MUTHAYAMMAL ENGINEERING COLLEGE, RASIPURAM 637408
(Autonomous)

DEPARTMENT OF
ELECTRICAL AND ELECTRONICS ENGINEERING

19EEC10 - POWER SYSTEM ANALYSIS

COURSE FACULTY
Dr.N.MOHANANTHINI,
ASP/EEE

COURSE CONTENTS

UNIT I

INTRODUCTION

UNIT II

POWER FLOW ANALYSIS

UNIT III

FAULT ANALYSIS – BALANCED FAULTS

UNIT IV

FAULT ANALYSIS – UNBALANCED FAULTS

UNIT V

POWER SYSTEM STABILITY

COURSE OBJECTIVES

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- To impart knowledge on the need for power system analysis and model various power system components
- To apply numerical methods to solve the power flow problem.
- To analyze the power system under faulted conditions - balanced faults.
- To analyze the power system under faulted conditions - unbalanced faults
- To model and analyze the stability of power system by equal area criterion, Modified Euler and Runge - Kutta fourth order method

COURSE OUTCOMES

- Study about the components of the power system.
- Apply numerical methods to solve the power flow problem.
- Evaluate the system under faulted conditions-balanced faults.
- Evaluate the system under faulted conditions-unbalanced faults
- Compute the stability of power system with the help of equal area criterion, Modified Euler and Runge - Kutta fourth order method

UNIT I	**INTRODUCTION**	9
Need for system planning and operational studies – Introduction to restructuring – Single line diagram - Per unit representation – Per unit impedance and reactance diagram – Bus incidence Matrix - Primitive network – Formation of Y – bus by two rule method - Gaussian elimination method - Formation of Y – bus using singular transformation method		
UNIT II	**POWER FLOW ANALYSIS**	9
Importance of power flow analysis in planning and operation of power systems – statement of power flow problem – classification of buses – development of power flow model in complex variables form and Polar variable form - Power flow solution using Newton Raphson, Gauss seidel and Fast decoupled method		
UNIT III	**FAULT ANALYSIS – BALANCED FAULTS**	9
Importance of short circuit analysis - assumptions in fault analysis – analysis using Thevenin's theorem – Z –bus building algorithm – fault analysis using Z-bus – computations of short circuit capacity, post fault voltage, currents and line flows		

UNIT IV	**FAULT ANALYSIS – UNBALANCED FAULTS**	9
Introduction to symmetrical components – sequence impedances – sequence circuits of synchronous machine, transformer and transmission lines - sequence networks analysis of single line to ground, line to line and double line to ground faults using Thevenin's theorem and Z-bus matrix.		
UNIT V	**POWER SYSTEM STABILITY**	9
Steady state and transient Stability – Introduction to voltage stability – Single Machine Infinite Bus (SMIB) system: Development of swing equation - step by step method - equal area criterion - solution of swing equation by modified Euler method and Runge - Kutta fourth order method		
		Total = 45 Periods

TEXT BOOKS:

Sl.No	Author(s)	Title of the Book	Publisher	Year of Publication
1.	NagrathI.J. and Kothari D.P	Modern Power System Analysis	Tata McGraw Hill	2011
2	John J.Grainger and W.D.Stevenson Jr.	Power System Analysis	Tata Mc Graw-Hill	2010

REFERENCE BOOKS:

Sl.No	Author(s)	Title of the Book	Publisher	Year of Publication
1.	Hadi Saadat	Power System Analysis	Tata McGraw Hill	2010
2.	P.Venkatesh, B.V.Manikandan, S.Charles Raja, A.Srinivasan	Electrical Power Systems- Analysis, Security and Deregulation	PHI Learning Private Limited	2012
3.	Kundur P	Power System Stability and Control	Tata McGraw Hill	2010
4.	J.DuncanGlover, Mulukutla S.Sarma, Thomas J.Overbye	Power System Analysis & Design	Cengage Learning,	2012
5.	Olle.I .Elgerd	Electric Energy Systems Theory–An Introduction	Tata Mc Graw Hill	2012

UNIT I

INTRODUCTION

INTRODUCTION TO POWER SYSTEM ANALYSIS

Contents:

- ✓ Overview
- ✓ Function of power system analysis
- ✓ Need for Power System Analysis

Overview of Power System Analysis

- A power system consists of generation, transmission and distribution system.
- The components of the power systems are generators, transformers, transmission lines, distribution lines, loads and compensating devices like shunt, series, and static VAR compensators.
- In order to maintain power system, the bulk power has to be transmitted through transmission and distribution lines to the consumers safely and economically.
- The evaluation of power system is called power system analysis
- In monitoring power system analysis, we are mainly dealing with power or load flow analysis, short circuit analysis and stability analysis.

The functions of power system analysis are:

- To monitor the voltage at various buses, real and reactive power flow between buses.
- To design the circuit breakers.
- To plan future expansion of the existing system.
- To analyze the system under different fault conditions (3 fault-G, L-L,L-L-G faults).
- To study the ability of the system for larger disturbances (sudden application of large load).
- To study the ability of the system for small disturbances (routine or small load changes).

Need For System Analysis in planning and operation of power system

- Operational planning covers the whole period ranging from the incremental stage of system development.
- The system operation engineers at various points like area, space, regional and national load despatch deals with the despatch of power.
- Power balance equation is

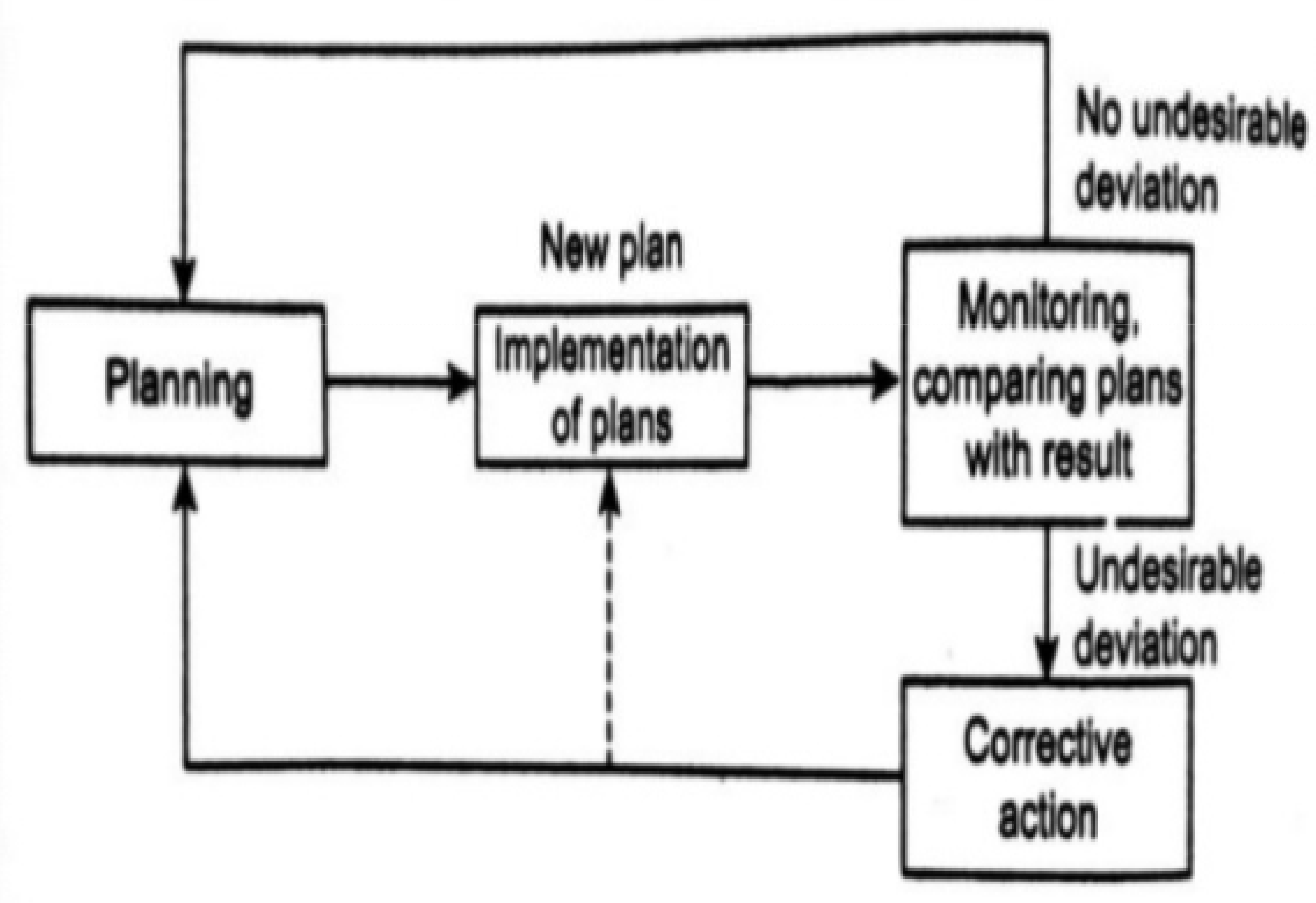
$$P_D = \sum_{i=1}^{N} P_{Gi}, \qquad i=1,2,3,\ldots N$$

Total demand=Sum of the real power generation

i.e., the generation should be such a way that to meet out the required demand.

- When this relation is satisfied, it gives good economy and security.

Block diagram for planning and operation of power system



Steps to be followed:

- ✓ Planning of power system
- ✓ Implementation of the plans
- ✓ Monitoring the system
- ✓ Compare with the results

- If no undesirable deviation occurs, then directly go to planning of system
- If undesirable deviation occurs, then take corrective action (i.e., increase or decrease) and then go to the planning of the system.
- For planning and operation of power system, the following analyses are more important.

    Load flow analysis

    Short circuit analysis

    Transient analysis

- To identify the potential deficiencies of the proposed system, the cause of the equipment failure and malfunction can be determined through a system study.

1.4.2 Need for operational studies:

The various operational studies of power system which helps the Power Engineers to evaluate the performance of power system involves three major studies. They are

(i) Load Flow analysis (or) Power flow analysis

(ii) Short circuit analysis (or) Fault analysis

(iii) Stability analysis (or) Transient analysis

(i) Load Flow analysis:

Definition:

Load flow analysis is one of the basic tools used in power system studies. It is concerned with the steady state analysis of the system when it is working under a normal balanced operating conditions.

Need for load flow studies:

(i) It is very important for planning, economic scheduling, control and operation of existing systems as well as planning its future expansion.

(ii) Solution of load flow gives bus voltages and line/transformer power flows for a given load condition.

(iii) It helps in long term planning and operational planning.

Objectives:

(i) Informations such as magnitude and phase angle of voltages of each bus, active and reactive power flow in each line and power loss in line can be calculated.

(ii) To determine the optimum size and location of the capacitors for power factor improvement.

(ii) Short circuit analysis:

Definition:

Short circuit analysis deals with analysing a system under faulty conditions. The fault may be symmetrical fault (3ϕ fault) (or) an unsymmetrical fault (L-L, L-G or L-L-G fault).

Need for short circuit studies:

(i) The system must be protected against heavy flow of short circuit currents by disconnecting the faulty section from the healthy section by means of circuit breaker. To estimate the magnitude of fault current for the proper choice of circuit breaker and protective relays short circuit study is essential.

(ii) It is more important to develop the protective schemes for various parts of the system.

Objectives:

(i) To have proper relay setting and co-ordination.

(ii) To obtain the rating of the protective switch gears.

(iii) To select the circuit breakers.

(iv) To perform whenever system expansion is planned.

(v) To select and set phase relays, while the line to ground fault is used for earth relays, a 3ϕ fault information is used.

(iii) Stability analysis:

Definition:

Stability analysis is termed as the analysis made on the ability of the system to bring it to a stable condition after a disturbance.

If the disturbance is "small" such as gradual infinitesimal variation in system variables like rotor angle voltage etc. Then the analysis is known as "steady-state stability analysis".

If the disturbance is "Large" such as sudden outage of line, sudden loss of excitation, sudden application or removal of loads etc. Then the analysis is known as "Transient stability analysis".

Need for stability analysis:

(i) Transient stability studies gives the information that the system can withstand the transient conditions like high magnitude of voltage and frequency.

(ii) It is more helpful in determining power transfer capability between two different systems.

Objectives:

(i) To determine the nature of relaying system needed.

(ii) Critical clearing time of circuit breakers can be determined.

Restructuring

Contents:

- ✓ Introduction
- ✓ Need for restructuring
- ✓ Structure of restructuring

Introduction

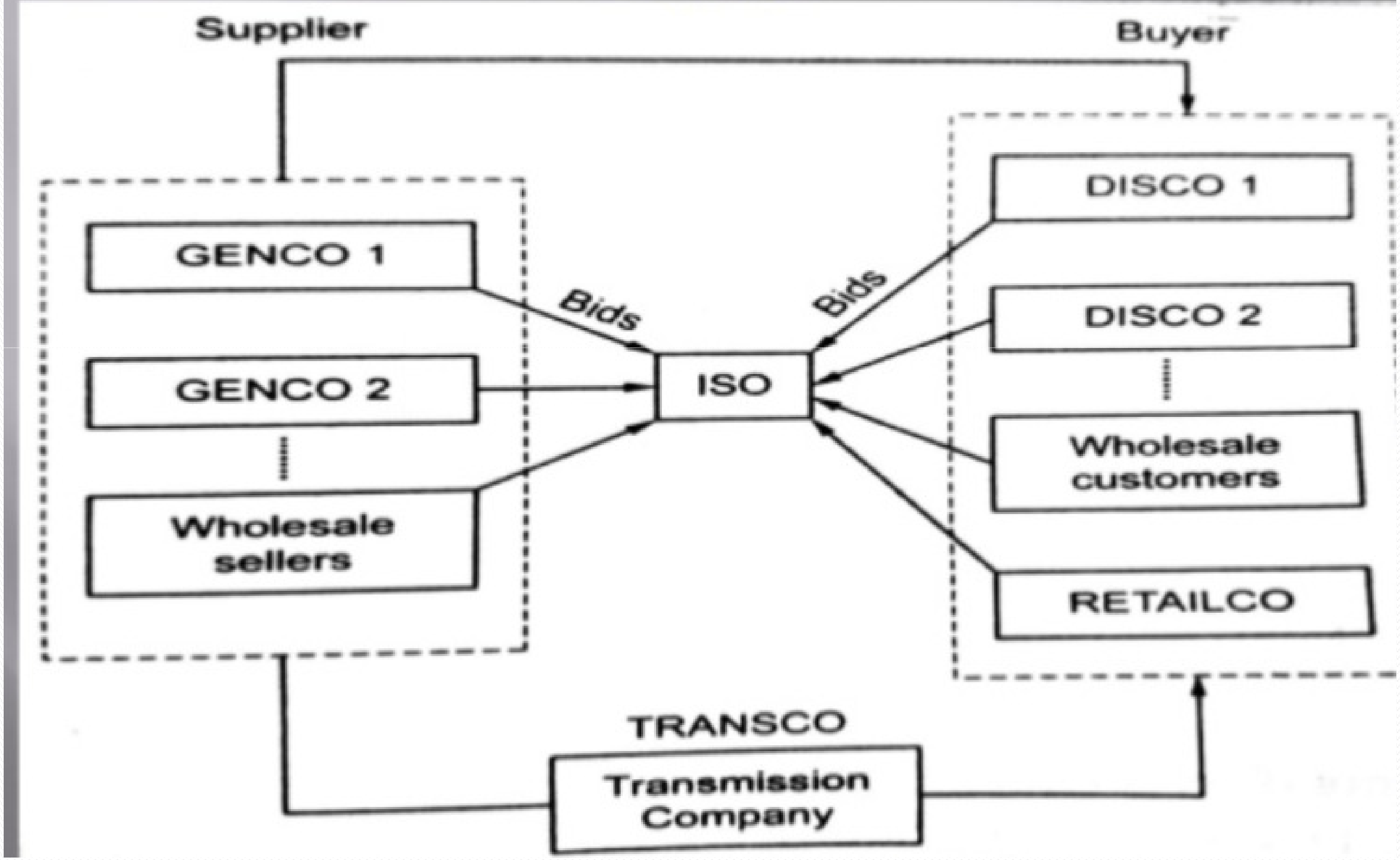
Restructuring is to separate the functions of power generation, transmission, distribution and electricity supply to consumers.

Power System restructuring gives

- ✓ Benefit of lower electricity price
- ✓ Better consumer service
- ✓ Improved system efficiency.

- As energy demand continues to grow in future, higher voltage levels are needed.
- Technical problems arise in the transmission system due to voltage stability and dynamic stability.
- This involves heavy pricing over the customers.
- The restructuring power system is to give opportunity to the customer to buy energy at a more favourable price and choose the generating company which gives energy at a lower price.

Structure of restructured Electricity market



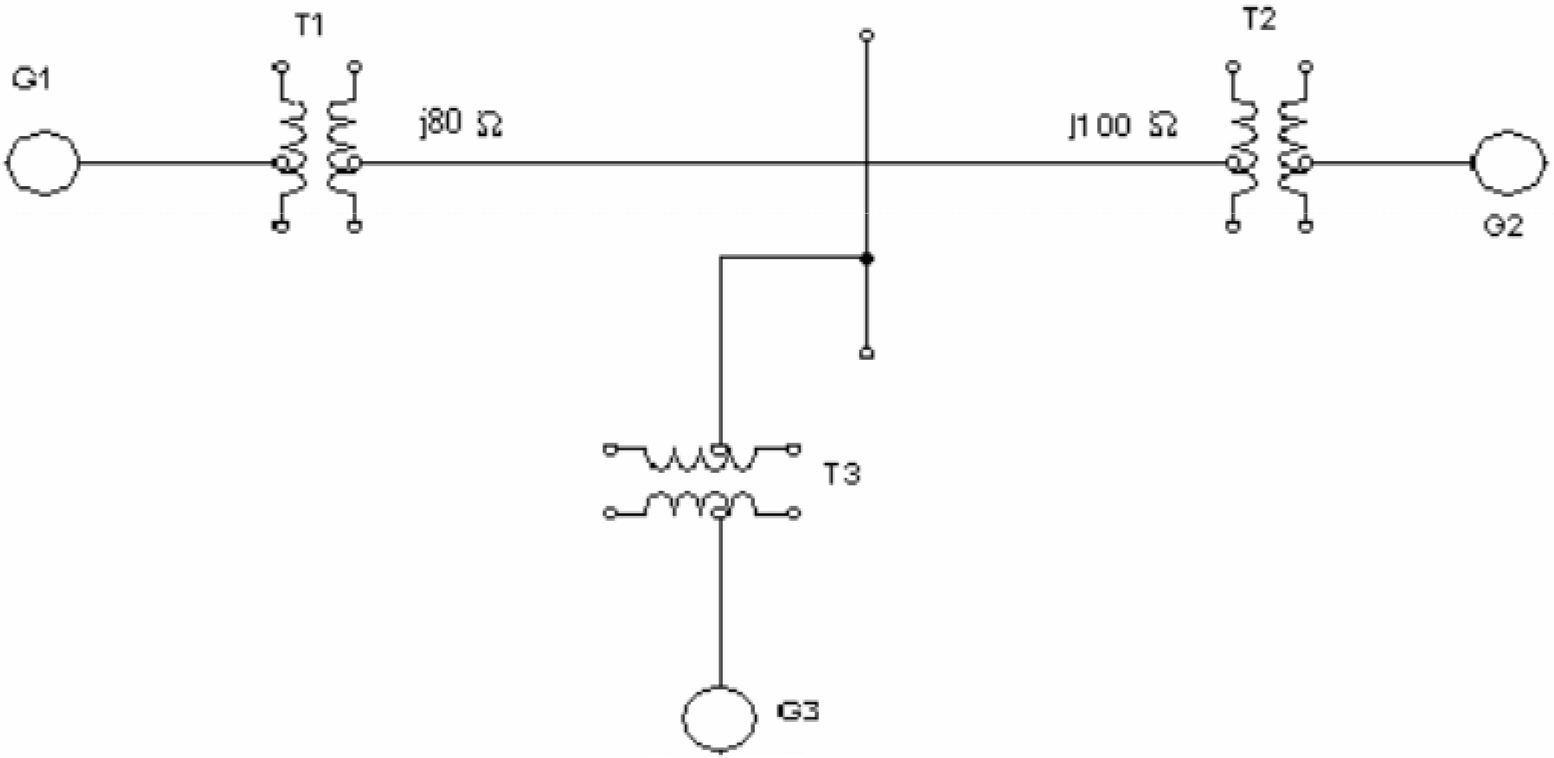
- The market participants are
  - ✓ Generating Companies(GENCO)
  - ✓ Transmission companies(TRANSCO)
  - ✓ Distribution Companies(DISCO)
  - ✓ Customers
  - ✓ Independent System Operator(ISO)
  - ✓ Retail Companies(RETAILCO)
- In ISO, sellers and buyers submit their bid to inject power into out of the pool. Winning bidders are paid the spot price that is equal to the highest bid of the winners.ISO balancing supply and demand in real-time and maintain reliability.

SINGLE LINE DIAGRAM

A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and interconnection between them are shown by a straight line even though the system is three phase system.

The ratings and the impedances of the components are also marked on the single line diagram

Example



Per unit value

The per unit value of any quantity is defined as the ratio of the actual value of the any quantity to the base value of the same quantity as a decimal.

per unit=actual value/base value

Need for base values

The components or various sections of power system may operate at different voltage and power levels.

It will be convenient for analysis of power system if the voltage, power, current and impedance rating of components of power system are expressed with reference to a common value called base value

Advantages of per unit system

i. Per unit data representation yields valuable relative magnitude information.

ii. Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.

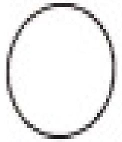
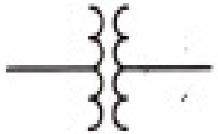
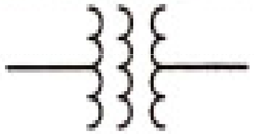
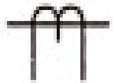
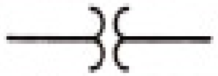
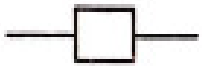
iii. The p.u systems are ideal for the computerized analysis and simulation of complex power system problems.

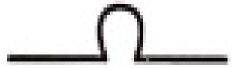
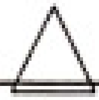
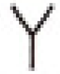
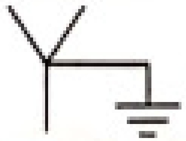
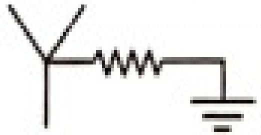
iv. Manufacturers usually specify the impedance values of equivalent in per unit of the equipments rating.

v. The ohmic values of impedances are refereed to secondary is different from the value as referee to primary. However, if base values are selected properly, the p.u impedance is the same on the two sides of the transformer.

vi. The circuit laws are valid in p.u systems, and the power and voltages equations are simplified since the factors of $\sqrt{3}$ and 3 are eliminated

Standard Symbols for Single Line Diagram

Alternator or Synchronous motor	
Two winding power transformer	
Three winding power transformer	
Current transformer	
Potential transformer	 or 
Transmission line	
Power circuit breaker (oil or liquid)	

Air blast circuit breaker	
3ϕ, 3 wire delta connection	
3ϕ, Y connection, neutral ungrounded	
3ϕ, Y connection, neutral solidly grounded	
3ϕ, Y connection, neutral solidly grounded through resistor	
3ϕ, Y connection, neutral solidly grounded through reactor	

1.9 IMPEDANCE DIAGRAM AND REACTANCE DIAGRAM :-

1.9.1 Impedence Diagram :

In order to calculate the performance of a system under load conditions or upon the oc. curance of a Fault (short circuit studies), the one line diagram is used to draw the single phase (or) per phase equivalent circuit of the system which combines the equivalent circuits of vari. ous components present in the particular system. Such a per phase equivalent circuit of the system is known as "Impedance diagram."

1.9.1.1 Assumptions :

(i) Generators are represented as voltage sources with series resistance and inductiv reactance.

(ii) Shunt branches of the transformer which are responsible for core loss and magnetiza tion are neglected.

(iii) Shunt capacitance of the π - model of Transmission lines are neglected for simplic ity.

(iv) Static loads are represented by resistance and inductive reactance.

(v) Neutral grounding impedances are neglected.

(vi) Induction motor loads are neglected since their current dies out within a few cycle after the instant of fault.

