MUTHAYAMMAL ENGINEERING COLLEGE

RASIPURAM-637408

COURSE CODE & TITLE – 19ECC10/Digital Communication Systems

Contents

Introduction, sampling process.

Quantization, quantization noise, conditions for optimality of quantizer, encoding.

Pulse-Code Modulation (PCM),Line codes, Differential encoding, Regeneration, Decoding & Filtering.

Noise considerations in PCM systems.

Time-Division Multiplexing (TDM), Synchronization.

Delta modulation (DM).

Differential PCM (DPCM), Processing gain Adaptive DPCM (ADPCM)

Comparison of the above systems.

Introduction

- **Source: analog or digital**
- **Transmitter: transducer, amplifier, modulator, oscillator, power amp., antenna**
- **Channel: e.g. cable, optical fibre, free space**
- **Receiver: antenna, amplifier, oscillator, demodulator, power amplifier, transducer**
- **Recipient: e.g. person, (loud) speaker, computer**

➢ Types of information:

Voice, data, video, music, email etc.

➢ Types of communication systems:

Public Switched Telephone Network
(voice,fax,modem)
Satellite systems
Radio,TV broadcasting Cellular phones
Computer networks (LANs, WANs, WLANs)

Information Representation

- **Communication system converts information into electrical electromagnetic/optical signals appropriate for the transmission medium.**
- **Analog systems convert analog message into signals that can propagate through the channel.**
- **Digital systems convert bits(digits, symbols) into signals**

- Computers naturally generate information as characters/bits
- Most information can be converted into bits
- Analog signals converted to bits by sampling and quantizing (A/D conversion)

WHY DIGITAL?

- **Digital techniques need to distinguish between discrete symbols allowing regeneration versus amplification**

- **Good processing techniques are available for digital signals, such as medium.**
  - Data compression (or source coding)
  - Error Correction (or channel coding)(A/D conversion)
  - Equalization
  - Security

- **Easy to mix signals and data using digital techniques**

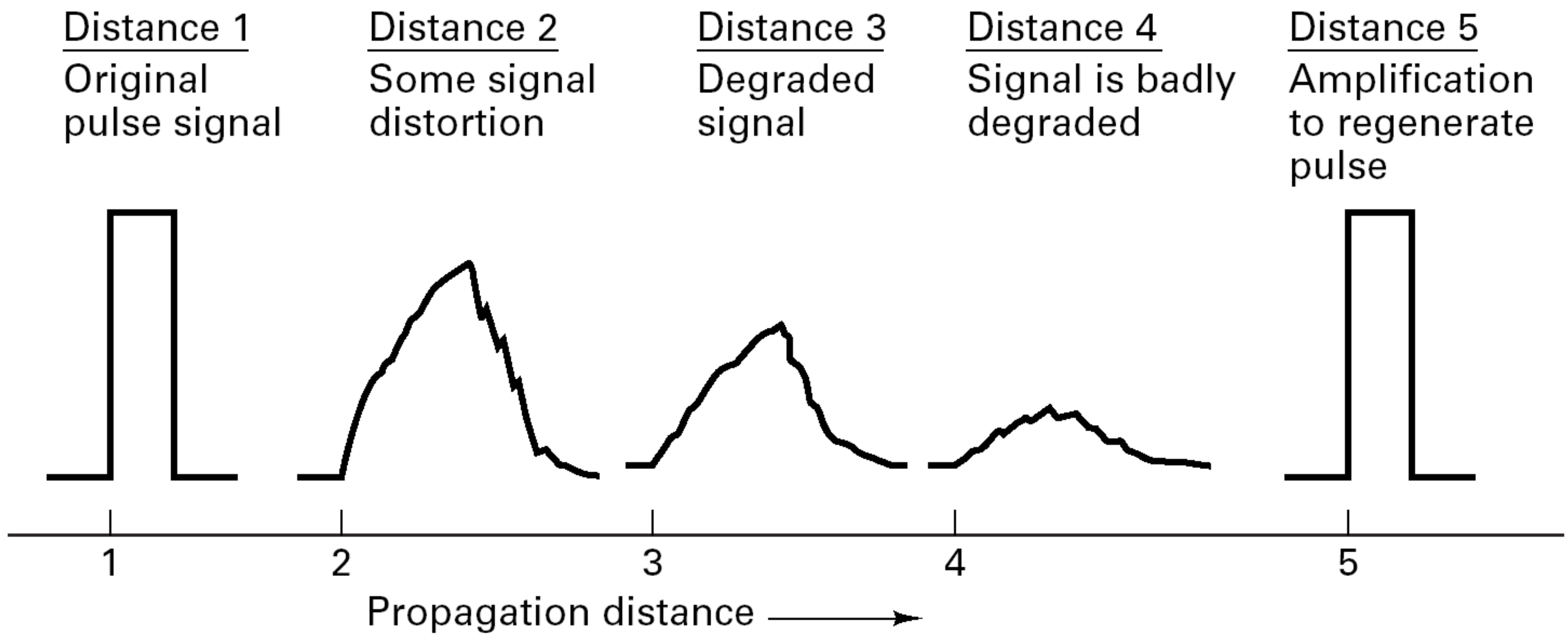
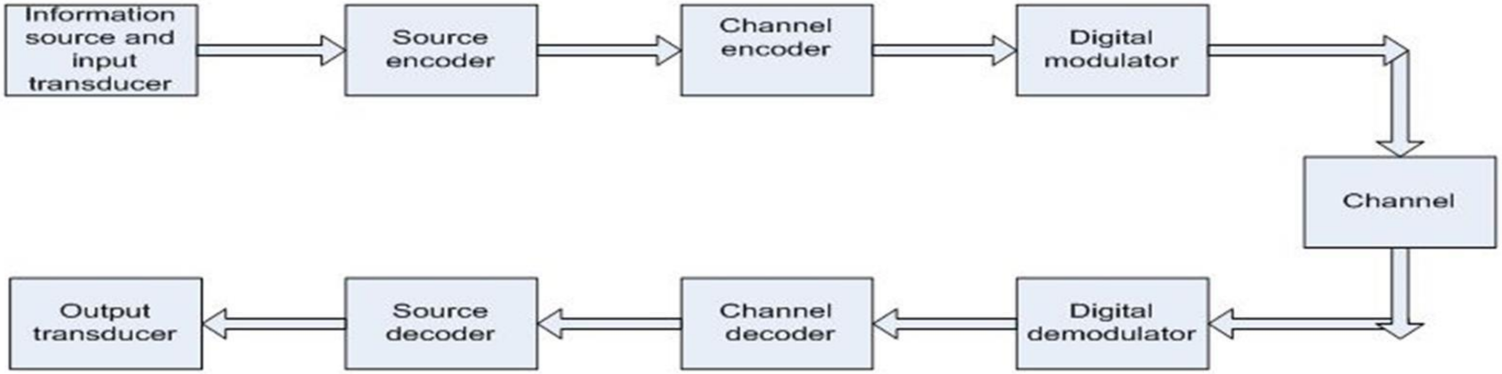


Figure 1.1 Pulse degradation and regeneration.

Elements of a Digital Communication System

- Block diagram of a communication system:



Information Source and Sinks

Information Source and Input Transducer:

- The source of information can be analog or digital,
  - Analog: audio or video signal,
  - Digital: like teletype signal.
- In digital communication the signal produced by this source is converted into digital signal consists of 1′s and 0′s.

Output Transducer:

The signal in desired format analog or digital at the output

Channel

- The communication channel is the physical medium that is used for transmitting signals from transmitter to receiver
  - Wireless channels: Wireless Systems
  - Wired Channels: Telephony
- Channel discrimination on the basis of their property and characteristics, like AWGN channel etc.

Source Encoder And Decoder

Source Encoder

- In digital communication we convert the signal from source into digital signal.
- ***Source Encoding or Data Compression:*** the process of efficiently converting the output of wither analog or digital source into a sequence of binary digits is known as source encoding.

Source Decoder

- At the end, if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm.

Channel Encoder And Decoder

Channel Encoder:

- The information sequence is passed through the channel encoder. The purpose of the channel encoder is to introduce, in controlled manner, some redundancy in the binary information sequence that can be used at the receiver to overcome the effects of noise and interference encountered in the transmission on the signal through the channel.

Channel Decoder:

- Channel decoder attempts to reconstruct the original information sequence from the knowledge of the code used by the channel encoder and the redundancy contained in the received data

Digital Modulator And Demodulator

Digital Modulator:

- The binary sequence is passed to digital modulator which in turns convert the sequence into electric signals so that we can transmit them on channel. The digital modulator maps the binary sequences into signal wave forms.

Digital Demodulator:

- The digital demodulator processes the channel corrupted transmitted waveform and reduces the waveform to the sequence of numbers that represents estimates of the transmitted data symbols.

Why Digital Communications?

- Easy to regenerate the distorted signal
- Regenerative repeaters along the transmission path can detect a digital signal and retransmit a new, clean (noise free) signal
- These repeaters prevent accumulation of noise along the path

This is not possible with analog communication systems

- Two-state signal representation

The input to a digital system is in the form of a sequence of bits (binary or M-ary)

- Immunity to distortion and interference
- Digital communication is rugged in the sense that it is more immune to channel noise and distortion

- Hardware is more flexible
- Digital hardware implementation is flexible and permits the use of microprocessors, mini-processors, digital switching and VLSI

Shorter design and production cycle

- Low cost

The use of LSI and VLSI in the design of components and systems have resulted in lower cost

- Easier and more efficient to multiplex several digital signals

Digital multiplexing techniques – Time & Code Division Multiple Access - are easier to implement than analog techniques such as Frequency Division Multiple Access

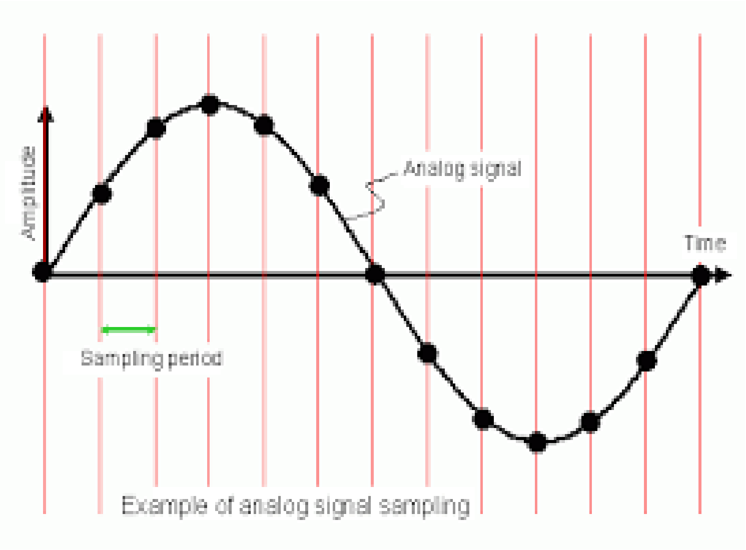
- Can combine different signal types – data, voice, text, etc.
- Data communication in computers is digital in nature whereas voice communication between people is analog in nature
- **Using digital techniques, it is possible to combine both format for transmission through a common medium**
- **Encryption and privacy techniques are easier to implement**
- Better overall performance
- Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth. Digital signals can be coded to yield extremely low rates and high fidelity as well as privacy.

Disadvantages:

- **Requires reliable “synchronization”**
- **Requires A/D conversions at high rate**
- **Requires larger bandwidth**
- **Nongraceful degradation**
- **Performance Criteria**
- **Probability of error or Bit Error Rate**

Sampling Process

Sampling is converting a *continuous time* signal into a *discrete time* signal.

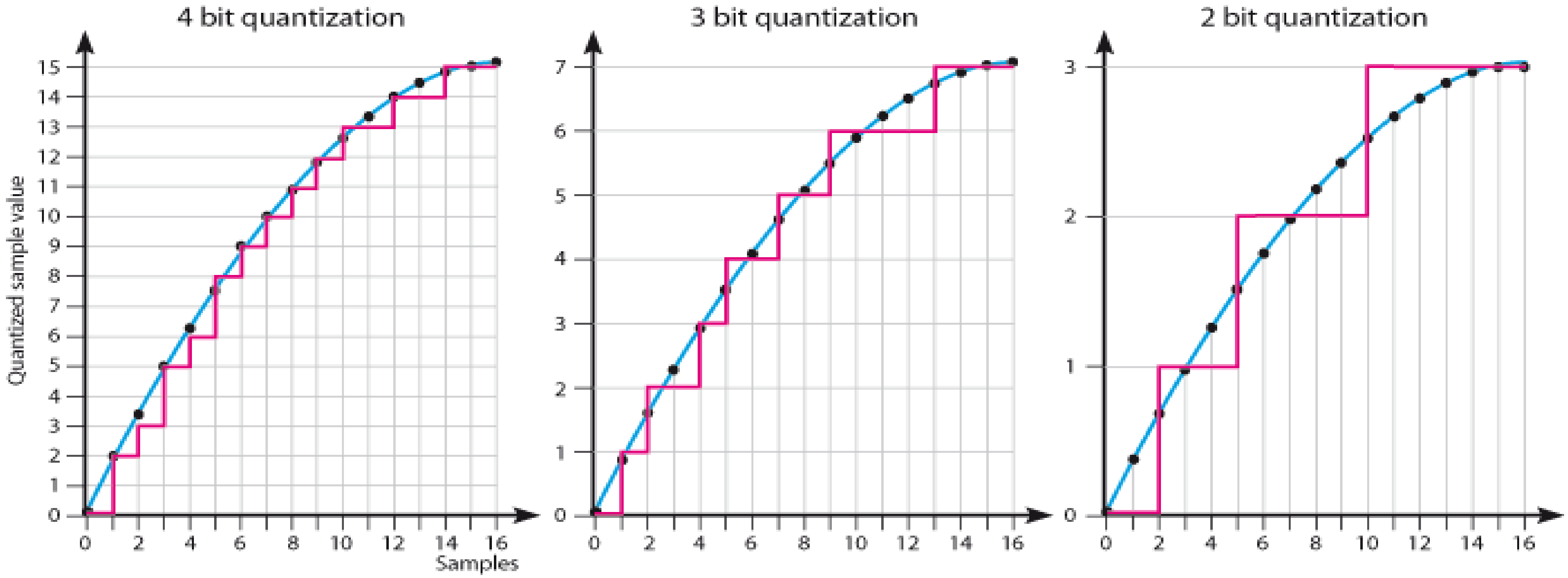


There three types of sampling

- Impulse (ideal) sampling
- Natural Sampling
- Sample and Hold operation

Quantization

Quantization is a non linear transformation which maps elements from a continuous set to a finite set.

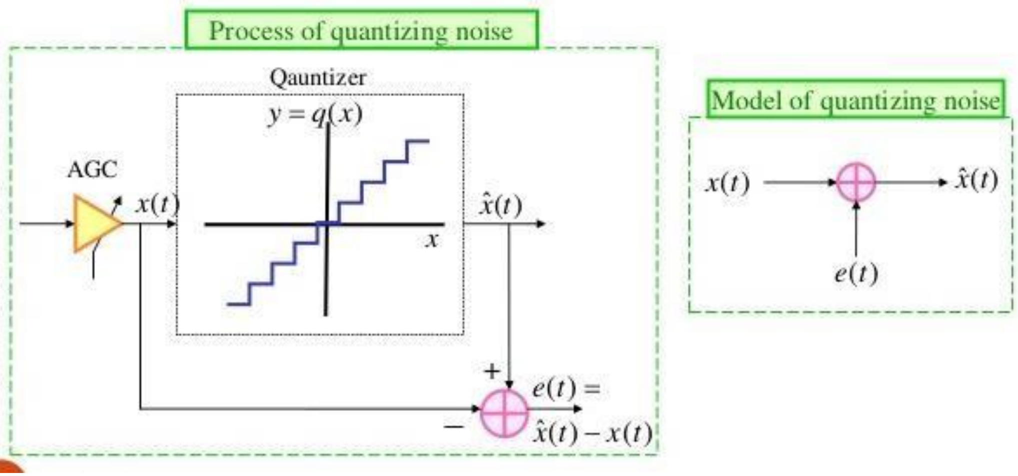


Quantization Noise

Quantization error

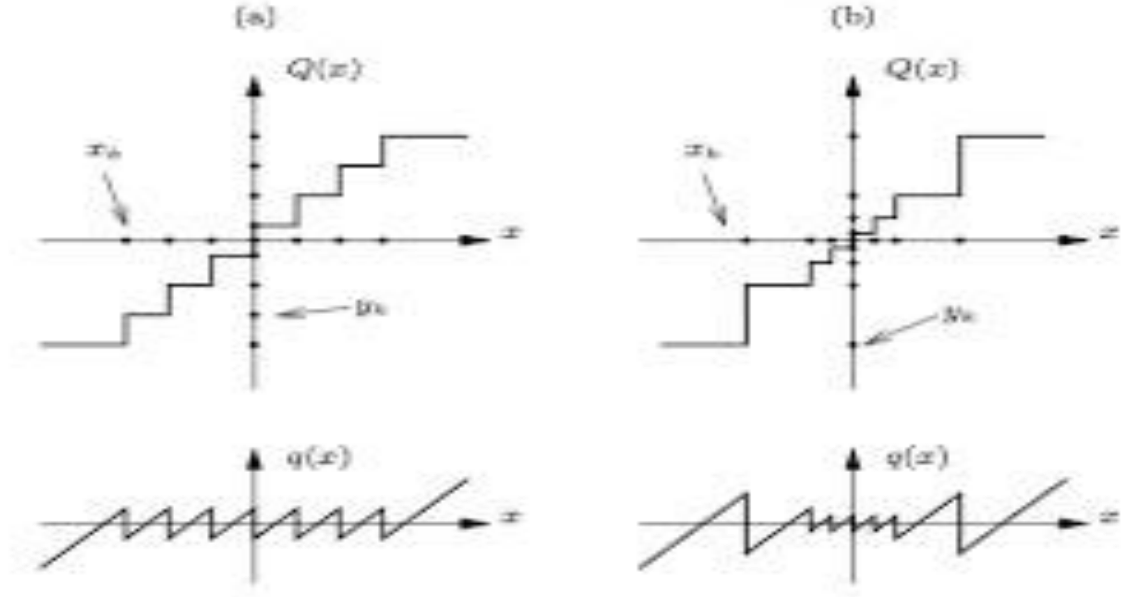
- Quantizing error: The difference between the input and output of a quantizer

$$\Longrightarrow e(t) = \hat{x}(t) - x(t)$$



Uniform & Non- Uniform Quantization

Non uniform Quantization Used to reduce quantization error and increase the dynamic range when input signal is not uniformly distributed over its allowed range of values.



Uniform quantization

Non-Uniform quantization

Condition for Optimal Quantizer

Find the two sets $\{v_k\}_{k=1}^{L}$ and $\{\mathcal{J}_k\}_{k=1}^{L}$, that minimize the average distortion

$$D = \sum_{k=1}^{L} \int_{m \in \mathcal{J}_k} d(m, v_k) f_M(m) dm$$

where $f_M(m)$ is the pdf, which is known.

The mean-square distortion is used commonly

$$d(m, v_k) = (m - v_k)^2$$

The optimization is a nonlinear problem which may not have closed form solution. However the quantizer consists of two components: an encoder characterized by $\mathcal{J}_k$, and a decoder characterized by v_k

Scalar Quantization

- Depending on the measure of distortion employed, we can define the average distortion resulting from quantization.
- A popular measure of distortion, used widely in practice, is the *squared error distortion* defined as $(x - \hat{x})^2$.
  - In this expression x is the sampled signal value and $\hat{x}$ is the quantized value, i.e., $\hat{x} = Q(x)$.
  - If we are using the squared error distortion measure, then
    $$d(x, \hat{x}) = (x - Q(x))^2 = \tilde{x}^2$$
    where $\tilde{x} = x - \hat{x} = x - Q(x)$.
  - Since X is a random variable, so are $\hat{X}$ and $\tilde{X}$; therefore, the average (mean squared error) distortion is given by
    $$D = E[d(x, \hat{x})] = E(x - Q(x))^2$$
    - Mean squared distortion, or *quantization noise* as the measure of performance.
    - A more meaningful measure of performance is a normalized version of the

Encoding

PCM encoding example

Level	Code word
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Levels are encoded using this table

Table: Quantization levels with belonging code words

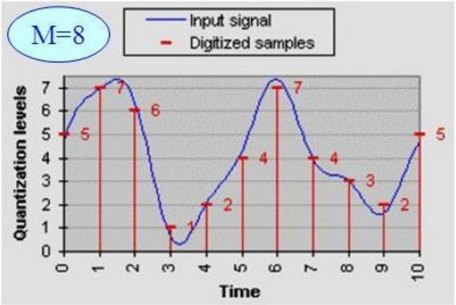


Chart 1. Quantization and digitalization of a signal.

