# 19CAB01 - COMPUTER ORGANIZATION AND ARCHITECTURE 

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## 20MC102/COMPUTER ORGANIZATION AND ARCHITECTURE

## Objectives:

- To impart the knowledge in the field of digital electronics
- To impart knowledge about the various components of a computer and its internals.
- To design and realize the functionality of the computer hardware with basic gates and other components using combinational and sequential logic.
- To understand the importance of the hardware-software interface


## UNIT-I

## DIGITAL FUNDAMENTALS

Number Systems and Conversions - Boolean Algebra and Simplification

- Minimization of Boolean Functions - Karnaugh Map, Logic Gates NAND - NOR Implementation.


## UNIT-II COMBINATIONAL AND SEQUENTIAL CIRCUITS

Design of Combinational Circuits - Adder - Subtractor - Encoder Decoder - MUX - DEMUX - Comparators, Flip Flops - Triggering

- Master - Slave Flip Flop - State Diagram and Minimization Counters - Registers.


## UNIT-III BASIC STRUCTURE OF COMPUTERS

Functional units - Basic operational concepts - Bus structures Performance and Metrics - Instruction and Instruction sequencing Addressing modes - ALU Design - Fixed point and Floating point operations.

UNIT-IV PROCESSOR DESIGN
Processor basics - CPU Organization - Data path design - Control design - Basic concepts - Hardwired control - Micro programmed control - Pipeline control - Hazards - Super scalar operations.

## UNIT-V MEMORY,I/O SYSTEM AND PARALLEL PROCESSING

Memory technology - Memory systems - Virtual memory Caches - Design methods - Associative memories - Input/output system - Programmed I/O - DMA and Interrupts - I/O Devices and Interfaces - Multiprocessor Organization - Symmetric multiprocessors - Cache Coherence.

## Outcomes:

- Able to design digital circuits by simplifying the Boolean functions
- Able to Understand the organization and working principle of computer hardware components
- Able to understand mapping between virtual and physical memory
- Acquire knowledge about multiprocessor organization and parallel processing
- Able to trace the execution sequence of an instruction through the processor


## TEXT BOOKS :

1. Morris Mano, "Digital Design", Prentice Hall of India, Fourth Edition 2010.
2. Carl Hamacher, Zvonko Vranesic, SafwatZaky and Naraig Manjikian, "Computer organization and Embedded Systems", Sixth Edition, Tata McGraw Hill, 2012.
3. William Stallings, "Computer Organization \& ArchitectureDesigning for Performance" 9th Edition 2012.

## REFERENCES:

1. Charles H. Roth, Jr., "Fundamentals of Logic Design", Jaico Publishing House, Mumbai, Fourth Edition, 2002.
2. David A. Patterson and John L. Hennessy, "Computer Organization and Design: The Hardware/Software Interface", Fourth Edition, Morgan Kaufmann/Elsevier,2009.
3. John P. Hayes, "Computer Architecture and Organization", Third Edition, Tata McGraw Hill, 2000

## What is COA(Computer Organization and Architecture)?

- It is the study of internal working, structuring and implementation of a computer system.
- Architecture in computer system refers to the externally visual attributes of the system.
- Organization of computer system results in realization of architectural specifications of a computer system.


## Overview

- Computer organization
- physical aspects of computer systems.
- E.g., circuit design, control signals, memory types.
- How does a computer work?
- Computer architecture
- Logical aspects of system as seen by the programmer.
- E.g., instruction sets, instruction formats, data types, addressing modes.
- How do I design a computer?


## Why study computer organization and architecture?

- Design better programs, including system software such as compilers, operating systems, and device drivers.
- Optimize program behavior.
- Evaluate (benchmark) computer system performance.
- Understand time, space, and price tradeoffs.