1.9.2 Reactance diagram :

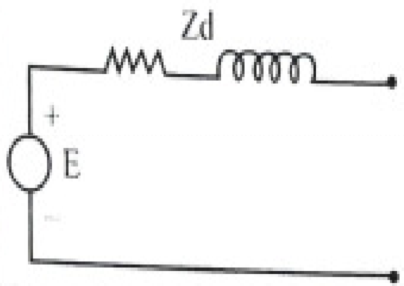
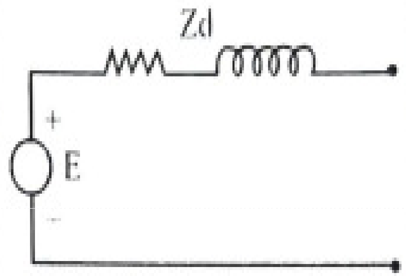
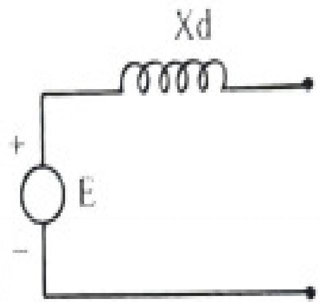
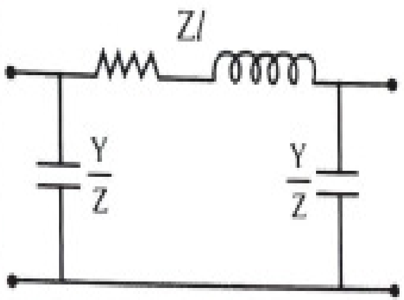
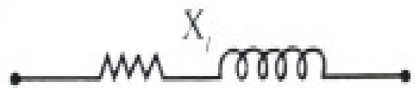
1.9.2.1 Assumptions :

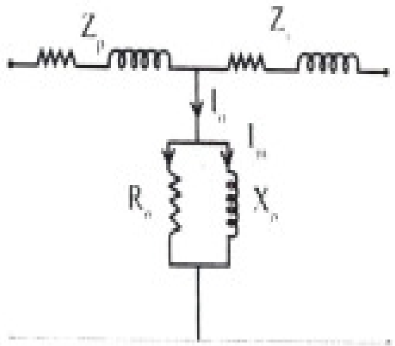
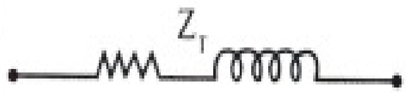
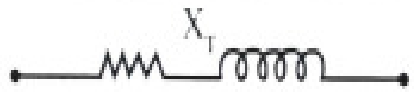
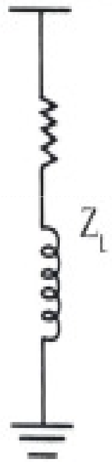
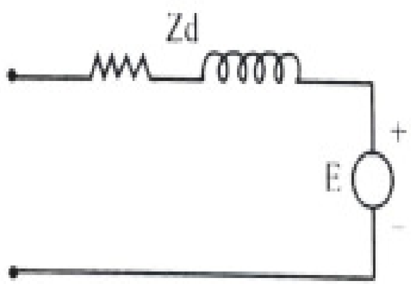
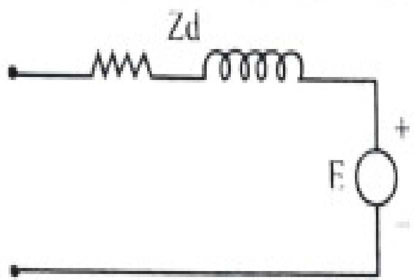
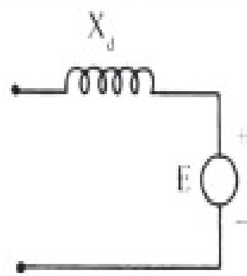
(i) All the resistances present in the impedance diagram are neglected.

(ii) Static loads are neglected since their current has only a little effect on the total line current during the fault conditions.

Ofcourse the omission of resistances introduces some error, but the results may be satisfactory since the inductive reactance of a system is much larger than its resistance.

$X_L >>>> R$, R can be omitted.

Component	Equivalent circuit (i.e.) Impedance diagram (Before approximations)	Final impedance diagram (after Approximations)	Reactance diagram (After approximations)
1. Alternator			
2. Transmission line			

3. Transformer			
4. Load			Neglected
5. Synchronous motor			

2.8.2. SAMPLE ONE-LINE DIAGRAM

The sample one line diagram of the power system is as shown in Fig.2.10.

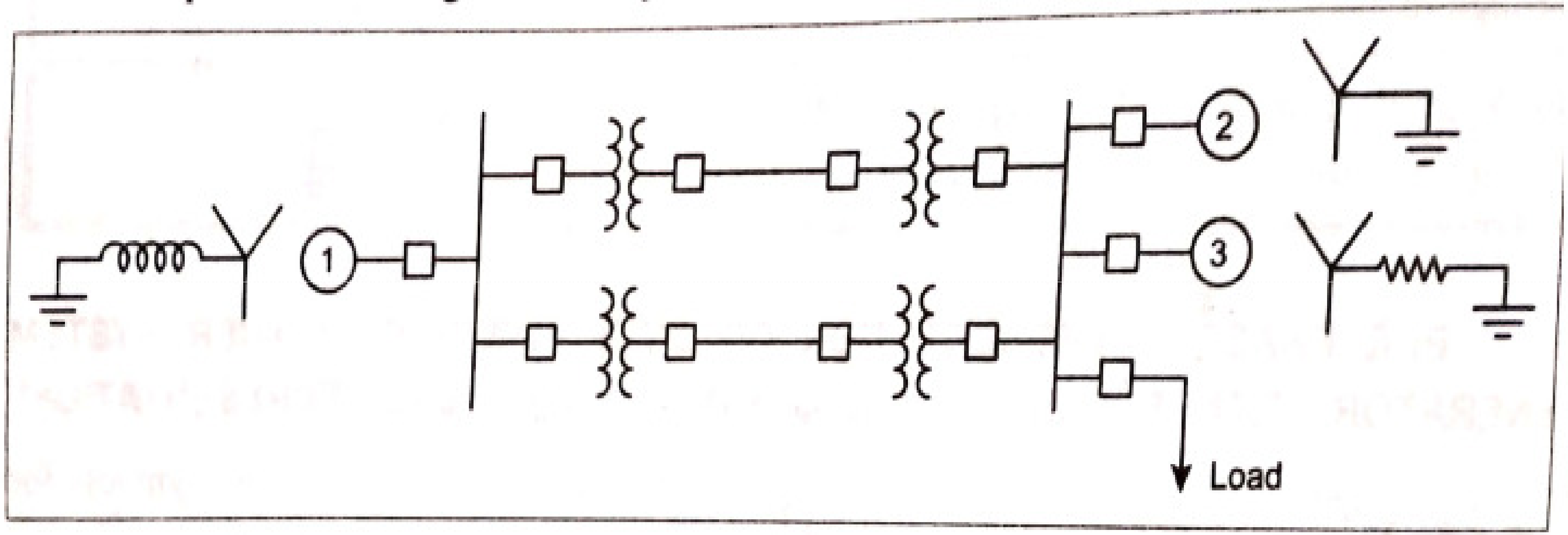


Fig. 2.10. Sample one line diagram

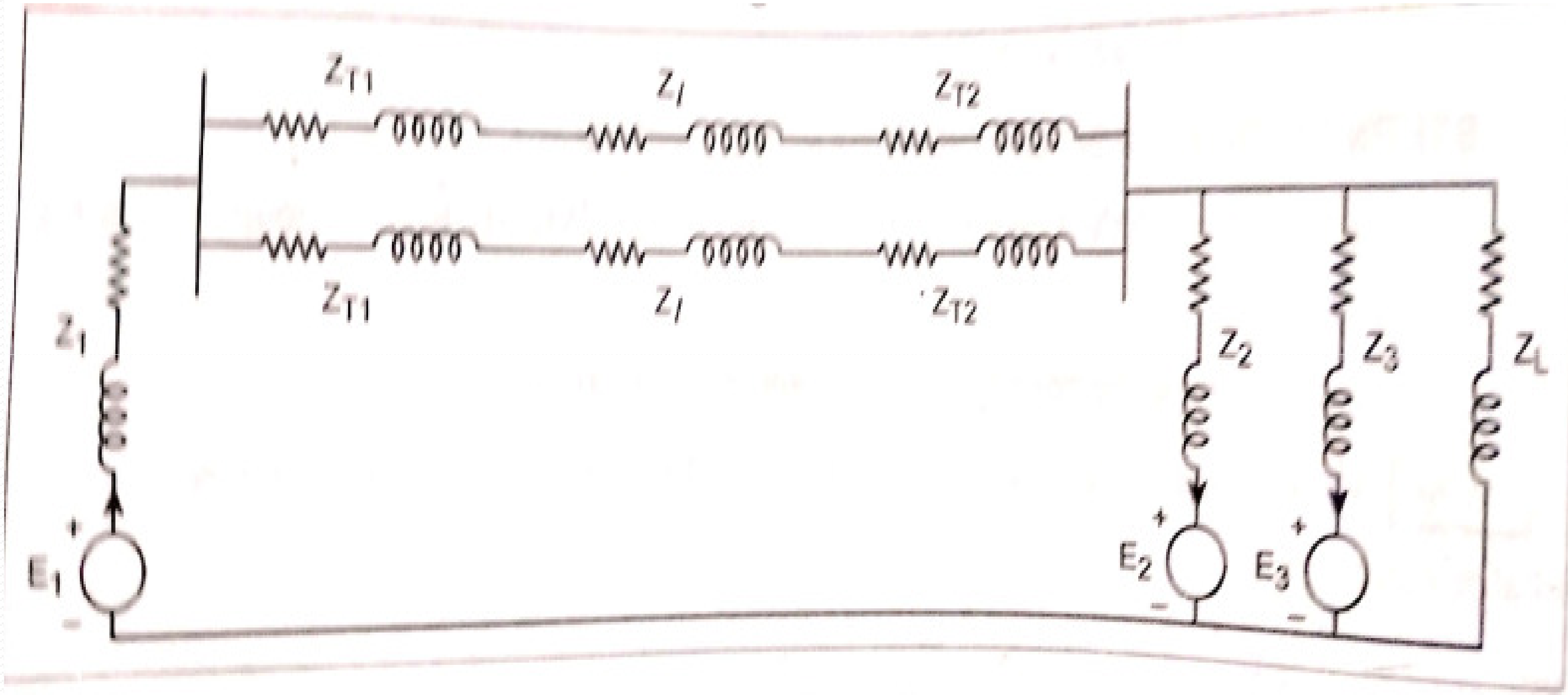


Fig. 2.13. Impedance diagram

2.8.5. REACTANCE DIAGRAM

The reactance diagram on single phase basis under balanced condition can be drawn from impedance diagram as shown in Fig.2.14.

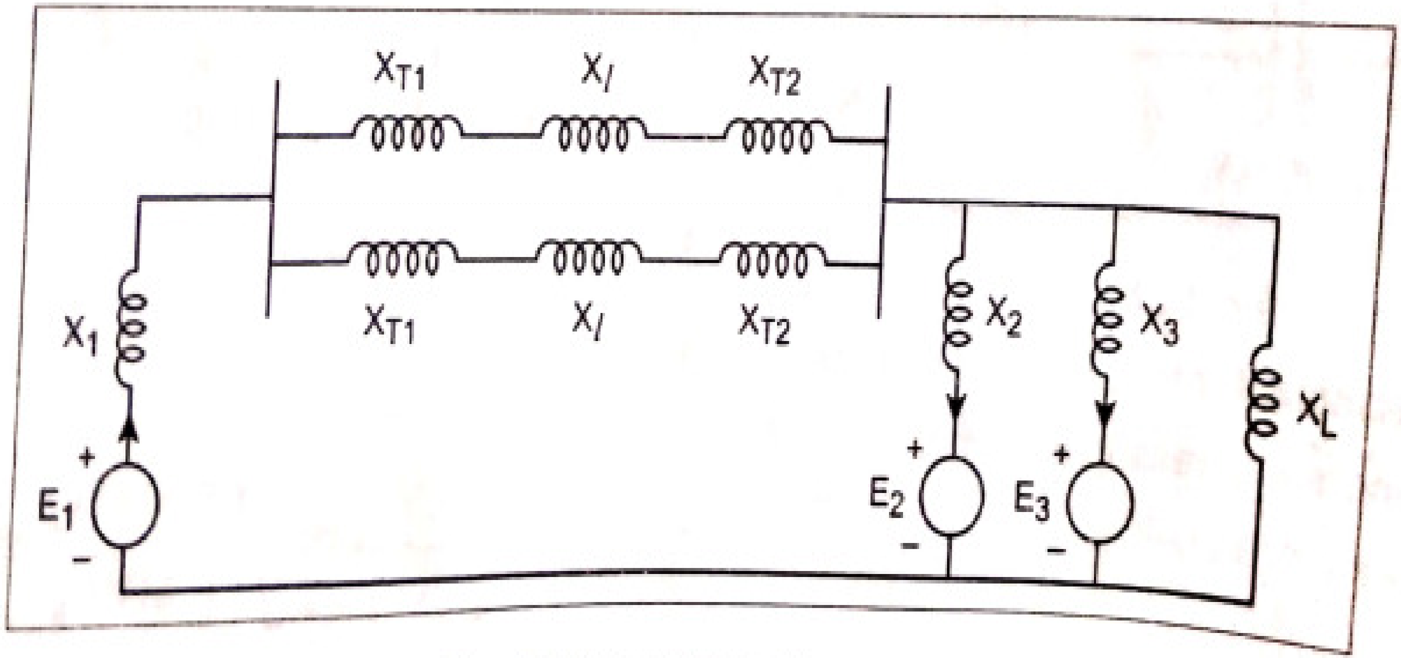
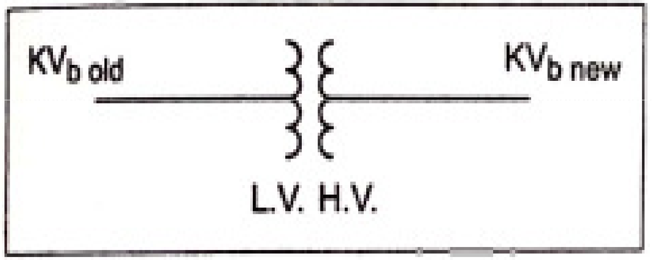


Fig. 2.14. Reactance diagram

2.8.3. STEPS TO DRAW PER UNIT IMPEDANCE DIAGRAM

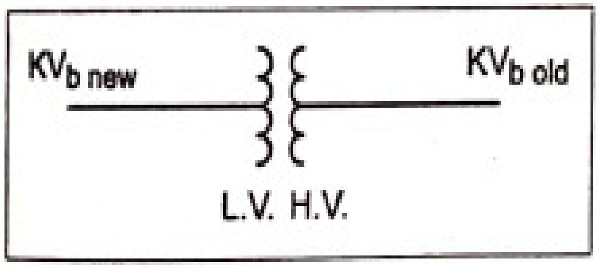
- Choose a common MVA or base MVA for the system (Mostly highest generator rating is taken).
- Choose an appropriate base KV for each and every section.

Note In any side change occurs or presence of transformer, recalculate new KV_b using transformation ratio.



$$KV_{b\ new} = KV_{b\ old} \times \frac{\text{HT side rating}}{\text{LT side rating}} \qquad \ldots (2.27)$$

Fig. 2.11.



$$KV_{b\ new} = KV_{b\ old} \times \frac{\text{LT side rating}}{\text{HT side rating}} \qquad \ldots (2.28)$$

Fig. 2.12.

- Calculate per unit impedance in each section.

For generator, transformer, motor

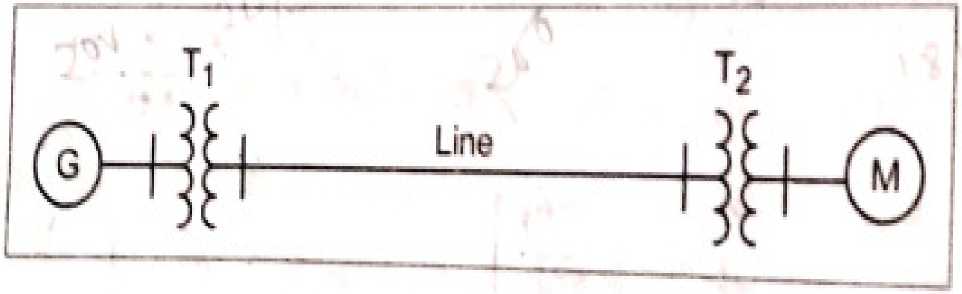
$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

For transmission line,

$$Z_{p.u.} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{(KV_b)^2} \times MVA_b \quad \ldots (2.29)$$

- Draw the impedance diagram from the one-line diagram.

Example 2.4 *The three phase power and line-line voltage rating of the electric power system shown in Fig. are given below.*



G_1 :	60 MVA,	20 KV,	$X'' = 9\%$
T_1 :	50 MVA,	20/200 KV,	$X = 10\%$
T_2 :	50 MVA,	200/20 KV,	$X = 10\%$
M :	43.2 MVA,	18 KV,	$X'' = 8\%$
Line :		200 KV,	$Z = 120 + j200\ \Omega$

Draw an impedance diagram showing all impedances in p.u. on a 100 MVA base. Choose 20 KV as the base voltage for generator.

Solution :

$$KV_{b\ new} = 20$$

$$MVA_{b\ new} = 100$$

Generator : $KV_{b\ given} = 20, \quad MVA_{b\ given} = 60\ MVA$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.09 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{100}{60}\right] = j0.15 \quad p.u.$$

Transformer T_1 (Primary) :

$$KV_{b\ new} = 20$$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.1 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{100}{50}\right] = j0.2 \quad p.u.$$

Transmission Line :

$$Z_{actual} = 120 + j200\ \Omega$$

Transformer secondary side change occurs, so calculate $KV_{b\ new}$ as

$$KV_{b\ new} = KV_{b\ old} \times \left[\frac{\text{H.T. side voltage rating of } T_1}{\text{L.T. side voltage rating of } T_1}\right]$$

$$= 20 \times \left[\frac{200}{20}\right] = 200\ \ KV$$

$$Z_{p.u.} = \frac{Z_{actual}}{KV_{b\ new}^2} \times MVA_{b\ new} = \frac{120 + j200}{200^2} \times 100$$

$$= 0.3 + j0.5\ \ p.u.$$

Transformer T_2 (Primary) :

$$KV_{b\,new} = 200\ KV$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.1 \times \left[\frac{200}{200}\right]^2 \times \left[\frac{100}{50}\right] = j0.2\ \text{p.u.}$$

Motor : Transformer secondary side change occurs, so calculate $KV_{b\,new}$ as

$$KV_{b\,new} = KV_{b\,old} \times \left[\frac{\text{L.T. side voltage rating of } T_2}{\text{H.T. side voltage rating of } T_2}\right]$$

$$= 200 \times \frac{20}{200} = 20\ \ KV$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.08 \times \left[\frac{18}{20}\right]^2 \times \left[\frac{100}{43.2}\right] = j0.15\ \ \text{p.u.}$$

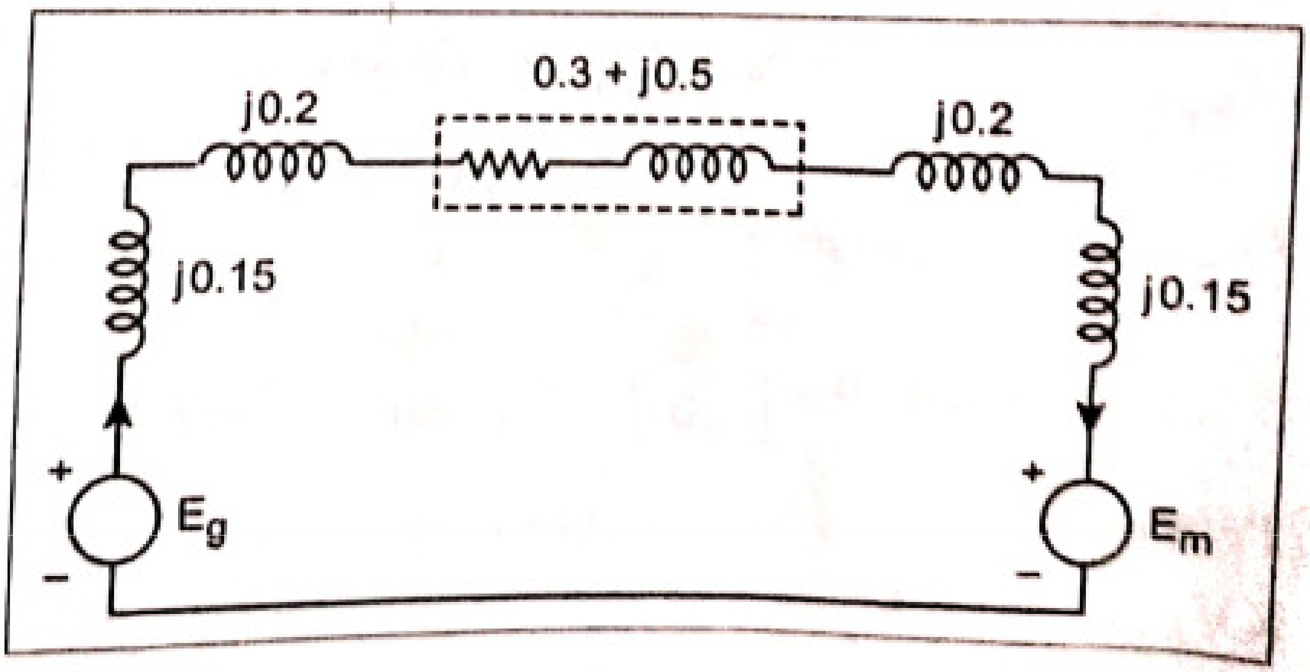
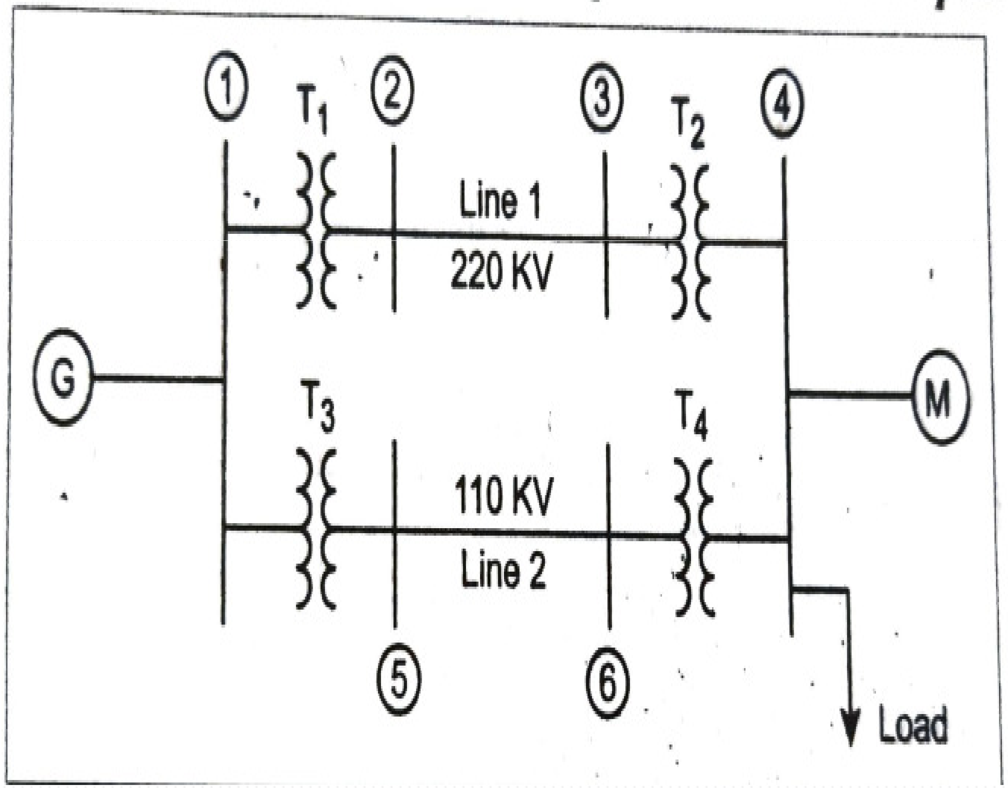


Fig. Impedance diagram

Example 2.7 *The one line diagram of a three phase power system is shown in Fig. Select a common base of 100 MVA and 20 KV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in p.u.*



G :	85 MVA,	20 KV,	$X'' = 16\%$
T_1 :	60 MVA,	20/220 KV,	$X = 10\%$
T_2 :	50 MVA,	220/11 KV,	$X = 5\%$
T_3 :	50 MVA,	20/110 KV,	$X = 7\%$
T_4 :	40 MVA,	110/11 KV,	$X = 9\%$
M :	65 MVA,	10.5 KV,	$X'' = 17\%$

The three phase load at bus 4 absorbs 62 MVA, 0.8 power factor lagging at 10.5 KV. Line 1 and Line 2 have reactances of 45 Ω and 60 Ω respectively.