Signal is quantized in 11 time points & 8 quantization segments.

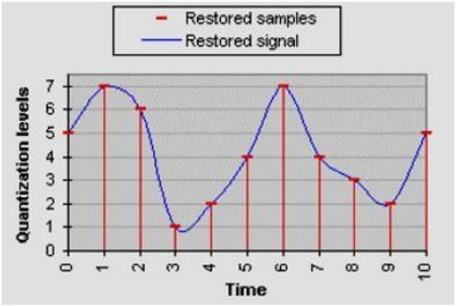
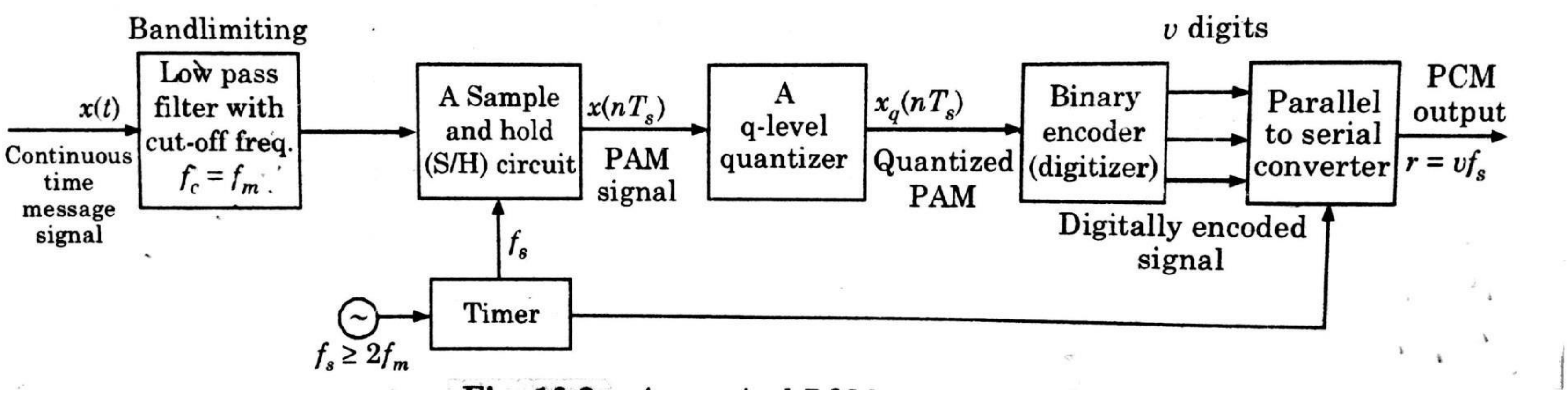


Chart 2. Process of restoring a signal.
PCM encoded signal in binary form:
101 111 110 001 010 100 111 100 011 010 101
Total of **33** bits were used to encode a signal

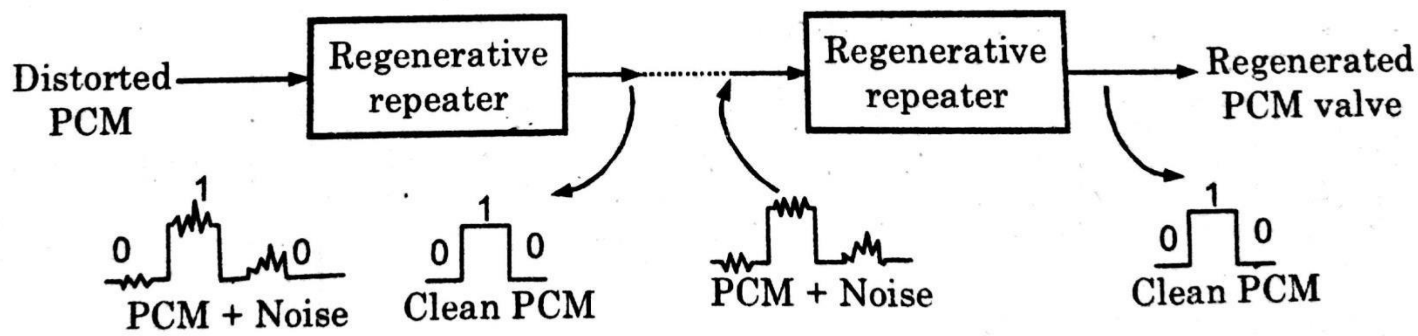
Pulse Code Modulation

- **Pulse Code Modulation (PCM) is a special form of A/D conversion.**
- **It consists of sampling, quantizing, and encoding steps.**
  1.Used for long time in telephone systems
  2. Errors can be corrected during long haul transmission
  3. Can use time division multiplexing
  4. In expensive

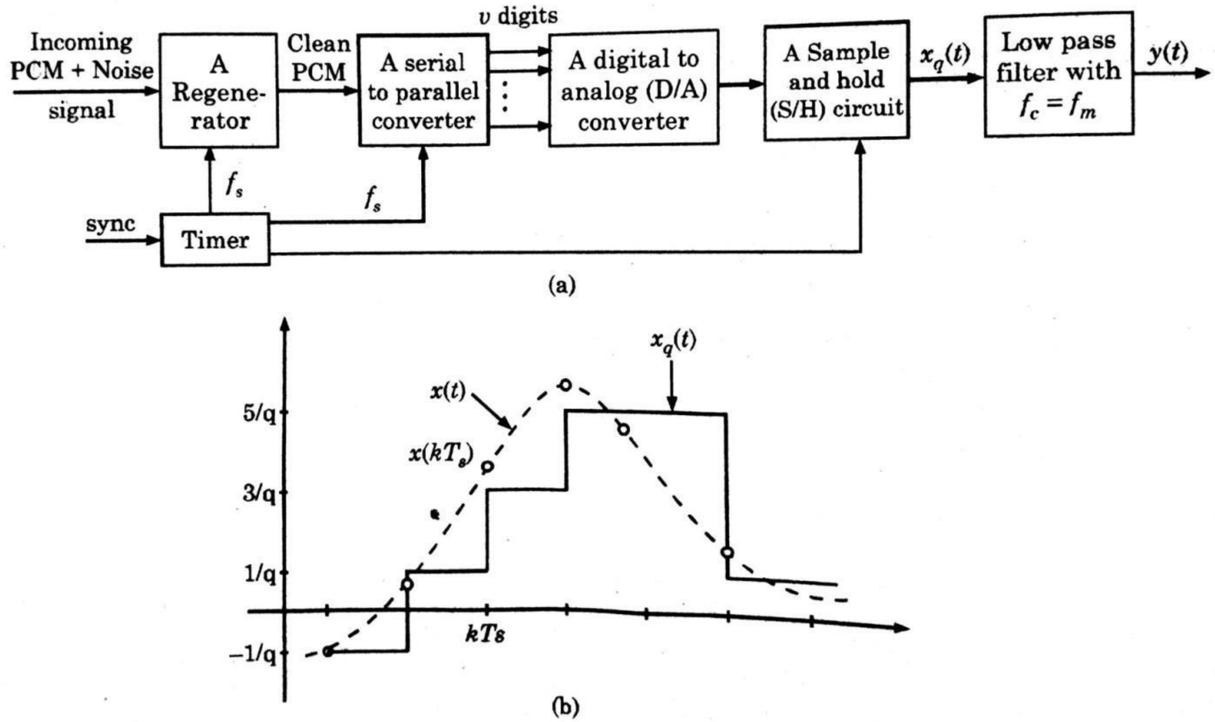
PCM Transmitter



PCM Transmission Path



PCM Receiver



Reconstructed waveform

Bandwidth of PCM

Assume *w(t)* is band limited to *B* hertz.

Minimum sampling rate = *2B* samples / second

A/D output = *n* bits per sample (quantization level $M=2^n$)

Assume a simple PCM without redundancy.

Minimum channel bandwidth = bit rate /2

➢**Bandwidth of PCM signals:**

$B_{PCM} \geq nB$ **(with sinc functions as orthogonal basis)**

$B_{PCM} \geq 2nB$ **(with rectangular pulses as orthogonal basis)**

➢ **For any reasonable quantization level *M*, PCM requires much higher bandwidth than the original *w(t)*.**

Advantages of PCM

Relatively inexpensive. Easily multiplexed.

Easily regenerated.

Better noise performance than analog system.

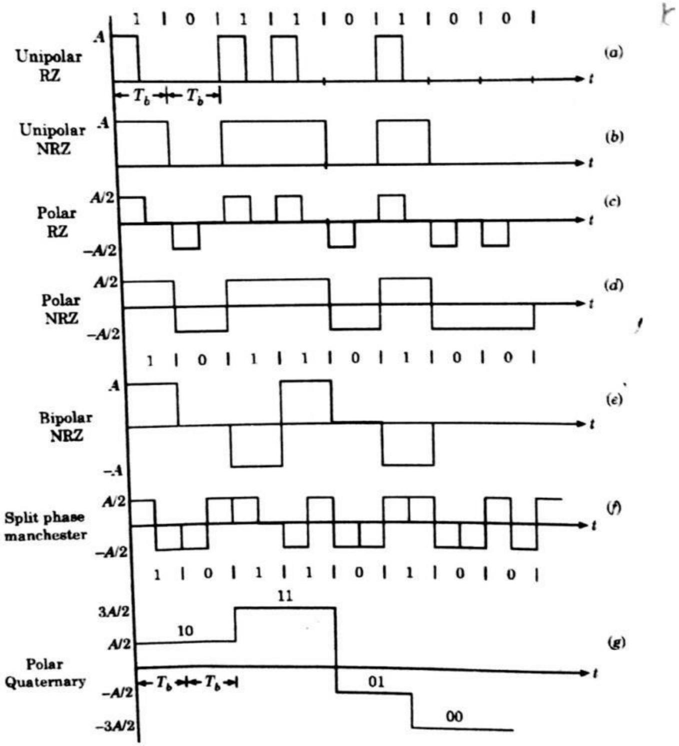
Signals may be stored and time-scaled efficiently.

Efficient codes are readily available.

Disadvantage

Requires wider bandwidth than analog signals

Line Codes


1 0 1 1 0 1 0 0
Unipolar RZ
A
T_b T_b
(a)
Unipolar NRZ
(b)
Polar RZ
A/2
−A/2
(c)
Polar NRZ
(d)
Bipolar NRZ
−A
(e)
Split phase manchester
(f)
3A/2
10
11
01
00
Polar Quaternary
−3A/2
(g)
t

Categories of Line Codes

- **Polar** - Send pulse or negative of pulse
- **Unipolar** - Send pulse or a 0
- **Bipolar** - Represent 1 by alternating signed pulses

Generalized Pulse Shapes

- **NRZ** -Pulse lasts entire bit period
  - Polar NRZ
  - Bipolar NRZ
- **RZ** - Return to Zero - pulse lasts just half of bit period
  - Polar RZ
  - Bipolar RZ
- **Manchester Line Code**

  Send a 2- ϕ pulse for either 1 (high$\rightarrow$ low) or 0 (low$\rightarrow$ high)

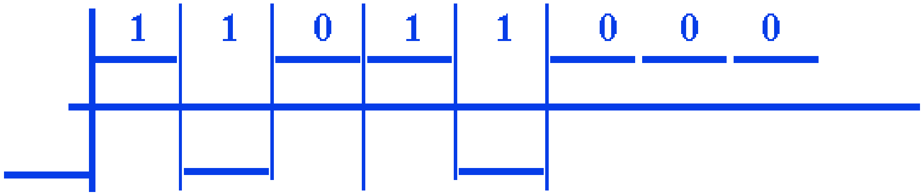
  Includes rising and falling edge in each pulse

  No DC component

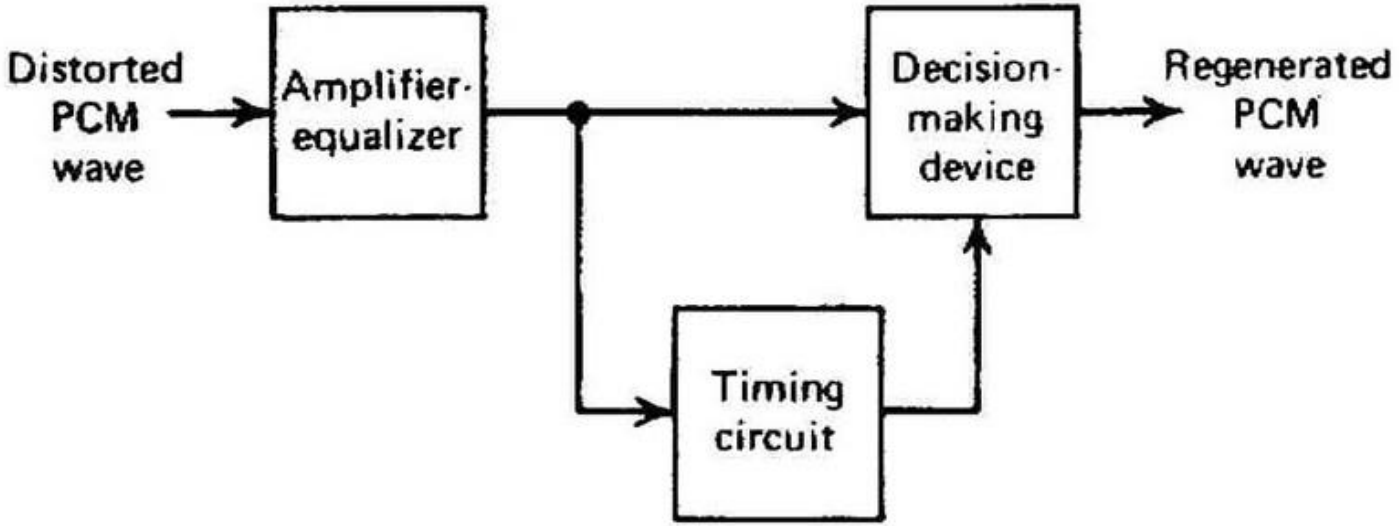
Differential Encoding

Differential Encoding

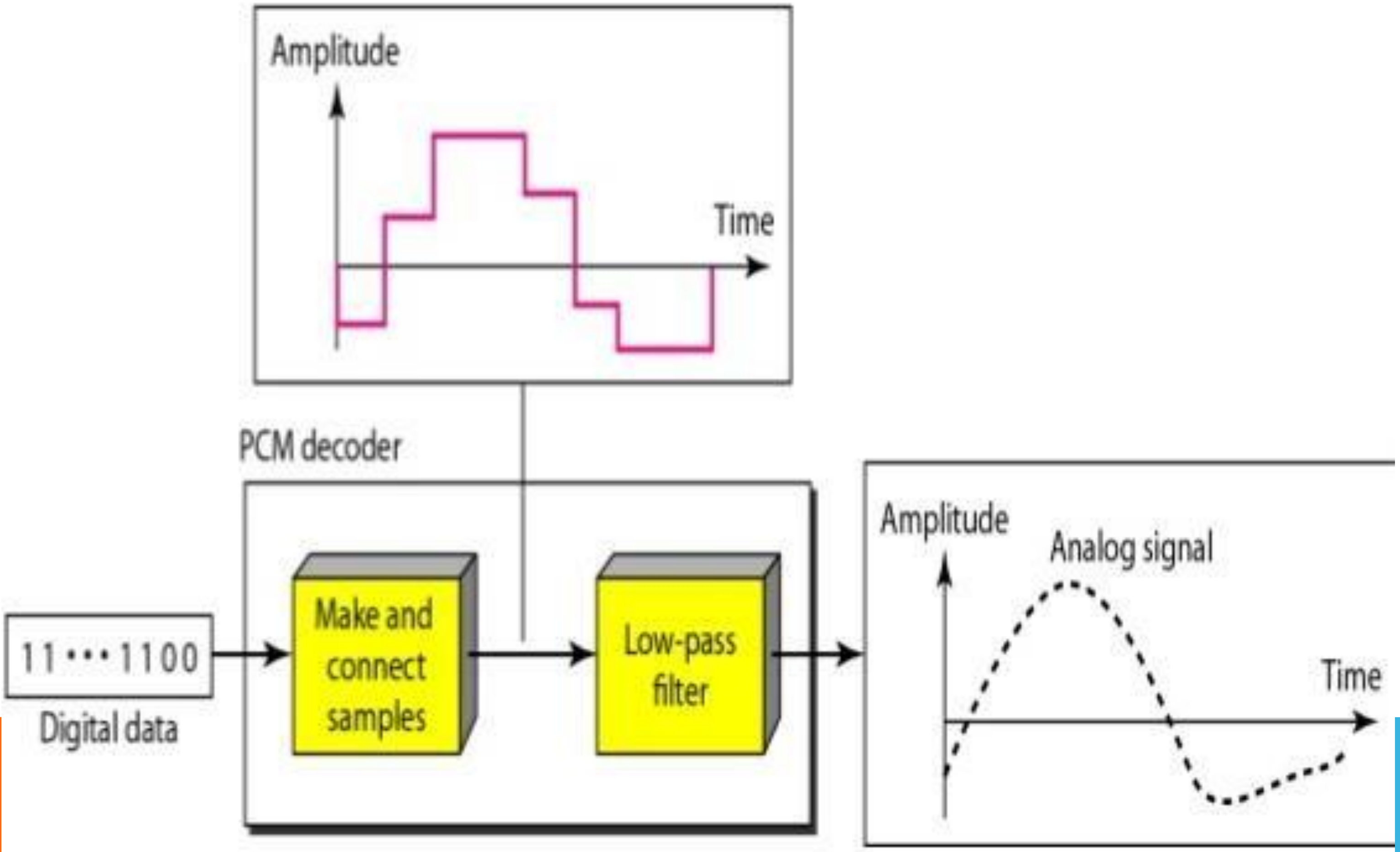
- change from previous signal is important, rather than its absolute value
- advantages:
  - not sensitive to loss of sense of polarity
  - easier to detect a transition rather than an absolute value
- example is NRZ-M



Pulse-Code Modulation (Regeneration)



Components of a PCM decoder



Noise Considerations In PCM

- Two main effects produce the noise or distortion in the PCM output:
  - Quantizing noise that is caused by the M-step quantizer at the PCM transmitter.
  - Bit errors in the recovered PCM signal, caused by channel noise and improper filtering.