## Computer Architecture VS Computer Organization

| Computer Architecture | Computer Organization |
| :--- | :--- |
| Concerned with the way hardware components are <br> connected together to form a computer system. | Concerned with the structure and behaviour of a <br> computer system as seen by the user. |
| Acts as the interface between hardware and <br> software. | Deals with the components of a connection in a <br> system. |
| Helps us to understand the functionalities of a <br> system. | Tells us how exactly all the units in the system are <br> arranged and interconnected. |
| A programmer can view architecture in terms of <br> instructions, addressing modes and registers. | Whereas Organization expresses the realization of <br> architecture. |
| While designing a computer system architecture is <br> considered first. | An organization is done on the basis of <br> architecture. |
| Deals with high-level design issues. | Deals with low-level design issues. |
| Architecture involves Logic (Instruction sets, <br> Addressing modes, Data types, Cache <br> optimization) | Organization involves Physical Components <br> (Circuit design, Adders, Signals, Peripherals) |

## NUMBER SYSTEM

- It is defined as a system of writing to express numbers.
- It is the mathematical notation for representing numbers of a given set by using digits or other symbols in a consistent manner.
- The value of any digit in a number can be determined by:
- The digit
- Its position in the number
- The base of the number system


## Characteristics of Numbering Systems

1) The digits are consecutive.
2) The number of digits is equal to the size of the base.
3) Zero is always the first digit.
4.)

The base number is never a digit.
When 1 is added to the largest digit, a sum of zero and a carry of one results.
6) Numeric values are determined by the implicit positional values of the digits.

## Base-N Number System

- Base N
- N Digits: 0, 1, 2, 3, 4, 5, ..., N-1
- Example: $1045_{\mathrm{N}}$
- Positional Number System

$$
\begin{aligned}
& N^{n-1} \cdots N^{4} N^{3} N^{2} N^{1} N^{0} \\
& d_{n-1} \cdots d_{4} d_{3} d_{2} d_{1} d_{0}
\end{aligned}
$$

- Digit $d_{0}$ is the least significant digit (LSD).
- Digit $d_{n-1}$ is the most significant digit (MSD).
- The most common number systems used in digital technology. They are,
$>$ Decimal number system
$>$ Binary number system
$>$ Octal number system
> Hexadecimal number system


## Decimal Number System

- Composed of ten symbols ( $0,1,2,3,4,5,6,7,8,9$ ) known as Base 10 system. Example: 1045ı
- Positional Value System in which value of digit depends on its position.
- Example: 8247.324 is represented as
$\left(8 \times 10^{3}\right)+\left(2 \times 10^{2}\right)+\left(4 \times 10^{1}\right)+\left(7 \times 10^{0}\right)+\left(3 \times 10^{-1}\right)+\left(2 \times 10^{-2}\right)+\left(4 \times 10^{-3}\right)$



## Binary Number System

- Composed of two symbols ( 0,1 ) also called as "Base 2 system"
- Used to model the series of electrical signals computers use to represent information
- o represents no voltage or an off state and 1 represents the presence of voltage or an on state. Binary numbers o and 1 known as bits.
- Example: 1011.101 is represented as

$$
\begin{aligned}
& \left(1 \times 2^{3}\right)+\left(\mathrm{OX2}^{2}\right)+\left(1 \times 2^{1}\right)+\left(1 \times 2^{0}\right)+\left(1 \times 2^{-1}\right)+\left(\mathrm{OX}^{-2}\right)+\left(1 \times 2^{-3}\right) \\
& \begin{array}{lllllll}
2^{3} & 2^{2} & 2^{1} & 2^{0} & 2^{-1} & 2^{-2} & 2^{-3}
\end{array} \\
& \text { MSD Decimal Point LSD }
\end{aligned}
$$

## Octal Number System

- Composed of eight symbols (0,1,2,3,4,5,6,7) also known as the Base 8 System
- Groups of three (binary) digits can be used to represent each octal digit
- Compared with binary system this is compact and occupies less space for data.
- Example: 5432 is represented as

$$
\left.5^{x} 8^{3}\right)+\left(4 \times 8^{2}\right)+\left(3 \times 8^{1}\right)+\left(2 \times 8^{\circ}\right)
$$