☺ ***Solution :***

$$MVA_{b\ new} = 100$$

$$KV_{b\ new} = 20\ KV \text{ on generator side}$$

Generator G_1 :

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.16 \times \left[\frac{20}{20}\right]^2 \times \frac{100}{85} = j0.188 \text{ p.u.}$$

Transformer T_1 referred to primary (LV side) :

$$KV_{b\ new} = 20\ KV$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.1 \times \left[\frac{20}{20}\right]^2 \left[\frac{100}{60}\right] = j0.167 \text{ p.u.}$$

Transformer T_3 referred to primary (LV side) :

$$KV_{b\,new} = 20 \text{ KV}$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.07 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{100}{50}\right] = j0.14 \text{ p.u.}$$

Line 1 : Transformer T_1 secondary side change occurs, so calculate $KV_{b\,new}$ as

$$KV_{b\,new} = KV_{b\,old} \times \left[\frac{\text{H.T side rating of } T_1}{\text{L.T side rating of } T_1}\right]$$

$$KV_{b\,new} = 20 \times \frac{220}{20} = 220 \text{ KV}$$

$$Z_{p.u.\,new} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{KV^2_{b\,new}} \times MVA_{b\,new}$$

$$= \frac{j45}{220^2} \times 100 = j0.093 \text{ p.u.}$$

Transformer T_2 referred to primary side :

$$KV_{b\,new} = 220\ KV$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.05 \times \left[\frac{220}{220}\right]^2 \times \left[\frac{100}{50}\right] = j0.1\ \text{p.u.}$$

Line 2 : Transformer T_3 secondary side change occurs, so calculate $KV_{b\,new}$ as

$$KV_{b\,new} = 20 \times \frac{110}{20} = 110\ KV$$

$$Z_{p.u.\,new} = \frac{Z_{actual}}{KV_b^2} \times MVA_b = \frac{j60}{110^2} \times 100 = j0.496\ \text{p.u.}$$

Transformer T_4 referred to primary :

$$KV_{b\,new} = 110\ KV$$

$$Z_{p.u.\,new} = Z_{p.u.\,given} \times \left[\frac{KV_{b\,given}}{KV_{b\,new}}\right]^2 \times \left[\frac{MVA_{b\,new}}{MVA_{b\,given}}\right]$$

$$= j0.09 \times \left[\frac{110}{110}\right]^2 \times \left[\frac{100}{40}\right] = j0.225\ \text{p.u.}$$

Motor M **:** Transformer T_4 secondary side change occurs, so calculate $KV_{b\ new}$ as

$$KV_{b\ new} = 110 \times \frac{11}{110} = 11\ KV$$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.17 \times \left[\frac{10.5}{11}\right]^2 \times \left[\frac{100}{65}\right] = j0.238\ \text{p.u.}$$

Load at bus 4 :

Load apparent power at 0.8 power factor lagging is given by

$$S_{L(3\phi)} = 62 \angle \cos^{-1}(0.8) = 62 \angle 36.87°$$

$$\text{Actual load impedance } Z_L = \frac{V_{LL}^2}{S^*_{L(3\phi)}} = \frac{10.5^2}{62 \angle -36.87°} = 1.4226 + j1.07\,\Omega$$

$$\text{Base impedance } Z_b = \frac{KV_b^2}{MVA_b} = \frac{11^2}{100} = 1.21\,\Omega$$

$$Z_{L\,p.u.} = \frac{Z_{actual}}{Z_{base}}$$

$$= \frac{1.4226 + j1.07}{1.21} = 1.176 + j0.884 \text{ p.u.}$$

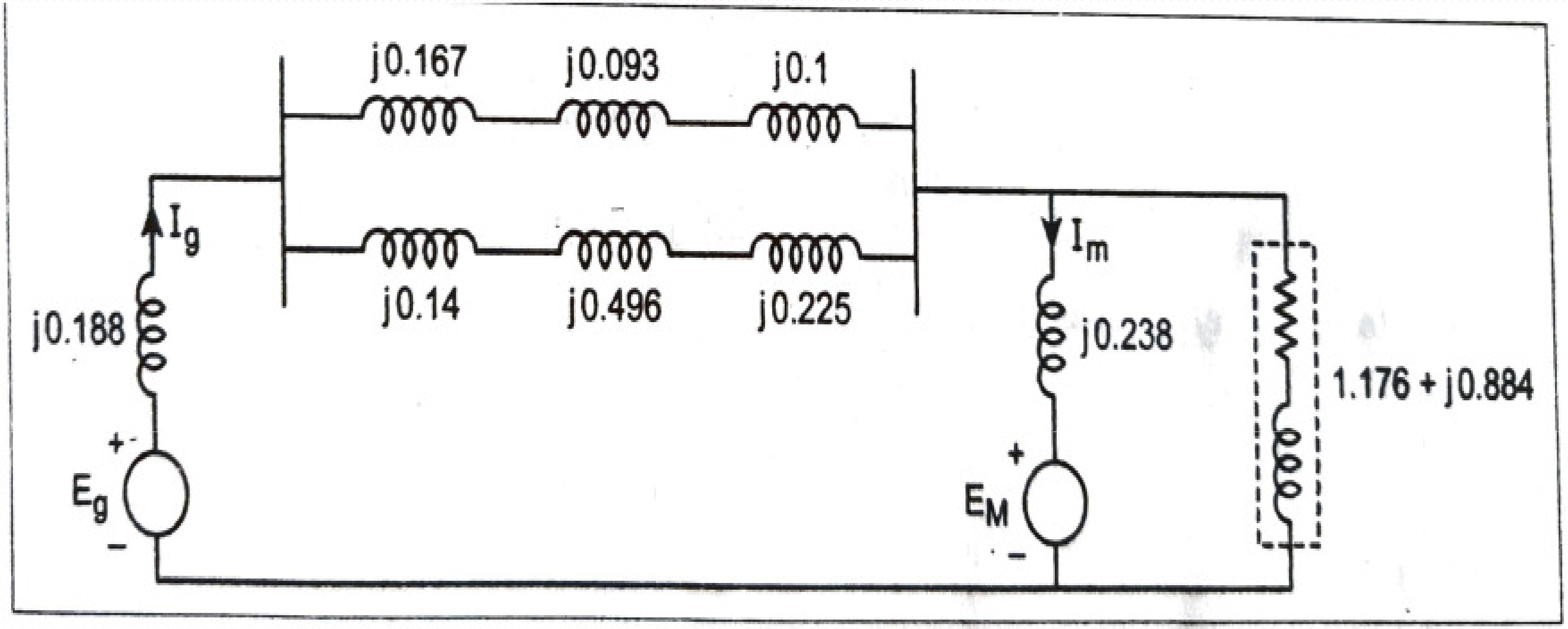


Fig. Impedance diagram

Primitive Network and Bus Admittance Matrix

The injected bus currents and bus voltages of a power system under steady state condition can be related through these matrices as

$$[Y][V] = [I] \quad \ldots (3.1)$$

$$[Z][I] = [V] \quad \ldots (3.2)$$

These matrices are important building blocks of power system modelling and analysis.

3.2. FORMATION OF Y-BUS BY TWO-RULE METHOD OR INSPECTION METHOD

Consider a three bus power system shown in Fig.3.1.

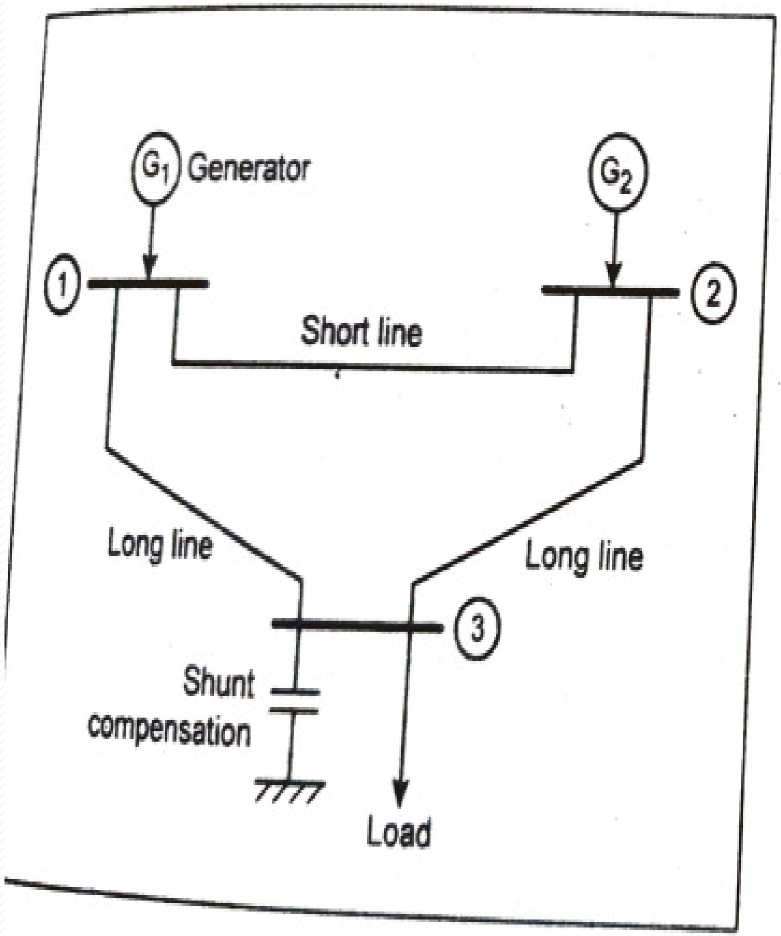


Fig. 3.1. 3-Bus sample system

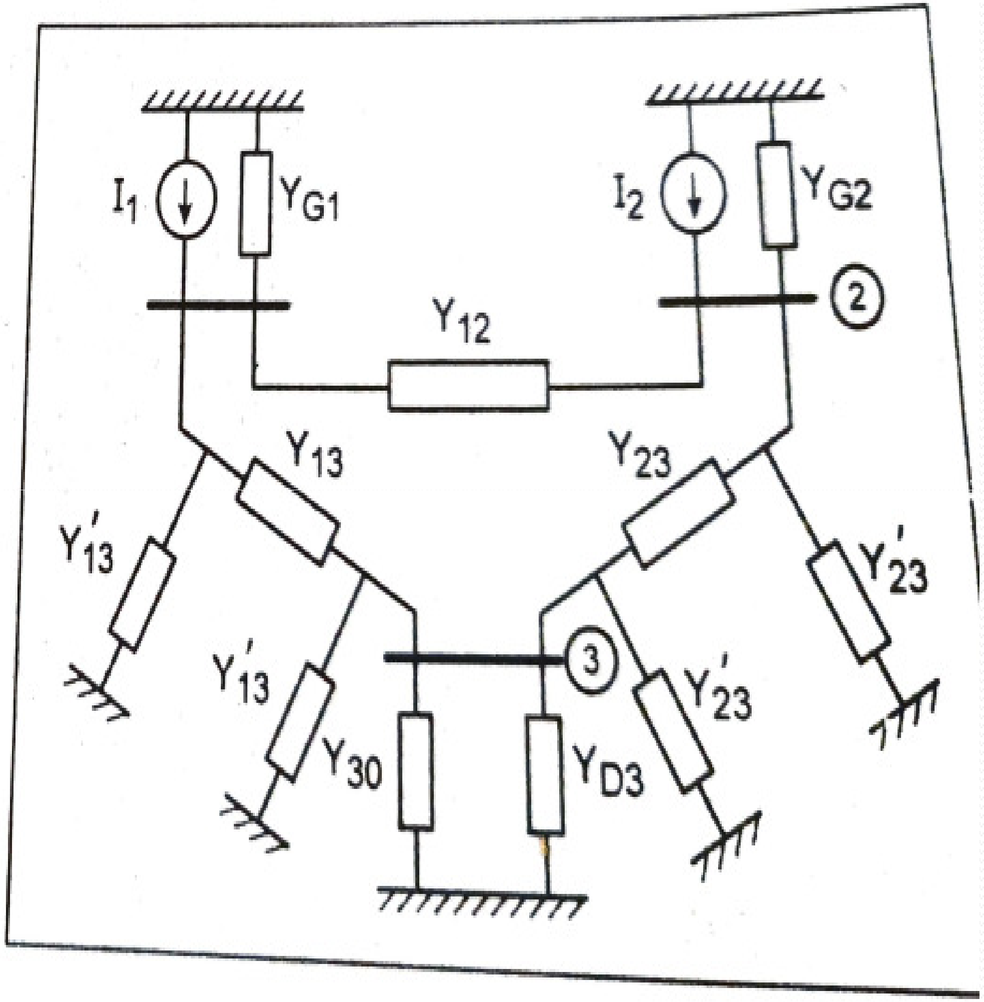


Fig. 3.2. Equivalent power network

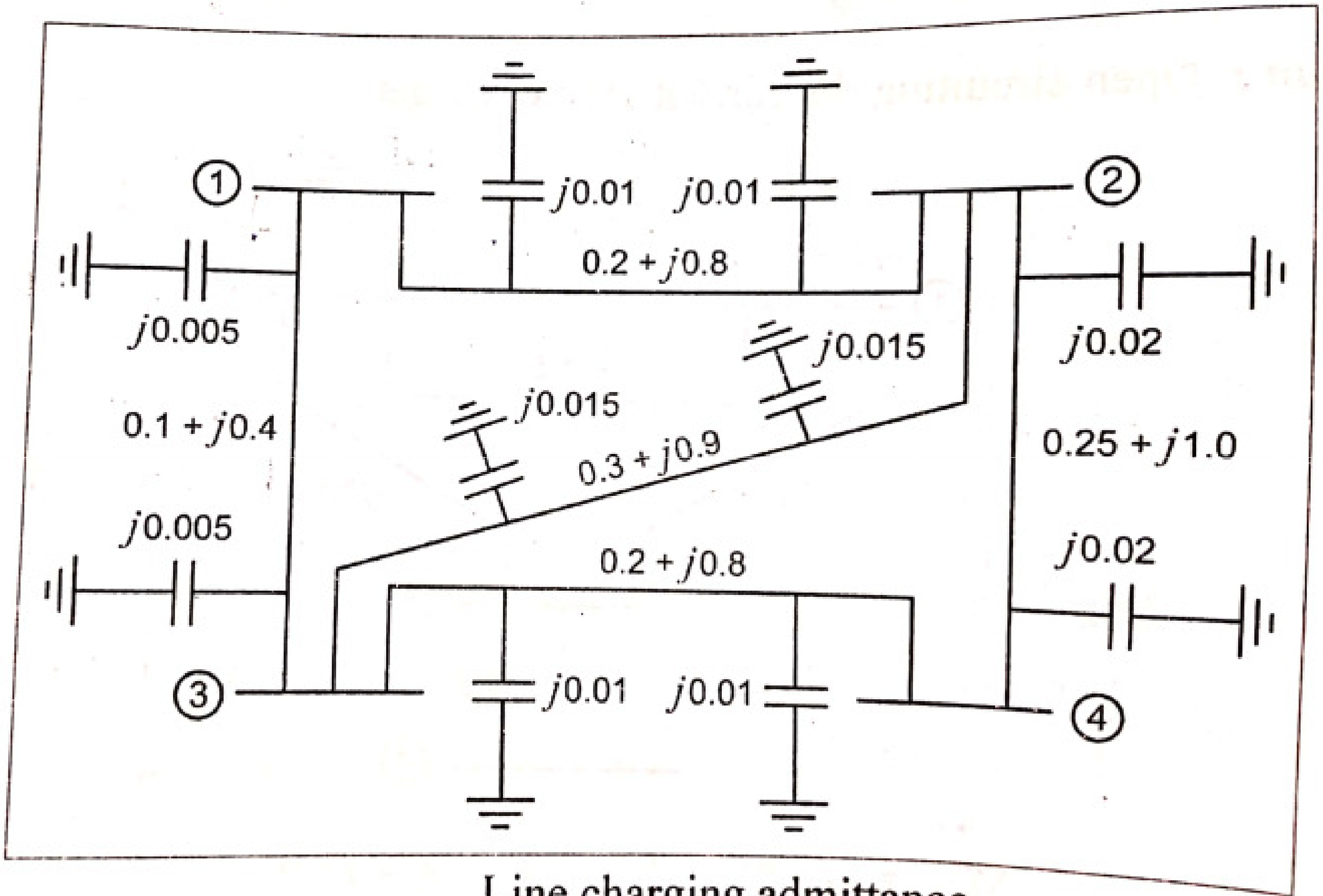
Formation of Y Bus by Two Rule Method or Inspection Method

Example 3.8 *The parameters of 4 bus system are as follows :*

Bus code	Line impedance (p.u)	Line charging admittance (p.u)
1 – 2	0.2 + j0.8	j0.02
2 – 3	0.3 + j0.9	j0.03
2 – 4	0.25 + j1.0	j0.04
3 – 4	0.2 + j0.8	j0.02
1 – 3	0.1 + j0.4	j0.01

Draw the network and find bus admittance matrix.

☺ ***Solution :***



$$\text{Half line charging admittance} = \frac{\text{Line charging admittance}}{2}$$

$$Y_{11} = Y_{12} + Y_{13} + Y_{10}$$

$$= \frac{1}{0.2 + j0.8} + \frac{1}{0.1 + j0.4} + j0.01 + j0.005$$

$$= 0.294 - j1.176 + 0.588 - j2.353 + j0.015$$

$$= 0.882 - j3.514$$

$$Y_{21} = Y_{12} = \frac{-1}{0.2 + j0.8} = -0.294 + j1.176$$

$$Y_{31} = Y_{13} = \frac{-1}{0.1 + j0.4} = -0.588 + j2.353$$

$$Y_{41} = Y_{14} = 0$$

$$Y_{22} = Y_{21} + Y_{24} + Y_{23} + Y_{20}$$

$$= \frac{1}{0.2+j0.8} + \frac{1}{0.25+j1.0} + \frac{1}{0.3+j0.9} + j0.01 + j0.02 + j0.015$$

$$= 0.294 - j1.176 + 0.235 - j0.941 + 0.333 - j1 + j0.045$$

$$= 0.862 - j3.072$$

$$Y_{32} = Y_{23} = \frac{-1}{0.3+j0.9} = -0.333 + j1$$

$$Y_{42} = Y_{24} = \frac{-1}{0.25+j1} = -0.235 + j0.941$$

$$Y_{33} = Y_{31} + Y_{34} + Y_{32} + Y_{30}$$

$$= \frac{1}{0.1+j0.4} + \frac{1}{0.2+j0.8} + \frac{1}{0.3+j0.9} + j0.005 + j0.015 + j0.01$$

$$= 0.588 - j2.353 + 0.294 - j1.176 + 0.333 - j1 + j0.03$$

$$= 1.215 - j4.499$$

$$Y_{34} = Y_{43} = \frac{-1}{0.2+j0.8} = -0.294 + j1.176$$

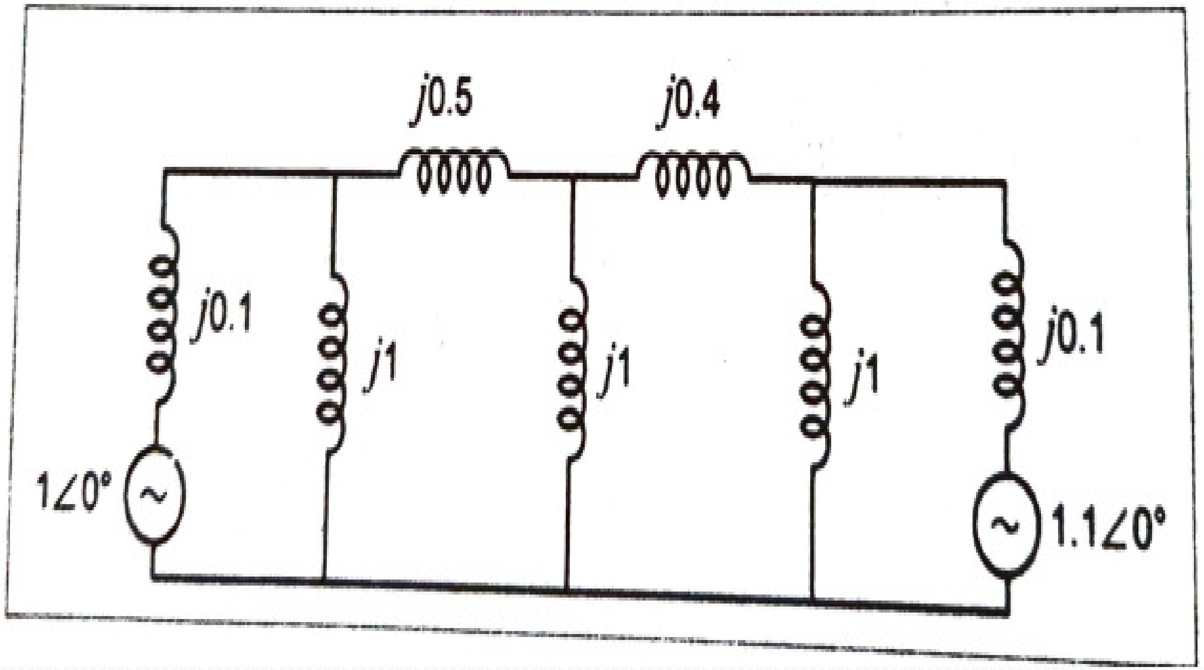
$$Y_{44} = Y_{42} + Y_{43} + Y_{40} = \frac{1}{0.25 + j1} + \frac{1}{0.2 + j0.8} + j0.02 + j0.01$$

$$= 0.235 - j0.941 + 0.294 - j1.176 + j0.03 = 0.5294 - j2.088$$

$$\text{Y-bus} = \begin{bmatrix} 0.882 - j3.514 & -0.294 + j1.176 & -0.588 + j2.353 & 0 \\ -0.294 + j1.176 & 0.862 - j3.072 & -0.333 + j1 & -0.235 + j0.941 \\ -0.588 + j2.353 & -0.333 + j1 & 1.215 - j4.499 & -0.294 + j1.176 \\ 0 & -0.235 + j0.941 & -0.294 + j1.176 & 0.529 - j2.088 \end{bmatrix}$$

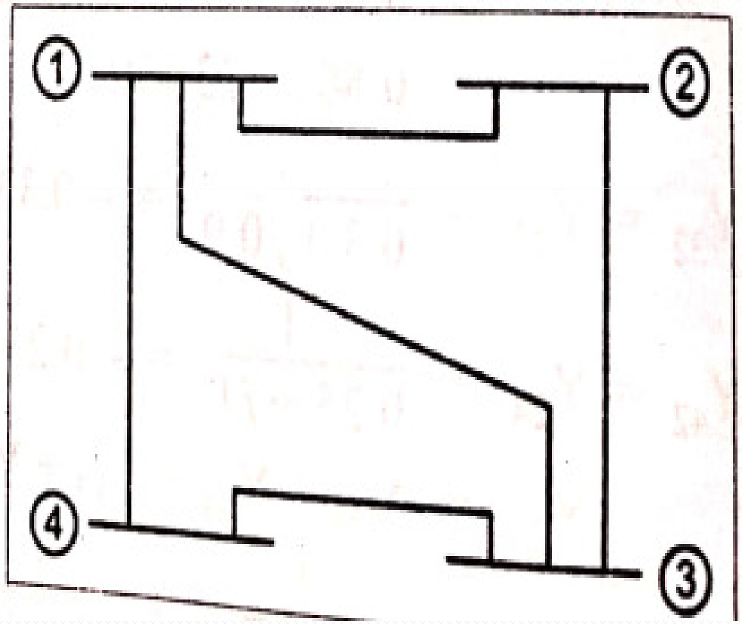
Assignment Problem 1

Example 3.6 *For the network shown in Fig., write the elements of bus admittance matrix directly by inspection method.*



3.3. ELIMINATION OF A NODE OR BUS (GAUSSIAN ELIMINATION OR KRON REDUCTION METHOD)

In power system, many substations, generating stations and load centres are present. So, the size of the Y-bus matrix is very large. Solving the nodal equation to find unknown bus voltages is very difficult. To minimize computational effort and computer storage, successive elimination or Gaussian elimination method is applicable.



general

$$Y_{ij}^{1} = Y_{ij} - \frac{Y_{in} Y_{nj}}{Y_{nn}}$$

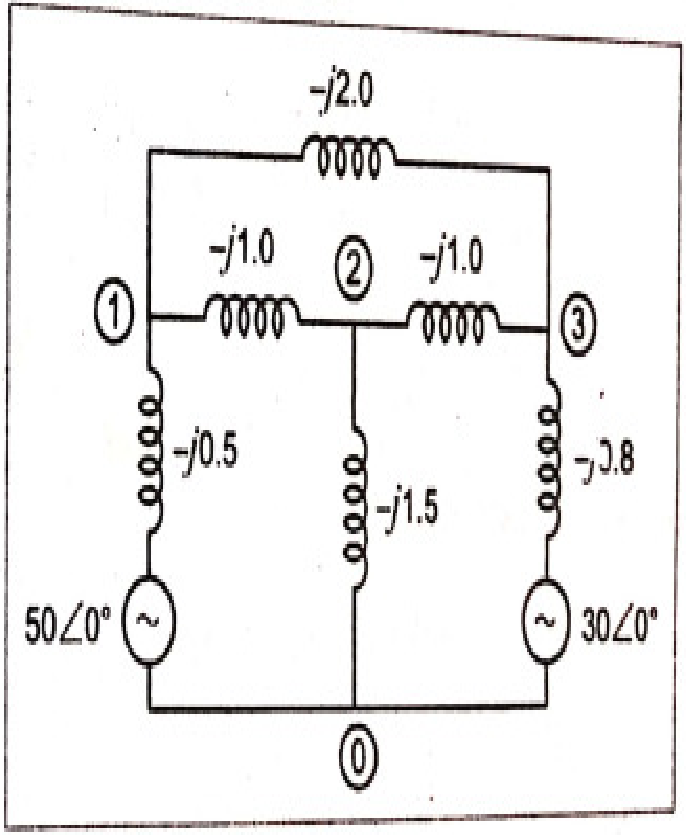
$$Y_{ij\,new} = Y_{ij\,old} - \frac{Y_{in} \cdot Y_{nj}}{Y_{nn}}$$

$$i = 1, 2, \ldots\ldots\ldots, n, \quad i \neq n$$

$$j = 1, 2, \ldots\ldots\ldots, n, \quad j \neq n$$

e n is the node which is to be removed.