- If the input analog signal is band limited and sampled fast enough so that the aliasing noise on the recovered signal is negligible, the ratio of the recovered analog peak signal power to the total average noise power is:

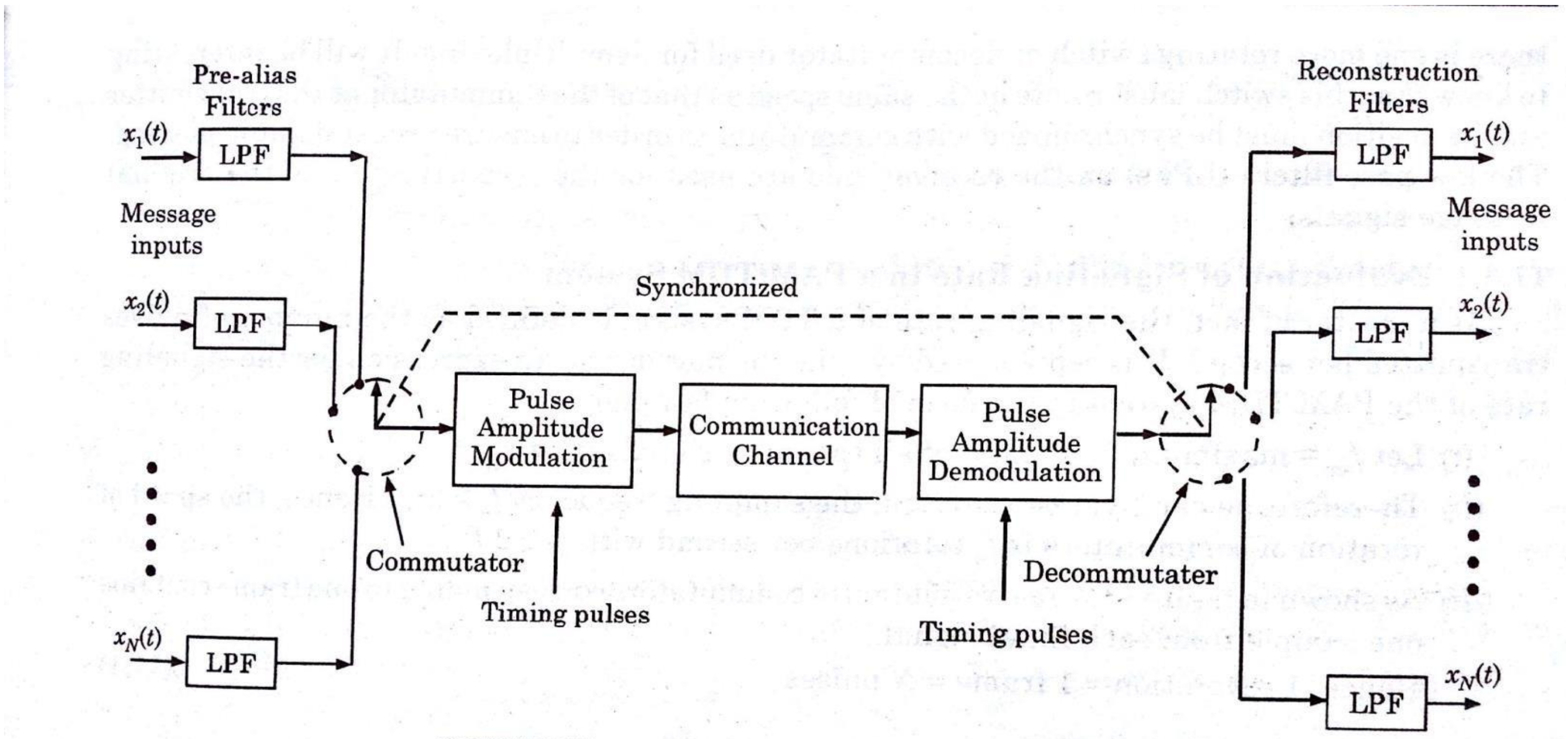
$$\boxed{\left(\frac{S}{N}\right)_{pk\ \ out} = \frac{3M^2}{1+4\left(M^2-1\right)P_e}}$$

- The ratio of the average signal power to the average noise power is

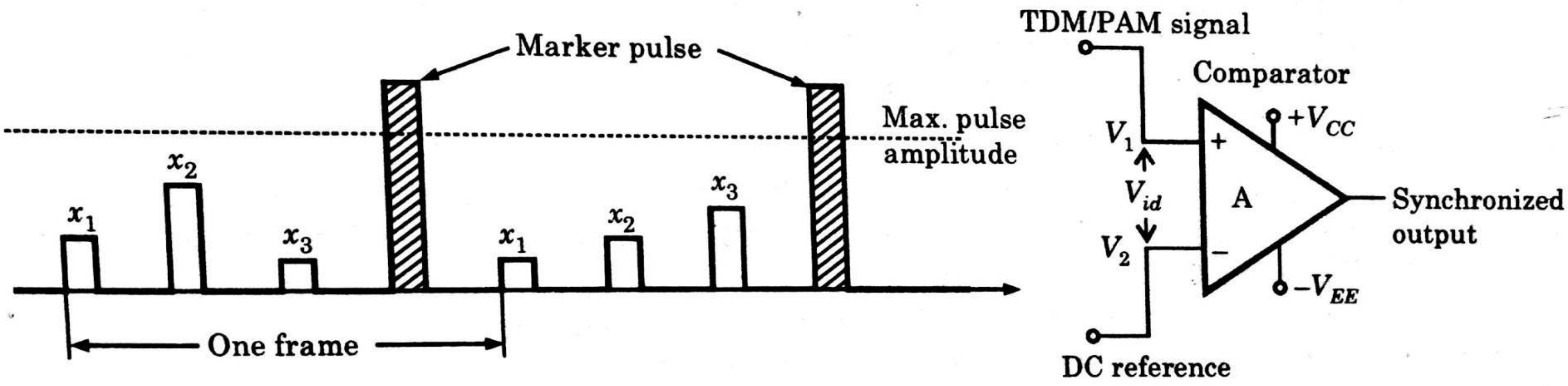
$$\boxed{\left(\frac{S}{N}\right)_{out} = \frac{M^2}{1+4\left(M^2-1\right)P_e}}$$

  - **M** is the number of quantized levels used in the PCM system.
  - P_e is the probability of bit error in the recovered binary PCM signal at the receiver DAC before it is converted back into an analog signal.

Time Division Multiplexing(TDM)



Synchronization



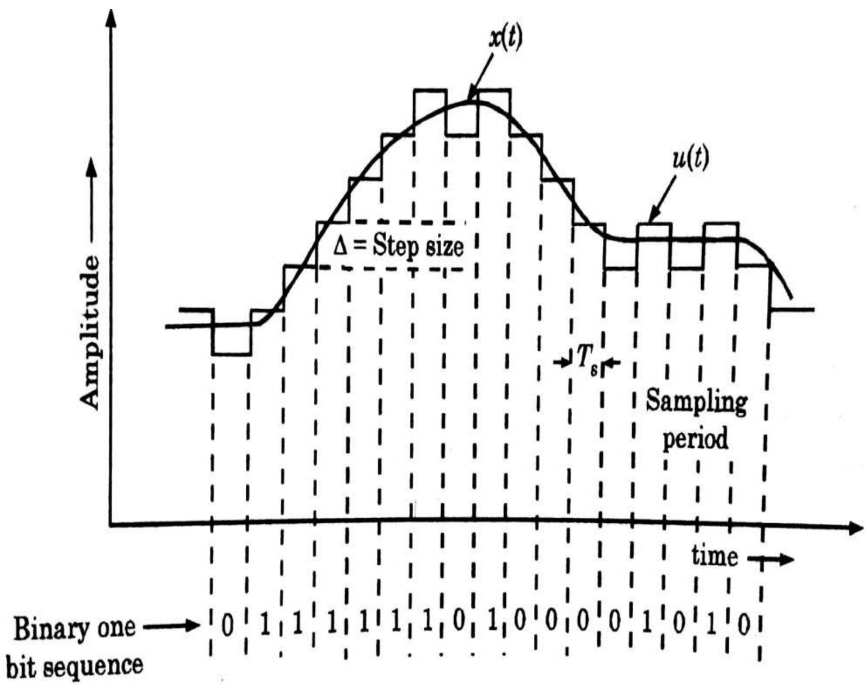
Delta Modulation

Types of noise

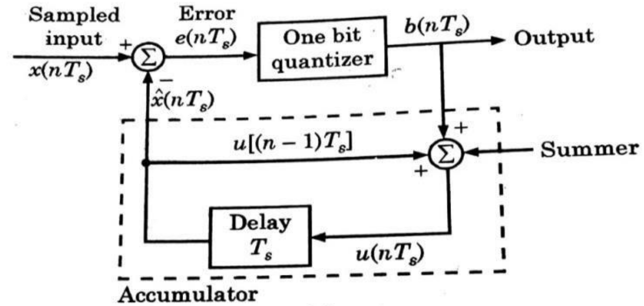
➢Quantization noise: step size δ takes place of smallest quantization level.

➢δ too small: slope overload noise

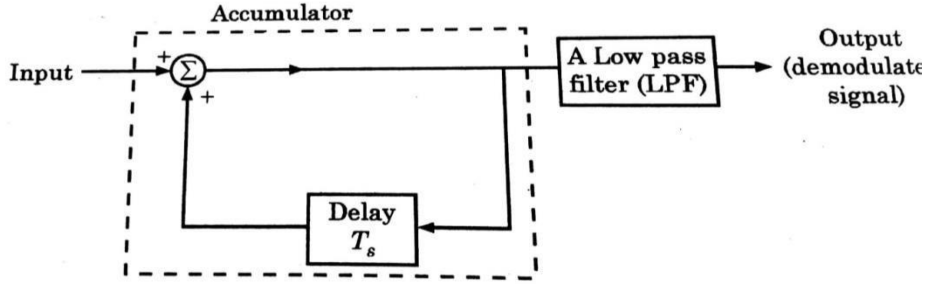
too large: quantization noise and granular noise



Delta Modulator Transmitter & Receiver

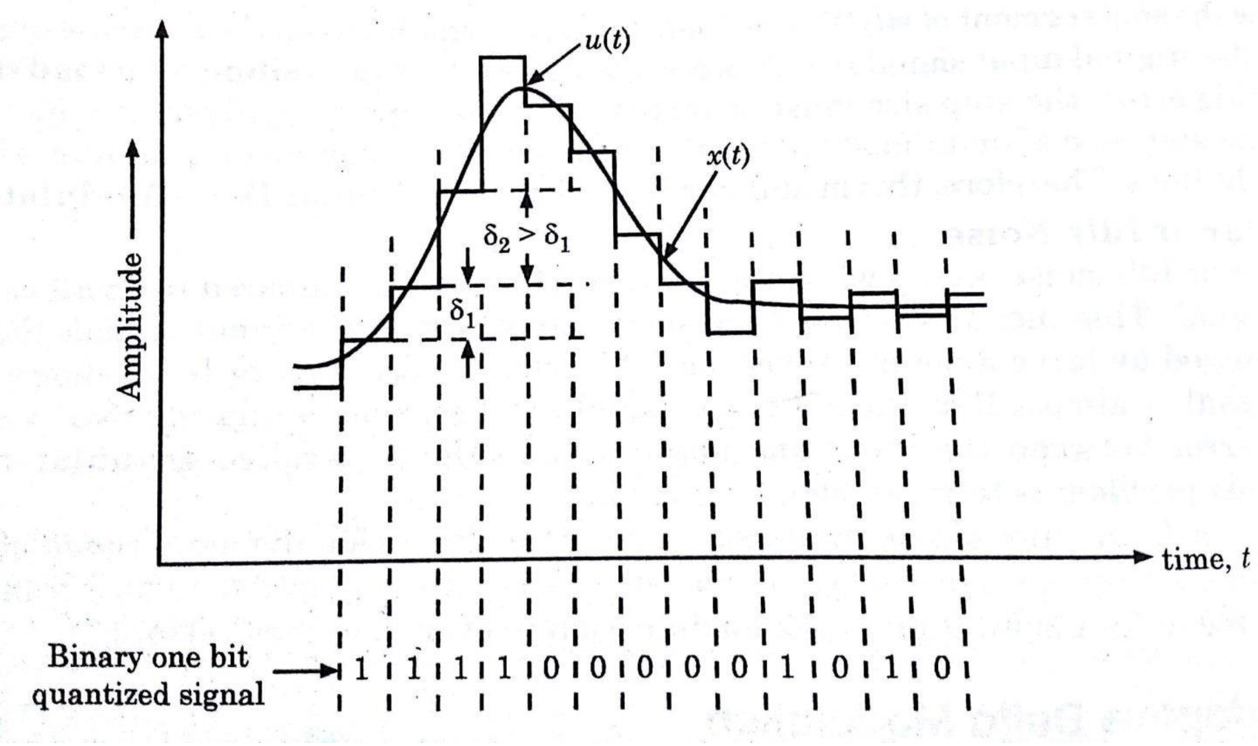


(a)



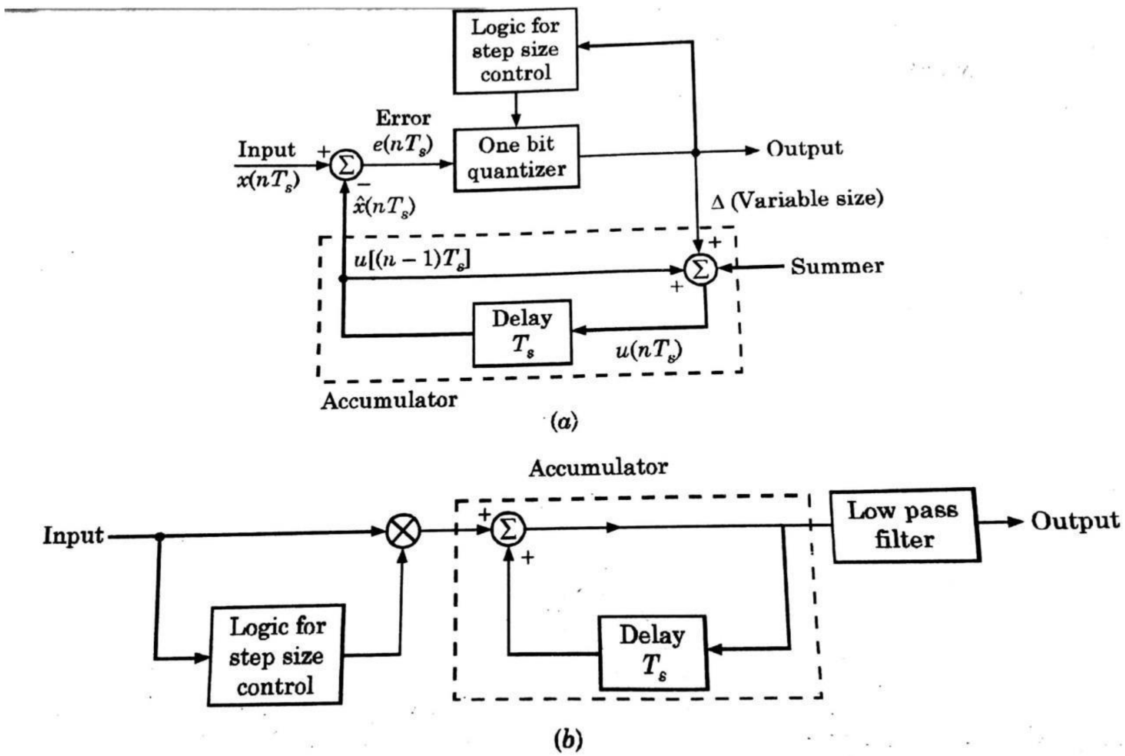
(b)

ADM



u(t)
x(t)
Amplitude
δ2 > δ1
δ1
time, t
Binary one bit quantized signal
1 1 1 1 0 0 0 0 0 1 0 1 0

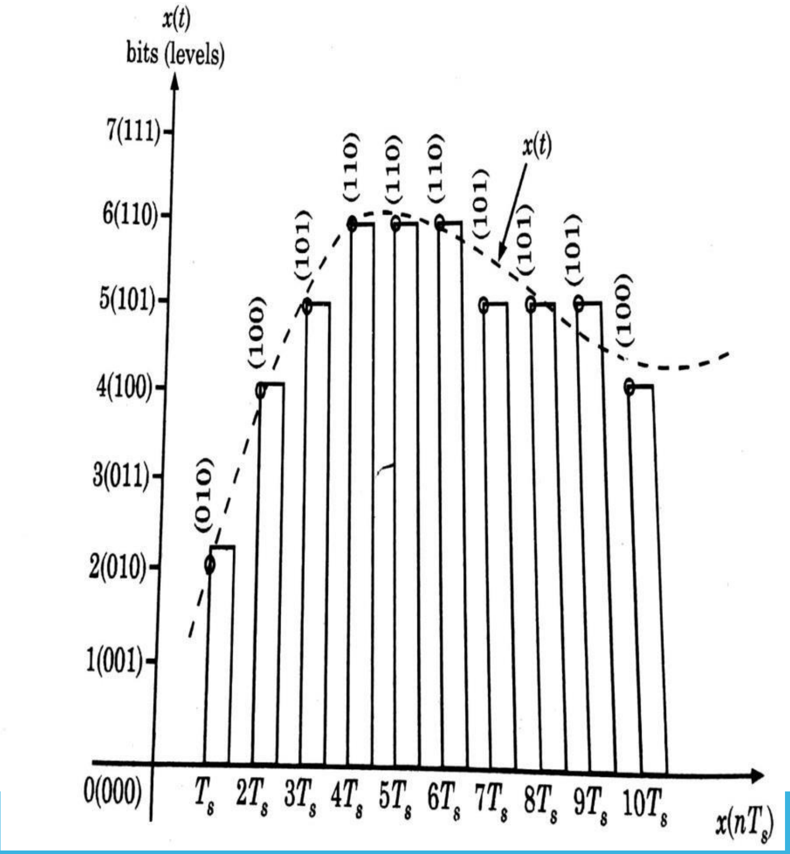
ADM Transmitter & Receiver



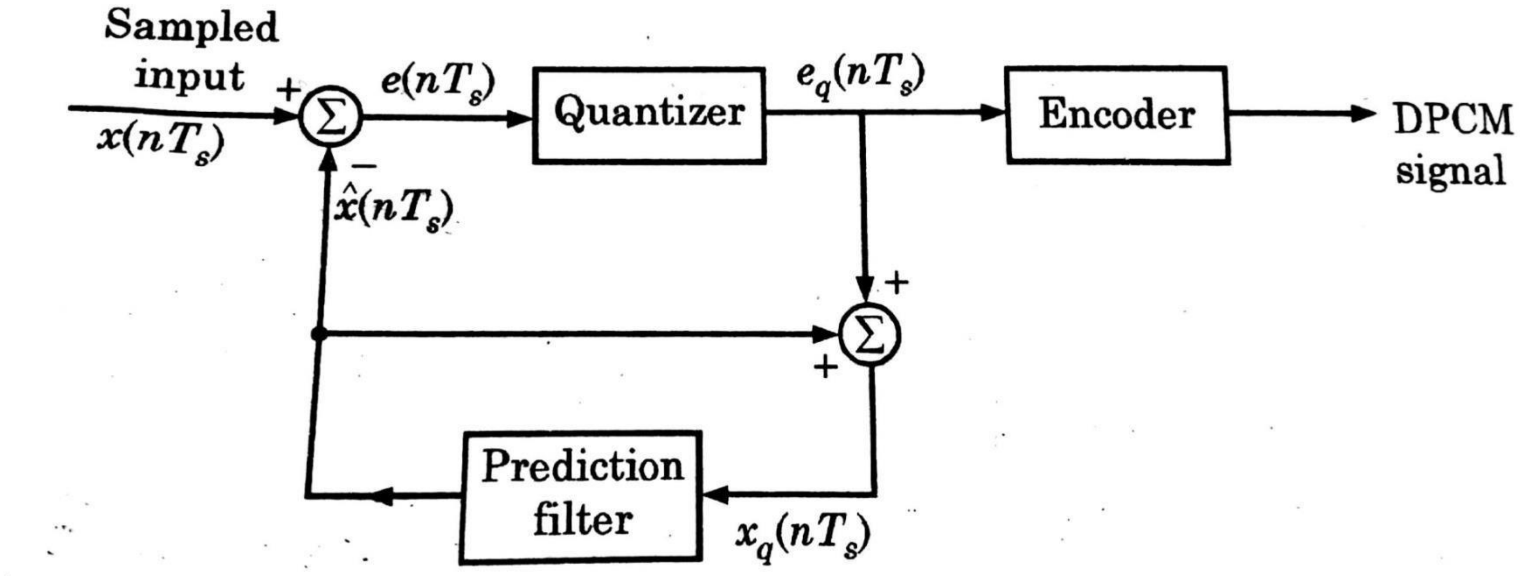
DPCM

Often voice and video signals do not change much from one
sample to next.
-Such signals has energy concentrated in lower frequency.
-Sampling faster than necessary generates redundant information.
Can save bandwidth by not sending all samples.