## Hexa Decimal Number System

- Base 16 system
- Uses 16 symbols (digits o-9 \& letters A,B,C,D,E,F)
- Groups of four bits represent each base 16 digit

Example: 306B.1C is represented as
$\left(3 \times 16^{3}\right)+\left(\mathrm{ox}_{1} 6^{2}\right)+\left(6 \times 16^{1}\right)+\left(\mathrm{Bx16}^{\mathrm{o}}\right)+\left(\mathrm{xx16}^{-1}\right)+\left(\mathrm{Cx16}^{-2}\right)$

## FOUR BASIC NUMBER SYSTEM DIGITS TABLE

| Decimal | Binary | Octal | Hexadecimal |
| :---: | :---: | :---: | :---: |
| 0 | 0000 | 0 | 0 |
| 1 | 0001 | 1 | 1 |
| 2 | 0010 | 2 | 2 |
| 3 | 0011 | 3 | 3 |
| 4 | 0100 | 4 | 4 |
| 5 | 0101 | 5 | 5 |
| 6 | 0110 | 6 | 6 |
| 7 | 0111 | 7 | 7 |
| 8 | 1000 | 10 | 8 |
| 9 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

## CONVERSIONS

- Conversion of a number system are, decimal to any base, any base to decimal, binary to any base, any base to binary, octal to hexa, hexa to octal.
Decimal to any base conversion
a) Decimal to binary conversion
i) Converting a decimal number by 2 until quotient of $o$ is obtained.
ii) Binary number is obtained by taking reminder after each division in reverse order. (double-dabble method)
- Example: 89.625


Fraction Part
$(89.625)_{10}=(-)_{2}$

| 2 | 89 |  |
| ---: | ---: | ---: |
| 2 | 44 | 1 |
| 2 | 22 | 0 |
| 2 | 11 | 0 |
| 2 | 5 | 1 |
| 2 | 2 | 1 |
| 2 | 1 | 0 |
|  | 0 | 1 |$\uparrow$

$$
\begin{aligned}
& 0.625 \times 2=1.250 \Rightarrow 1 \downarrow \\
& 0.250 \times 2=0.500 \Rightarrow 0 \downarrow \\
& 0.500 \times 2=1.0 \quad \Rightarrow \quad 1 \downarrow
\end{aligned}
$$

$$
\therefore(89.625)_{10}=(1011001.101)_{2}
$$

- Binary number is obtained by multiplying number continuously by 2 , recording a carry each time is known as fraction conversion.
b) Decimal to Octal conversion
- Procedure is same as that of double dabble method

Example: $(1032.6875)_{10}$
$(1032.6875)_{10} \rightarrow(?)_{8}$

| 8 | 1032 |
| :--- | :---: |
| 8 | 129,0 |
| 8 | 16,1 |
|  | 2,0 |

## For Real Part-

- We convert the real part from base 10 to base 8 using division method same as above.
- So, $(1032)_{10}=(2010)_{8}$


## For Fractional Part-

- We convert the fractional part from base 10 to base 8 using multiplication method.


## Step-o1:

- Multiply 0.6875 with 8 . Result $=5.5$.
- Write 5 in real part and 0.5 in fractional part.