Example 3.10 Determine the bus admittance matrix of the system is shown in Fig. Admittances are given in per unit. Determine the reduced bus admittance matrix after eliminating node 3.



$$\text{Y-bus} = \begin{matrix}1\\2\\3\end{matrix}\begin{bmatrix} -j0.5-j1.0-j2.0 & j1.0 & j2.0 \\ j1.0 & -j1.0-j1.0-j1.5 & j1.0 \\ j2.0 & j1.0 & -j2.0-j1.0-j0.8 \end{bmatrix}$$

$$= \begin{bmatrix} -j3.5 & j1.0 & j2.0 \\ j1.0 & -j3.5 & j1.0 \\ j2.0 & j1.0 & -j3.8 \end{bmatrix}$$

n=3 3x3

i=1, j=1

$$Y_{11\,new} = Y_{11\,old} - \frac{Y_{13}\,Y_{31}}{Y_{33}} = -j3.5 - \frac{j2.0 \times j2.0}{-j3.8} = -j2.45$$

i=1, j=2

$$Y_{12\,new} = Y_{12\,new} = Y_{12\,old} - \frac{Y_{13} \cdot Y_{32}}{Y_{33}} = j1.0 - \frac{j2.0 \times j1.0}{-j3.8} = j1.526$$

$$Y_{22\,new} = Y_{22\,old} - \frac{Y_{23} \cdot Y_{32}}{Y_{33}} = -j3.5 - \frac{j1.0 \times j1.0}{-j3.8} = -j3.237$$

$$\text{Reduced Y-bus} = \begin{bmatrix} -j2.45 & j1.526 \\ j1.526 & -j3.237 \end{bmatrix}$$

2x2

Assignment Problem 2

Example 3.11 *Eliminate buses 3 and 4 in given bus admittance and form new bus admittance matrix.*

$$
Y\text{-bus} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (4) \end{array}
\begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\
\begin{bmatrix} -j9.0 & 0.0 & j4.0 & j5.0 \\ 0.0 & -j7.5 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14.5 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix} \end{array}
$$

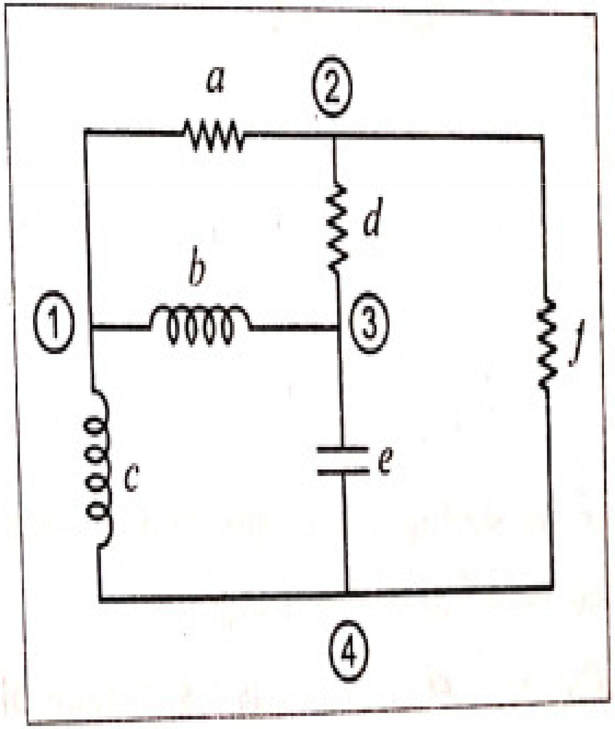
3.4. FORMATION OF Y-BUS BY SINGULAR TRANSFORMATION

3.4.1. GRAPH THEORY

Y-bus can be found by another method called singular transformation. We are using graph theory to determine bus admittance matrix.

Network : Network is an interconnection of elements in various branches at different nodes. A network is as shown in Fig.3.4.

Example :



Graph : A graph is a representation of network obtained by replacing every element of network by a line segment and every junction point by a node. A graph corresponding to the network is as shown in Fig.3.5.

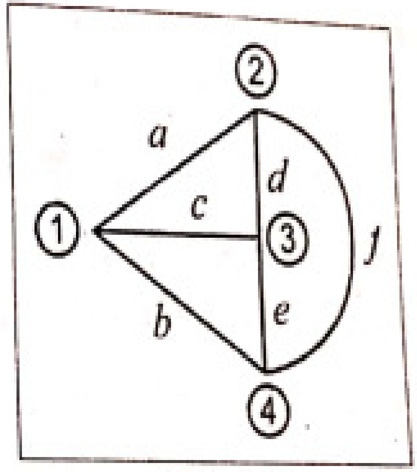


Fig. 3.5.

Oriented Graph : If every branch of a graph has direction, then the graph is called as a directed graph or oriented graph. A oriented graph is as shown in Fig.3.6.

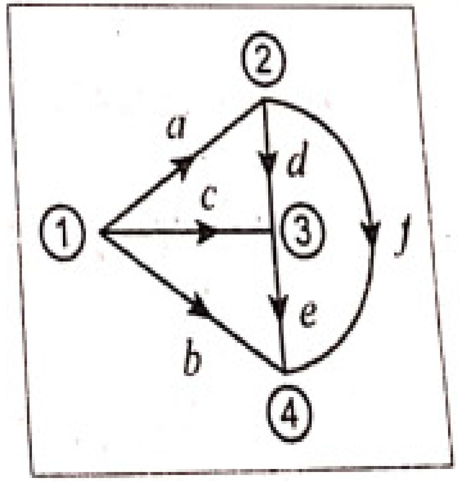


Fig. 3.6.

Branch or Edge :

Fig. 3.7. Branch

A branch is represented by a line segment in the graph of a network as shown in Fig.3.7.

Node or Bus or Vertices :

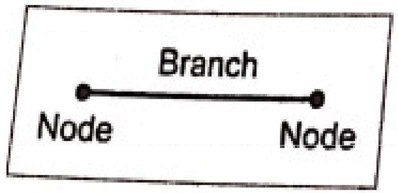


Fig. 3.8. Node

A node is a terminal of a branch which is represented by a point as shown in Fig.3.8.

Loop or Closed Path :

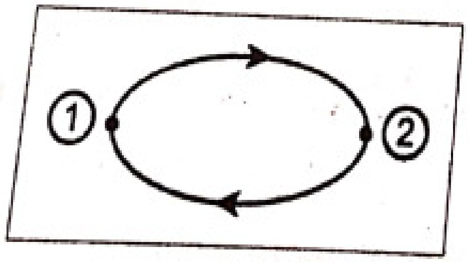


Fig. 3.9. Closed loop

Properties of a Tree :

- Number of nodes in a graph = Number of nodes in the tree of that graph.
- All the nodes must be connected by elements called the tree branches.
- Tree branches must not form any loop or closed path in the subgraph.
- Every connected graph has at least one tree.
- Rank of tree = Rank of a graph.
- Number of tree branches = Number of nodes – one ($n - 1$).

Examples of tree :

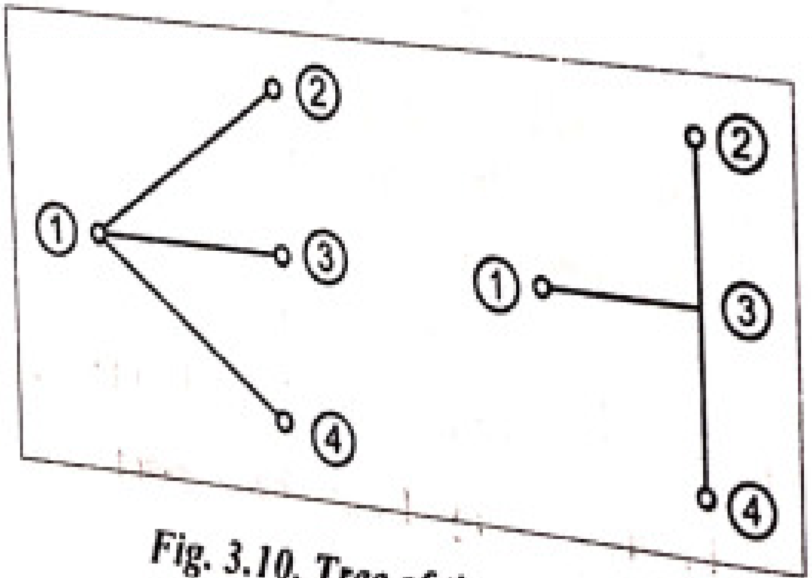


Fig. 3.10. Tree of the graph

Link or Chord : The removal branches of the tree is called links. The branches of cotree is called link or chord. The different links of the graph is as shown in Fig.3.11.

Example :

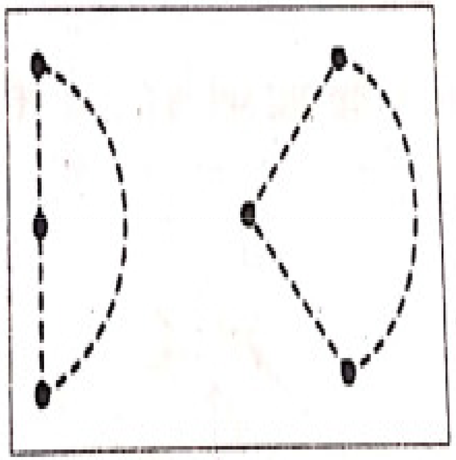


Fig. 3.11. Link of the graph

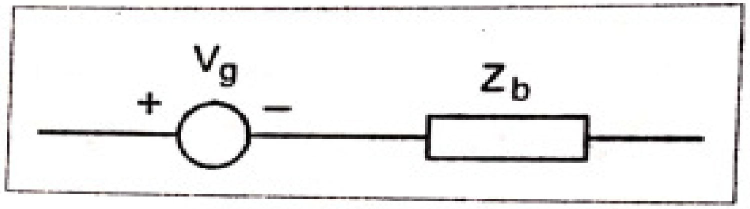
Cotree : The set of all links of a given tree is called the cotree of the graph.

3.4.2. PRIMITTIVE IMPEDANCE MATRIX [$Z_{PRIMITTIVE}$]

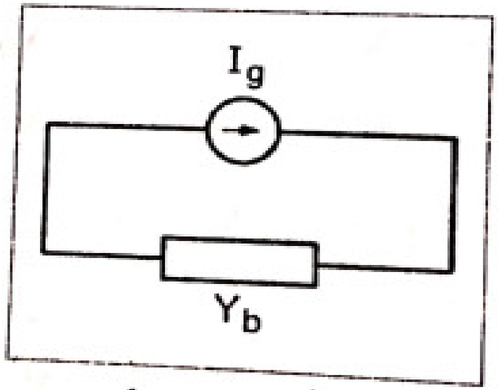
Primitive network:

A network element may contain active and passive components.

Impedance form:



Admittance form:



$$Y_b = \frac{1}{Z_b}$$

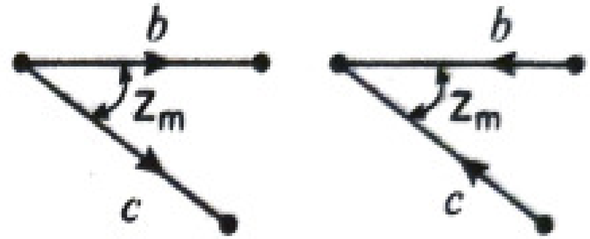
A set of unconnected elements is defined as a primitive network.

Example : For the given

$$Z_{primittive} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} Z_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_b & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_h \end{bmatrix} \end{array}$$

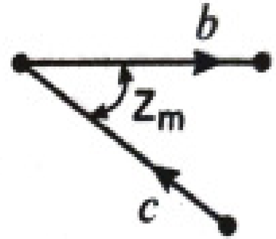
Assumption : Direction of current is same,

then the direction of Z_m is positive.



Direction of current is opposite,

then the direction of Z_m is negative.



$$Z_{\text{primittive}} = \begin{bmatrix} Z_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_b & Z_m & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_m & Z_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_h \end{bmatrix} \quad \ldots (3.22)$$

3.4.3. PRIMITTIVE ADMITTANCE MATRIX

Matrix which contain information about the transmission line (admittance) is called primittive admittance matrix.

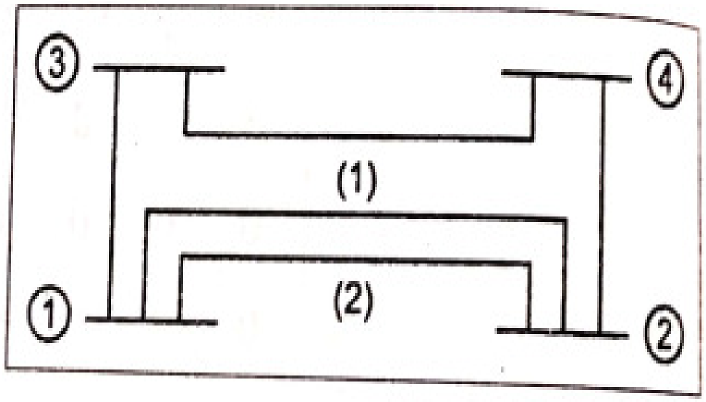
Primittive admittance matrix is the inverse of primittive impedance matrix.

$$Y_{primittive} = [Z_{primittive}]^{-1} \quad \text{... (3.23)}$$

$$Y_{bus} = \text{Bus admittance matrix} = [A][Y_b][A]^T$$

$$= [A][Y_{primittive}][A]^T$$

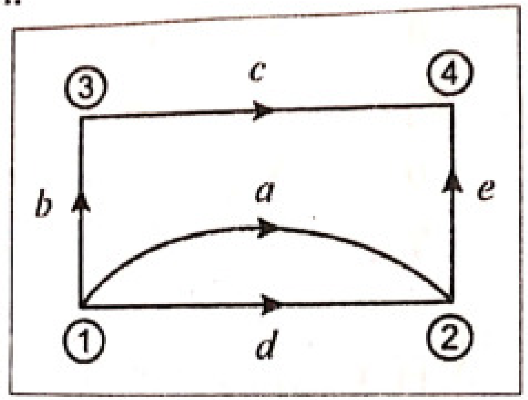
Example 3.14 *Form Y_{bus} by singular transformation for the network shown in Fig. The impedance data is given in Table. Take (1) as reference node.*



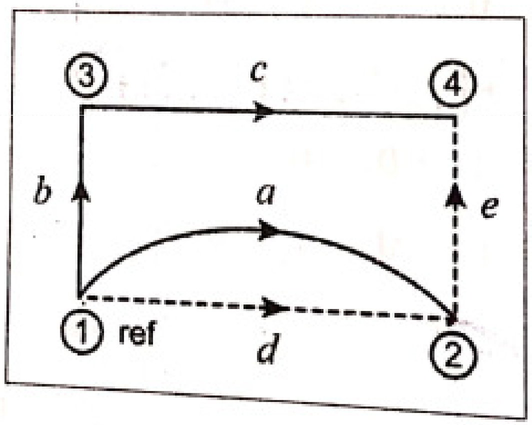
Element No.	Self	
	Bus code	Impedance
1	1 – 2 (1)	0.6
2	1 – 3	0.5
3	3 – 4	0.5
4	1 – 2 (2)	0.4
5	2 – 4	0.2

☺ ***Solution :*** Oriented Graph.

☺ ***Solution :*** Oriented Graph.



Take (1) as reference. Draw a tree.



Incidence matrix

$$[A] = \begin{array}{c} \\ (2) \\ (3) \\ (4) \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

$$[A]^T = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccc} (2) & (3) & (4) \end{array} \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

$$\text{Primittive impedance matrix } [Z_{\text{Primittive}}] = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} j0.6 & 0 & 0 & 0 & 0 \\ 0 & j0.5 & 0 & 0 & 0 \\ 0 & 0 & j0.5 & 0 & 0 \\ 0 & 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & 0 & j0.2 \end{bmatrix} \end{array}$$

Primittive admittance matrix $[Y_{Primittive}] = [Z_{Primittive}]^{-1}$

$Z = 1/Y$

$Y = 1/Z$

$$= \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} -j1.667 & 0 & 0 & 0 & 0 \\ 0 & -j2.0 & 0 & 0 & 0 \\ 0 & 0 & -j2 & 0 & 0 \\ 0 & 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & 0 & -j5 \end{bmatrix} \end{array}$$

Bus admittance matrix $[Y_{bus}] = [A][Y_{Primittive}][A]^T$

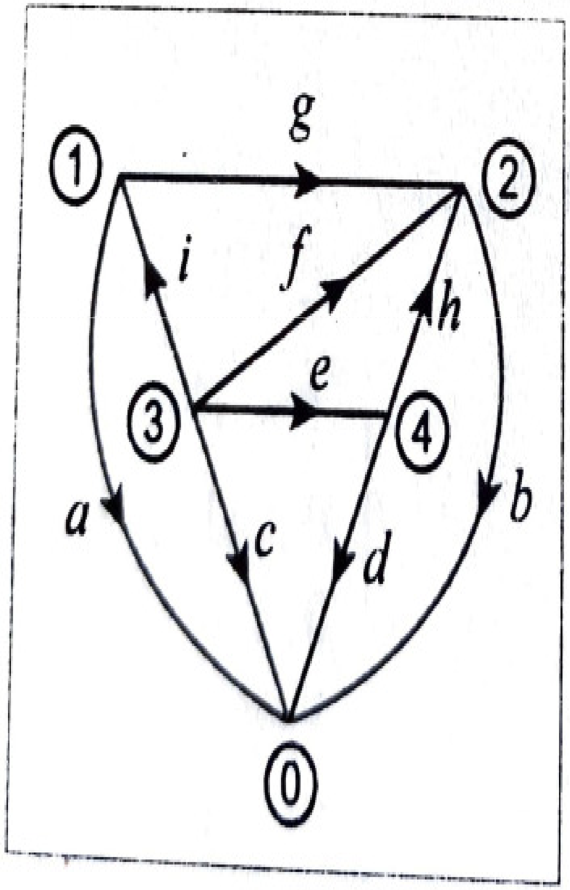
$$[Y_{Primittive}][A]^T = \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{bmatrix} j1.667 & 0 & 0 \\ 0 & j2 & 0 \\ 0 & -j2 & j2 \\ j2.5 & 0 & 0 \\ -j5 & 0 & j5 \end{bmatrix}$$

$$[Y_{bus}] = [A][Y_{Primittive}][A]^T = \begin{matrix} \\ (1) \\ (2) \\ (3) \end{matrix} \begin{matrix} (1) & (2) & (3) \\ \left[\begin{matrix} -j1.667 - j2.5 - j5 \\ 0 \\ j5 \end{matrix}\right. & \begin{matrix} 0 \\ -j2 - j2 \\ j2 \end{matrix} & \left.\begin{matrix} j5 \\ j2 \\ -j2 - j5 \end{matrix}\right] \end{matrix}$$

$$= \begin{bmatrix} -j9.167 & 0 & j5 \\ 0 & -j4 & j2 \\ j5 & j2 & -j7 \end{bmatrix}$$

Assignment Problem 3

Example 3.13 *Find Y_{bus} using singular transformation for the system of Fig. shown.*



MUTHAYAMMAL ENGINEERING COLLEGE, RASIPURAM 637408
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DEPARTMENT OF
ELECTRICAL AND ELECTRONICS ENGINEERING

POWER SYSTEM ANALYSIS

UNIT II

POWER FLOW ANALYSIS

Load Flow Study (Or) Power Flow Study

The study of various methods of solution to power system network is referred to as load flow study. The solution provides the voltages at various buses, power flowing in various lines and line-losses.

The following work has to be performed for a load flow study.

(i) Representation of the system by single line diagrams.

(ii) Determining the impedance diagram using the information in single line diagram.

(iii) Formulation of network equations.

(iv) Solution of network equations.

5.2. NEED FOR LOAD FLOW ANALYSIS (OR) IMPORTANCE OF POWER FLOW ANALYSIS IN PLANNING AND OPERATION OF POWER SYSTEMS

Load flow analysis is performed on a symmetrical steady-state operating condition of a power system under normal mode of operation. The solution of load flow gives bus voltages and line/transformer power flows for a given load condition. This information is essential for long term planning and operational planning.

Long Term Planning

Load flow analysis helps in investigating the effectiveness of alternative plans and choosing the best plan for system expansion to meet the projected operating state.

Operational Planning

It helps in choosing the best unit commitment plan and generation schedules to run the system efficiently for the next day's load condition without violating the bus voltages and line flow operating limits.

5.3. CLASSIFICATION OF BUSES

In the network of power system, buses become nodes and a voltage can be specified for each bus.

The power flow equation is

$$P_i + j\,Q_i = V_i \sum_{j=1}^{N} Y_{ij}^* \, V_j^*, \quad i = 1, 2, \ldots\ldots, N \qquad \ldots (5.1)$$

Complex bus voltage $V_i = |V_i| \angle \delta_i$ ... (5.2)

From equation (5.1) and (5.2), we know that power system is associated with four quantities and they are real power (P), Reactive power (Q), Voltage magnitude | V |, and phase angle of voltage (δ). In load flow problem, two quantities are specified for each bus and the remaining two quantities are obtained by solving the load flow equations. The buses are classified based on the variables specified. There are three types of buses.

There are three types of buses.

1. Slack bus or swing bus or reference bus.
2. Generator bus or voltage controlled bus or P-V bus or regulated bus.
3. Load bus or P-Q bus.

The following table gives the quantities specified and the quantities to be specified for each bus.

S.No.	Bus	Quantities specified	Quantities to be specified
1.	Slack bus	$\|V\|, \delta$	P, Q
2.	P-V bus (or) Generator bus	$P, \|V\|$	Q, δ
3.	P-Q bus (or) Load bus	P, Q	$\|V\|, \delta$

where

$$P = P_G - P_L$$

$$Q = Q_G - Q_L$$

P_G = Real power generated by generator connected to the bus.

Q_G = Reactive power generated by generator connected to the bus.

P_L = Real power drawn by the load.

Q_L = Reactive power drawn by the load.

Slack Bus

In slack bus, voltage magnitude and phase angle of voltages are specified pertaining to a generator bus usually a large capacity generation bus is chosen. We assume voltage (V) as reference phasor,

$$i.e., \quad \delta = 0$$

$$\text{where} \quad \delta = \text{Phase angle of voltage.}$$

This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the network.

Obtain (N – 1) complex bus voltages from the (N – 1) load flow equations. Incidentally the specification of $|V_{slack}|$ helps us to fix the voltage level of the system. In power flow study, at all buses net complex flow into the network is not known in advance.

While specifying a generation schedule for a given system demand, one can fix up the generation setting of all the generation buses except one bus because of the limitation of not knowing the transmission loss in advance. Therefore, it is necessary to have one bus called ***slack bus.***

Power Balance equation is

$$\underbrace{P_L}_{\text{Real power loss}} = \sum_{i=1}^{N} P_i = \underbrace{\sum_{i=1}^{N} P_{Gi}}_{\text{Total generation}} - \underbrace{\sum_{i=1}^{N} P_{Di}}_{\text{Total load}} \qquad \ldots (5.3)$$

P_L depends on I^2R loss in the transmission line and transformers of the network. The individual currents in the various lines of the network cannot be calculated until after the voltage magnitude and angle are known at every bus of the system. Therefore P_L is initially unknown. Real and reactive power are not specified for slack bus.

Generator Bus or P-V Bus or Voltage Controlled Bus or Regulated Bus

At these buses, the real power and voltage magnitudes are specified. The phase angles of the voltages and the reactive power are to be determined. The limits on the value of the reactive power are also specified.

In order to maintain a good voltage profile over the system, Automatic Voltage Regulator (AVR) is used.

Static VAR compensator buses are called as P-V buses because real power and voltage magnitudes are specified at these buses.

Load Bus or P-Q Bus

At these buses, the active and reactive powers are specified. The magnitude and phase angle of the voltage are unknown. These are called as load bus.

5.4.2. PRACTICAL LOAD FLOW PROBLEM OR STATEMENT OF PRACTICAL POWER FLOW PROBLEM

Practical load flow problem can be stated as follows :

Given : *The network configuration, complex power demands for all buses, real power generation schedules and voltage magnitudes of all the P-V buses and voltage magnitude of the slack bus.*

To determine :

- Bus admittance matrix.
- Bus voltage phase angles of all buses except the slack bus and bus voltage magnitudes of all the P-Q buses.

$$\text{State vector } X = [V_1, V_2, \ldots\ldots\ldots, V_N, \delta_1, \delta_2, \ldots\ldots, \delta_N]$$

5.5. POWER FLOW EQUATION (PFE) / DEVELOPMENT OF POWER FLOW MODEL IN COMPLEX VARIABLE FORM AND POLAR VARIABLE FORM

The power flow or load flow model in complex form is obtained by writing one complex power matching equation at each bus for the Fig.5.2.

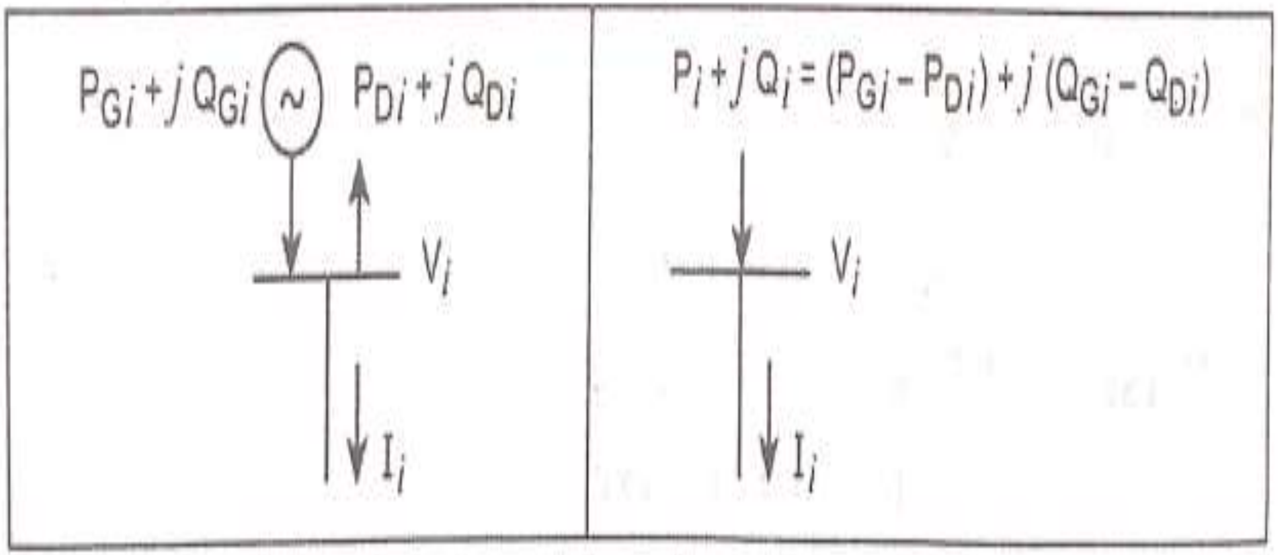


Fig. 5.2. Complex power balancing at a bus

Net power injected into the bus i.

$$S_i = S_{Gi} - S_{Di}$$

$$= P_{Gi} + j\,Q_{Gi} - (P_{Di} + j\,Q_{Di})$$

$$= P_{Gi} - P_{Di} + j\,(Q_{Gi} - Q_{Di})$$

$$= P_i + j\,Q_i$$

We know, $$P_i + j\,Q_i = V_i\,I_i^* \quad \dots (5.5)$$

Consider two bus system as shown in Fig.5.3.