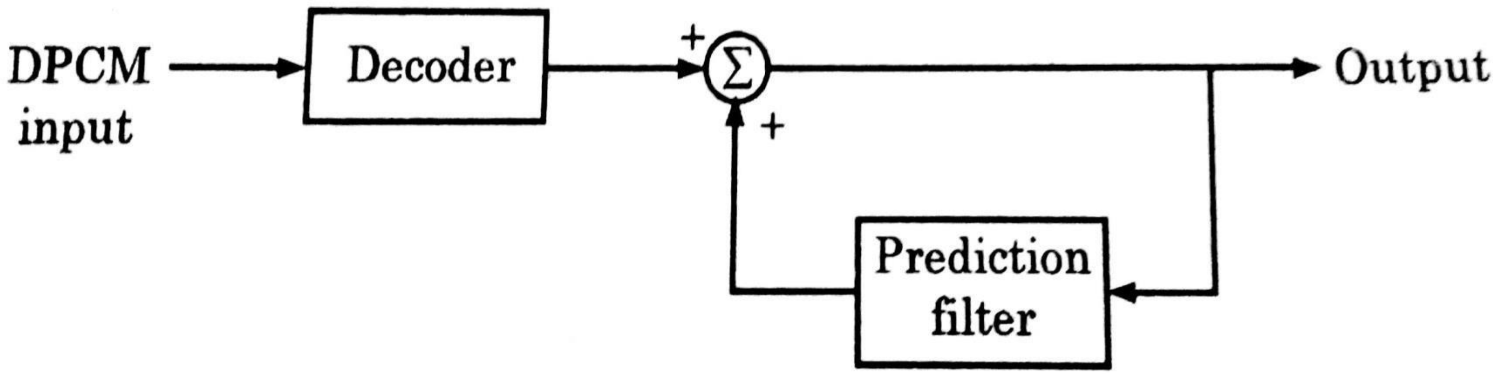
* Send true samples occasionally.
* In between, send only change from previous value.
* Change values can be sent using a fewer number of bits than true samples.



DPCM Transmitter



DPCM Receiver

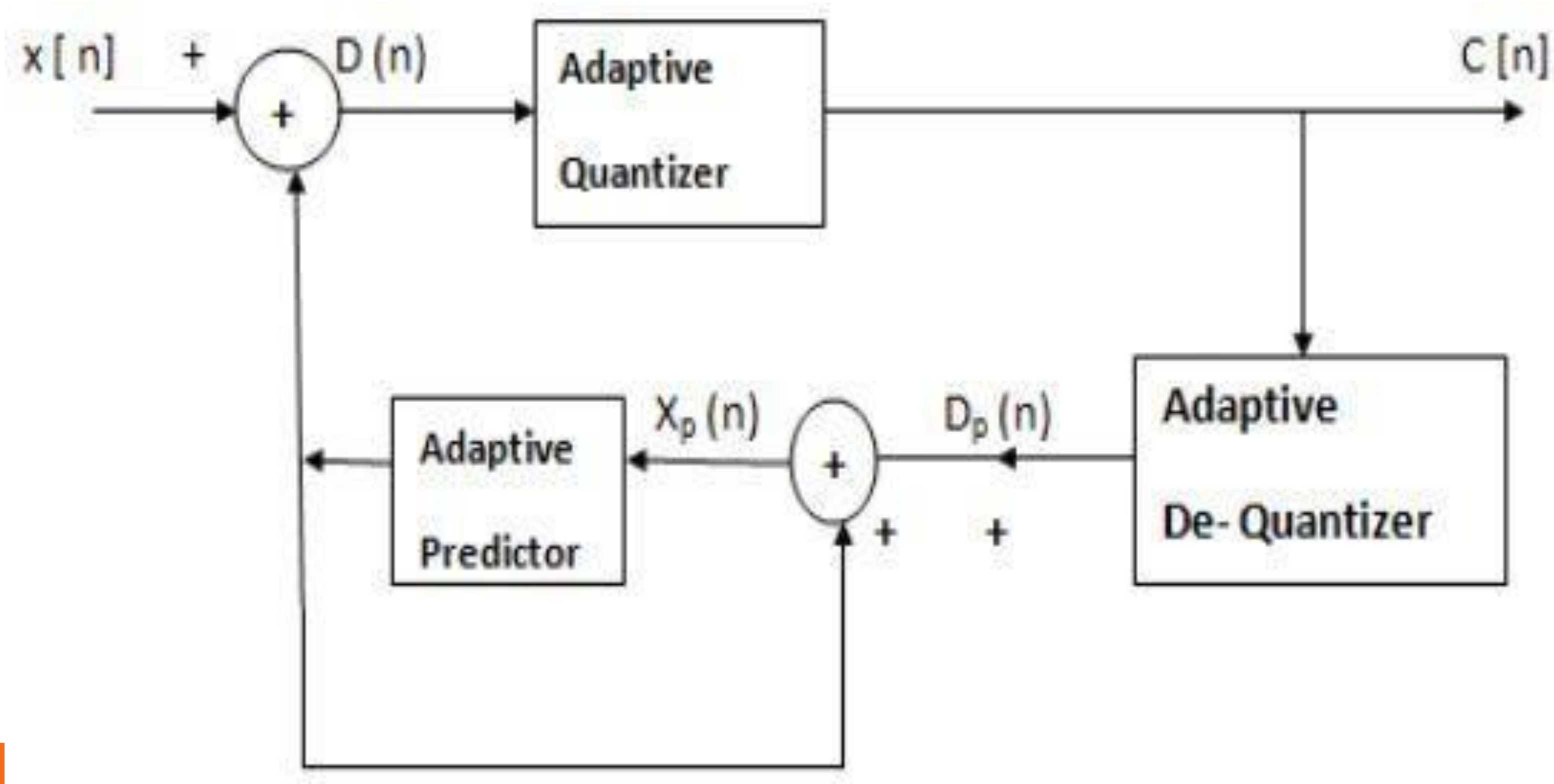


Processing Gain

In a spread-spectrum system, the process gain (or "processing gain") is the ratio of the spread (or RF) bandwidth to the unspread (or baseband) bandwidth. It is usually expressed in decibels (dB).

For example, if a 1 kHz signal is spread to 100 kHz, the process gain expressed as a numerical ratio would be 100000/1000 = 100. Or in decibels, 10 $\log_{10}(100)$ = 20 dB.

Adaptive DPCM



Comparisons

S. No.	*Parameter of comparison*	*Pulse Code Modulation (PCM)*	*Delta modulation (DM)*	*Adaptive Delta Modulation (ADM)*	*Differential Pulse Code Modulation (DPCM)*
1.	Number of bits.	It can use 4, 8 or 16 bits per sample.	It uses only one bit for one sample.	Only one bit is used to encode one sample.	Bits can be more than one but are less than PCM.
2.	Levels and step size	The number of levels depend on number of bits. Level size is kept fixed.	Step size is kept fixed and cannot be varied.	According to the signal variation, step size varies (*i.e.* Adapted).	Here, Fixed number of levels are used.
3.	Quantization error and distortion	Quantization error depends on number of levels used.	Slope overload distor-tion and granular noise are present.	Quantization noise is present but other errors are absent.	Slope overload distor-tion and quantization noise is present.
4.	Transmission bandwidth	Highest bandwidth is required since number of bits are high	Lowest bandwidth is required.	Lowest bandwidth is required.	Bandwidth required is lower than PCM.
5.	Feedback	There is no feedback in transmitter or receiver.	Feedback exists in transmitter.	Feedback exists.	Here, Feedback exists.
6.	Complexity of implementation	System complex.	Simple.	Simple.	Simple

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(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
P.Kothakota, Puthalapattu (M), Chittoor Dist – 517 112 AP, India

Department of Electronics and Communication Engineering

Digital Communication Systems

Unit-2
Baseband Pulse Transmission

By
B SAROJA
Associate Professor Dept. of . ECE

Contents

Introduction

Matched filter, Properties of Matched filter,Matched filter for rectangular pulse, Error rate due to noise

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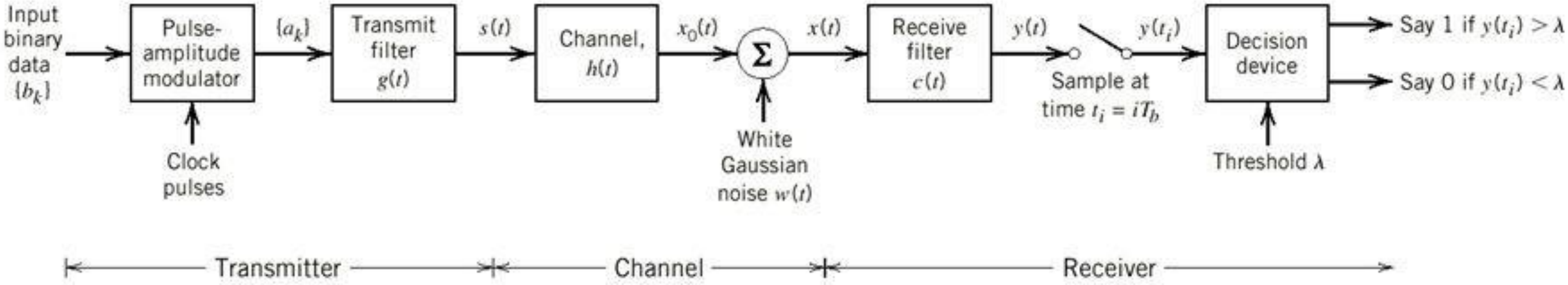
transmission Ideal Nyquis tchannel

Raised cosine filter & its spectrum

Correlative coding–Duo binary & Modified duo binary signaling schemes, Partial response signaling

Baseband M-array PAM transmission Eye diagrams

Baseband binary data transmission system.



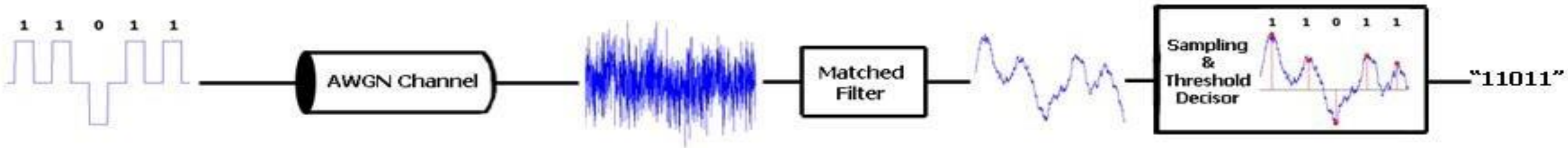
Matched Filter

It passes all the signal frequency components while suppressing any frequency components where there is only noise and allows to pass the maximum amount of signal power.

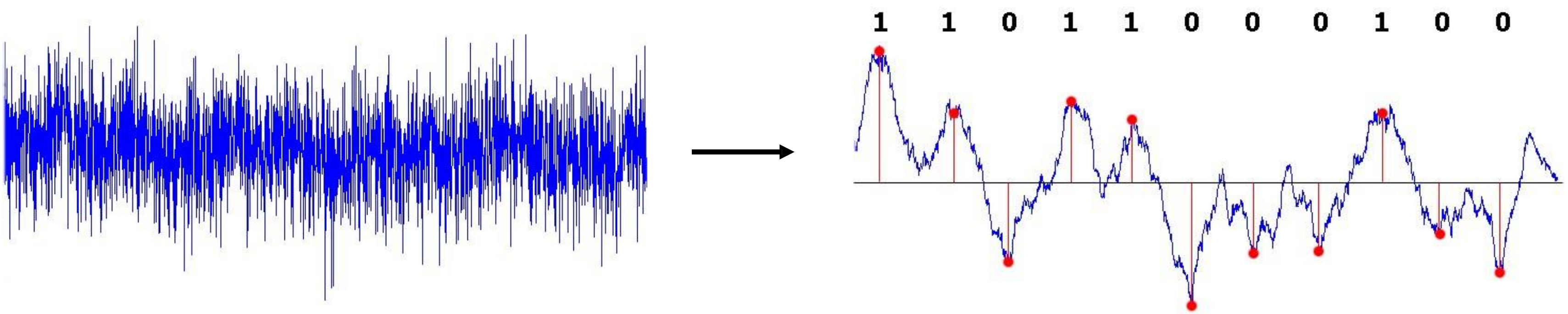
The purpose of the matched filter is to maximize the signal to noise ratio at the sampling point of a bit stream and to minimize the probability of undetected errors received from a signal.

To achieve the maximum SNR, we want to allow through all the signal frequency components.

Matched Filter:



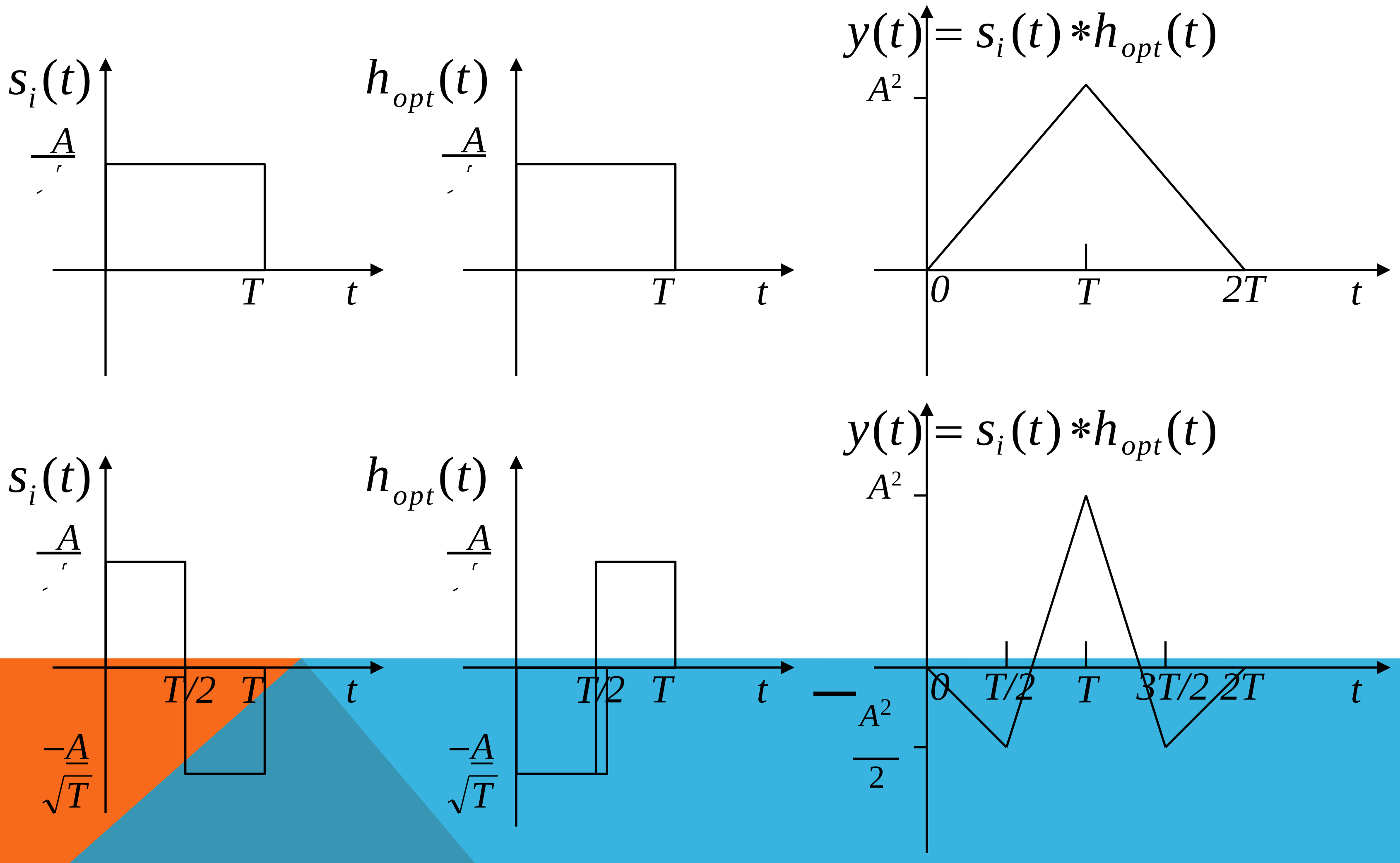
- **Consider the received signal as a vector r, and the transmitted signal vector as s**
- **Matched filter "projects" the r onto signal space spanned by s ("matches" it)**



Filtered signal can now be safely sampled by the receiver at the correct sampling instants, resulting in a correct interpretation of the binary message

Matched filter is the filter that <u>maximizes the signal-to-noise ratio</u> it can be shown that it also <u>minimizes the BER: *it is a simple projection operation*</u>

Example Of Matched Filter (Real Signals)



Properties of the Matched Filter

1. The Fourier transform of a matched filter output with the matched signal as input is, except for a time delay factor, proportional to the ESD of the input signal.

$$Z(f) = |S(f)|^2 \exp(-j2\pi fT)$$

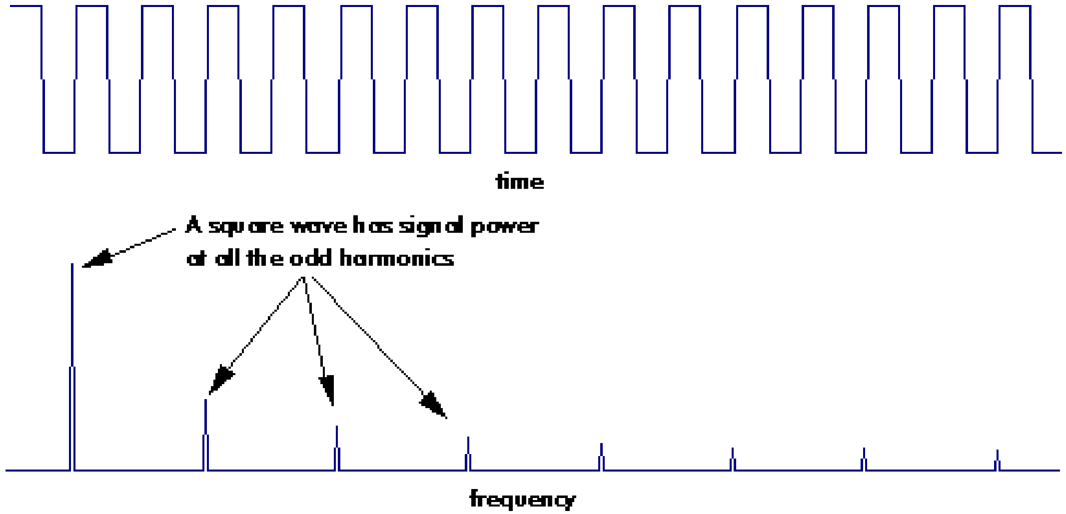
2.The output signal of a matched filter is proportional to a shifted version of the autoc is n

$$z(t) = R_s(t - T) \Rightarrow z(T) = R_s(0) = E_s$$

3.The output SNR of a matched filter depends only on the ratio of the signal energy to the PSD of the white noise at the filter input.