Step-o2:

- Multiply o. 5 with 8 . Result $=4.0$.
- Write 4 in real part and o.o in fractional part.
- Since fractional part becomes o, so we stop.

|  | Real part | Fractional Part |
| :---: | :---: | :---: |
| $0.6875 \times 8$ | 5 | 0.5 |
| $0.5 \times 8$ | 4 | 0.0 |

- From here, $(0.6875)_{10}=(0.54)_{8}$
- Combining the result of real and fractional parts, we have,

$$
(1032.6875)_{10}=(2010.54)_{8}
$$

c) Decimal to Hexa conversion Same as double-dabble method.
Example: (2020) ${ }_{10}$

- Using division method, we have-


Any base to Decimal conversion
a) Binary to Decimal

Example: $(\mathbf{1 0 1 1 1 1 . 1 1 0 1})_{2} \rightarrow(?)_{10}$

- Converted into decimal by multiplying 1 or o by their weight adding the two integer conversion.
Integer Part : 101111
$=\left(1 \times 2^{5}+0 \times 2^{4}+1 \times 2^{3}+1 \times 2^{2}+1 \times 2^{1}+1 \times 2^{0}\right)$
$=(32+0+8+4+2+1)$
$=(47)$

Fraction Part 1101
$=\left(1 \times 2^{-1}+1 \times 2^{-2}+0 \times 2^{-3}+1 \times 2^{-4}\right)$
$=(0.5000+0.2500+0.0000+0.0625)$
$=0.8125$

Result $=(101111.1101)_{2} \rightarrow(47.8125)_{10}$

```
b) Octal to Decimal
Example:
\((36.4)_{8}\)
Integer Conversion : 36
\(=3 \times 8^{1}+6 \times 8^{0}\)
\(=24+6\)
\(=30\)
```

Fraction Conversion : 0.4
$=4 \times 8^{-1}$
$=4 \times 0.125$
$=0.500$
Result : $(36.4)_{8}=(30.5)_{10}$

## c) Hexa to Decimal

## Example:

$\left(\mathrm{A}_{3} \mathrm{~B}\right)_{16}$
$=\mathbf{A x} 16^{2}+3 \times 16^{1}+\mathrm{B} \times 16^{0}$
$=2560+048+011$
$=2619$
Result : $(\mathrm{A} 3 \mathrm{~B})_{16}=(2619)_{10}$

Binary to any Base Conversion
a) Binary to Octal

Reverse of Octal to Binary
Procedure: Form 3-bit combination moving from right most position to left position
Example: (10011010101) ${ }_{2}$

| 010 | 011 | 010 | 101 |
| :--- | :---: | :---: | :---: |
| 2 | 3 | 2 | 5 |
| $(10011010101$ | $)_{2}$ | $=$ | $(2325)_{8}$ |

## b) Binary to Hexa

Procedure: Write binary number starting from right most position, the group binary number into groups of four bits.
Example:
$(10100110)_{2}$
$1010 \quad 0110$
A 6
$(10100110)_{2}=(\mathrm{A} 6)_{16}$

## Any Base to Binary Conversion

a) Octal to Binary

Reverse method of binary to octal.
Example:
$(532)_{8}$

| 5 | 3 | 2 |
| :--- | :--- | :--- |
| 101 | 011 | 010 |

$(532)_{8}=(101011010)_{2}$
b) Hexa to Binary

Reverse method of binary to hexa conversion.
Example:

| $(2 D 6)_{16}$ |  |  |
| :---: | :---: | :---: |
| 2 | D | 6 |
| 0010 | 1101 | 0110 |

$(2 \mathrm{D} 6)=(\mathrm{oo1011010110})_{2}$

## Hexadecimal Conversion

a) Hexa to Octal

Convert the given hexa decimal number to binary equivalent and then from binary to octal.
Example:
$(5 \mathrm{C} 2)_{16}$

| (Hexa to Binary) | 5 | $C$ | 2 |  |
| :--- | :--- | :--- | :---: | :---: |
|  | 0101 | 1100 | 0010 |  |
| (Binary to Octal) | 010 | 111 | 000 | 010 |
|  | 2 | 7 | 0 | 2 |

$$
(5 \mathrm{C} 2)=(2702)_{8}
$$

b) Octal to Hexa

- Convert octal to binary number and binary into equivalent hexadecimal number.