Let I_1 be the net or bus current entering into bus 1.

Let I_2 be the net current entering into bus 2.

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = y_{10} + y_{12}$$

$$Y_{22} = y_{20} + y_{21}$$

$$Y_{12} = Y_{21} = -y_{21}$$

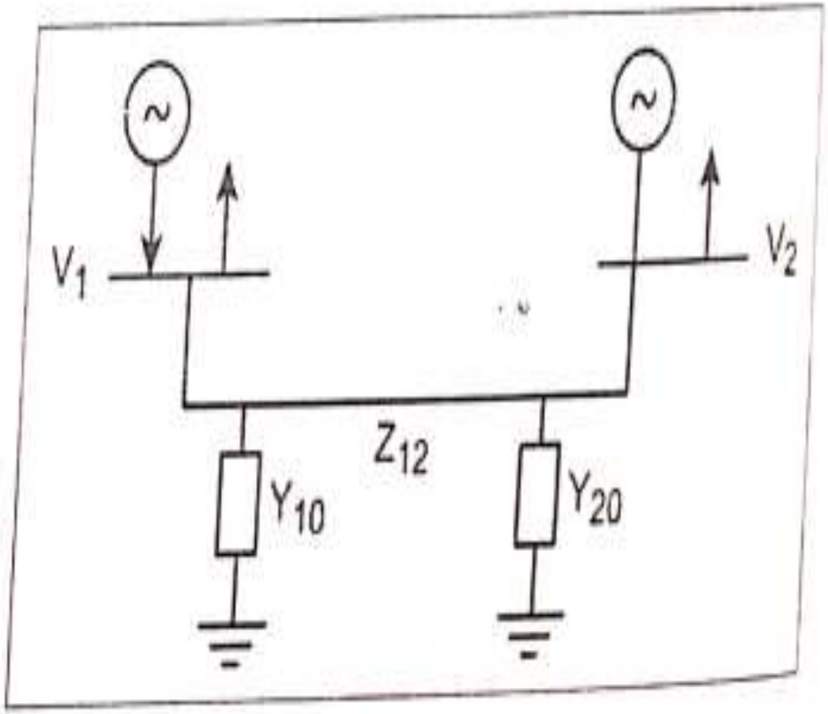


Fig. 5.3. Two bus system

In general $\quad Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \cos \theta_{ij} + j |Y_{ij}| \sin \theta_{ij}$ ... (5.6)

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In general, the net current entering into i^{th} bus

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \dots\dots\dots + Y_{iN} V_N = \sum_{j=1}^{N} Y_{ij} V_j \qquad \dots (5.7)$$

Substituting I_i from equation (5.7) in equation (5.1), we get

$$S_i = P_i + j\, Q_i = V_i\, I_i^*$$

$$S_i = P_i - j\, Q_i = V_i^*\, I_i$$

$$P_i - j\, Q_i = V_i^* \sum_{j=1}^{N} Y_{ij} V_j, \quad \text{where } i = 1, 2, \dots\dots\dots, N$$

There are N complex variable equations from which the N unknown complex variables $V_1, V_2, \dots\dots, V_N$ can be determined.

Substituting Y_{ij} from equation (5.6), we get,

$$P_i - j\, Q_i = V_i^* \sum_{j=1}^{N} |Y_{ij}| \angle \theta_{ij}\, V_j$$

$$\text{where } V_i = |V_i| \angle \delta_i, \quad V_i^* = |V_i| \angle -\delta_i$$

$$V_j = |V_j| \angle \delta_j; \quad \delta_i = \text{Phase angle of voltage}$$

$$\therefore P_i - j Q_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \angle (\theta_{ij} + \delta_j - \delta_i)$$

Equating real and reactive parts, we obtain

$$P_i = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$$

$$Q_i = -\sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$$

The P_i and Q_i equations are called as polar form of the power flow equations (or) static load flow equations.

For an 'N' bus system, there will be 2N power flow equations. Each bus is characterized by four variables P_i, Q_i, V_i and δ_i resulting in a total of 4N variables. The power flow equations can be solved for 2N variables if the remaining 2N variables are specified.

5.6. SOLUTION TO LOAD FLOW PROBLEM

A number of methods are available for solving load flow problem. In all these methods, voltage solution is initially assumed and then improved upon using some iterative process until convergence is reached.

The load flow methods are given by,

(*i*) Gauss-Seidel Load Flow Method (GSLF).

(*ii*) Newton-Raphson Load Flow Method (NRLF).

(*iii*) Fast-decoupled Load Flow Method (FDLF).

5.7. ITERATIVE SOLUTION USING GAUSS-SEIDEL METHOD TO LOAD FLOW PROBLEM – INCLUDING Q-LIMIT CHECK FOR VOLTAGE CONTROLLED BUSES

5.7.5. ALGORITHM FOR ITERATION METHOD

Step 1 : Form Y-bus matrix.

Step 2 : Assume $V_k = V_{k\,(spec)} \angle 0°$ at all generator buses.

Step 3 : Assume $V_k = 1 \angle 0° = 1 + j0$ at all load buses.

Step 4 : Set iteration count = 1 (iter = 1).

Step 5 : Let bus number $i = 1$.

Step 6 : If 'i' refers to generator bus go to step no. 7, otherwise go to step 8.

Step 7(a) : If 'i' refers to the slack bus go to step 9. Otherwise go to step 7(b).

Step 7(b) : Compute Q_i using,

$$Q_i^{cal} = -\mathrm{Im}\left[\sum_{j=1}^{N} V_i^{*} Y_{ij} V_j\right]$$

$$Q_{Gi} = Q_i^{cal} + Q_{Li}$$

Check for Q limit violation.

If $Q_{i\,(min)} < Q_{Gi} < Q_{i\,(max)}$, then $Q_{i\,(spec)} = Q_i^{cal}$.

If $Q_{i\,(min)} < Q_{Gi}$, then $Q_{i\,(spec)} = Q_{i\,(min)} - Q_{Li}$

If $Q_{i\,(max)} < Q_{Gi}$, then $Q_{i\,(spec)} = Q_{i\,(max)} - Q_{Li}$

If Q_{limit} is violated, then treat this bus as P–Q bus till convergence is obtained.

Step 8 : Compute V_i using the equation,

$$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i\,(spec)} - Q_{i\,(spec)}}{V_i^{old\,*}} - \sum_{i=1}^{j-1} Y_{ij} V_j^{new} - \sum_{i=j+1}^{n} Y_{ij} V_i^{old}\right]$$

Step 9 : If i is less than number of buses, increment i by 1 and go to step 6.

Step 10 : Compare two successive iteration values for V_i.

If $V_i^{new} - V_i^{old} <$ tolerance, go to step 12.

Step 11 : Update the new voltage as

$$V^{new} = V^{old} + \alpha\,(V^{new} - V^{old})$$

$$V^{old} = V^{new}$$

iter = iter + 1 ; go to step 5

Step 12 : Compute relevant quantities.

$$\text{Slack bus power, } S_1 = P_i - j\,Q_i = V^* I = V_i^* \sum_{j=1}^{N} Y_{ij} V_j$$

$$\text{Line flows } S_{ij} = P_{ij} + j\,Q_{ij}$$

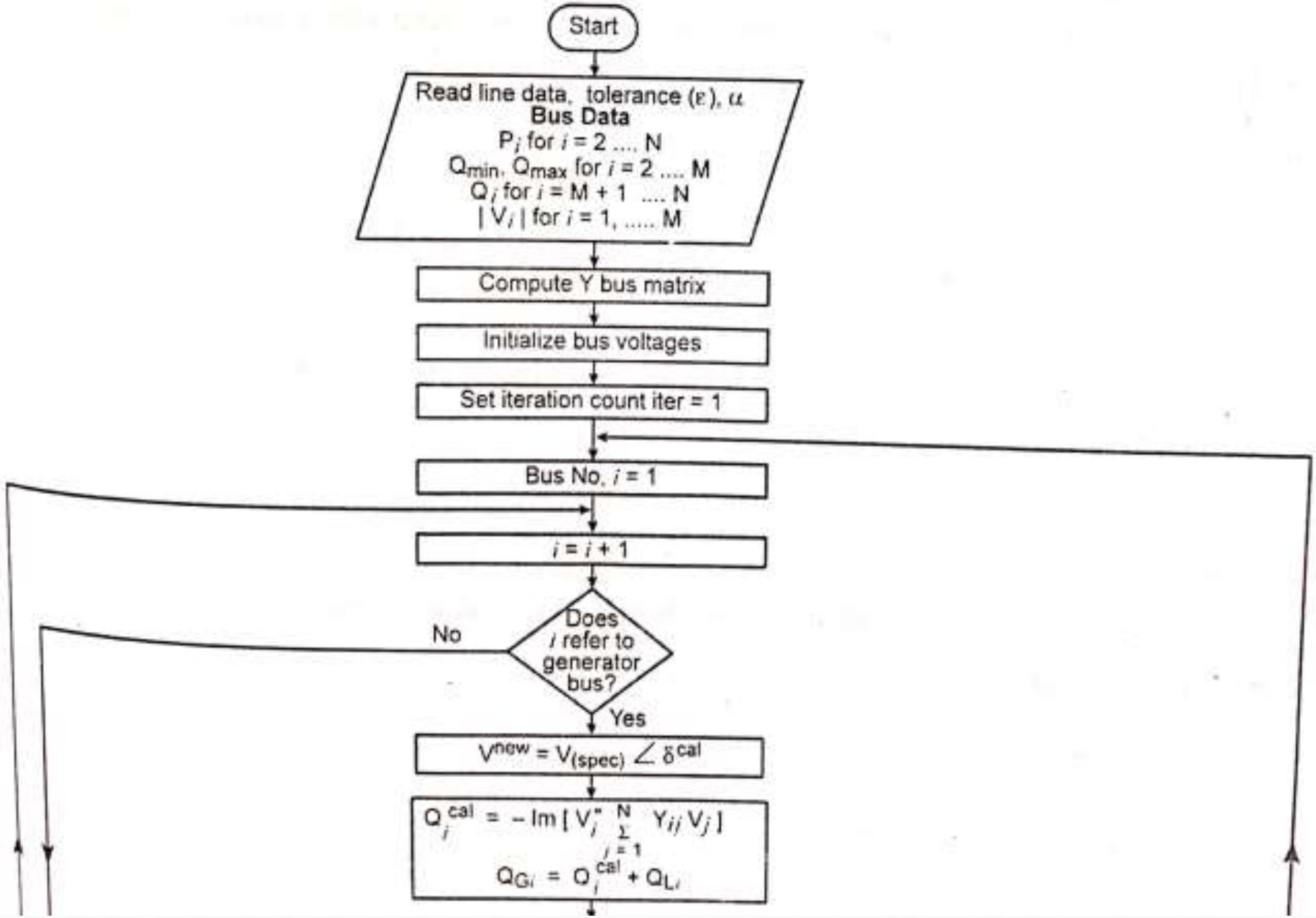
$$= V_i [\,V_i^* - V_j^*\,]\, Y_{ij\ \text{series}}^* + |V_i|^2\, Y_{ii}^*$$

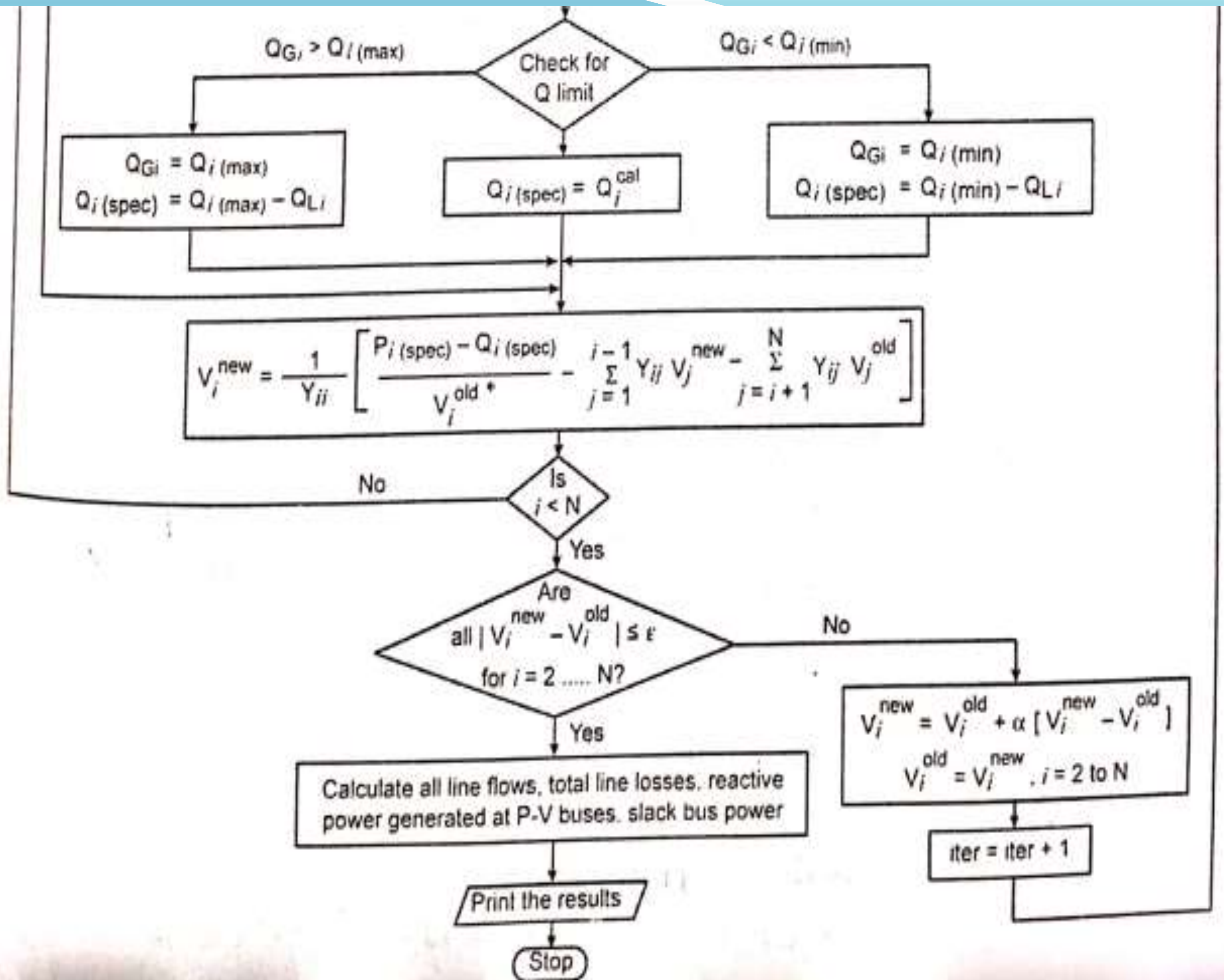
$$P_{\text{Loss}} = P_{ij} + P_{ji}$$

$$Q_{\text{Loss}} = Q_{ij} + Q_{ji}$$

Step 13 : Stop the execution.

‍5.7.6. FLOW CHART FOR GAUSS-SEIDEL METHOD INCLUDING PV BUS ADJUSTMENT





$Q_{Gi} > Q_{i(max)}$
Check for Q limit
$Q_{Gi} < Q_{i(min)}$
$Q_{Gi} = Q_{i(max)}$
$Q_{i(spec)} = Q_{i(max)} - Q_{Li}$
$Q_{i(spec)} = Q_i^{cal}$
$Q_{Gi} = Q_{i(min)}$
$Q_{i(spec)} = Q_{i(min)} - Q_{Li}$
$V_i^{new} = \frac{1}{Y_{ii}}\left[\frac{P_{i(spec)} - Q_{i(spec)}}{V_i^{old*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{N} Y_{ij} V_j^{old}\right]$
No
Is $i < N$
Yes
Are all $|V_i^{new} - V_i^{old}| \le \varepsilon$ for $i = 2 ..... N$?
No
$V_i^{new} = V_i^{old} + \alpha [V_i^{new} - V_i^{old}]$
$V_i^{old} = V_i^{new}$, $i = 2$ to N
iter = iter + 1
Yes
Calculate all line flows, total line losses, reactive power generated at P-V buses, slack bus power
Print the results
Stop

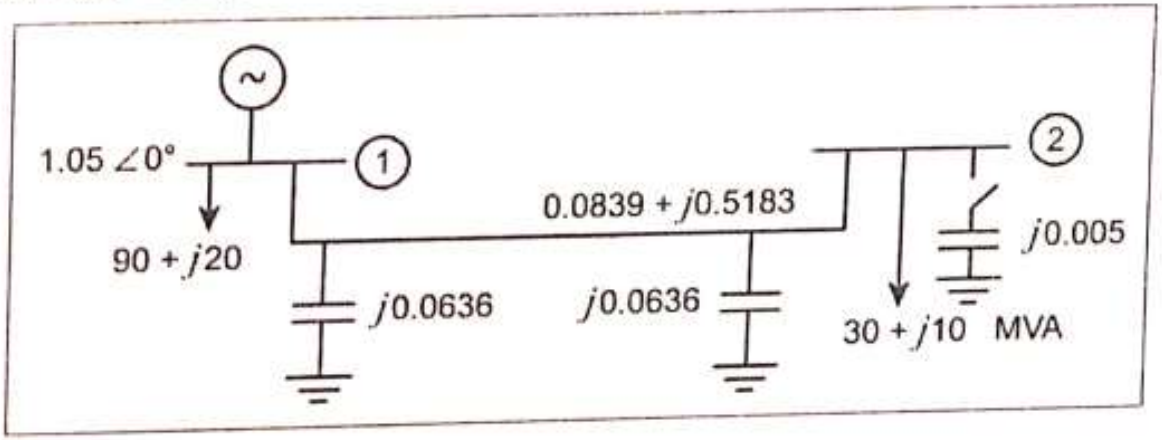
Advantages of G.S. Method

1. Calculations are simple and so the programming task is less.
2. The memory requirement is less.
3. Useful for small size system.

Disadvantages of G.S. Method

1. Requires large number of iterations to reach convergence.
2. Not suitable for large systems.
3. Convergence time increases with size of the system.

Example 5.1 ***Perform power flow of one iteration for the system as shown in Fig., using Gauss-Seidal method. Determine slack bus power, line flows and line losses. Take base MVA as 100. ($\alpha = 1.1$).***



☺ ***Solution :***

Step 1 : Formulate Y_{bus}.

When the switch is open, there is no connection of capacitor at bus 2.

Take the bus 2 as load bus.

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

Step 2 : Initialize bus voltages.

$$V_1^{old} = 1.05 \angle 0° \quad \text{p.u}$$

$$V_2^{old} = 1.0 \angle 0° \quad \text{p.u}$$

Step 3 : Calculate V_2^{new} .

$$P_2 = -30 \text{ MW} = \frac{-30}{100} \text{ p.u} = -0.3 \text{ p.u}$$

$$Q_2 = -10 \text{ MVAR} = \frac{-10}{100} \text{ p.u} = -0.1 \text{ p.u}$$

$$V_2^{new} = \frac{1}{Y_{22}} \left[\frac{P_2 - j\,Q_2}{V_2^{old\,*}} - Y_{21}\,V_1^{new} \right]$$

$$= \frac{1}{0.3044 - j1.816} \left[\frac{-0.3 + j0.1}{1.0 \angle -0^\circ} - (-0.3044 + j1.88)\,1.05 \right]$$

$$= 1.0054 - j0.1577 = 1.018 \angle -8.915^\circ$$

Step 4 : Calculate V_2^{new} Using acceleration factor.

$$V_{2\,acc}^{new} = V_2^{old} + \alpha\,[\,V_2^{new} - V_2^{old}\,]$$

$$= 1.0 + 1.1\,[1.0054 - j0.1577 - 1]$$

$$= 1.0059 - j0.173 = 1.0207 \angle -9.78^\circ$$

Step 5 : Slack bus power.

$$S_1 = P_1 - jQ_1 = V_1^* [Y_{11} V_1 + Y_{12} V_2]$$

$$= 1.05 \angle -0° [(0.3044 - j1.816)\, 1.05 + (-0.3044 + j1.88)(1.0207 \angle -9.78°)]$$

$$= 0.3556 + j0.0388 \text{ p.u} = 35.56 + j3.88 \text{ MVA}$$

$$P_1 = 35.56 \text{ MW}, \; Q_1 = -3.88 \text{ MVAR}$$

$$\text{Real power generation } P_{G1} = P_1 + P_{L1}$$

$$= 35.56 + 90 = 125.56 \text{ MW}$$

$$\text{Reactive power generation } Q_{G1} = Q_1 + Q_{L1}$$

$$= -3.88 + 20 = 16.12 \text{ MVAR}$$

Step 6 : Line flows.

Bus		$S_{ij} = P_{ij} + j\,Q_{ij} = V_i\,[V_i^* - V_j^*]\,Y_{ij\ \text{series}}^* + \lvert V_i \rvert^2\,Y_{Pi}^*$
From	To	
1	2	$S_{12} = V_1\,[V_1^* - V_2^*]\,Y_{12\ \text{series}}^* + \lvert V_1 \rvert^*\,Y_{10}^*$ $= 1.05\,[1.05\,\angle{-0°} - (1.0059 + j0.173)] \times (0.3044 + j1.88) + 1.05^2 \times (-j0.0636)$ $= 0.3556 - j0.0383$ p.u $P_{12} = 0.3556$ p.u $= 35.56$ MW $Q_{12} = -0.0383$ p.u $= -3.83$ MVAR
2	1	$S_{21} = V_2\,[V_2^* - V_1^*]\,Y_{12\ \text{series}}^* + \lvert V_2 \rvert^2\,Y_{20}^*$ $= (1.0059 - j0.173)\,[1.0059 + j0.173 - 1.05] \times (0.3044 + j1.88) + 1.0207^2 \times (-j0.0636)$ $= -0.3459 - j0.038$ p.u $P_{21} = -0.3459$ p.u $= -34.59$ MW $Q_{21} = -0.038$ p.u $= -3.8$ MVAR

Step 7 : Transmission line loss $(S_{ij\ Loss} = S_{ij} + S_{ji})$

$$P_{12\ Loss} = P_{12} + P_{21} = 35.56 - 34.59 = 0.97 \quad MW$$

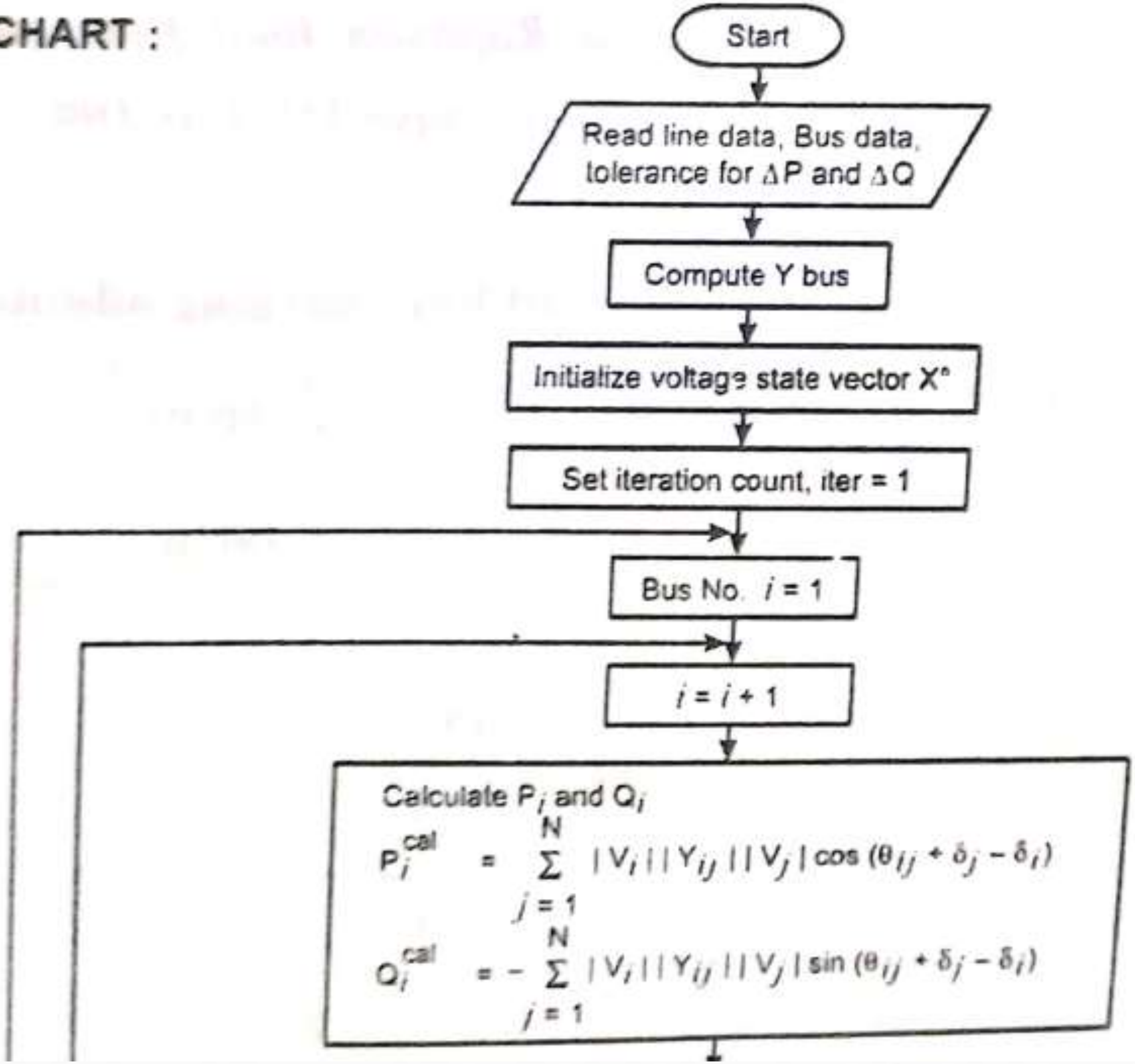
$$Q_{12\ Loss} = Q_{12} + Q_{21} = -3.83 + (-3.8) = -7.63 \quad MVAR$$