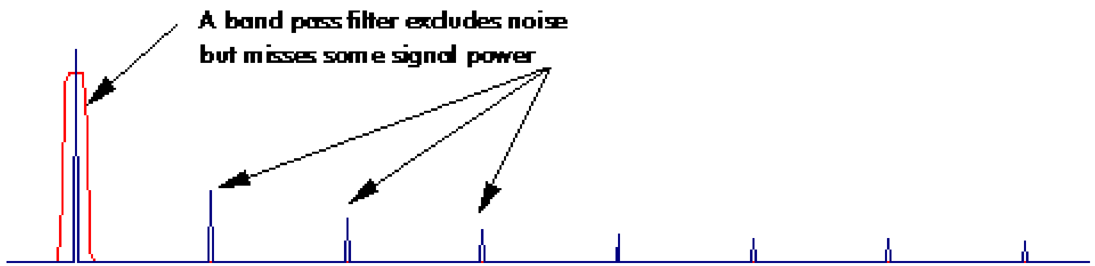
$$\max\left(\frac{S}{N}\right)_T = \frac{E_s}{N_0/2}$$

Matched Filter: Frequency domain View



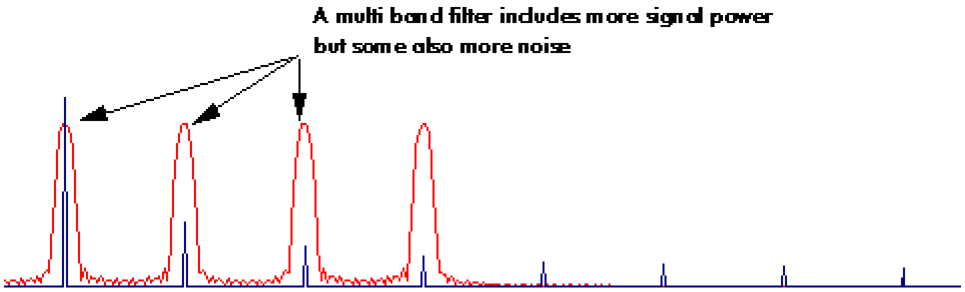
Simple Bandpass Filter:

excludes noise, but misses some signal power

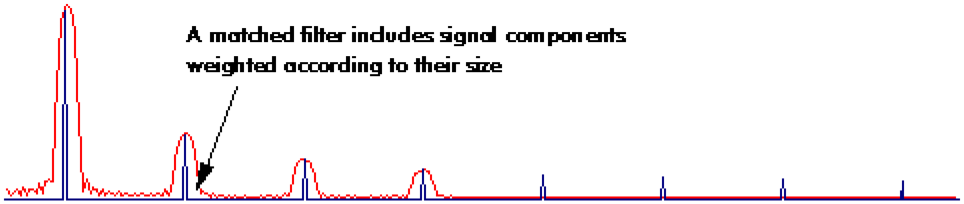


Matched Filter: Frequency Domain View (Contd

<u>Multi-Bandpass Filter:</u> includes more signal power, but adds more noise also!



<u>Matched Filter:</u> includes more signal power, weighted according to size
=> maximal noise rejection!



Matched Filter For Rectangular Pulse

Matched filter for causal rectangular pulse has an impulse response that is a causal rectangular pulse

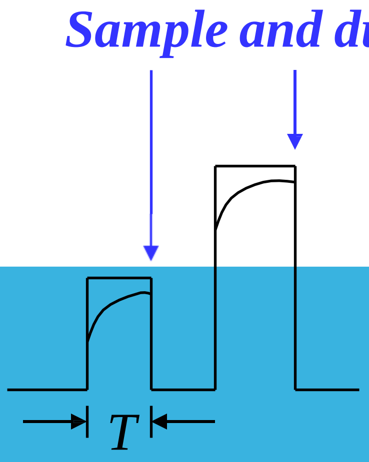
Convolve input with rectangular pulse of duration *T* sec and sample result at *T* sec is same as to

First, integrate for *T* sec

Second, sample at symbol period *T* sec

Third, reset integration for next time period

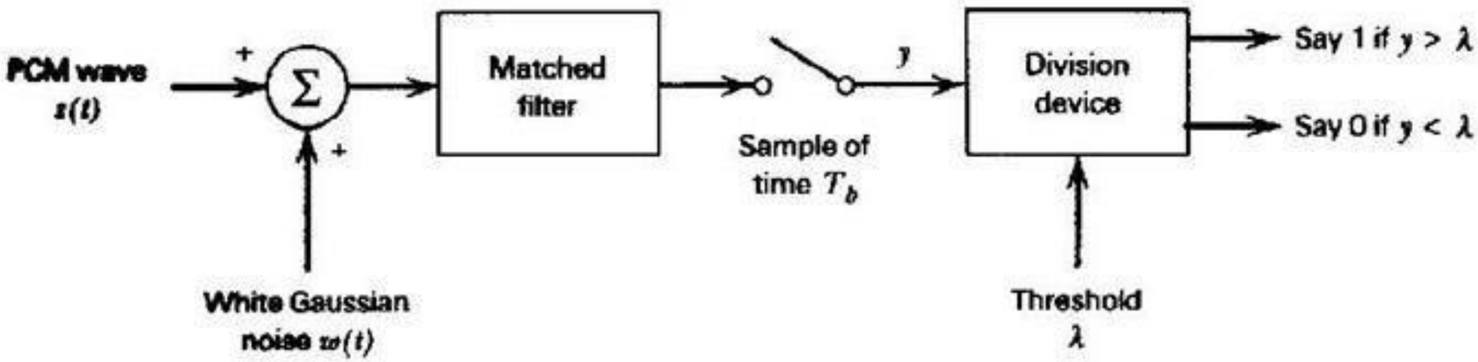
Integrate and dump circuit



Error Rate Due to Noise

Components of Receiver

- matched filter
- sampler
- decision device



Receiver for baseband transmission of binary-encoded PCM wave using NRZ signaling.

Inter-symbol Interference (ISI)

ISI in the detection process due to the filtering effects of the system

Overall equivalent system transfer function

$$H(f) = H \qquad (f)\, H$$

- creates echoes and hence time dispersion
- causes ISI at sampling time

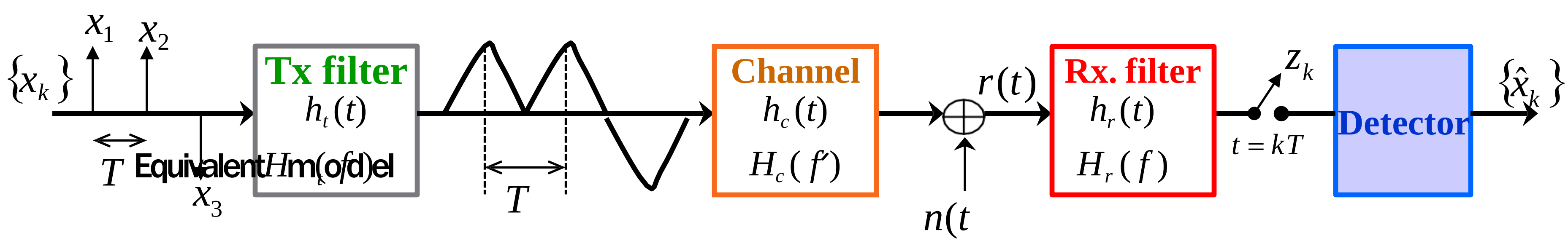
ISI effect

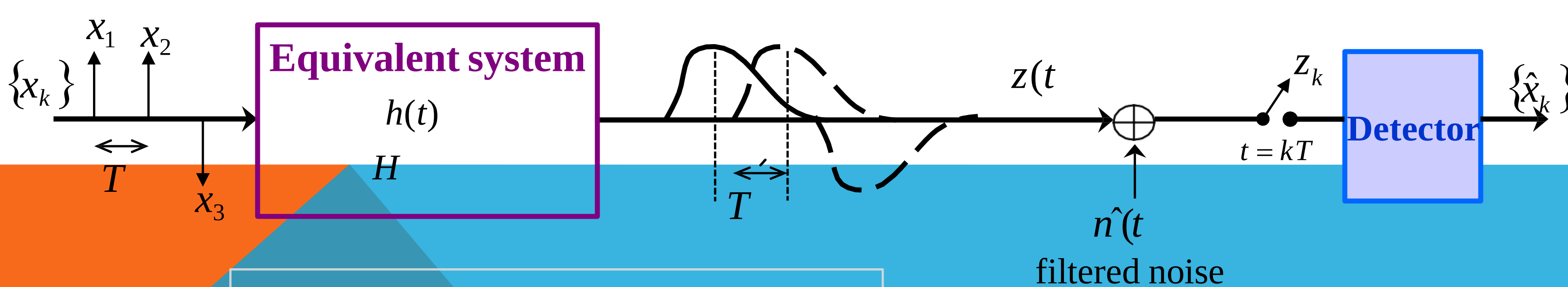
$$z_k = s_k + n_k + \sum \alpha_i s_i$$

$i \neq k$

Inter-symbol Interference (ISI): MODEL

Baseband system model





$$H(f) = H_t(f)H_c(f)H_r(f)$$

Nyquist's Criterion For Distortionless Baseband Binary Transmission

- According to the solution described b Eqs.(7.54) and (7.55), no frequencies of absolute value exceeding half the bit rate are needed. Hence, one signal waveform that produces zero intersymbol interferen is defined by the ***sinc function***

$$p(t) = \frac{\sin(2f\pi Wt)}{2\pi Wt}$$
$$= \sin c(2Wt)$$

the special value of the bit rate $R_b = 2W$ is called the ***Nyquist rate*** and W is itself *called the* ***Nyquist bandwidth***

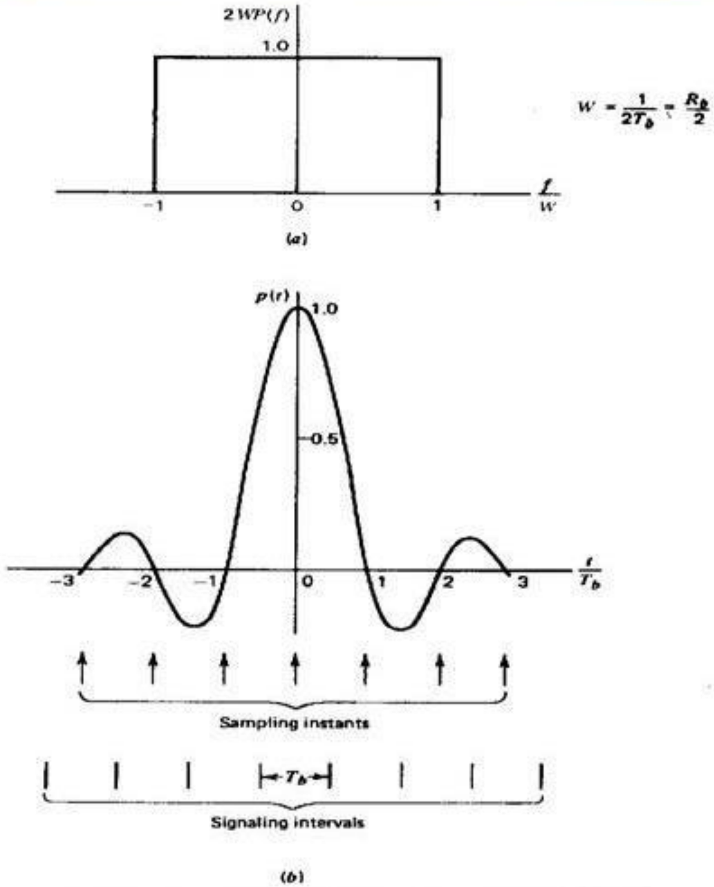


Figure 7.8 (a) Ideal amplitude response. (b) Ideal basic pulse shape.

Nyquist's Criterion for Distortionless Baseband Binary Transmission

$$P(f) = \begin{cases} \dfrac{1}{2B_o} & |f| < f_1 \\ \dfrac{1}{4B_o}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2B_o - 2f_1}\right]\right\}, & f_1 \le |f| < 2B_o - f_1 \\ 0 & |f| \ge 2B_o - f_1 \end{cases}$$

The frequency f_1 and bandwidth B_o are related by

$$\alpha = 1 - \frac{f_1}{B_o}$$

The parameter α is called the *rolloff factor*. For $\alpha = 0$, that is, $f_1 = B_o$, we get the minimum bandwidth solution described earlier.

The frequency response $P(f)$, normalized by multiplying it by $2B_o$, is shown plotted in Fig. 6.7*a* for three values of α namely, 0, 0.5, and 1. We see that for $\alpha = 0.5$ or 1, the rolloff characteristic of $P(f)$ cuts off gradually as compared

Nyquist's Criterion for Distortionless Baseband Binary Transmission

(1) IDEAL SOLUTION

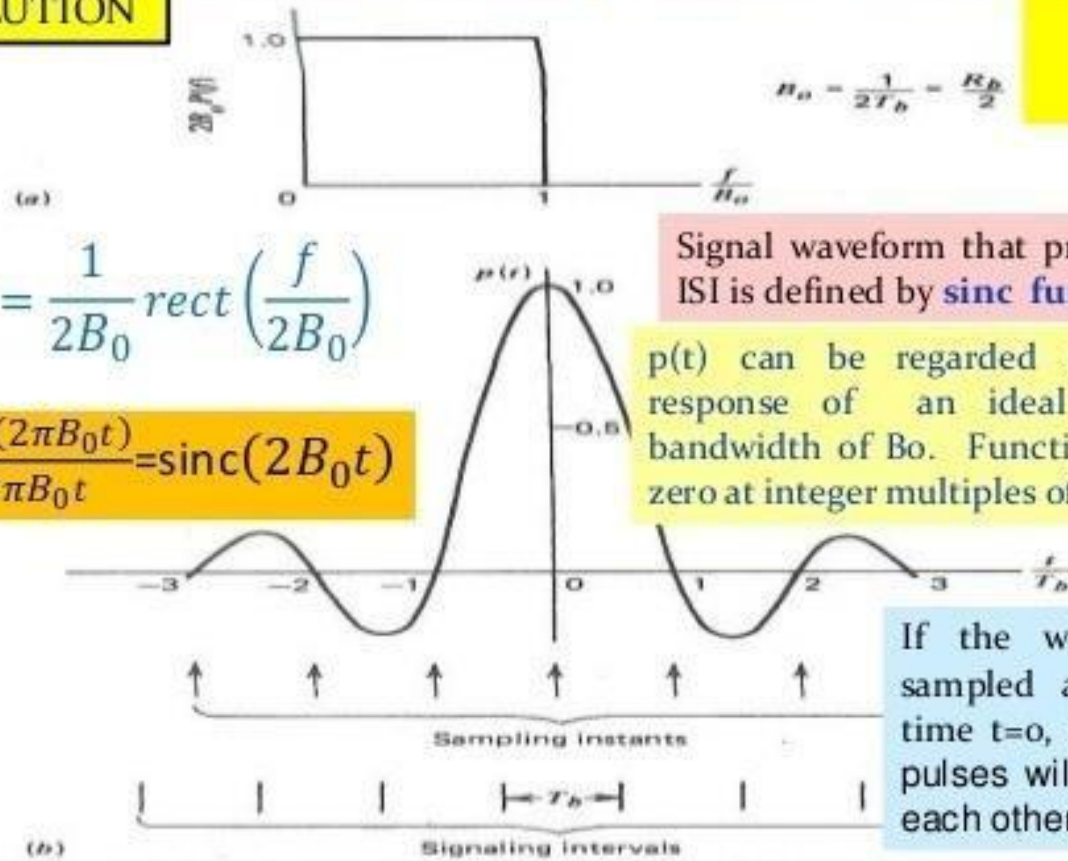


Figure 6.6 (a) Ideal amplitude response. (b) Ideal basic pulse shape.

$$B_0 = \frac{1}{2T_b} = \frac{R_b}{2}$$

$$B_0 = \frac{R_b}{2}$$

$$P(f) = \frac{1}{2B_0} rect\left(\frac{f}{2B_0}\right)$$

$$p(t) = \frac{sin(2\pi B_0 t)}{2\pi B_0 t} = \text{sinc}(2B_0 t)$$

Signal waveform that produces zero ISI is defined by **sinc function.**

p(t) can be regarded as the impulse response of an ideal LPF with a bandwidth of Bo. Function goes through zero at integer multiples of Tb.

If the waveform y(t) is sampled at the instants of time t=0, ±Tb, ±2Tb, .. then pulses will not interfere with each other

IDEAL NYQUIST CHANNEL

One way to satisfy Equ. (4.53) is to specify the frequency function $P(f)$ to be a rectangular function:

$$P(f) = \begin{cases} 1/2W, & -W < f < W \\ 0, & |f| > W \end{cases}$$

$$= (1/2W)\ \mathrm{rect}(f/2W)$$

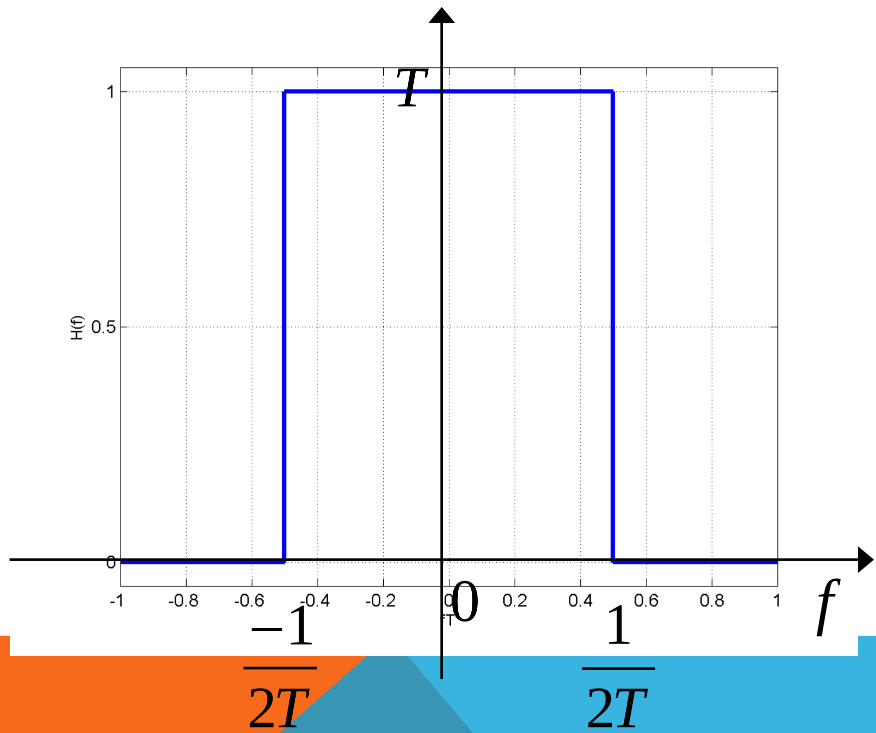
where rect(f) stands for a rectangular function of unit amplitude and centered on $f = 0$, and the overall system bandwidth W is defined by

$$W = R_b/2 = 1/2T_b$$

Equiv System: Ideal Nyquist Pulse (FILTER)

Ideal Nyquist filter

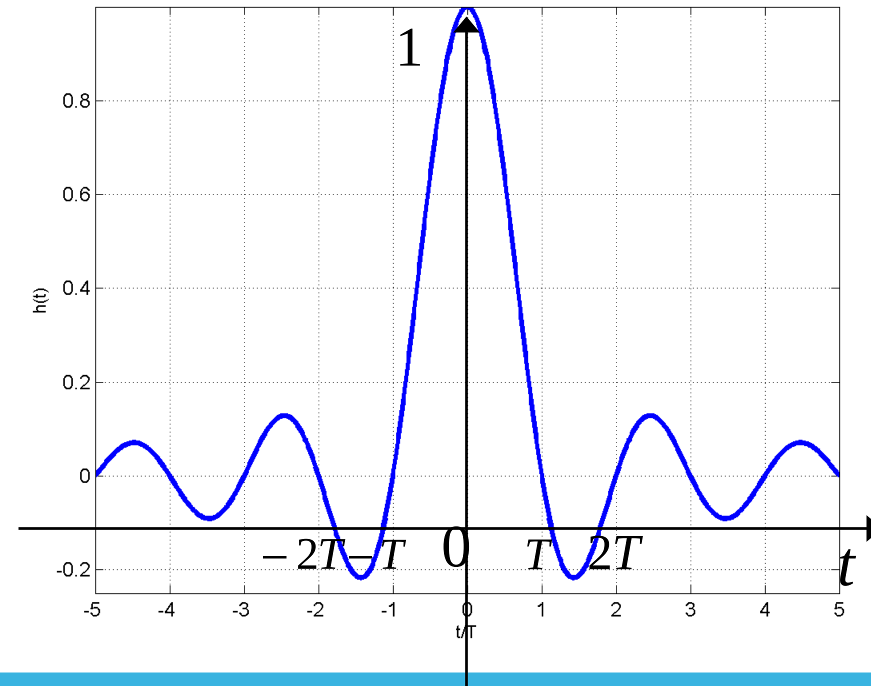
$H(f)$



$$W = \frac{1}{2T}$$

Ideal Nyquist pulse

$$h(t) = \mathrm{sinc}(t/T)$$



Nyquist Pulses (FILTERS)

Nyquist pulses (filters):

- Pulses (filters) which result in no ISI at the <u>sampling time</u>.