## Example:

$$
(321)_{8}
$$

| Octal to Binary | 3 | 2 | 1 |
| :--- | :--- | :--- | :--- |
|  | 011 | 010 | 001 |
| Binary to Hexa | 1101 | 0001 |  |
|  | $D$ | 1 |  |
|  | $(321)_{8}=(D 1)_{16}$ |  |  |

## BOOLEAN ALGEBRA

- The boolean algebra is used to simplify the design the logic circuits and involves lengthy mathematical operations.
- Binary logic deals with two values o and 1.(o-False, 1True)
- Karnaugh Map- Simplification of boolean equation upto four equations. (Difficult if more than five input variables).
- Quine Mcclusky method- Tabular method of minimization(reduces the requirement of hardware)


## Boolean Logic Operations

- Boolean Function- Algebraic expression formed using constant, binary variables and basic logic operation symbols.
- Logical Operation
- AND (Multiplication)
- OR (Addition)
- NOT (Logical Complementation)


## Logical AND

$$
\mathrm{Y}=\mathrm{A} . \mathrm{B}
$$

## Truth Table

| Input |  | Output |
| :---: | :---: | :---: |
| A | B | Y=A.B |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Logical OR

$$
\mathrm{Y}=\mathrm{A}+\mathrm{B}
$$

Truth Table

| Input |  | Output |
| :---: | :---: | :---: |
| A | B | $\mathrm{Y}=\mathrm{A}+\mathrm{B}$ |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## Logical NOT (Inverter)

Convert logical 1 to o and vice versa.

## Truth Table

| Input | Output |
| :---: | :---: |
| $\mathbf{A}$ | Y |
| 0 | 1 |
| 1 | 0 |

Basic Laws of Boolean Algebra
Logical operations expressed and minimized mathematically using the rules, laws and theorems of boolean algebra.
Boolean Addition:
$\mathrm{A}+0=\mathrm{A}$
$\mathrm{A}+1=1$
$\mathrm{A}+\mathrm{A}=\mathrm{A}$
$\mathrm{A}+\overline{\mathrm{A}}=1$
$\bar{A}=\bar{A}$
$1+\overline{\mathrm{A}}=1$

## Boolean Multiplication:

A. $1=\mathrm{A}$
A. $0=0$
A. $\mathrm{A}=\mathrm{A}$
A. $\bar{A}=0$
$\left((\mathrm{A})^{\prime}\right)^{\prime}=\mathrm{A}$

## Properties of Boolean Algebra

- Commutative Properties
- Associative Properties
- Idempotent Properties
- Identity Properties
- Null Properties
- Distributive Properties
- Negation Properties
- Double Negation Properties
- Absorption Properties
- Demorgan's Theorem


## 1 ) Commutative Properties

- Boolean Addition

$$
A+B=B+A
$$

## Proof

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{A}+\mathbf{B}$ | $\mathbf{B}+\mathbf{A}$ |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

- Boolean Multiplication A.B=B.A


## Proof

| A | B | A.B | B.A |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

## 2) Associative Properties

| $A+(B+C)=(A+B)+C$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proof |  |  |  |  |  |  |
| A | B | C | A+B | B+C | $(\mathrm{A}+\mathrm{B})+\mathrm{C}$ | $\mathrm{A}+(\mathrm{B}+\mathrm{C})$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

- Similarly, the associative law of multiplication is given by,
A.(B.C)=(A.B).C

3) Distributive Properties
i) Boolean Addition

It is distributive over boolean multiplication.

$$
\mathrm{A}+\mathrm{BC}=(\mathrm{A}+\mathrm{B}) \cdot(\mathrm{A}+\mathrm{C})
$$