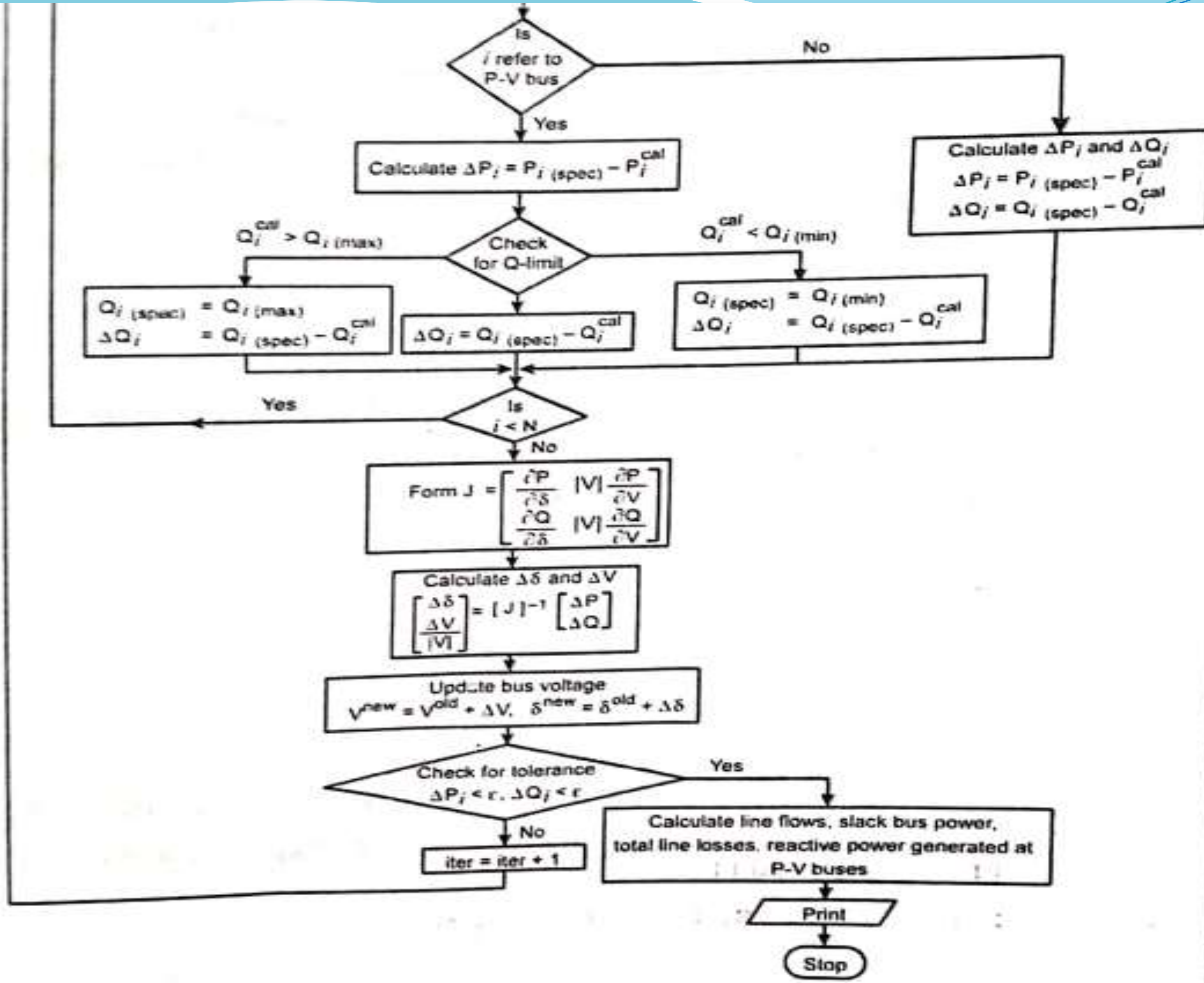
5.8. NEWTON-RAPHSON METHOD

The Gauss-Seidel algorithm is very simple but the convergence becomes increasingly slow as the system size grows. The Newton-Raphson technique, converges equally fast for large as well as small systems, usually in less than 4 to 5 iterations but more functional evaluations are required. It has become very popular for large system studies.

The most widely used method for solving simultaneous non-linear algebraic equations is the N-R method. This method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor's series expansion.

FLOW CHART :


Start
Read line data, Bus data, tolerance for ΔP and ΔQ
Compute Y bus
Initialize voltage state vector X°
Set iteration count, iter = 1
Bus No. i = 1
i = i + 1
Calculate P_i and Q_i
$P_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$
$Q_i^{cal} = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$



Is
i refer to
P-V bus
No
Yes
Calculate $\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_i^{cal} > Q_{i\,(max)}$
Check
for Q-limit
$Q_i^{cal} < Q_{i\,(min)}$
$Q_{i\,(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_{i\,(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
Yes
Is
$i < N$
No
Form $J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & |V|\frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & |V|\frac{\partial Q}{\partial V} \end{bmatrix}$
Calculate $\Delta\delta$ and ΔV
$\begin{bmatrix} \Delta\delta \\ \frac{\Delta V}{|V|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$
Update bus voltage
$V^{new} = V^{old} + \Delta V, \quad \delta^{new} = \delta^{old} + \Delta\delta$
Check for tolerance
$\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$
Yes
No
iter = iter + 1
Calculate line flows, slack bus power,
total line losses, reactive power generated at
P-V buses
Print
Stop

Algorithm:

1. Formulate Y-bus matrix.
2. Assume flat start for starting voltage solution.

$$\delta_i^{\circ} = 0, \text{ for } i = 1, \ldots\ldots, N \qquad \text{for all buses except slack bus.}$$

$$|V_i^{\circ}| = 1.0, \qquad \text{for } i = M+1, M+2, \ldots\ldots, N \text{(for all PQ buses).}$$

$$|V_i| = |V_i|_{(spec)} \qquad \text{for all PV buses and slack bus.}$$

3. For load buses, calculate P_i^{cal} and Q_i^{cal}.
4. For PV buses, check for Q-limit violation.

If $Q_{i\,(min)} < Q_i^{cal} < Q_{i\,(max)}$, the bus acts as P-V bus.

If $Q_i^{cal} > Q_{i\,(max)}$, $Q_{i\,(spec)} = Q_{i\,(max)}$.

If $Q_i^{cal} < Q_{i\,(min)}$, $Q_{i\,(spec)} = Q_{i\,(min)}$, the P-V bus will act as P-Q bus.

5. Compute mismatch vector using

$$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$$

$$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$$

6. Compute $\Delta P_{i\,(max)} = \max |\Delta P_i|;$ $i = 1, 2, \ldots\ldots, N$ except slack

$\Delta Q_{i\,(max)} = \max |\Delta Q_i|,$ $i = M + 1 \ldots\ldots N$

7. Compute Jacobian matrix using $J = \begin{bmatrix} \frac{\partial P_i}{\partial \delta} & |V| \frac{\partial P_i}{\partial |V|} \\ \frac{\partial Q_i}{\partial \delta} & |V| \frac{\partial Q_i}{\partial |V|} \end{bmatrix}$

8. Obtain state correction vector $\begin{bmatrix} \Delta \delta \\ \frac{\Delta V}{|V|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$

9. Update state vector using

$$V^{new} = V^{old} + \Delta V = V^{old} + \frac{\Delta V}{|V^{old}|} = V^{old} \left[1 + \frac{\Delta V}{|V^{old}|} \right]$$

$$\delta^{new} = \delta^{old} + \Delta \delta$$

10. This procedure is continued until

$|\Delta P_i| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3.

Advantages of N-R Method

1. The N-R method is faster, more reliable and the results are accurate.
2. Requires less number of iterations for convergence.
3. The number of iterations are independent of the size of system (number of buses).
4. Suitable for large size systems.

Disadvantages of N-R Method

1. The programming logic is more complex than G.S. Method.
2. The memory requirement is more.
3. Number of calculations per iteration are higher than G.S. method.

Example 5.13 *Perform two iteration of Newton Raphson load flow method and determine the power flow solution for the given system. Take base MVA as 100.*

☺ ***Solution : Line Data :***

Line	Bus		R (p.u)	X (p.u)	Half line charging admittance $\left(\frac{Y_P}{2}\ \text{(p.u)}\right)$
	From	To			
1	1	2	0.0839	0.5183	0.0636

Bus Data :

Bus	P_L	Q_L
1	90	20
2	30	10

$$Y_{bus} = \begin{bmatrix} 0.3044 - j1.816 & -0.3044 + j1.88 \\ -0.3044 + j1.88 & 0.3044 - j1.816 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 1.842 \angle -1.405 & 1.904 \angle 1.7314 \\ 1.904 \angle 1.7314 & 1.842 \angle -1.405 \end{bmatrix}$$

[*Note :* Use in rad mode]

Assume the initial value *i.e.*, $\delta = 0$, $V = 1.0$.

$$[X] = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \end{bmatrix}$$

Step 3 : Calculate P_2^{cal}, Q_2^{cal}, ΔP_2 and ΔQ_2.

$$P_2^{cal} = |V_2| \{|V_1||Y_2|\cos(\theta_{12}+\delta_2-\delta_1)+|V_2||Y_{22}|\cos(\theta_{22}+\delta_2-\delta_2)\}$$

$$= 1.0\,[1.05 \times 1.904\cos(1.7314) + 1.842\cos(-1.405)]$$

$$= 1.05 \times 1.904(-0.15991) + 1.842\,(0.16503)$$

$$= -0.015 \text{ p.u}$$

$$P_{2\,(spec)} = P_{G2} - P_L$$

$$= 0 - \frac{30}{100} = -0.3 \text{ p.u}$$

$$\Delta P_2 = P_{2\,(spec)} - P_2^{cal}$$

$$= -0.3 - (-0.015) = -0.285$$

$$Q_2^{cal} = -V_2\{|V_1||Y_{21}|\sin(\theta_{12}+\delta_1-\delta_2)+|V_2||Y_{22}|\sin(\theta_{22}+\delta_2-\delta_2)\}$$

$$= -1.0\,[1.05 \times 1.904\sin(1.7314) + 1.0 \times 1.842\sin(-1.405)]$$

$$= -0.157 \text{ p.u}$$

$$\Delta Q_2 = Q_{2\,(spec)} - Q_2^{cal} = -0.1 - (-0.157)$$

$$= 0.057$$

Step 4 : Form Jacobian matrix.

$$\begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & V_2 \dfrac{\partial P_2}{\partial V_2} \\ \dfrac{\partial Q_2}{\partial \delta_2} & V_2 \dfrac{\partial Q_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \dfrac{\Delta |V_2|}{|V_2|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{12}| \sin(\theta_{12} + \delta_1 - \delta_2) + |V_2|^2 |Y_{22}| \times 0$$

$$= 1.0 \times 1.05 \times 1.904 \sin(1.7314)$$

$$= 1.973$$

$$|V_2| \frac{\partial P_2}{\partial V_2} = |V_1||V_2||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) + 2|V_2|^2 |Y_{22}| \cos(\theta_{22})$$

$$= 1.05 \times 1.904 \cos(1.7314) + 2 \times 1.842 \cos(-1.405)$$

$$= 0.289$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \cos(\theta_{12} + \delta_1 - \delta_2) - |V_2|^2 |Y_{22}| \times 0$$

$$= 1.05 \times 1.904 \times \cos(1.7314)$$

$$= -0.3197$$

$$|V_2| \frac{\partial Q_2}{\partial V_2} = -|V_1||V_2||Y_{21}| \sin(\theta_{12} + \delta_1 - \delta_2) - 2|V_2|^2 Y_{22}| \sin(\theta_{22})$$

$$= -1.05 \times 1.904 \sin(1.7314) - 2 \times 1.842 \sin(-1.405)$$

$$= 1.66$$

Step 5 : Compute Δx,

$$\begin{bmatrix} \Delta\delta_2 \\ \dfrac{\Delta V_2}{|V_2|} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & |V_2|\dfrac{\partial P_2}{\partial V_2} \\ \dfrac{\partial Q_2}{\partial \delta_2} & |V_2|\dfrac{\partial Q_2}{\partial V_2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.973 & 0.289 \\ -0.3196 & 1.66 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix}$$

$$= \begin{bmatrix} 0.493 & -0.086 \\ 0.0949 & 0.586 \end{bmatrix} \begin{bmatrix} -0.285 \\ 0.057 \end{bmatrix} = \begin{bmatrix} -0.145 \\ 0.0064 \end{bmatrix}$$

$$\delta_2^1 = \delta_2^{\circ} + \Delta\delta_2 = 0 + (-0.145) = -0.145$$

$$V_2^1 = V_2 + |V_2| \cdot \frac{\Delta V_2}{|V_2|} = 1.0 + 0.0064 = 1.0064$$

Iteration 2 : Compute mismatch vectors.

$$P_2^{cal} = 1.0064\,[1.05 \times 1.904 \cos(1.7314 + 0 + (-0.145)) + 1.0064 \times 1.842 \cos(-1.405)]$$

$$= -0.297$$

$$\Delta P_2 = P_{2\,(spec)} - P_2^{cal} = -0.3 - (-0.297) = -0.003$$

$$Q_2^{cal} = -\{1.0064\,[1.05 \times 1.904 \times \sin(1.7314 + 0 - (-0.145)) + 1.0064 \times 1.842 \times \sin(-1.405)]\}$$

$$= -0.078$$

$$\Delta Q_2 = Q_{2\,(spec)} - Q_2^{cal}$$

$$= -0.1 - (-0.078) = -0.021$$

Compute Jacobian matrix.

$$\frac{\partial P_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \sin(1.7314 + 0.145) = 1.919$$

$$|V_2| \frac{\partial P_2}{\partial V_2} = 1.05 \times 1.0064 \times 1.904 \cos(1.7314 + 0.145) + 2 \times 1.0064^2 \times 1.842 \times \cos(-1.405) = 0.015$$

$$\frac{\partial Q_2}{\partial \delta_2} = 1.0064 \times 1.05 \times 1.904 \times \cos(1.7314 + 0.145) = -0.605$$

$$|V_2| \frac{\partial Q_2}{\partial V_2} = -1.05 \times 1.904 \sin(1.7314 + 0.145) - 2 \times 1.0064 \times 1.842 \times \sin(-1.405) = 1.76$$

$$\begin{bmatrix} \Delta\delta_2 \\ \dfrac{\Delta V_2}{|V_2|} \end{bmatrix} = \begin{bmatrix} 1.919 & 0.015 \\ -0.605 & 1.76 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \frac{1}{3.3865} \begin{bmatrix} 1.76 & -0.015 \\ 0.605 & 1.919 \end{bmatrix} \begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5197 & -0.0044 \\ 0.1786 & 0.566 \end{bmatrix} \begin{bmatrix} -0.003 \\ -0.021 \end{bmatrix} = \begin{bmatrix} -0.0015 \\ -0.0124 \end{bmatrix}$$

$$\delta_2 = \delta_2^{old} + \Delta\delta_2$$

$$= -0.145 + (-0.0015) = -0.1465 \text{ rad}$$

$$V_2 = V_2^{old} + |V_2^{old}| \cdot \frac{\Delta V_2}{|V_2^{old}|}$$

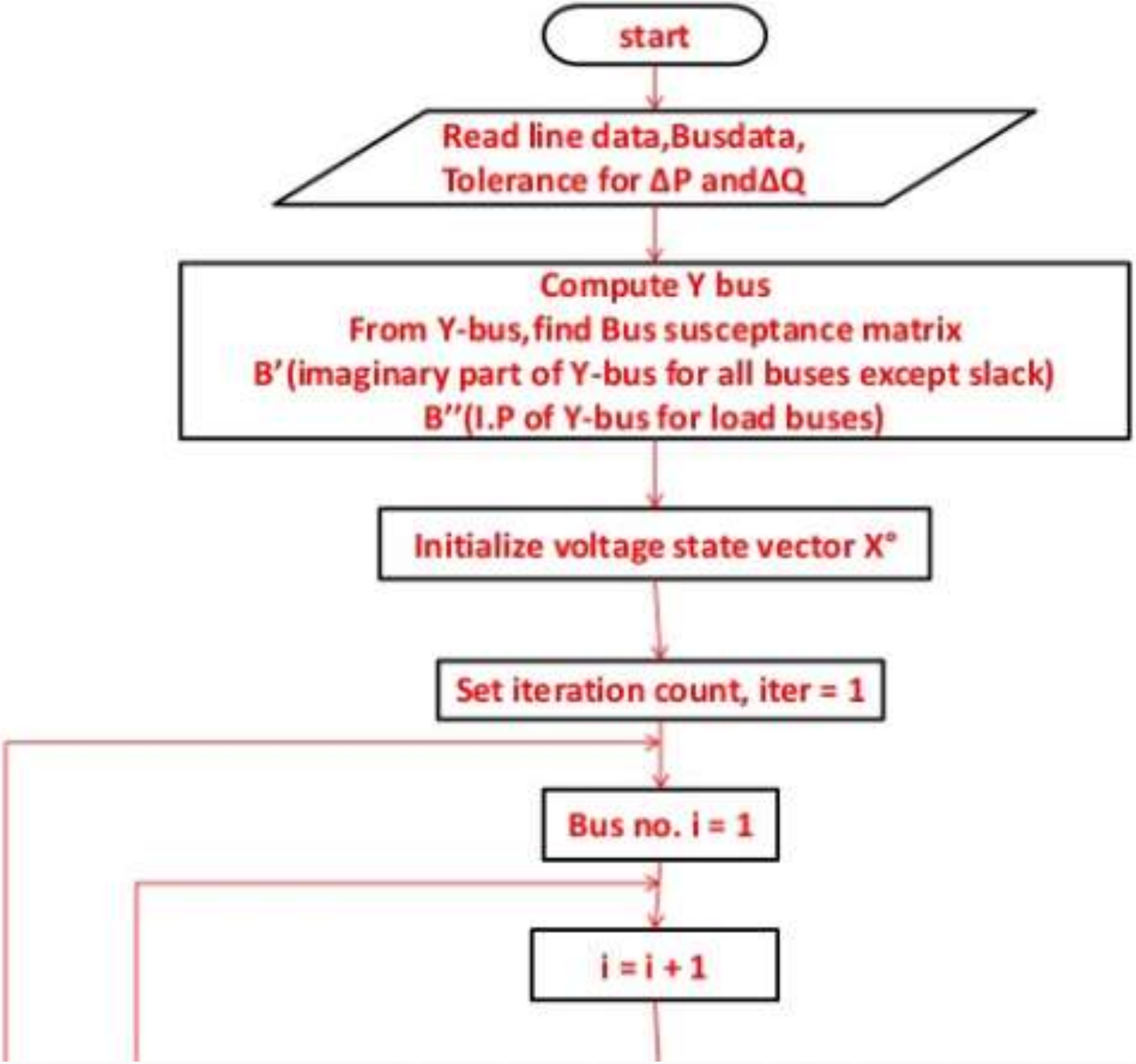
$$= 1.0064 + 1.0064 \times (-0.0124) = 0.994 \text{ p.u}$$

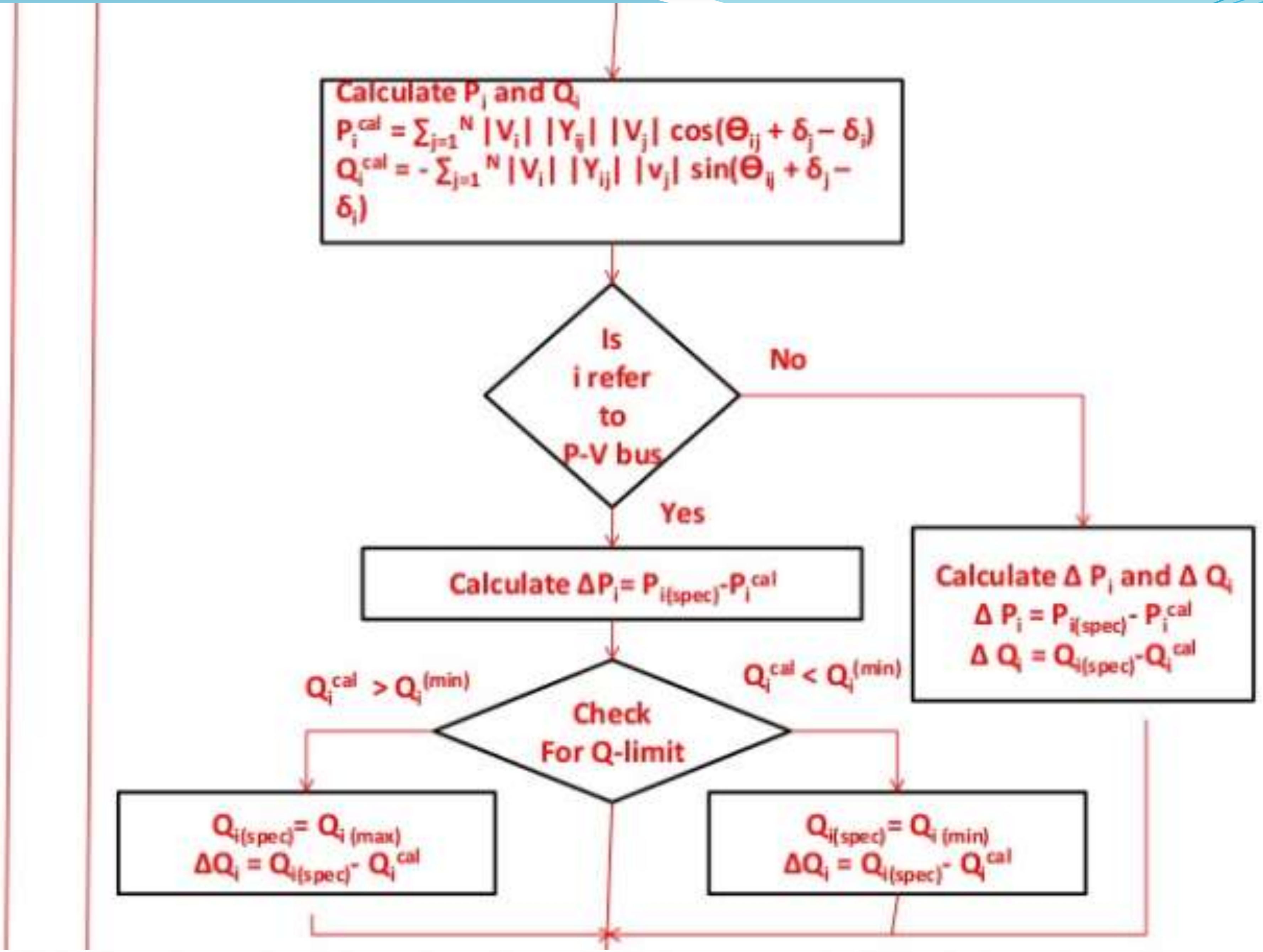
Fast Decoupled Method to Solve Power Flow Problem

The fast decoupled power flow method is a very fast and efficient method of obtaining power flow problem solution.