Nyquist filter:

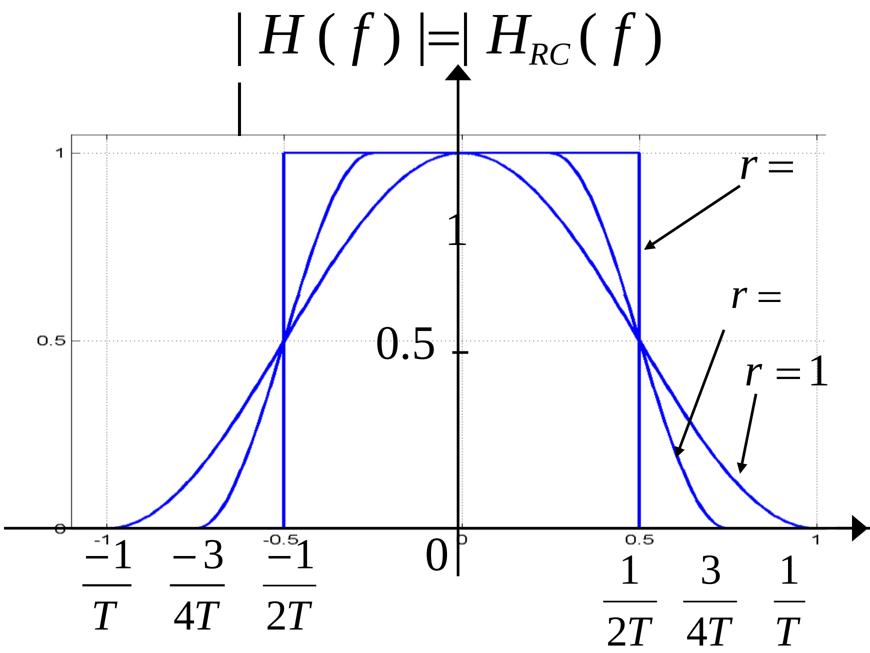
- Its transfer function in frequency domain is obtained by convolving a rectangular function with any real even-symmetric frequency function

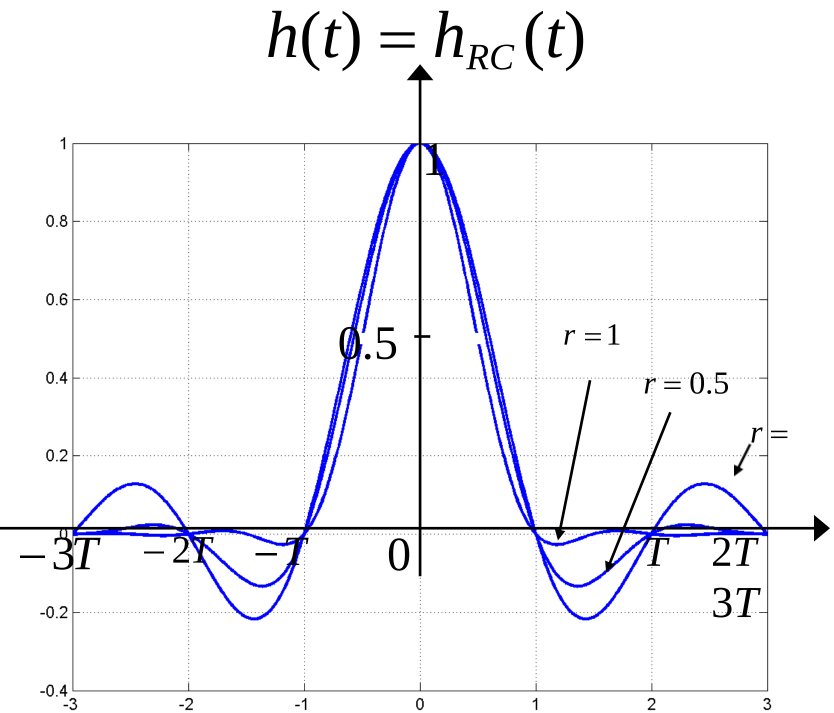
Nyquist pulse:

- Its shape can be represented by a sinc(t/T) function multiply by another time function.

Example of Nyquist filters: Raised-Cosine filter

Raised Cosine Filter & Its Spectrum





Baseband $W_{sSB} = (1+r)\dfrac{R_s}{2}$

Passband $W_{DSB} = (1+r)R_s$

RAISED COSINE FILTER & ITS SPECTRUM

Raised-Cosine Filter

- A Nyquist pulse (No ISI at the sampling time)

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left[\dfrac{\pi}{4}\,\dfrac{|f| + W - 2W_0}{W - W_0}\right] & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases}$$

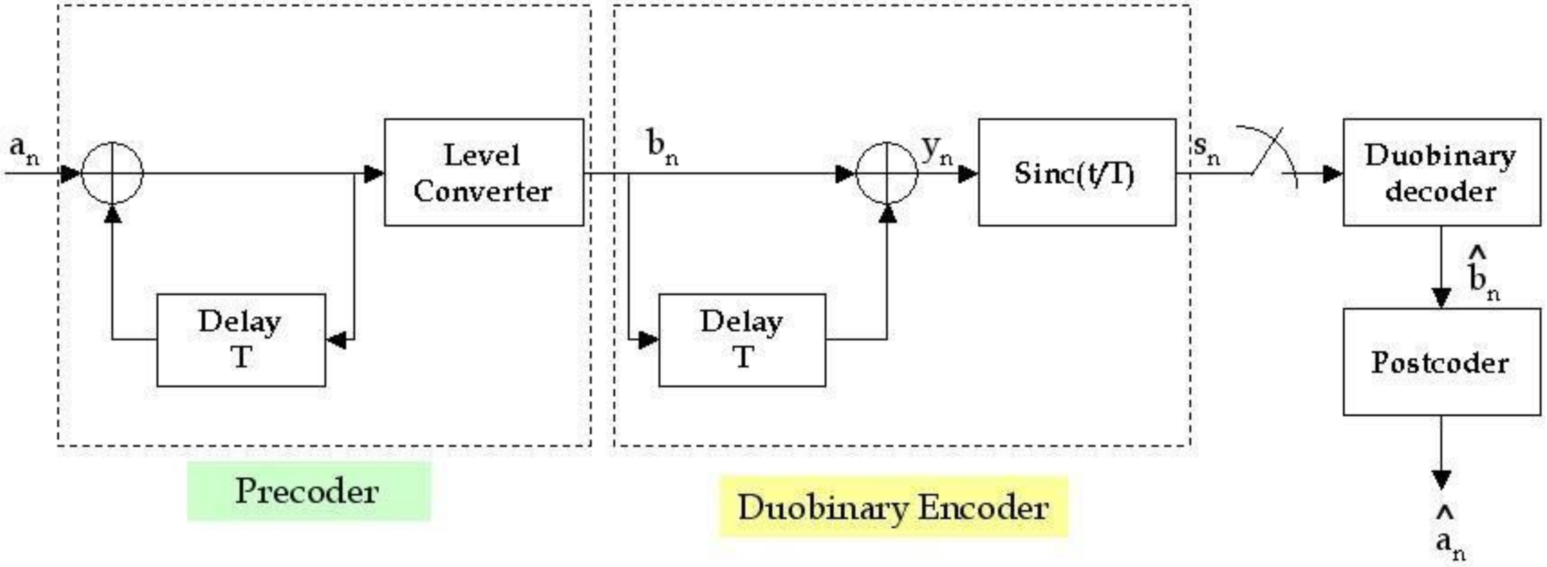
$$h(t) = 2W_0(\mathrm{sinc}(2W_0 t))\,\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

$\boxed{\textbf{Excess bandwidth: } \boxed{W - W_0}}$

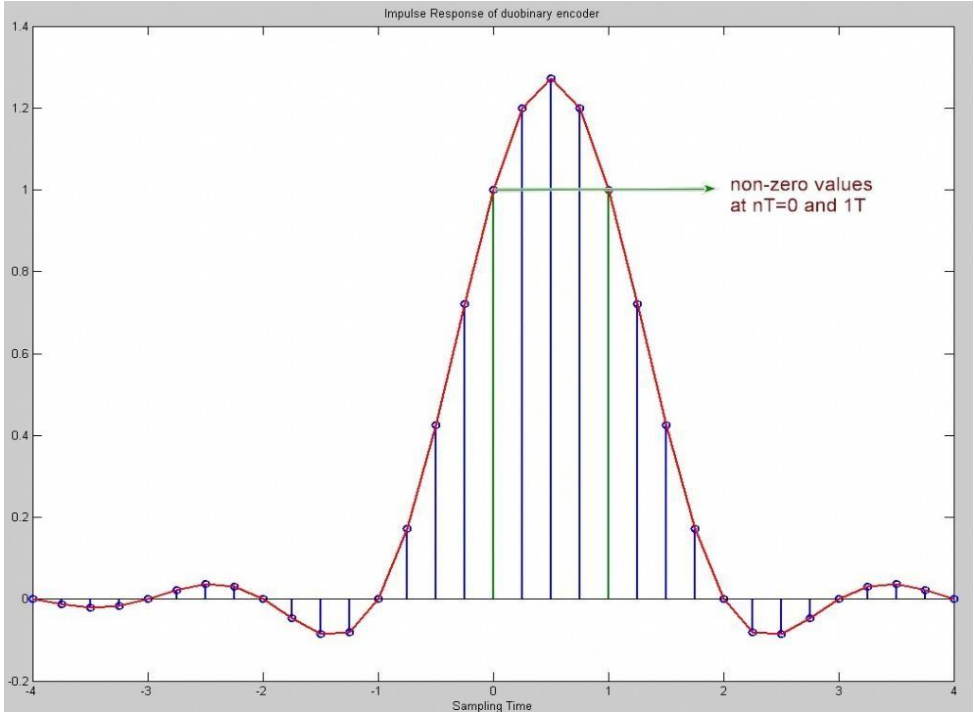
Roll-off factor $r = \frac{W - W}{}$

$0 \le r \le 1$

Correlative Coding – DUO



Impulse Response of Duobinary Encoder



Encoding Process

1) a_n = binary input bit; $a_n \in \{0,1\}$.

2) b_n = NRZ polar output of Level converter in the precoder and is given by,

bn={−d,if an=0+d,if an=1

3) y_n can be represented as

The duobinary encoding correlates present sample a_n and the previous input sample a_{n-1}.

Decoding Process

The receiver consists of a duobinary decoder and a postcoder.

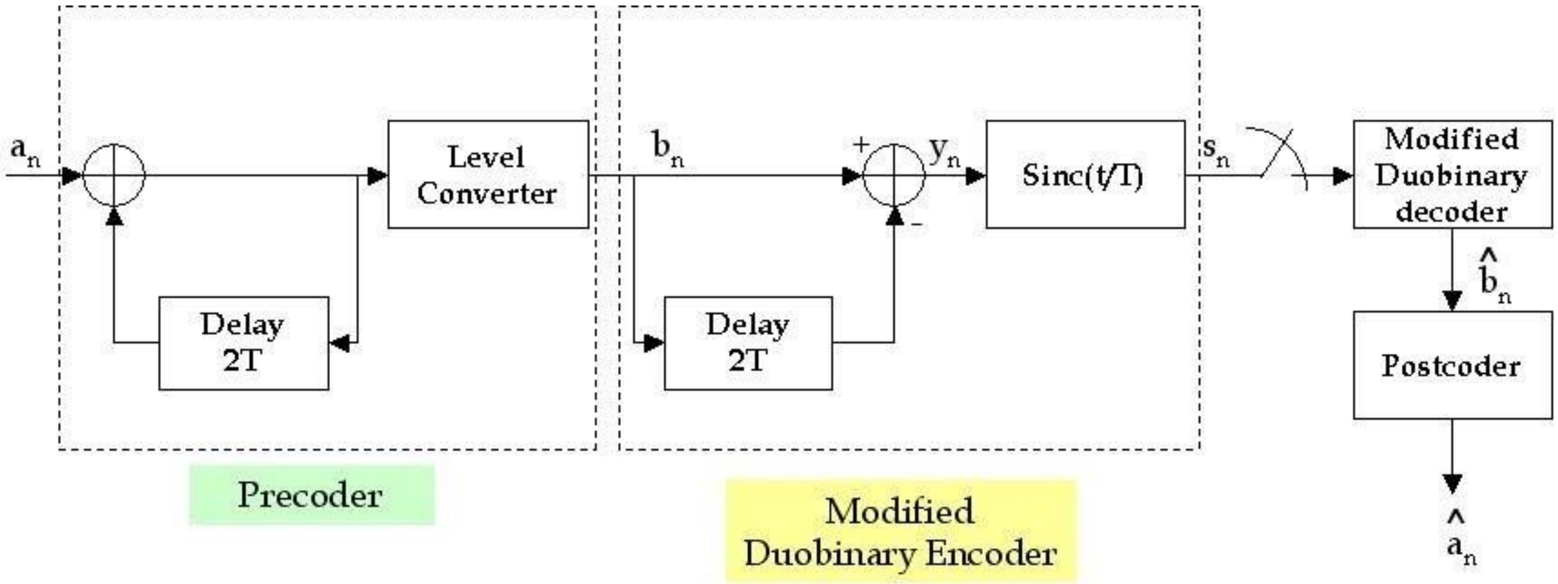
$\hat{b}_n = y_n - \hat{b}_{n-1}$

This equation indicates that the decoding process is prone to error propagation as the estimate of present sample relies on the estimate of previous sample.

This error propagation is avoided by using a precoder before duobinary encoder at the transmitter and a postcoder after the duobinary decoder.

The precoder ties the present sample and previous sample and the postcoder does the reverse process.

Correlative Coding – Modified Duobinary Signaling



Modified Duobinary Signaling is an extension of duobinary signaling.

Modified Duobinary signaling has the advantage of zero PSD at low frequencies which is suitable for channels with poor DC response.

It correlates two symbols that are 2T time instants apart, whereas in duobinary signaling, symbols that are 1T apart are correlated.

The general condition to achieve zero ISI is given by

p(nT)={1,n=00,n≠0

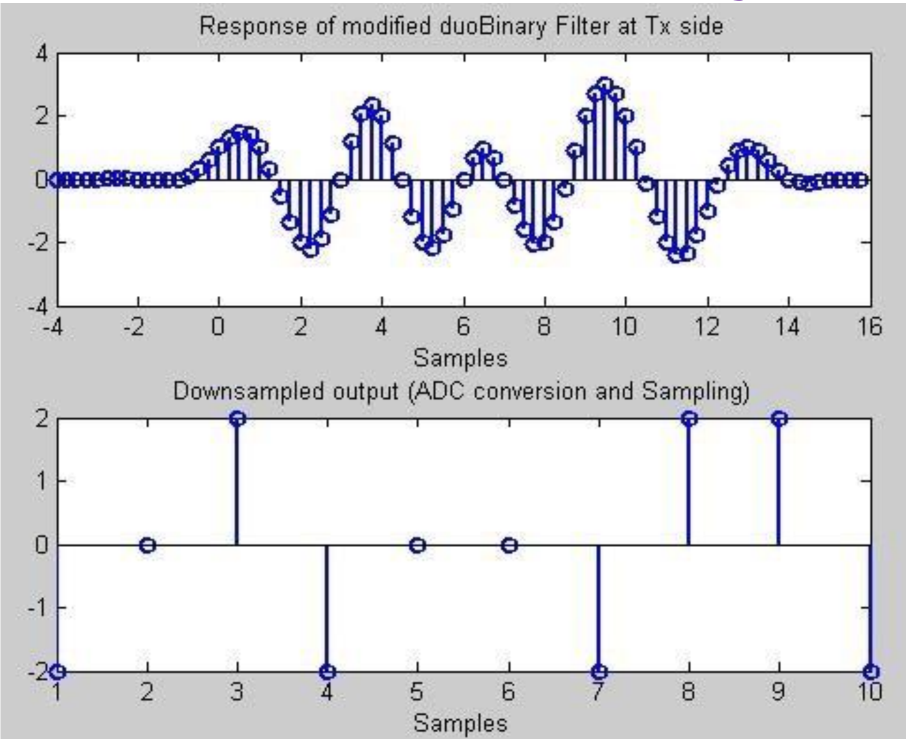
In the case of modified duobinary signaling, the above equation is modified as

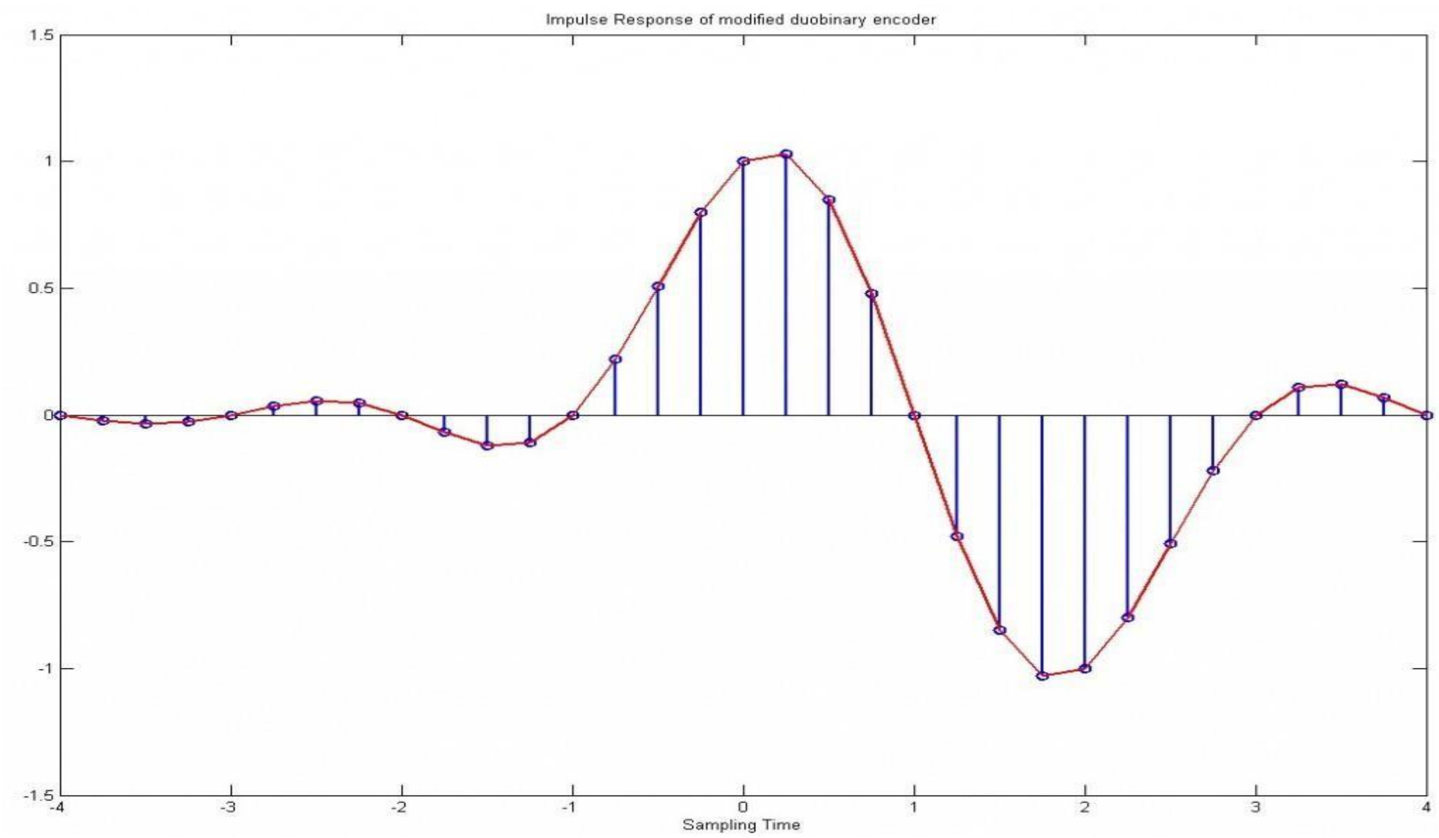
p(nT)={1,n=0,20,otherwise

which states that the ISI is limited to two alternate samples.