- Proof

LHS:
$\mathrm{A}+\mathrm{BC}=\mathrm{A} .1+\mathrm{BC}$

$$
=\mathrm{A} \cdot(1+\mathrm{B})+\mathrm{BC}
$$

$$
=\mathrm{A} .1+\mathrm{AB}+\mathrm{BC}
$$

$$
=\mathrm{A}(1+\mathrm{C})+\mathrm{AB}+\mathrm{BC}
$$

$$
=\mathrm{A} .1+\mathrm{AC}+\mathrm{AB}+\mathrm{BC}
$$

$$
=A+A C+A B+B C
$$

$$
=A \cdot A+A C+A B+B C
$$

[Since A.1=A]
[Since $1+\mathrm{B}=1$ ]
[Since $1+\mathrm{C}=1$ ]
[Since A.A=A]

$$
=\mathrm{A}(\mathrm{~A}+\mathrm{C})+\mathrm{B}(\mathrm{~A}+\mathrm{C})
$$

$$
=(\mathrm{A}+\mathrm{C})(\mathrm{A}+\mathrm{B})
$$

=RHS
ii) Boolean Multiplication

Distributive over boolean addition
A. $(\mathrm{B}+\mathrm{C})=\mathrm{A} . \mathrm{B}+\mathrm{A} . \mathrm{C}$

Proof
RHS

$$
\begin{aligned}
\mathrm{A} \cdot \mathrm{~B}+\mathrm{A} \cdot \mathrm{C} & & =\mathrm{A} \cdot \mathrm{~B} \cdot 1+\mathrm{A} \cdot \mathrm{C} \cdot 1 & \\
& =\mathrm{A} \cdot \mathrm{~B}(1+\mathrm{C})+\mathrm{A} \cdot \mathrm{C}(1+\mathrm{B}) & & {[\text { Since } 1+\mathrm{C}=1,} \\
& =\mathrm{A} \cdot \mathrm{~B}+\mathrm{ABC}+\mathrm{A} \cdot \mathrm{C}+\mathrm{A} \cdot \mathrm{~B} \cdot \mathrm{C} & & \\
& =\mathrm{A} \cdot \mathrm{~B}+\mathrm{A} \cdot \mathrm{C}+\mathrm{ABC} & & {[\text { Since } \mathrm{A}+\mathrm{A}=\mathrm{A}} \\
& =\mathrm{AB}(1+\mathrm{C})+\mathrm{AC} & & \text { [Since } 1+\mathrm{C}=1] \\
& =\mathrm{AB}+\mathrm{AC} & & \\
& =\mathrm{A} \cdot(\mathrm{~B}+\mathrm{C}) & & \\
& =\mathrm{LHS} & &
\end{aligned}
$$

## 4) Idempotent Properties

i) $\mathrm{A}+\mathrm{A}=\mathrm{A}$

Proof
If $\mathrm{A}=\mathrm{o}$ then $\mathrm{O}+\mathrm{O}=\mathrm{O}=\mathrm{A}$
If $A=1$ then $1+1=1=A$
ii) $\mathrm{A} \cdot \mathrm{A}=\mathrm{A}$

Proof
If $\mathrm{A}=0$ then $0.0=0=\mathrm{A}$
If $\mathrm{A}=1$ then $1.1=1=\mathrm{A}$

## 5) Identity Properties

i) $\mathrm{A} .1=\mathrm{A}$

Proof
If $\mathrm{A}=\mathrm{o}$ then $0.1=0=\mathrm{A}$
If $\mathrm{A}=1$ then $1.1=1=\mathrm{A}$
ii) $\mathrm{A}+1=1$

Proof
If $\mathrm{A}=\mathrm{o}$ then $\mathrm{O}+\mathrm{l}=1$
If $A=1$ then $1+1=1$

## 6) Null Properties

i) $\mathrm{A} . \mathrm{o}=\mathrm{o}$

Proof
If $A=0$ then $0.0=0$
If $A=1$ then $1.0=0$
ii) $\mathrm{A}+\mathrm{o}=\mathrm{A}$

Proof
If $\mathrm{A}=\mathrm{o}$ then $\mathrm{O}+\mathrm{O}=\mathrm{O}=\mathrm{A}$
If $\mathrm{A}=1$ then $\mathrm{o}+1=1=\mathrm{A}$