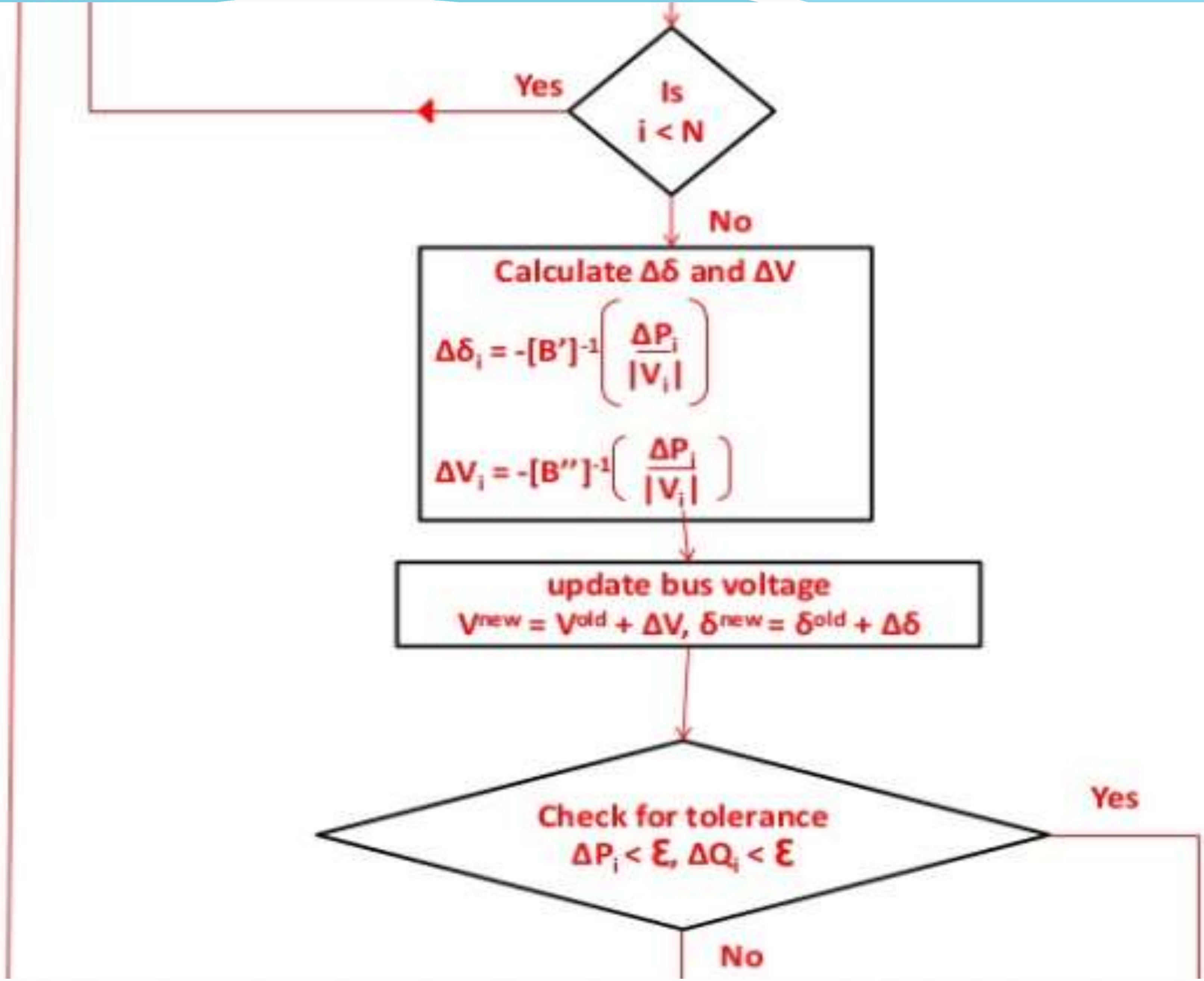
In this method, both, the speeds as well as the sparsity are exploited.

This is actually an extension of Newton-Raphson method formulated in polar coordinates with certain approximations which result into a fast algorithm for power flow solution.

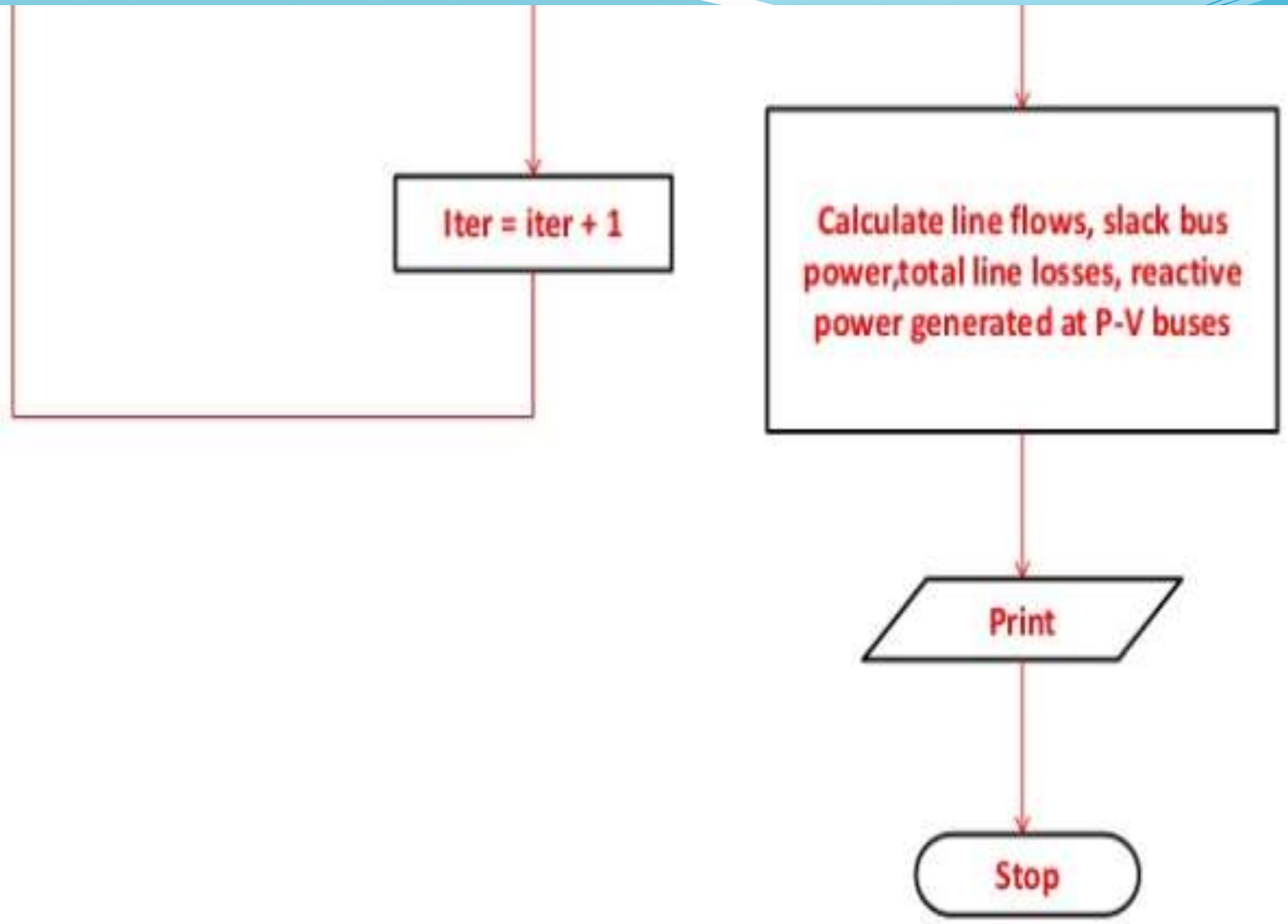

start
Read line data,Busdata,
Tolerance for ΔP andΔQ
Compute Y bus
From Y-bus,find Bus susceptance matrix
B'(imaginary part of Y-bus for all buses except slack)
B''(I.P of Y-bus for load buses)
Initialize voltage state vector X°
Set iteration count, iter = 1
Bus no. i = 1
i = i + 1



Calculate P_i and Q_i
$P_i^{cal} = \sum_{j=1}^{N} |V_i| |Y_{ij}| |V_j| \cos(\Theta_{ij} + \delta_j - \delta_i)$
$Q_i^{cal} = - \sum_{j=1}^{N} |V_i| |Y_{ij}| |v_j| \sin(\Theta_{ij} + \delta_j - \delta_i)$
Is i refer to P-V bus
No
Yes
Calculate $\Delta P_i = P_{i(spec)} - P_i^{cal}$
Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$
$Q_i^{cal} > Q_i^{(min)}$
$Q_i^{cal} < Q_i^{(min)}$
Check For Q-limit
$Q_{i(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$
$Q_{i(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i(spec)} - Q_i^{cal}$



Yes
Is
i < N
No
Calculate Δδ and ΔV
$\Delta\delta_i = -[B']^{-1}\left(\frac{\Delta P_i}{|V_i|}\right)$
$\Delta V_i = -[B'']^{-1}\left(\frac{\Delta P_i}{|V_i|}\right)$
update bus voltage
$V^{new} = V^{old} + \Delta V, \delta^{new} = \delta^{old} + \Delta\delta$
Yes
Check for tolerance
$\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$
No



Iter = iter + 1
Calculate line flows, slack bus power,total line losses, reactive power generated at P-V buses
Print
Stop

S.No	G.S	N.R	FDLF
1	Require large number of iterations to reach convergence	Require less number of iterations to reach convergence.	Require more number of iterations than N.R method
2	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3	It has linear convergence characteristics	It has quadratic convergence characteristics	....
4	The number of iterations required for convergence increases with size of the system	The number of iterations are independent of the size of the system	The number of iterations are does not dependent of the size of the system
5	Less memory requirements	More memory requirements.	Less memory requirements than N.R.method.


Thank You

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

POWER SYSTEM ANALYSIS

UNIT III

FAULT ANALYSIS – BALANCED FAULTS

Analy…

Fault analysis is an important part of power system analysis. Short circuit studies are performed to determine bus voltages and currents flowing in the lines during various types of faults. Faults are classified as follows:

(*i*) Symmetrical or balanced faults

(*ii*) Unsymmetrical or unbalanced faults

- L – G (Line to ground)
- L – L (Line to line)
- L – L – G (Double line to ground)

When the network is symmetrically faulted, the phase currents and phase voltages possess three phase symmetry.

The magnitude of fault current depends on the internal (Thevenin's) impedance of generator plus the fault impedance. The internal impedance of generator under short circuit condition is not constant. For a 3ϕ short circuit occurs on the unloaded generator, behaviour can be divided into three periods as shown in Fig.6.1.

APPROXIMATIONS

FAULTS

Series Fault

1. Open conductor fault.
2. Two open conductor fault.

[illegible]

... system, loads are spec... the load currents are unknown.

- Determine current contributed by the two generators.

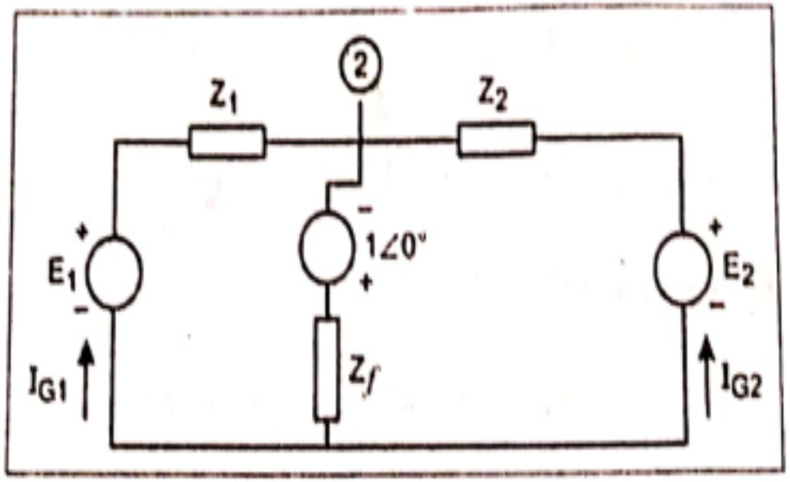


Fig. 6.13. Circuit for finding current contribution by generators

$$I_{G1} = I_f \times \frac{Z_2}{Z_1 + Z_2} \; ; \; I_{G2} = I_f \times \frac{Z_1}{Z_1 + Z_2} \quad \ldots (6.13)$$

[illegible]

... ...

$$\text{Prefault voltage} \approx 1 \angle 0^\circ$$

$$\therefore \text{SCC} = \frac{1}{X_{Th}} \text{ p.u. MVA}$$

$$\text{SCC} = \frac{1}{X_{Th}} \times MVA_b \text{ MVA} \qquad \ldots (6.17)$$

where MVA_b = Base MVA

The SCC have a tendency to grow as new generators are added and additional lines are built. The SCC is reduced by introducing artificial series reactors.

Solution :

Step 2 : D

sources should be short

generators :

[illegible]

[illegible] fault [illegible]


Thank You

UNIT -IV

Outlines

- Fault, classification and symmetrical fault.
- Percentage reactance and Short Circuit Current
- Per-Unit System and Base KVA
- Calculation of Short Circuit KVA
- Control techniques of Short Circuit Currents
- Location of Reactors in Power System
- Steps to symmetrical fault calculation
- Tutorial

Faults and Classification

- Symmetrical and unsymmetrical faults
- Symmetrical Fault: That fault on power system which gives rise to symmetrical currents (i.e. equal fault currents in the lines with 120 degree displacement) I called a symmetrical fault
- Why symmetrical fault is important??

- Though rarely occurs, but most severe and IMPOSE MORE HEAVY DUTY ON THE CIRCUIT BREAKER

- How to limit fault current: By impedance of the system

Percentage Reactance

- Percentage reactance (%X): It is the % of the total phase voltage dropped in the circuit when full load current is flowing

$$\%X = \frac{I\,X}{V} \times 100$$

Where I = full load current

V= phase voltage

X = reactance in ohms per phase

$$\boldsymbol{\%X = \frac{(kVA)X}{10(kV)^2}}$$

or $I_{SC} = \frac{V}{X} = I \times \left(\frac{100}{\%X}\right)$

Illustration on SC current

Illustration: If the % reactance of an element (transformer or generator or TL) is 20% and the full load current is 50 Amp. Calculate the short circuit current

Ans: Short circuit current= $I_{SC} = 50 \times \left(\frac{100}{20}\right) = 250\ Amp$

- % reactance of an equipment depends upon its kVA rating.
- It is necessary to find the % reactances of all the elements on a common kVA rating in power system because kVA rating of elements may different
- This common kVA rating is known as the base kVA which may be equal to that of the largest plant, or equal to the total plant capacity or any arbitrary value.

Base kVA and SC current

- **% reactance at base kVA**

$$= \left(\frac{Base\ kVA}{Rated\ kVA}\right) \times \%\ reactance\ at\ rated\ kVA$$

- **Base kVA does not affect the short circuit current calculation**

- **Short Circuit kVA:** The product of normal system voltage and short circuit current at the point of fault expressed in kVA is known as short circuit kVA

Short Circuit kVA for 3-phase circuit

$$= \frac{3\ VI_{SC}}{1000}$$

$$= \frac{3\ VI}{1000} \times \frac{100}{\%X}$$

SC Current $= \boldsymbol{Base\ kVA} \times \frac{\mathbf{100}}{\boldsymbol{\%X}}$

Steps for Symmetrical Fault Calculations

- Draw a single line diagram of the given network including rating, voltage and % reactance of each element
- Choose base kVA and convert all % reactance to this base value
- Draw the reactance diagram showing one phase of the system and the neutral . Indicate the %X on the base kVA in diagram.
- Find the total %X of the network up to the point of fault
- Find the full load current corresponding to the selected base kVA and the normal system voltage at the fault point
- Apply the formula to determine the Short Circuit Current

Tutorial

Q7.1. A circuit of a 3-phase system is given- see in Figure. The %X of each alternator is based on its own capacity. Find the short circuit current that will flow into a complete 3-phase short circuit at point F.

Answer: 4330 Amp

Tutorial

Q7.2. A 3-phase, 20 MVA, 10 kV alternator has internal reactance of 5% and negligible resistance. Find the external reactance per phase to be connected in series with the alternator so that steady current on short circuit does not exceed 8 times the full load current.

Answer: $X = 0.375\ \Omega$

Tutorial

Q7.3. A 3-phase transmission line operating at 10 kV and having a resistance of 1Ω and reactance of 4Ω is connected to the generating station bus bar through 5 MVA step-up transformer having a reactance of 5%. The bus bar supplied by a 10 MVA alternator having 10% reactance. Calculate the short circuit kVA fed to symmetrical fault between phases if it occurs

(i) At the load end of transmission line

(ii) (ii) at the high voltage terminals of the transformer

Answer: (i) Short circuit kVA = 16,440 kVA

(ii) Short circuit kVA = 50,000 kVA

Tutorial

Q7.4. The plant capacity of a 3-phase generating station consists of two 10000 kVA generators of reactance 12% each and one 5000 kVA generator of reactance 18%. The generators are connected to the station bus bar from which load is taken through three 5000 kVA step up transformer each having a reactance of 5%. Determine the maximum fault MVA which the circuit breakers on (i) low voltage side and (ii) high voltage side may have to deal with.

Answer: (i) Fault MVA = 194.5

(ii) Fault MVA = 66

Transient and Small Signal Stability Analysis

CHAPTER 11

Transient and Small Signal Stability Analysis

– Single Machine Infinite Bus System

11.1. INTRODUCTION

Power system stability is the property of the system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.

11.1.1. Distinction Between Steady State and Transient State

S.No.	Steady State	Transient State
1.	A power system is in steady state if all the measured physical quantities describing the operating conditions are constant for the analysis.	A power system is in transient state if the measured quantities are not constant.
2.	A system is said to be in steady state stable, when it follows small disturbance, and it returns to the same steady state condition.	A system is said to be transient state, when it follows large disturbance, and a significantly different but acceptable steady state operating condition is attained.
3.	It can be analyzed by using linear equations. The non-linear equations are replaced by linear equations.	It can be analyzed by using non-linear equations.
4.	***Examples of disturbance:*** Change in gain of the AVR (Automatic voltage regulator) in the excitation system of a large generating unit.	***Examples:*** Transmission system faults, sudden load changes, line switching, loss of generating units

11.2. DESCRIPTION OF POWER SYSTEM STABILITY PROBLEM

Stability problem is concerned with the behaviour of the synchronous machines after a disturbance.

Stability problem may be divided into steady-state stability and transient stability.

Steady-State Stability

It is the ability of the power system to bring it to a stable condition or remain in synchronism after a small disturbance such as gradual infinitesimal variations in system variables like rotor angle, voltage, *etc.*

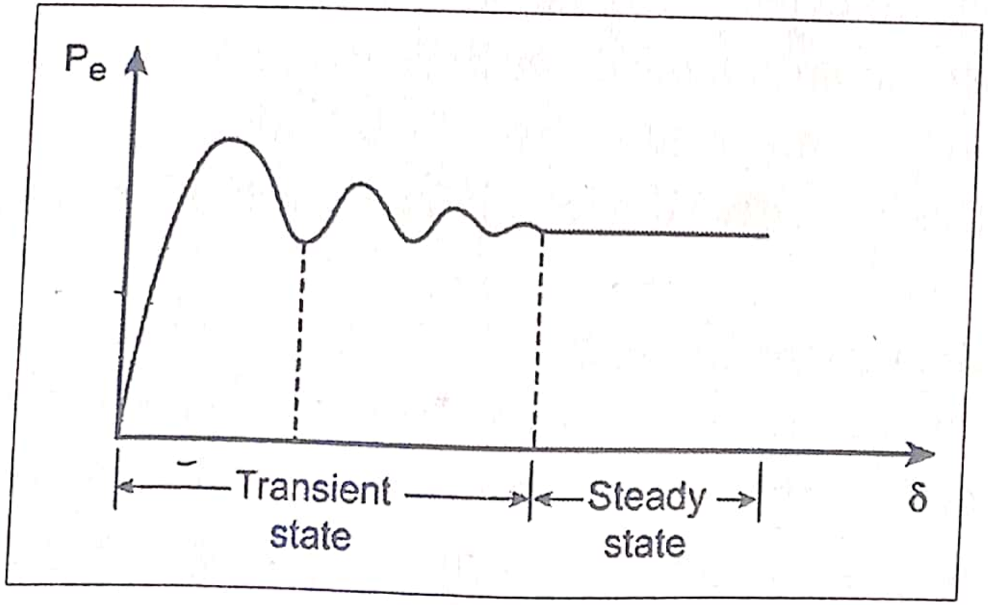


Fig. 11.1. Power Vs δ curve

Power Vs δ curve is as shown in Fig.11.1, which shows the transient and steady state.

Steady state stability is subdivided into static and dynamic stability.

STEADY STATE STABILITY

Static Stability	Dynamic Stability
It refers to inherent stability that prevails without the aid of automatic control devices.	It refers to inherently unstable system with automatic control devices.

Transient Stability

It is the ability of the system to bring it to a stable condition after a large disturbance. Large disturbances can occur due to the occurrence of fault, sudden outage of a line, sudden loss of excitation, sudden application or removal of loads, *etc.*

Transient stability studies are based on a first swing rather than multiswing. Here generator model is represented by a transient internal voltage E' behind transient reactance X_d'. In this, excitation, turbine-governor controls are not included.

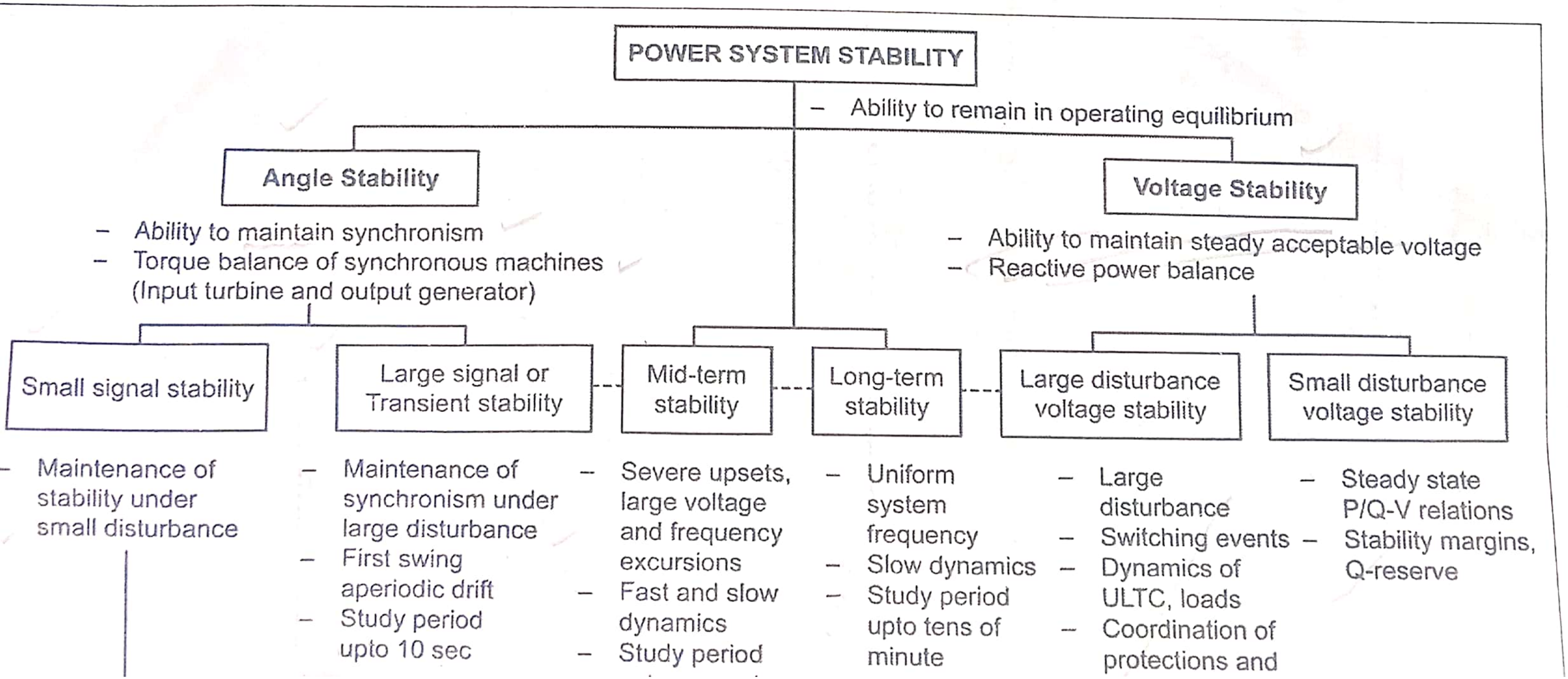
Multiswing stability studies extend over a longer study period. Here excitation and turbine-generator controls are included.

Dynamic Stability

It is the ability of the system to remain in synchronism when it is subjected to sustained oscillations followed by transient state.

11.3. IMPORTANCE OF STABILITY ANALYSIS IN POWER SYSTEM PLANNING AND OPERATION

- Transient stability studies deal with the effects of large, sudden disturbances such as the occurrence of a fault, sudden outage of a line or the sudden application or removal of loads.
- Transient stability studies give the information that the system can withstand the transient conditions like high magnitude of voltage and frequency.
- It deals with the stability of the system.
- Transient stability studies are needed when the new generating station and transmission facilities are planned.
- Stability studies are useful in determining the nature of relaying system needed, critical clearing time of circuit breakers *i.e.*, design of protection equipments.
- Stability studies are more helpful in determining power transfer capability between two different systems.


POWER SYSTEM STABILITY
– Ability to remain in operating equilibrium
Angle Stability
– Ability to maintain synchronism
– Torque balance of synchronous machines (Input turbine and output generator)
Voltage Stability
– Ability to maintain steady acceptable voltage
– Reactive power balance
Small signal stability
– Maintenance of stability under small disturbance
Large signal or Transient stability
– Maintenance of synchronism under large disturbance
– First swing aperiodic drift
– Study period upto 10 sec
Mid-term stability
– Severe upsets, large voltage and frequency excursions
– Fast and slow dynamics
– Study period
Long-term stability
– Uniform system frequency
– Slow dynamics
– Study period upto tens of minute
Large disturbance voltage stability
– Large disturbance
– Switching events
– Dynamics of ULTC, loads
– Coordination of protections and
Small disturbance voltage stability
– Steady state P/Q-V relations
– Stability margins, Q-reserve

problems in order to develop appropriate design and operating procedures.

11.5. SINGLE MACHINE INFINITE BUS (SMIB) SYSTEM

11.5.1. ROTOR DYNAMICS AND SWING EQUATION

Power or Torque Angle

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field are fixed. The angle between the two is known as the power angle or torque angle δ.

Swing Equation/Uses of Swing Equation

During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air-gap mmf, and a relative motion begins. The equation used to describe the 'behaviour of the synchronous machine during transient period' is known as the ***swing equation.***

After oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance is created by a change in generation, load or in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field.

Assumptions in Stability Studies

The assumptions in stability studies are :

1. Machine represented by classical model.
2. Controllers are not considered.
3. Loads are constants.
4. Voltage and currents are sinusoidal.

Consider a synchronous generator developing an electromagnetic torque T_e and running at the synchronous speed ω_{sm}.

