C_i \rightarrow \sum_i x_i = 1$$

- **Mixture Density:**

$$\rho = \sum_i \rho_i \rightarrow \sum_i m_i = 1$$

- **Mixture of Ideal Gases:**

$$C_i = \frac{p_i}{\mathfrak{R}_i T}$$

$$\rho_i = \frac{p_i}{R_i T}$$

$$p = \sum_i p_i$$

$$x_i = \frac{C_i}{C} = \frac{p_i}{p}$$

Molar and Mass Fluxes of Species A due to Diffusion in a Binary Mixture of Species A and B

- Molar Flux of Species A:

- By definition:

$$J_A^* = C_A (v_A - v^*)$$

$$v^* = x_A v_A + x_B v_B$$

- From **Fick's law** (mass transfer analog to Fourier's law):

$$J_A^* = -CD_{AB} \nabla x_A$$

↳ **Binary diffusion coefficient** or **mass diffusivity** (m²/s)

- Mass Flux of Species A:

- By definition:

$$j_A = \rho_A (v_A - v)$$

$$v = m_A v_A + m_B v_B$$

- From Fick's law:

$$j_A = -\rho D_{AB} \nabla m_A$$

Absolute Molar and Mass Fluxes of Species A in a Binary Mixture of Species A and B

- Molar Flux of Species A:

$$N_A'' = C_A v_A = J_A^* + C_A v^*$$

$$N_A'' = J_A^* + C_A (x_A v_A + x_B v_B)$$

$$N_A'' = -CD_{AB} \nabla x_A + x_A (N_A'' + N_B'')$$

- Mass Flux of Species A:

$$n_A'' = \rho_A v_A = j_A + \rho_A v$$

$$n_A'' = j_A + \rho_A (m_A v_A + m_B v_B)$$

$$n_A'' = -\rho D_{AB} \nabla m_A + m_A (n_A'' + n_B'')$$

- Special Case of **Stationary Medium**:

➤ $v^* = 0 \rightarrow N_A'' = J_A^*$

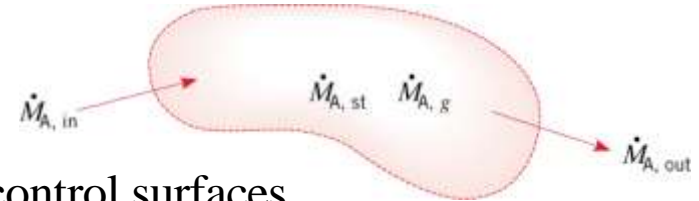
➤ $v = 0 \rightarrow n_A'' = j_A$

➤ Achieved to a good approximation for x_A (or m_A) \square 1 and N_B'' (or n_B'') \approx 0.

Conservation of Species

- Application to a Control Volume at an Instant of Time:

$$\dot{M}_{A,in} - \dot{M}_{A,out} + \dot{M}_{A,g} = \frac{dM_A}{dt} \equiv \dot{M}_{A,st}$$



$\dot{M}_{A,in}, \dot{M}_{A,out}$ → rate of transport across the control surfaces

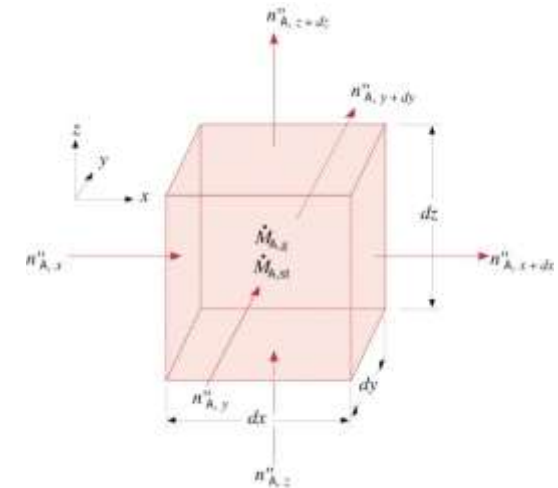
$\dot{M}_{A,g}$ → rate of generation of A due to **homogeneous chemical reactions** occurring in the control volume

$\dot{M}_{A,st}$ → rate of accumulation of A in the control volume

- Application in **Cartesian Coordinates** to a Differential Control Volume for a **Stationary Medium** of **Constant D_{AB}** and **C or ρ** :

- **Species Diffusion Equation on a Molar Basis:**

$$\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} + \frac{N_A}{D_{AB}} = \frac{1}{D_{AB}} \frac{\partial C_A}{\partial t}$$



- **Species Diffusion Equation on a Mass Basis:**

$$\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} + \frac{\partial^2 \rho_A}{\partial z^2} + \frac{n_A}{D_{AB}} = \frac{1}{D_{AB}} \frac{\partial \rho_A}{\partial t}$$

- **Boundary Conditions** (Molar Basis):

- Consider a Gas (A) / Liquid (B) or Gas (A) / Solid (B) Interface.