Here a controlled or “deterministic” amount of ISI is introduced and hence its effect can be removed upon signal detection at the receiver.

Impulse Response Of A Modified Duobinary Encoder

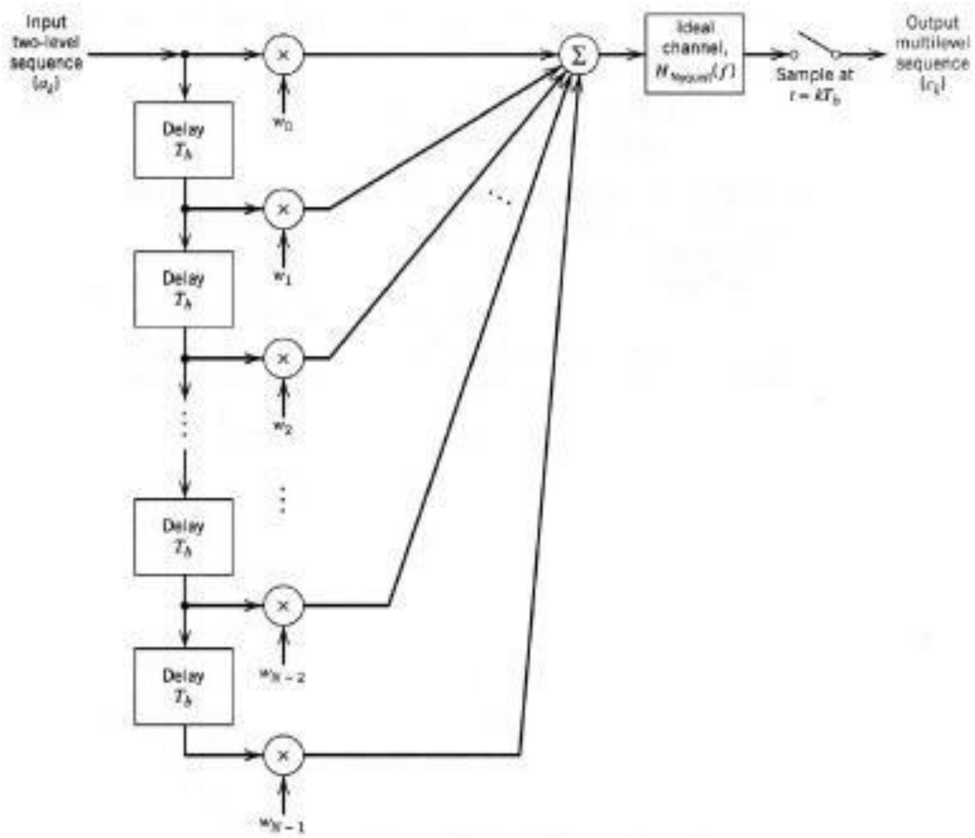




Impulse Response of modified duobinary encoder
1.5
1
0.5
0
-0.5
-1
-1.5
-4
-3
-2
-1
0
1
2
3
4
Sampling Time

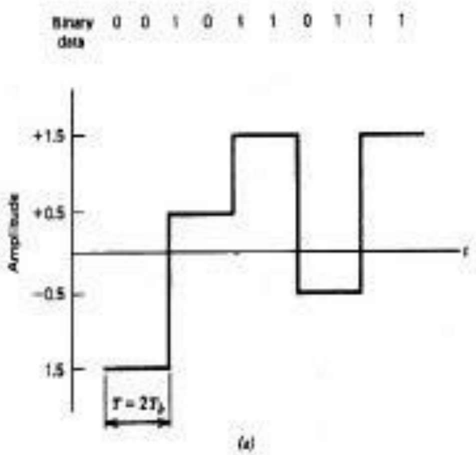
Partial Response Signalling

- Generalized form of correlative-level coding (partial response signaling)



Generalized correlative coding scheme.

Baseband M-ary PAM



Dibit	Amplitude
00	−1.5
01	−0.5
10	+0.5
11	+1.5

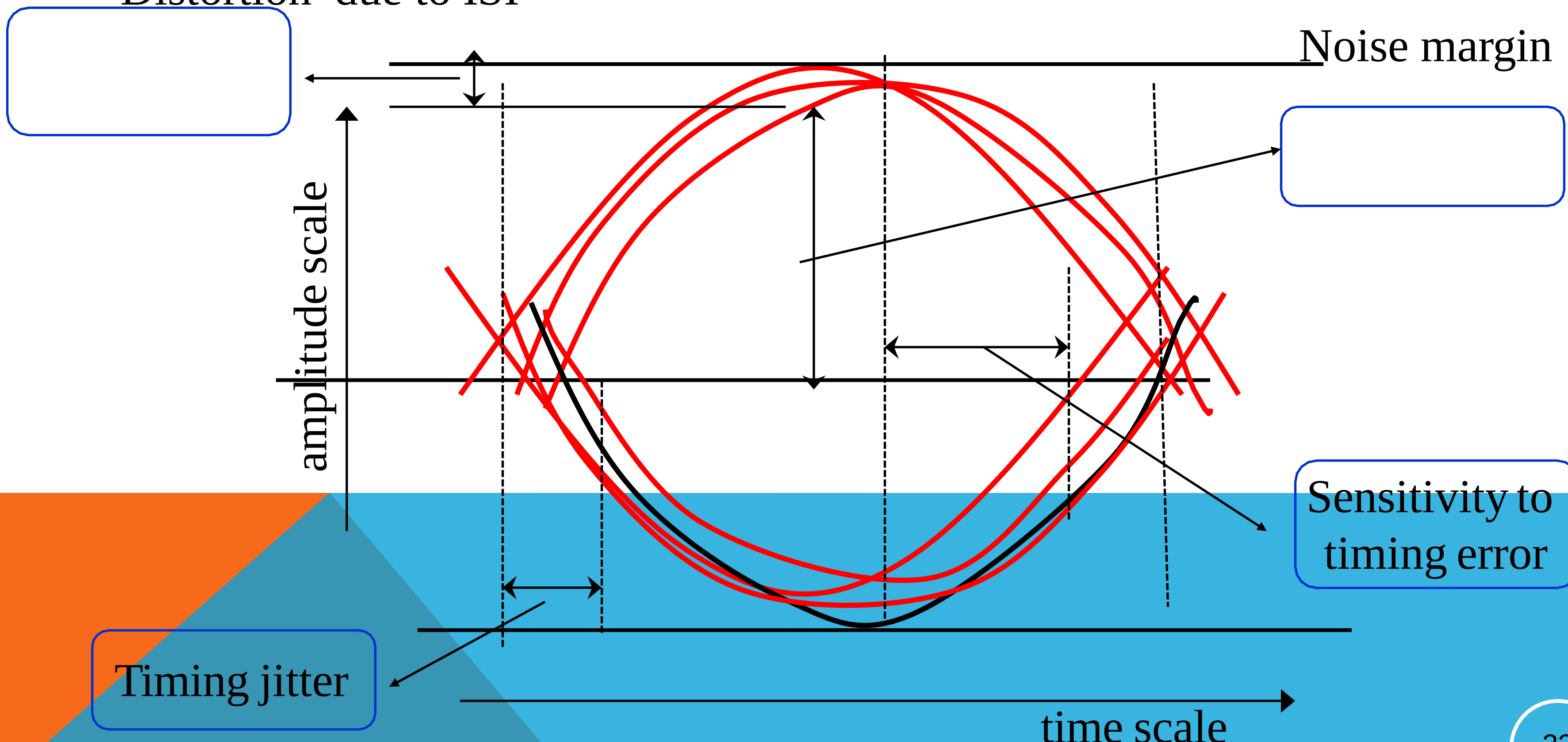
(b)

Output of a quaternary system. (a) Waveform. (b) Representation of the 4 possible dibits.

- Produce one of M possible amplitude level
- T : symbol duration
- 1/T: signaling rate, symbol per second, bauds
  - Equal to $\log_2 M$ bit per second
- T_b : bit duration of equivalent binary PAM : $T = T_b \log_2 M$
- To realize the same average probability of symbol error, transmitted power must be increased by a factor of $M^2/\log_2 M$ compared to binary PAM

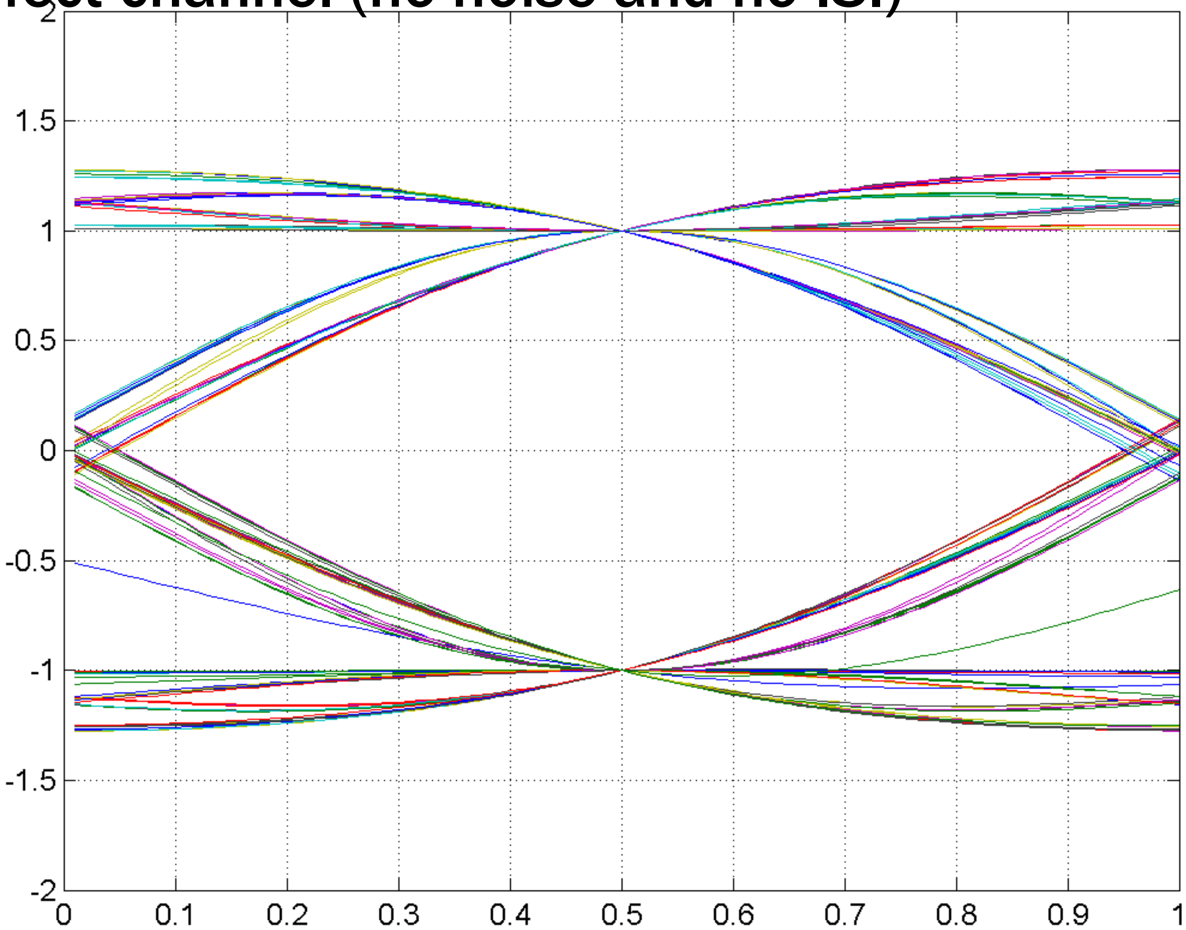
Eye Pattern

Eye pattern: **Display on an oscilloscope which sweeps the system response to a baseband signal at the rate *1/T* (*T* symbol duration)**

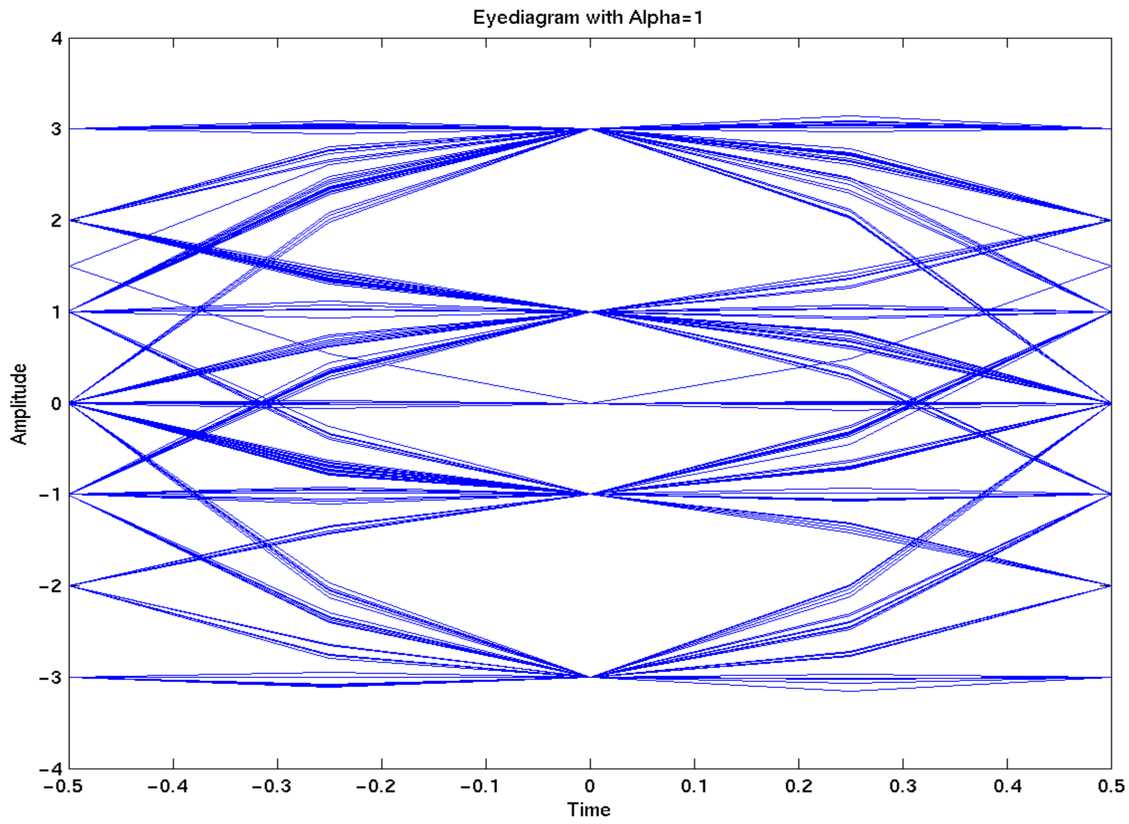


Example Of Eye Pattern:
BINARY-PAM, SRRC PULSE

Perfect channel (no noise and no ISI)



Eye Diagram For 4-PAM



VEMU INSTITUTE OF TECHNOLOGY
(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
P.Kothakota, Puthalapattu (M), Chittoor Dist – 517 112 AP, India

Department of Electronics and Communication Engineering

Digital Communication Systems

Unit-3
Signal Space Analysis

By
B SAROJA
Associate Professor
Dept. of . ECE

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Introduction

Geometric representation of signals

Gram-Schmidt orthogonalization procedure

Conversion of the Continuous AWGN channel into a vector channel Coherent detection of signals in noise

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Probability of error

Signal constellation diagram

Introduction:signal Space

What is a signal space?

- Vector representations of signals in an N-dimensional orthogonal space

Why do we need a signal space?

- It is a means to convert signals to vectors and vice versa.
- It is a means to calculate signals energy and Euclidean distances between signals.

Why are we interested in Euclidean distances between signals?

- For detection purposes: The received signal is transformed to a received vectors.
- The signal which has the minimum distance to the received signal
is estimated as the transmitted signal.

Transmitter takes the *symbol (data)* m_i (digital message source output) and encodes it into a *distinct signal* $s_i(t)$.

The *signal* $s_i(t)$ occupies the whole *slot T* allotted to *symbol* m_i.

$s_i(t)$ is a real valued energy signal (???)

$$E_i = \int_0^T s_i^2(t)dt, \quad i=1,2,....,M \qquad (5.2)$$

Transmitter takes the ***symbol (data)*** $\boldsymbol{m_i}$ **(digital message source output) and encodes it into a** ***distinct signal*** $\boldsymbol{s_i(t)}$**.**

The ***signal*** $\boldsymbol{s_i(t)}$ **occupies the whole** ***slot T*** **allotted to** ***symbol*** $\boldsymbol{m_i}$**.**

$\boldsymbol{s_i(t)}$ **is a real valued energy signal (*****signal with finite energy*****)**

$$\mathrm{E_i} = \int_0^T s_i^2(t)\,dt, \quad \mathrm{i=1,2,....,M} \qquad (5.2)$$

Geometric Representation of Signals

<u>Objective</u>: To represent any set of *M* *energy signals* *$\{s_i(t)\}$* as linear combinations of *N* *orthogonal basis functions*, where N ≤ M

Real valued energy signals $s_1(t)$, $s_2(t)$,..$s_M(t)$, each of duration T sec

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{Bmatrix} 0 \le t \le T \\ i==1,2,....,M \end{Bmatrix} \qquad (5.5)$$

Orthogonal basis function

coefficient

Energy signal

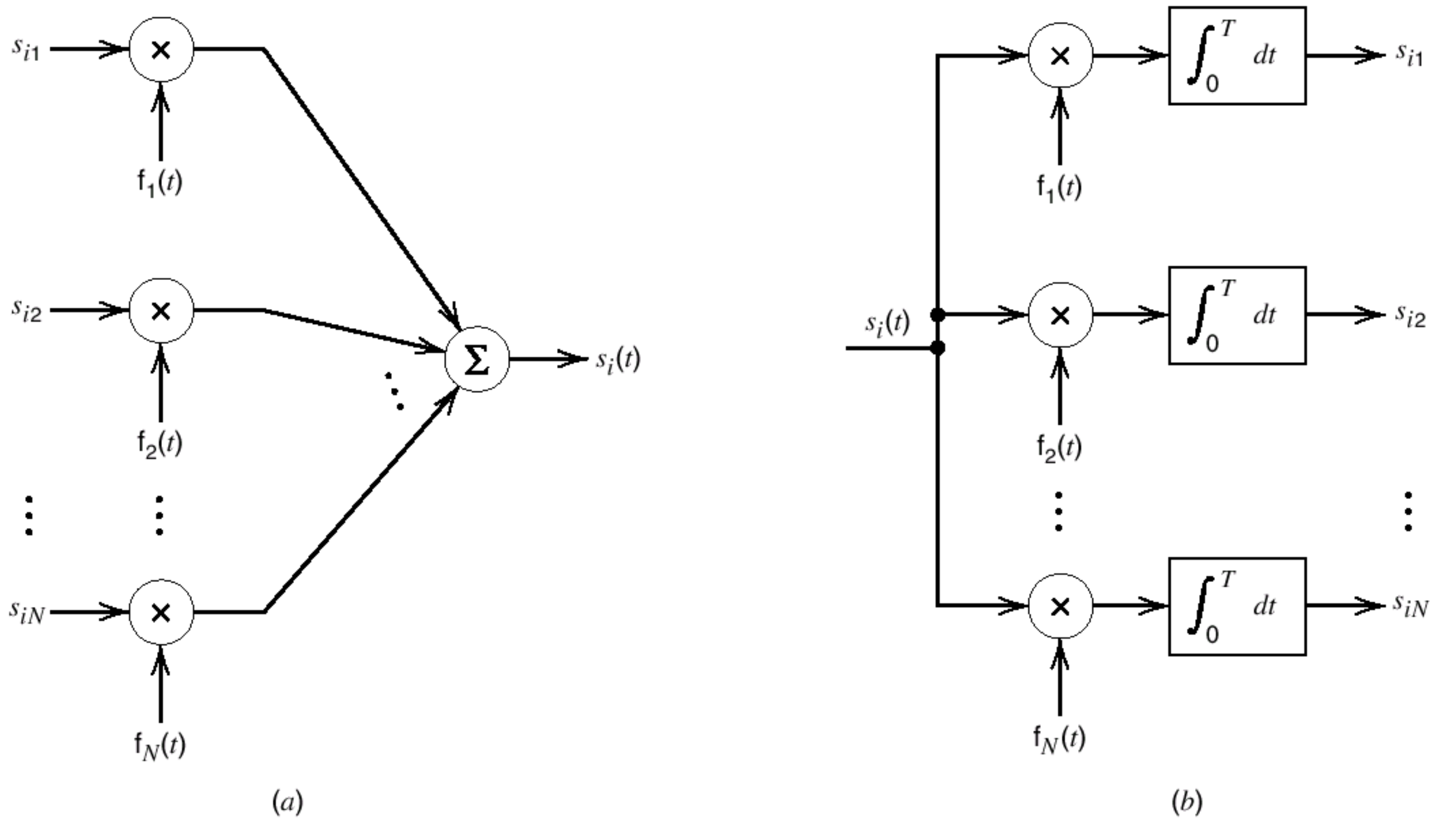
Coefficients:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \qquad \begin{Bmatrix} \text{i=1,2,....,M} \\ \text{j=1,2,....,M} \end{Bmatrix} \quad (5.6)$$

Real-valued basis functions:

$$\int_0^T \phi_\mathrm{i}(t)\phi_j(t)dt = \delta_{ij} = \begin{cases} 1 \ \text{ if } i = j \\ 0 \ \text{ if } i \neq j \end{cases} \quad (5.7)$$

A) SYNTHESIZER FOR GENERATING THE SIGNAL $S_I(T)$.
B) ANALYZER FOR GENERATING THE SET OF SIGNAL VECTORS $\{S_I\}$.