## 7) Negative Properties

i) $\mathrm{A} \cdot \mathrm{A}^{\prime}=\mathrm{o}$

Proof
If $A=0$ then $0.1=0$
If $A=1$ then $1.0=0$
ii) $\mathrm{A}+\mathrm{A}^{\prime}=1$

Proof
If $A=0$ then $0+1=1$
If $A=1$ then $1+0=1$

## 8) Double Negation Properties

i) $\left((A)^{\prime}\right)^{\prime}=A$

## Proof

If $A=0$ then $(A)^{\prime}=1,\left((A)^{\prime}\right)^{\prime}=0=A$
If $A=1$ then $(A)^{\prime}=0,\left((A)^{\prime}\right)^{\prime}=1=A$

## 9) Absorption Properties <br> i) $\mathrm{A}+\mathrm{AB}=\mathrm{A}$ <br> Proof

LHS: $\quad \mathrm{A}(1+\mathrm{B})=\mathrm{A} .1$

$$
\begin{aligned}
& =\mathrm{A} \\
& =\mathrm{RHS}
\end{aligned}
$$

ii) $\mathrm{A}(\mathrm{A}+\mathrm{B})=\mathrm{A}$

Proof
LHS: $\quad \mathrm{A}(\mathrm{A}+\mathrm{B})=\mathrm{A}(\mathrm{A}+\mathrm{B})$

$$
\begin{array}{ll}
=\mathrm{A} \cdot \mathrm{~A}+\mathrm{AB} & {[\text { Since } \mathrm{A} \cdot \mathrm{~A}=\mathrm{A}]} \\
=\mathrm{A}+\mathrm{AB} & \\
=\mathrm{A}(1+\mathrm{B}) & {[\text { Since } 1+\mathrm{B}=1]} \\
=\mathrm{A} & \\
=\mathrm{RHS} &
\end{array}
$$

[Since 1. $\mathrm{A}=\mathrm{A}$ ]
iii) $A+A^{\prime} B=A+B$

Proof
LHS: $\quad A+A^{\prime} B=\left(A+A^{\prime}\right)(A+B) \quad[$ Since $A+B C=(A+B)(A+C)]$

$$
=1(A+B) \quad\left[\text { Since } A+A^{\prime}=1\right]
$$

$=A+B$
$=$ RHS

## 10) Demorgan's Theorem

- First Theorem

States that complement of a product is equal to be sum of the complements.

$$
(\mathrm{A} . \mathrm{B})^{\prime}=\mathrm{A}^{\prime}+\mathrm{B}^{\prime}
$$

- Second Theorem

States that complement of a sum equal to product of the complements.

$$
(\mathrm{A}+\mathrm{B})^{\prime}=\mathrm{A}^{\prime} \cdot \mathrm{B}^{\prime}
$$

- DeMorgan's First Theorem using Truth Table

| Inputs |  | Truth Table Outputs For Each Term |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A.B | $\overline{\mathrm{A} . \mathrm{B}}$ | $\overline{\mathrm{A}}$ | $\overline{\mathrm{B}}$ | $\overline{\mathrm{A}}+\overline{\mathrm{B}}$ |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

$(A . B)^{\prime}=A^{\prime}+B^{\prime}$

- DeMorgan's Second Theorem using Truth Table

| Inputs |  | Truth Table Outputs For Each Term |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A+B | $\overline{\mathrm{A}+\mathrm{B}}$ | $\overline{\mathrm{A}}$ | $\overline{\mathrm{B}}$ | $\overline{\mathrm{A}} \cdot \overline{\mathrm{B}}$ |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
|  | $(\mathrm{~A}+\mathrm{B})^{\prime}=\mathrm{A}^{\prime} . \mathrm{B}^{\prime}$ |  |  |  |  |  |