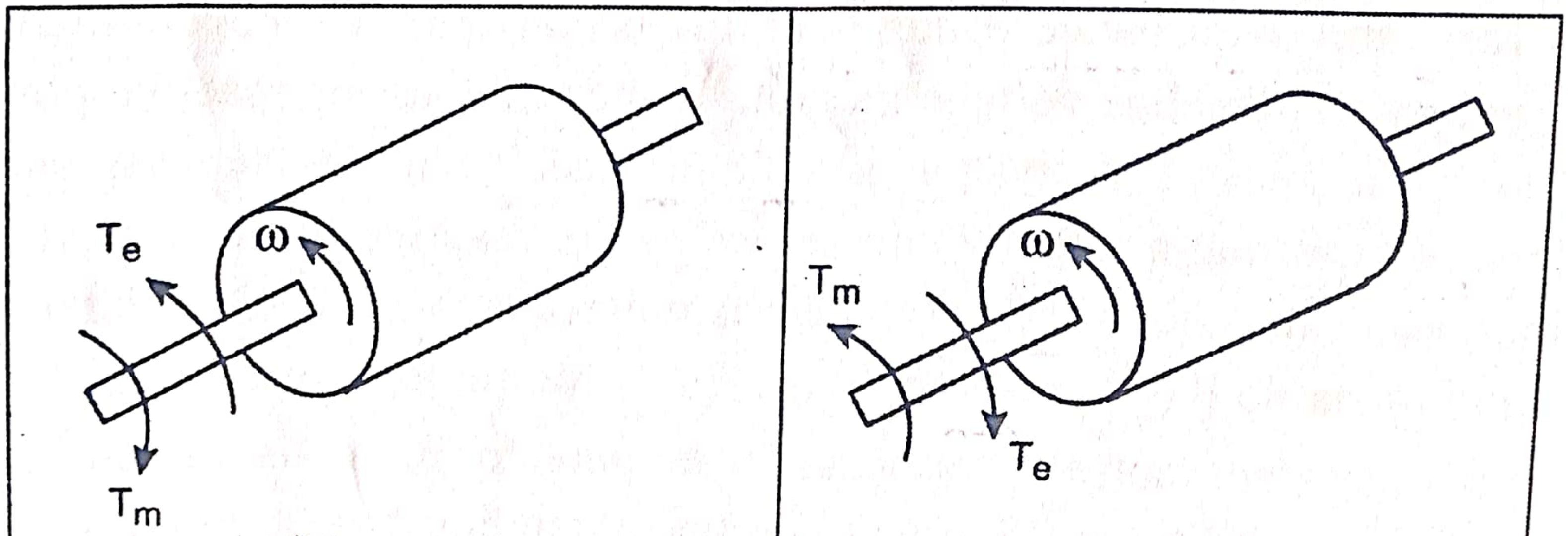
Let T_m be the driving mechanical torque.

T_e be the electrical torque.

The motor action and generator action is shown in Fig.11.5.

For generator action, T_m and T_e are positive.

θ_m is positive.



(a) Motor action *(b) Generator action*

Fig. 11.5.

Under steady-state with losses neglected.

$$T_m = T_e$$

$$\therefore \text{ Accelerating torque } T_a = T_m - T_e = 0$$

i.e., No acceleration or deceleration of rotor. Due to disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque on the rotor.

$$\text{Accelerating torque } T_a = T_m - T_e$$

Let J be the moment of inertia of the prime mover and generator.

From Law's of rotation,

$$\text{Acceleration } \alpha = \frac{d^2\theta_m}{dt^2}$$

$$\text{Accelerating torque } T_a = J\alpha$$

$$\therefore J\frac{d^2\theta_m}{dt^2} = T_m - T_e \quad \dots (11.1)$$

where θ_m is the angular displacement of the rotor with respect to the stationary reference axis on stator.

θ_m increases with time even at constant synchronous speed.

$$\therefore \theta_m = \omega_{sm} t + \delta_m \quad \dots (11.2)$$

where δ_m = Angular displacement of the rotor before disturbance in mechanical radians.

ω_{sm} = Constant angular velocity

Differentiating equation with (11.2) with respect to t, we get,

$$\text{Rotor angular velocity } \omega_m = \frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \quad \dots (11.3)$$

Differentiating equation (11.3) with respect to t, rotor acceleration is

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

Substituting in equation (11.1), we get,

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Multiplying by ω_m on both sides,

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e$$

Inertia Constant

M-constant or inertia constant is defined as the angular momentum at synchronous speed. If energy is measured in Joules and speed in mechanical radians per second. Unit of M is Joule-sec/Mechanical radian.

$$M = J\,\omega_m \text{ is the inertia constant.}$$

i.e., Angular momentum of the rotor at synchronous speed.

$$M \frac{d^2\delta_m}{dt^2} = P_m - P_e \qquad (P = \omega T) \qquad \ldots (11.4)$$

where P_m, P_e are mechanical and electrical powers.

This is the swing equation in terms of inertia constant

P.u. Inertia Constant

Kinetic Energy of the Rotating Masses $\quad W_K = \frac{1}{2} J\,\omega_m^2$

For stability studies, "Per unit inertia constant H" is defined as :

$$\text{P.u. Inertia constant, } H = \frac{\text{Stored kinetic energy in mega Joules of turbine, alternator and exciter rotor at synchronous speed}}{\text{Machine rating in MVA}}$$

$$H = \frac{\frac{1}{2} J\,\omega_{sm}^2}{S_B} \text{ sec}$$

$$J\,\omega_{sm} = \frac{2\,H\,S_B}{\omega_{sm}} = M$$

Substituting in equation (11.4),

$$\frac{2\,H\,S_B}{\omega_{sm}}\,\frac{d^2\delta_m}{dt^2} = P_m - P_e \qquad \dots (11.5)$$

Both sides of the above equation are dimensionless quantities. They are valid on the electrical side if the angle and the synchronous speed are expressed in electrical radians and electrical radians per second.

With angle and speed on electrical side,

$$\frac{2\,H\,S_B}{\frac{2}{P} \times \omega_{se}}\,\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

$$\frac{2\,H\,S_B}{\frac{2}{P} \times 2\pi f} \times \left(\frac{2}{P}\right)\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{H\,S_B}{\pi f}\,\frac{d^2\delta}{dt^2} = P_m - P_e \qquad \dots (11.6)$$

$$\omega_{se} = 2\pi f, \quad \omega_{sm} = \frac{2\pi N_S}{60}$$

$$= \frac{2\pi \times 120\,F}{60 \times P}$$

$$\omega_{sm} = \frac{2\,\omega_{se}}{P}$$

$$\int \omega_{sm} = \delta_m = \frac{2\delta}{P}$$

Dividing by MVA rating S_B on both sides of the equation (11.6),

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B}$$

$$\frac{P_m}{S_B} = \text{p.u mechanical power.}$$

$$\frac{P_e}{S_B} = \text{p.u electrical power.}$$

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{m\,(p.u)} - P_{e\,(p.u)} = P_{m\,(p.u)} - P_{max}\sin\delta \;.$$

$$M_{(p.u)} \cdot \frac{d^2\delta}{dt^2} = P_{m\,(p.u)} - P_{e\,(p.u)}$$

where $M_{(p.u)} = \frac{H}{\pi f}$, δ in radians

If δ is expressed in electrical degrees,

$$\frac{H}{180 f}\frac{d^2\delta}{dt^2} = P_{m\,(p.u)} - P_{e\,(p.u)}$$

These equations are called as ***Swing Equation.***

Swing Curve

We can write the equation (11.5) as two first order equations.

$$\frac{2H}{\omega_{sm}}\frac{d\Delta\omega}{dt} = P_{m\,(p.u)} - P_{e\,(p.u)}$$

11.5.2. SYNCHRONOUS MACHINE REPRESENTATION BY CLASSICAL MODEL

Consider a generator connected to an infinite bus through a double transmission line as shown in Fig.11.6.

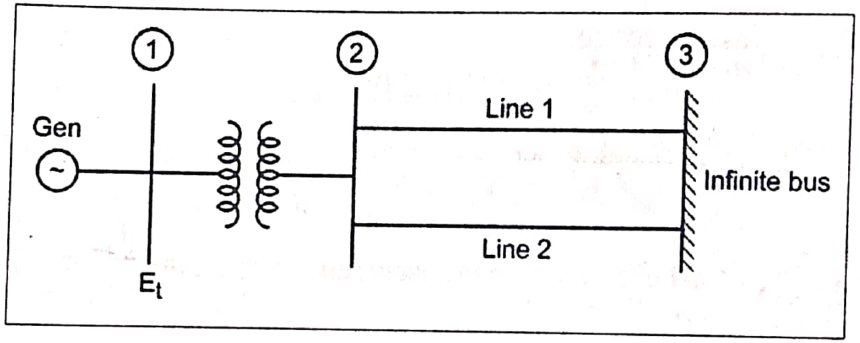


Fig. 11.6. Single machine connected to infinite bus system

Infinite Bus

The substation bus voltage and frequency is assumed to remain constant. This is called as infinite bus, since its characteristics do not change regardless of the power supplied or consumed by any device connected to it.

The generator model is as shown in Fig.11.7(*a*) and the equivalent circuit of Fig.11.6 is represented by classical model and all resistances neglected is as shown in Fig.11.7(*b*).

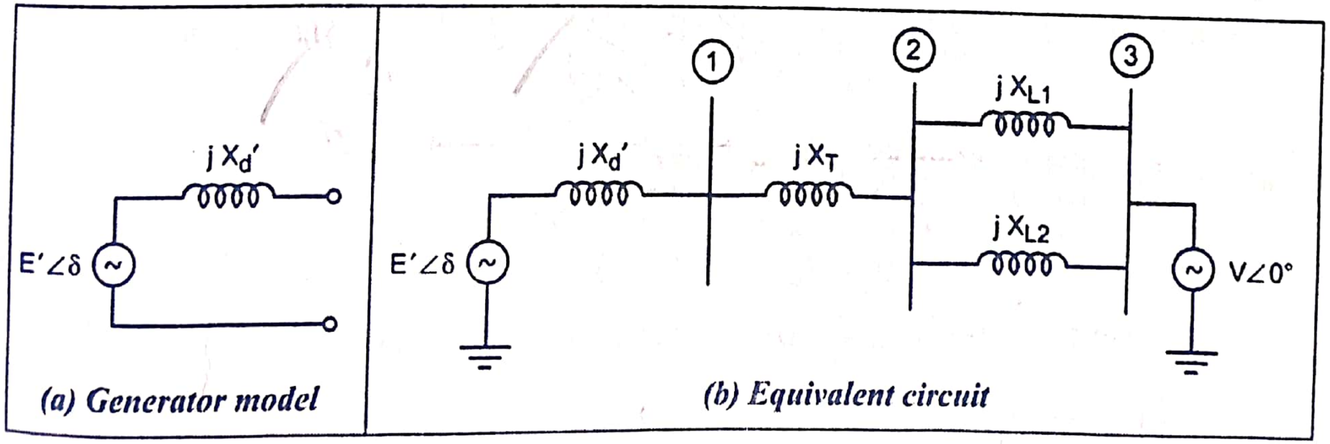


Fig. 11.7.

The simplified equivalent circuit is as shown in Fig.11.8.

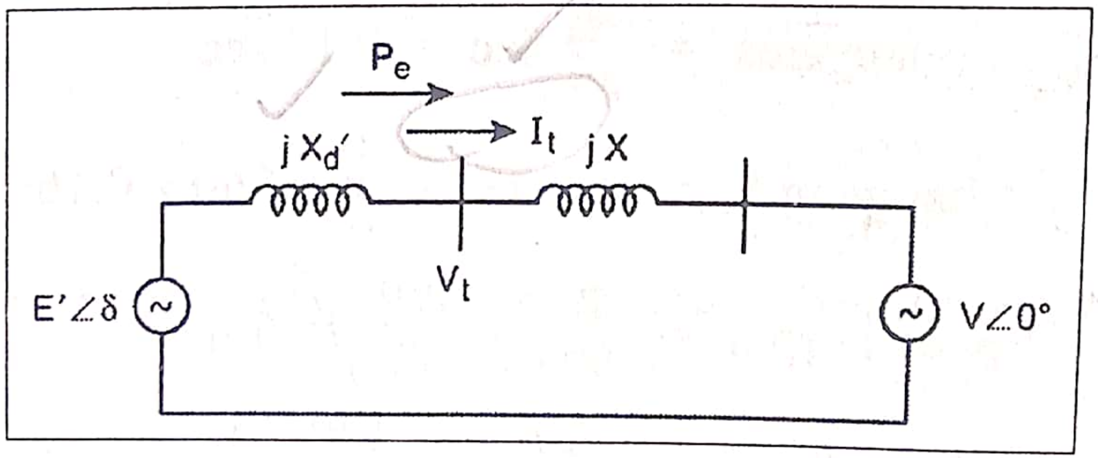


Fig. 11.8. Simplified equivalent circuit

$$\text{Now } X = \frac{X_{L1} \times X_{L2}}{X_{L1} + X_{L2}} + X_T$$

i.e., parallel combination of lines are connected in series with the transformer.

Let E_t' be the terminal voltage magnitude.

Let P_e, Q_e be the real and reactive power output of generator.

Case (i) **:** Assume V_t as reference.

$$i.e., \quad V_t = |V_t| \angle 0°$$

Voltage behind transient reactance E′.

$$E' = V_t + j X_d' I_t \quad \dots (11.8)$$

$$\text{where } I_t = \text{Stator current} = \frac{S^*}{V_t^*} = \frac{P_e - j Q_e}{|V_t| \angle 0°}$$

$$= \frac{P_e}{|V_t|} - j \frac{Q_e}{|V_t|} = I_{Re} - j I_{Im} \quad \dots (11.9)$$

Substituting equation (11.9) in (11.8), we get,

$$\therefore E' = V_t + j X_d' [I_{Re} - j I_{Im}] = |E'| \angle \beta \quad \dots (11.10)$$

Voltage of the infinite bus, $V = V_t - j X I_t$

$$= V_t - j X (I_{Re} - j I_{Im}) = |V| \angle \gamma \quad \dots (11.11)$$

Angle between E′ and V, $\delta = \beta - \gamma$ ... (11.12)

Voltage at bus (2) or voltage at high voltage side of transformer.

$$E_{HV} = V_t - j X_T \times I_t$$

Case (ii) **:** Assume infinite bus voltage V as reference.

$$V = |V| \angle 0°$$

$$E' = V + j X I_t; \quad E' = |E'| \angle \delta$$

where δ = Rotor angle with respect to synchronously rotating reference phasor $V \angle 0°$, E′ leads V by δ.

$$\text{Real power transfer } P_e = \frac{|E'||V|}{X} \sin \delta \quad \dots (11.13)$$

$$= P_{max} \sin \delta$$

11.6. POWER ANGLE EQUATION

The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation.

Power Angle Curve

All the elements are susceptance, then $G_{11} = 0$.

$$\therefore P_e = \frac{|E'||V|}{X_{12}} \sin \delta = P_{max} \sin \delta \quad \dots (11.18)$$

Power transmitted depends on the transfer reactance X_{12} and the angle between the voltages E′ and V *i.e.*, (δ). The curve P_e versus δ is known as power angle curve. The power angle curve is as shown in Fig.11.12.

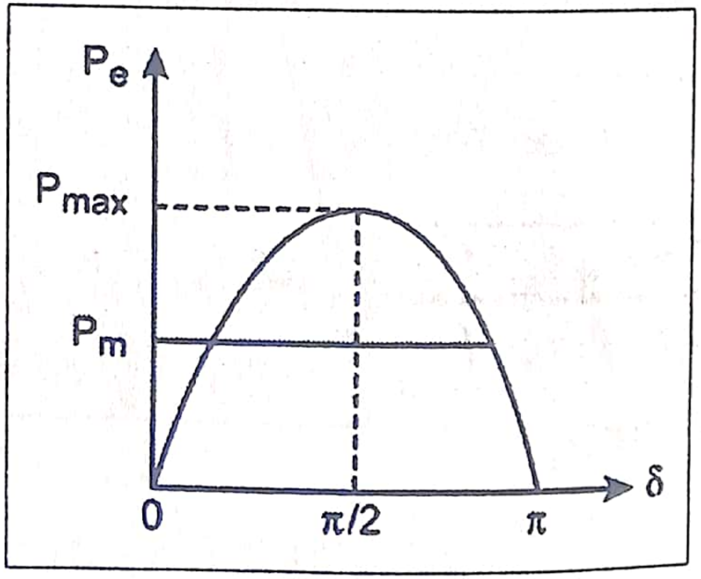
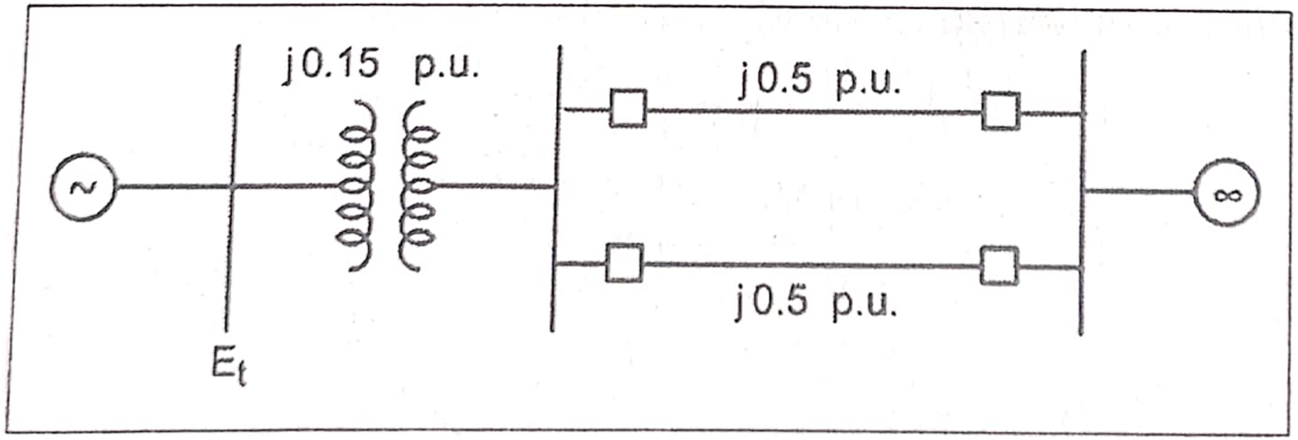
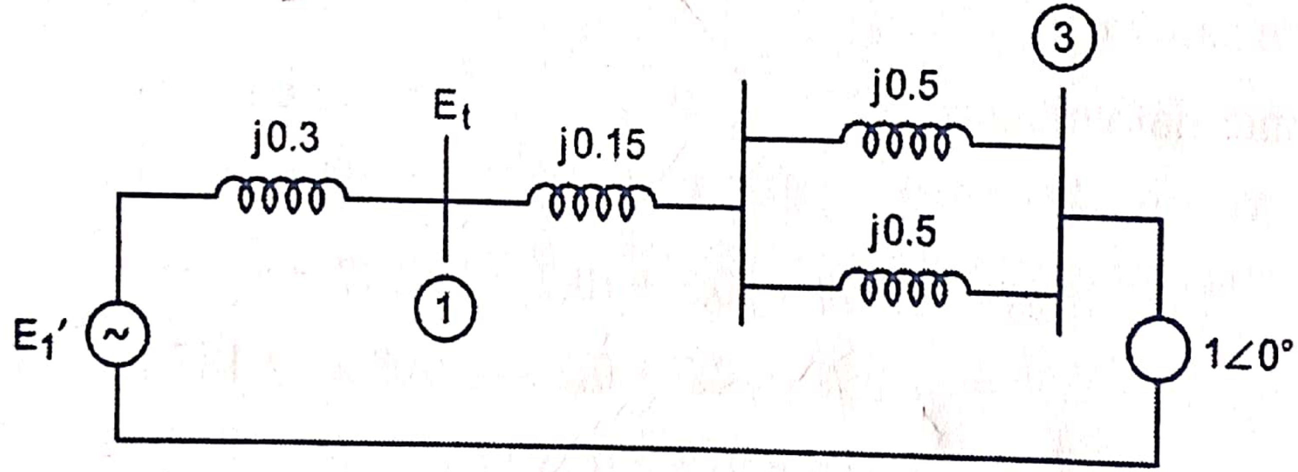


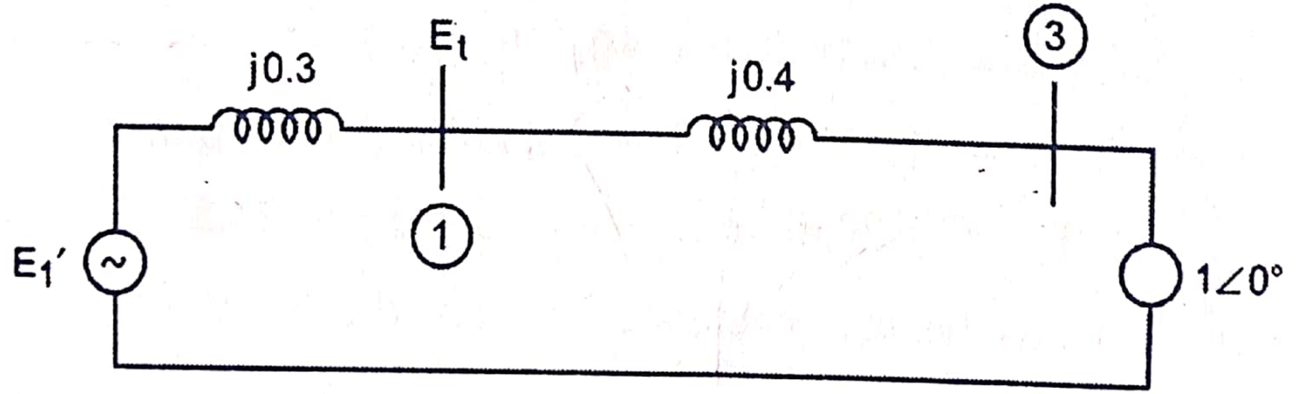
Fig. 11.12. Power angle curve

Example 11.5 The single line diagram of Fig. shows a generator connected through parallel transmission lines to a large metropolitan system considered as an infinite bus. The machine is delivering 0.9 p.u power, 0.8 p.f lagging, $E_t = 1.0$ p.u. The transient reactance of the generator is 0.3 p.u. Determine (a) the power angle equation for the given system. (b) Draw power angle curve, (c) Swing equation.



☺ **Solution :** $X'_d = 0.3$ p.u ; $P_e = 0.9$ p.u ; $E_t = 1.0$.





$$j0.15 + \frac{j0.5 \times j0.5}{j0.5 + j0.5} = j0.4$$

Power output of the Generator or Power transfer from terminal E_t.

$$P_e = \frac{|E_t| \cdot |V|}{X_{12}} \sin\theta$$

$$0.9 = \frac{1.0 \times 1.0}{0.4} \sin\theta$$

$$\sin\theta = 0.36$$

$$\theta = \sin^{-1} 0.36 = 21.1°$$

Terminal voltage $E_t = |E_t| \angle \theta$

$$= 1.0 \angle 21.1° = 0.933 + j0.36 \text{ p.u.}$$

Output current from the generator,

$$I = \frac{E_t - V}{z_{12}} = \frac{1.0 \angle 21.1° - 1.0 \angle 0°}{j0.4}$$

$$= 0.8999 + j0.1676 = 0.915 \angle 10.55° \text{ p.u.}$$

Transient internal voltage $E_1' = E_t + X_d' I$

$$= 1.0 \angle 21.1° + j0.3 \times 0.915\ 10.55°$$

$$= 0.883 + j0.63 = 1.084 \angle 35.52° \text{ p.u.}$$

$$= |E_1'| \angle \delta$$

Power angle equation is

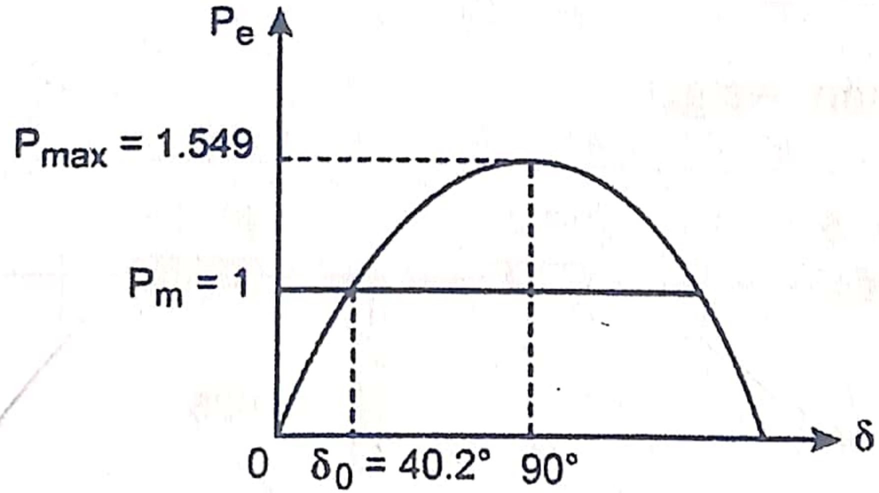
$$P_e = \frac{|E_1'| \times |V|}{X_{Total}} \sin\delta$$

$$= \frac{1.084 \times 1.0}{0.3 + 0.4} \times \sin\delta°$$

$$P_e = 1.549 \sin\delta$$

where δ is the machine rotor angle with respect to the infinite bus.

Power angle curve :



δ	P_e
0	0
90	1.549
40.2	1.0

Fig. Power angle curve

where δ_0 = Initial angular position of the generator rotor corresponding to the given operating conditions.

Swing equation is

$$\frac{H}{\pi f} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = 1.0 - 1.549 \sin \delta \quad \text{p.u.}$$