Known surface concentration:

$$x_A(0) = x_{A,s}$$

For weakly soluble conditions of a gas A in liquid B,

$$x_{A,s} = \frac{p_A}{H} \quad (\text{Henry's law})$$

$H \rightarrow$ **Henry's constant** (Table A.9)

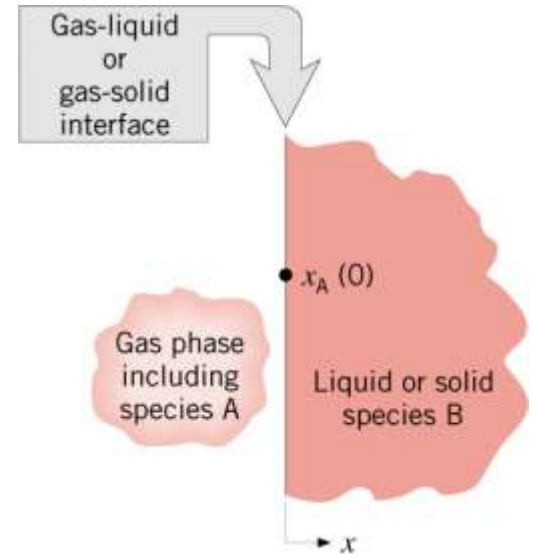
For gas A in a uniform solid B,

$$C_A(0) = Sp_A$$

$S \rightarrow$ **solubility** ($\text{kmol/m}^3 \cdot \text{bar}$) (Table A.10)

- **Heterogeneous** (surface) **reactions** (**Catalysis**)

$$N_A''(0) = N_A'' - CD_{AB} \left. \frac{dx_A}{dx} \right|_{x=0}$$



Special Cases for One-Dimensional , Steady-State Diffusion in a Stationary Medium

- Diffusion without Homogeneous Chemical Reactions

➤ For Cartesian coordinates, the molar form of the species diffusion equation is

$$\frac{d^2 x_A}{dx^2} = 0 \quad (1)$$

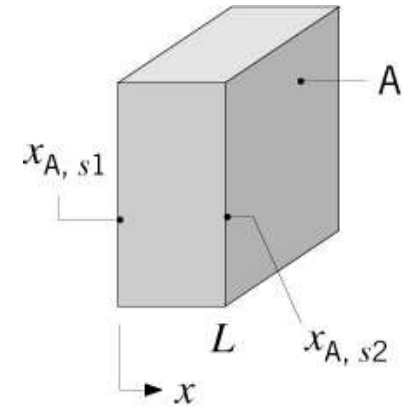
➤ Plane wall with known surface concentrations:

$$x_A(x) = (x_{A,s,2} - x_{A,s,1}) \frac{x}{L} + x_{A,s,1}$$

$$N''_{A,x} = J^*_{A,x} = -CD_{AB} \frac{dx_A}{dx} = \frac{D_{AB}(C_{A,s,1} - C_{A,s,2})}{L}$$

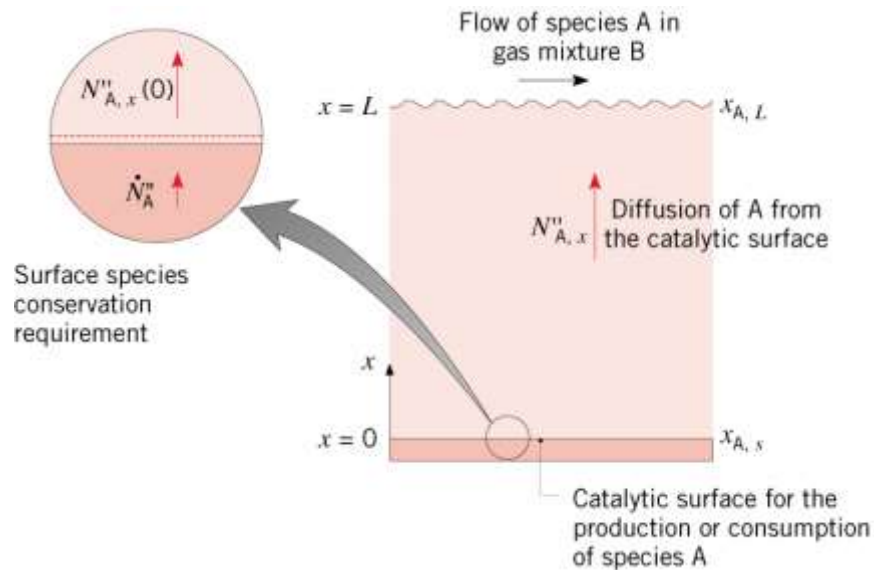
$$N_{A,x} = AN''_{A,x} = \frac{D_{AB}A}{L}(C_{A,s,1} - C_{A,s,2})$$

$$R_{m,diff} = \frac{L}{D_{AB}A}$$



Results for cylindrical and spherical shells → Table 14.1

- Planar medium with a **first-order catalytic surface**:



Assuming depletion of species A at the catalytic surface ($x = 0$),

$$N''_{A,x}(0) = \dot{N}''_A = -k_1'' C_A(0)$$

↳ Reaction rate constant (m/s)

$$-D_{AB} \left. \frac{dx_A}{dx} \right|_{x=0} = -k_1'' x_A(0)$$

Assuming knowledge of the concentration at a distance $x=L$ from the surface,

$$x_A(L) = x_{A,L}$$

Solution to the species diffusion equation (1) yields a linear distribution for $x_A(x)$:

$$\frac{x_A(x)}{x_{A,L}} = \frac{1 + \left(xk_1'' / D_{AB} \right)}{1 + \left(Lk_1'' / D_{AB} \right)}$$

Hence, at the surface,

$$\frac{x_A(0)}{x_{A,L}} = \frac{1}{1 + \left(Lk_1'' / D_{AB} \right)}$$

$$N_A''(0) = -CD_{AB} \left. \frac{dx_A}{dx} \right|_{x=0} = -\frac{k_1'' C x_{A,L}}{1 + \left(Lk_1'' / D_{AB} \right)}$$

Limiting Cases:

– Process is **reaction limited**:

$$k_1'' \rightarrow 0 \quad \left(Lk_1'' / D_{AB} \right) \ll 1$$

$$\frac{x_A(0)}{x_A(L)} \approx 1$$

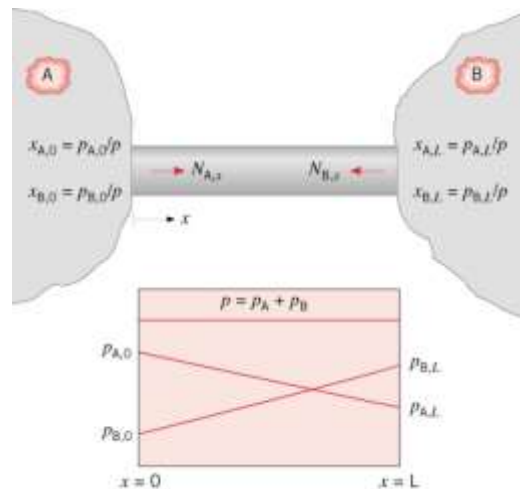
$$N_A''(0) \approx -k_1'' C x_{A,L}$$

– Process is **diffusion limited**:

$$k_1'' \rightarrow \infty \quad \left(Lk_1'' / D_{AB} \right) \gg 1$$

$$x_A(0) \approx 0 \quad N_A''(0) \approx -\frac{CD_{AB}x_{A,L}}{L}$$

➤ **Equimolar counterdiffusion**:



Occurs in an ideal gas mixture if p and T , and hence C , are uniform.

$$N_{A,x}'' = -N_{B,x}''$$

$$N_{A,x}'' = D_{AB} \frac{C_{A,0} - C_{A,L}}{L} = \frac{D_{AB}}{\mathfrak{R}T} \frac{p_{A,0} - p_{A,L}}{L}$$

- Diffusion with Homogeneous Chemical Reactions**

For Cartesian coordinates, the molar form of the species diffusion equation is

$$D_{AB} \frac{d^2 C_A}{dx^2} + \dot{N}_A = 0$$

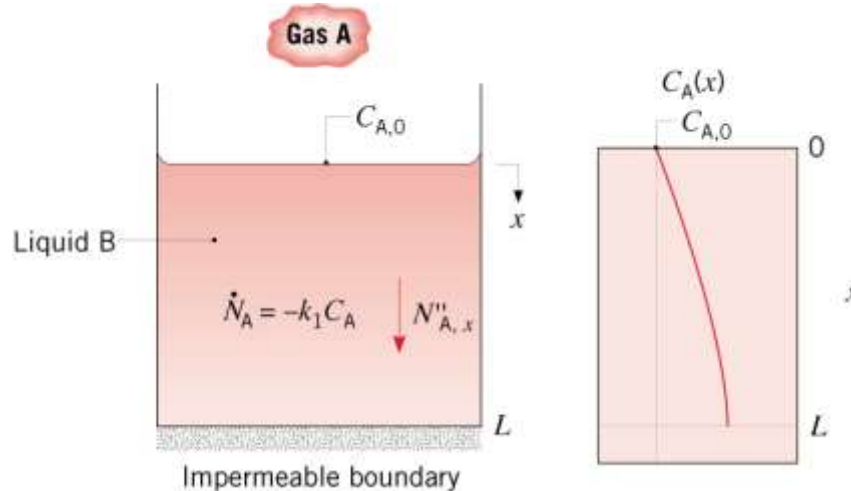
For a **first-order reaction** that results in **consumption** of species A,

$$\dot{N}_A = -k_1 C_A$$

and the general solution to the diffusion equation is

$$C_A(x) = C_1 e^{mx} + C_2 e^{-mx} \quad m = (k_1 / D_{AB})^{1/2}$$

Consider diffusion and homogeneous reaction of gas A in a liquid (B) container with an impermeable bottom:



Boundary conditions \longrightarrow

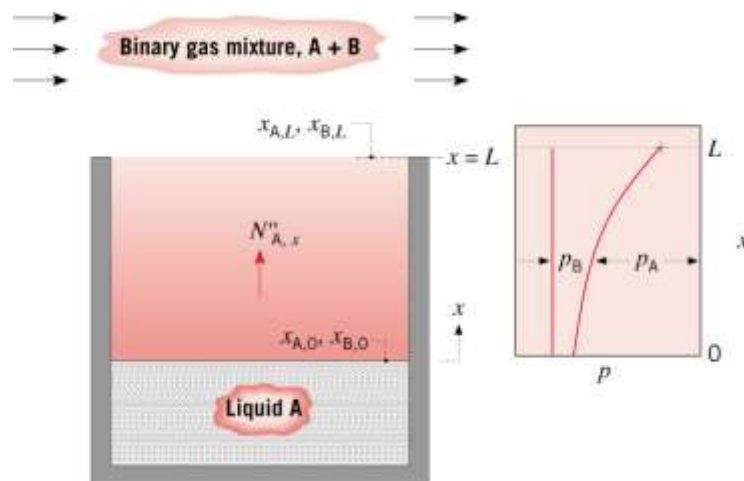
$$C_A(0) = C_{A,0} \qquad \left. \frac{dC_A}{dx} \right|_{x=L} = 0$$

Solution \longrightarrow

$$C_A(x) = C_{A,0} (\cosh mx - \tanh ml \sinh mx)$$

$$N''_{A,x}(0) = D_{AB} C_{A,0} m \tanh ml$$

Evaporation in a Column: A Nonstationary Medium



➤ Special Features:

- Evaporation of A from the liquid interface ($x_{A,0} = x_{A,sat(v)} > x_{A,L}$)
- Insolubility of species B in the liquid. Hence downward motion by diffusion must be balanced by upward bulk motion (advection) such that the absolute flux is everywhere zero.

$$N''_{B,x} = 0$$

- Upward transport of A by diffusion is therefore augmented by advection.

➤ Solution:

$$\frac{1-x_A}{1-x_{A,0}} = \left(\frac{1-x_{A,L}}{1-x_{A,0}} \right)^{x/L}$$

$$N''_{A,x} = \frac{CD_{AB}}{L} \ln \left(\frac{1-x_{A,L}}{1-x_{A,0}} \right)$$

One-Dimensional, Transient Diffusion in a Stationary Medium without Homogeneous Chemical Reactions

- Species Diffusion Equation in Cartesian coordinates

$$D_{AB} \frac{\partial^2 C_A}{\partial x^2} = \frac{\partial C_A}{\partial t}$$

- Initial and Boundary Conditions for a Plane Wall with Symmetrical Surface Conditions

$$C_A(x, 0) = C_{A,i}$$

$$C_A(L, t) = C_{A,s}$$

$$\left. \frac{\partial C_A}{\partial x} \right|_{x=0} = 0$$

- Nondimensionalization

$$\gamma^* \equiv \frac{C_A - C_{A,s}}{C_{A,i} - C_{A,s}}$$

$$x^* \equiv \frac{x}{L}$$

$$t_m^* \equiv \frac{D_{AB} t}{L^2} \equiv Fo_m$$

↳ Mass transfer Fourier number

➤ Species Diffusion Equation

$$\frac{\partial^2 \gamma^*}{\partial x^{*2}} = \frac{\partial \gamma^*}{\partial Fo_m}$$

➤ Initial and Boundary Conditions

$$\gamma^*(x^*, 0) = 1$$

$$\gamma^*(1, Fo_m) = 0$$

$$\left. \frac{\partial \gamma^*}{\partial x^*} \right|_{x^*=0} = 0$$

- Analogous to transient heat transfer by conduction in a plane wall with symmetrical surface conditions for which $Bi \rightarrow \infty$, and hence $T_s \rightarrow T_\infty$.

Hence, the corresponding one-term approximate solution for conduction may be applied to the diffusion problem by making the substitutions

$$\theta^* \leftrightarrow \gamma^*$$

$$Fo \leftrightarrow Fo_m$$

- Table 14.2 summarizes analogy between heat and mass transfer variables.