Each signal in the set $s_i(t)$ is completely determined by the vector of its coefficients

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \cdot \\ \cdot \\ \cdot \\ s_{iN} \end{bmatrix}, \qquad i = 1,2,\ldots,M \qquad (5.8)$$

The signal vector s_i concept can be extended to 2D, 3D etc. N-dimensional Euclidian space

Provides mathematical basis for the geometric representation of energy signals that is used in noise analysis

Allows definition of

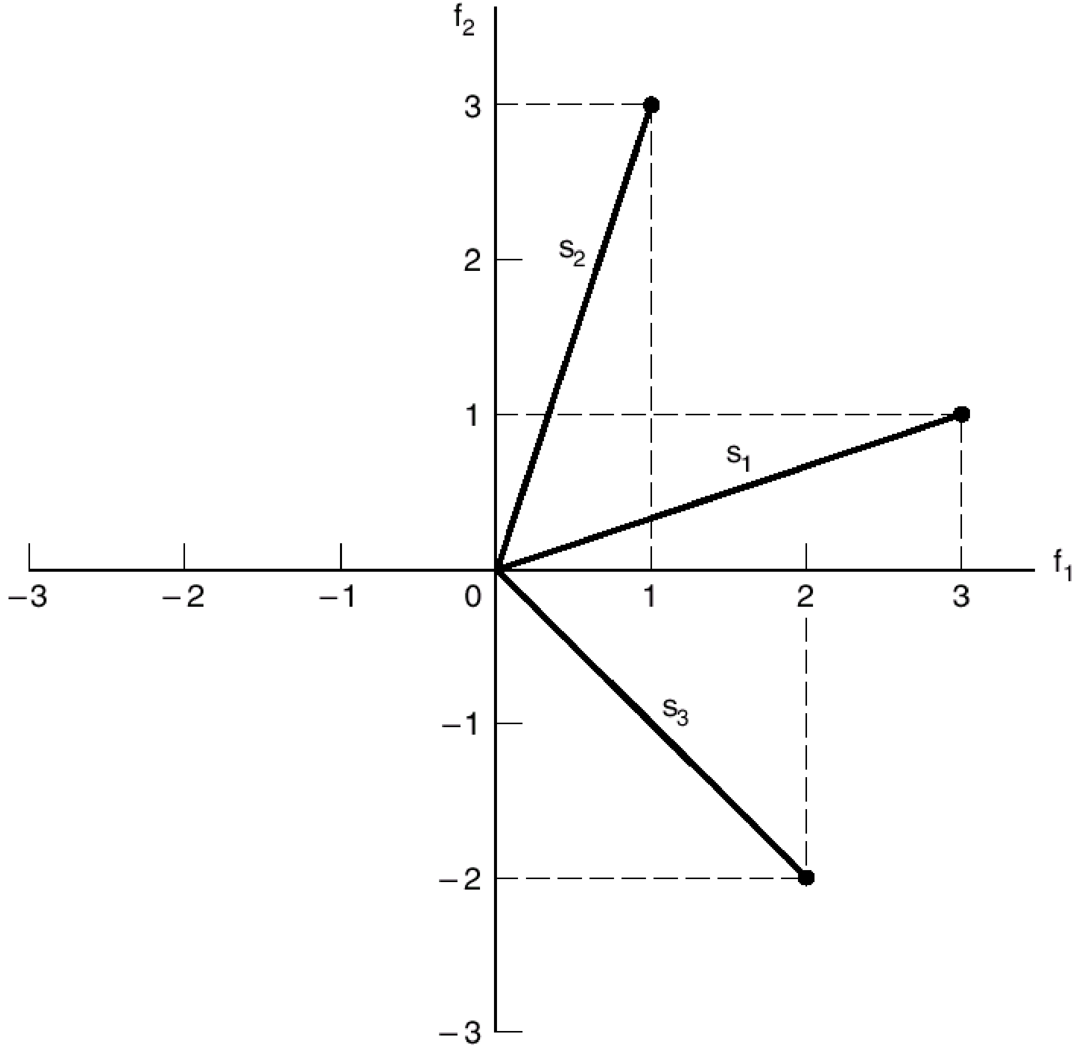
- Length of vectors (absolute value)
- Angles between vectors
- Squared value (inner product of s_i with itself)

$$\|\mathbf{s}_i\|^2 = \mathbf{s}_i^T \mathbf{s}_i$$

Matrix Transposition

$$= \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1,2,\ldots.,M \qquad (5.9)$$

ILLUSTRATING THE GEOMETRIC REPRESENTATION SIGNALS FOR THI WHEN N = 2 AND (TWO DIMENSION SPACE, THREE SIGNALS)



What is the relation between the *vector representation* of a signal and its *energy value?*

…start with the definition of average energy in a signal…(5.10)

$$\mathrm{E_i} = \int_0^T s_i^2(t)\,dt \qquad (5.10)$$

Where $s_i(t)$ is as in (5.5):

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad (5.5)$$

After substitution:

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t) \right] \left[\sum_{k=1}^{N} s_{ik}\phi_k(t) \right] dt$$

After regrouping:

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij} s_{ik} \int_0^T \phi_j(t)\phi_k(t)\,dt \qquad (5.11)$$

$\Phi_j(t)$ is orthogonal, so finally we have:

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2 \qquad (5.12)$$

The *energy of a signal* is equal to the *squared length of its vector*

Formulas for Two Signals

Assume we have a pair of signals: $s_i(t)$ and $s_j(t)$, each represented by its vector,

Then:

$$s_{ij} = \int_0^T s_i(t) s_k(t)\,dt = s_i^T \mathbf{s}_k \qquad (5.13)$$

Inner product of the signals is equal to the inner product of their vector representations [0,T]

Inner product is invariant to the selection of basis functions

Euclidian Distance

The Euclidean distance between two points represented by vectors (signal vectors) is equal to

$||s_i - s_k||$ and the squared value is given by:

$$\|s_i - s_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2 \qquad (5.14)$$

$$= \int_0^T (s_i(t) - s_k(t))^2 \, dt$$

ANGLE BETWEEN TWO SIGNALS

The *cosine of the angle Θ_{ik}* between two signal vectors s_i and s_k is equal to the inner product of these two vectors, divided by the product of their norms:

$$\cos\theta_{ik} = \frac{s_i^T s_k}{\|s_i\| \|s_k\|} \qquad (5.15)$$

So the two signal vectors are *orthogonal* if their inner product $s_i^T s_k$ is zero ($\cos \Theta_{ik} = 0$)

Schwartz Inequality

Defined as:

$$\left(\int_{-\infty}^{\infty} s_1(t)s_2(t)dt\right)^2 = \left(\int_{-\infty}^{\infty} s_1^2(t)dt\right)\left(\int_{-\infty}^{\infty} s_2^2(t)dt\right) \quad (5.16)$$

accept without proof…

Gram-schmidt Orthogonalization Procedure

Assume a set of M energy signals denoted by $s_1(t)$, $s_2(t)$, .. , $s_M(t)$.

1. Define the first basis function starting with s_1 as: (where E is the energy of the signal) (based on 5.12)	$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$ (5.19)
2. Then express $s_1(t)$ using the basis function and an energy related coefficient s_{11} as:	$s_1(t) = \sqrt{E_1}\phi_1(t) = s_{11}\phi_1(t)$ (5.20)
3. Later using s_2 define the coefficient s_{21} as:	$s_{21} = \int_0^T s_2(t)\phi_1(t)dt$ (5.21)

4. If we introduce the intermediate function g_2 as:

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \quad (5.22)$$

Orthogonal to $\varphi_1(t)$

5. We can define the second basis function φ_2 (t) as:

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)dt}} \qquad (5.23)$$

6. Which after substitution of g_2(t) using s_1(t) and s_2(t) it becomes:

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \qquad (5.24)$$

$$\int_0^T \phi_2^2(t)dt = 1 \qquad \text{(Look at 5.23)}$$

Note that φ_1(t) and φ_2(t) are orthogonal that means:

$$\int_0^T \phi_1(t)\phi_2(t)dt = 0$$

AND SO ON FOR N DIMENSIONAL SPACE…,

**In general a basis function can be defined using the following
formula:**

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij} \text{-} \phi_j(t) \qquad (5.25)$$

- where the coefficients can be defined using:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \quad , \qquad j = 1, 2, \ldots\ldots, i-1 \qquad (5.26)$$

Special Case:

For the special case of i = 1 $g_i(t)$ reduces to $s_i(t)$.

General case:

•Given a function $g_i(t)$ we can define a set of basis functions, which form an orthogonal set, as:

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}} \quad , \qquad i = 1, 2, \ldots.., N \qquad (5.27)$$

Conversion of the Continuous AW Channel into a Vector Channel

Suppose that the $s_i(t)$ is not any signal, but specifically the signal at the receiver side, defined in accordance with an AWGN channel:

$$x(t) = s_i(t) + w(t$$

$$\begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases}$$

So the output of the correlator (Fig. can be defined as:

$$x_i = \int_0^T x(t)\phi_j(t$$

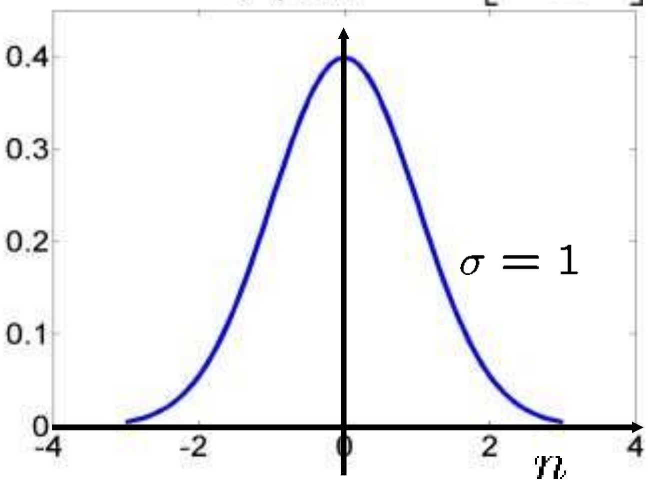
$$= s_{ij} + w_i,$$

$$j = 1, 2, \ldots, N$$

Additive White Gaussian Noise (AWGN)

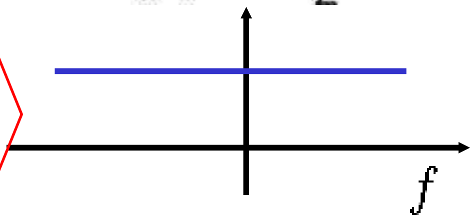
- Thermal noise is described by a zero-mean **Gaussian** random process, n(t) that ADDS on to the signal => “**additive**”
- Its <u>PSD is flat, </u>hence, it is called <u>*white noise*</u>.
  - Autocorrelation is a spike at 0: uncorrelated at any non-zero lag

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{n^2}{2\sigma^2}\right]$$



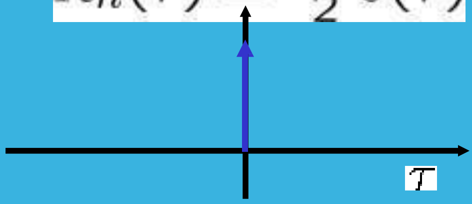
Probability density function (gaussian)

$$G_n(f) = \frac{N_0}{2} \text{ [w/Hz]}$$



Power spectral Density (flat => “white”)

$$R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$



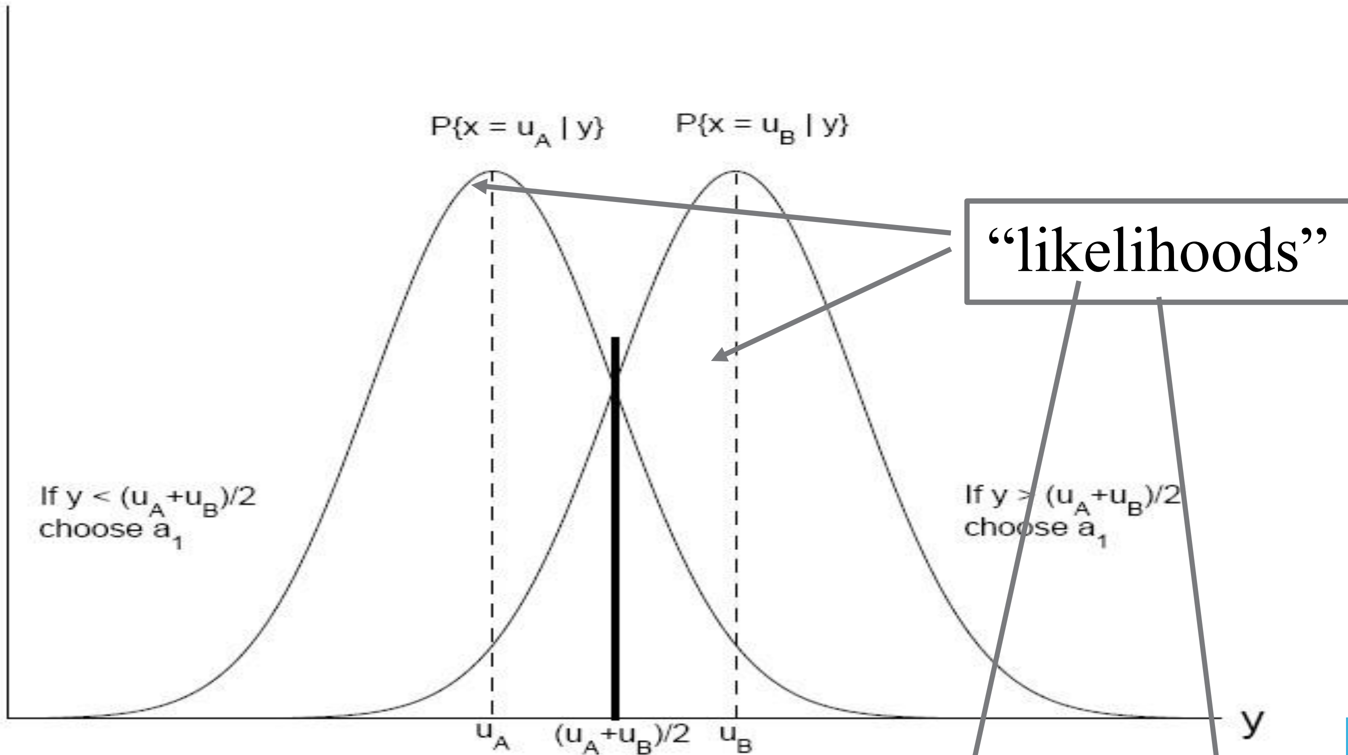
Autocorrelation Function (uncorrelated)

Finally,

- The AWGN channel, is equivalent to an N-dimensional vector channel, described by the observation vector

$$x = s_i + w, \qquad i = 1, 2, \ldots\ldots, M$$

Coherant Detection of Signals in Noise:



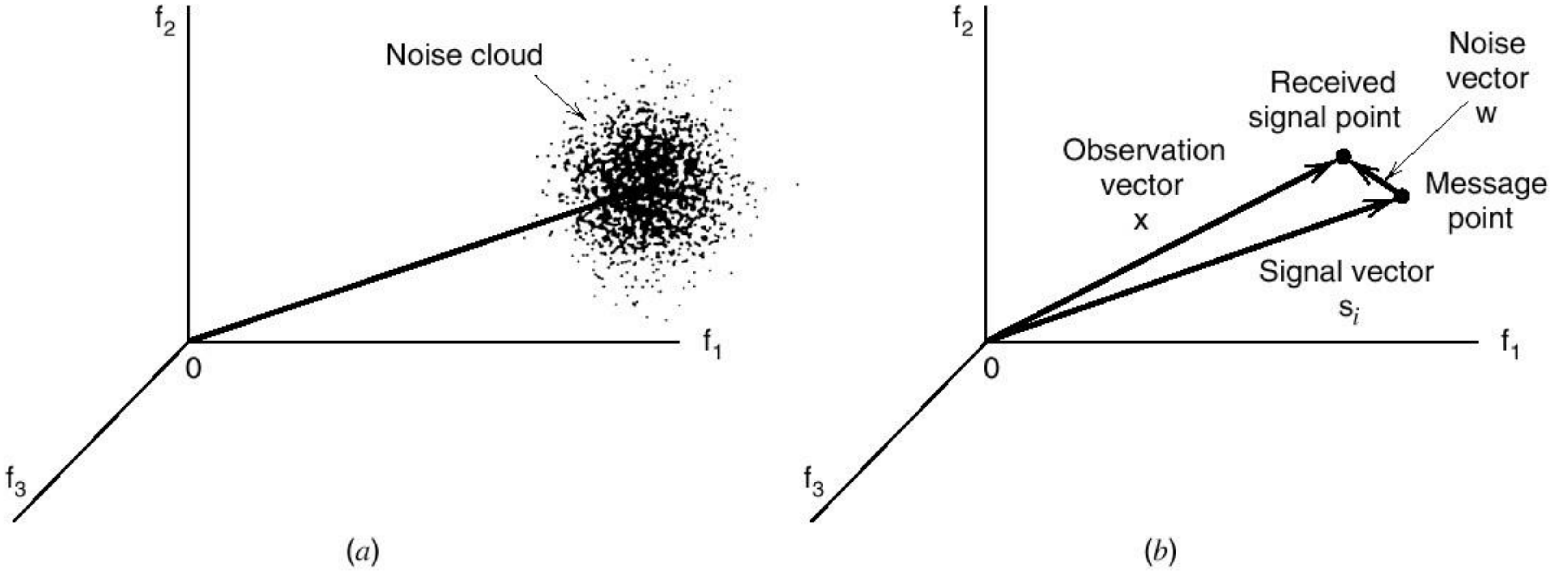
Assuming both symbols equally likely: u_A is chosen if $\mathbb{P}\{u = u_A|y\} \geq \mathbb{P}\{u = u_B|y\}$

Log-Likelihood => $|y - u_A| < |y - u_B|.$ **A simple distance criterion!**

Effect of Noise In Signal Space

The cloud falls off exponentially (gaussian).

Vector viewpoint can be used in signal space, with a random noise vector w



Correlator Receiver

The matched filter output at the <u>sampling time</u>, can be realized as the correlator output.

- Matched filtering, i.e. convolution with $s_i^*(T-\tau)$ simplifies to integration w/ $s_i^*(\tau)$, i.e. correlation or inner product!

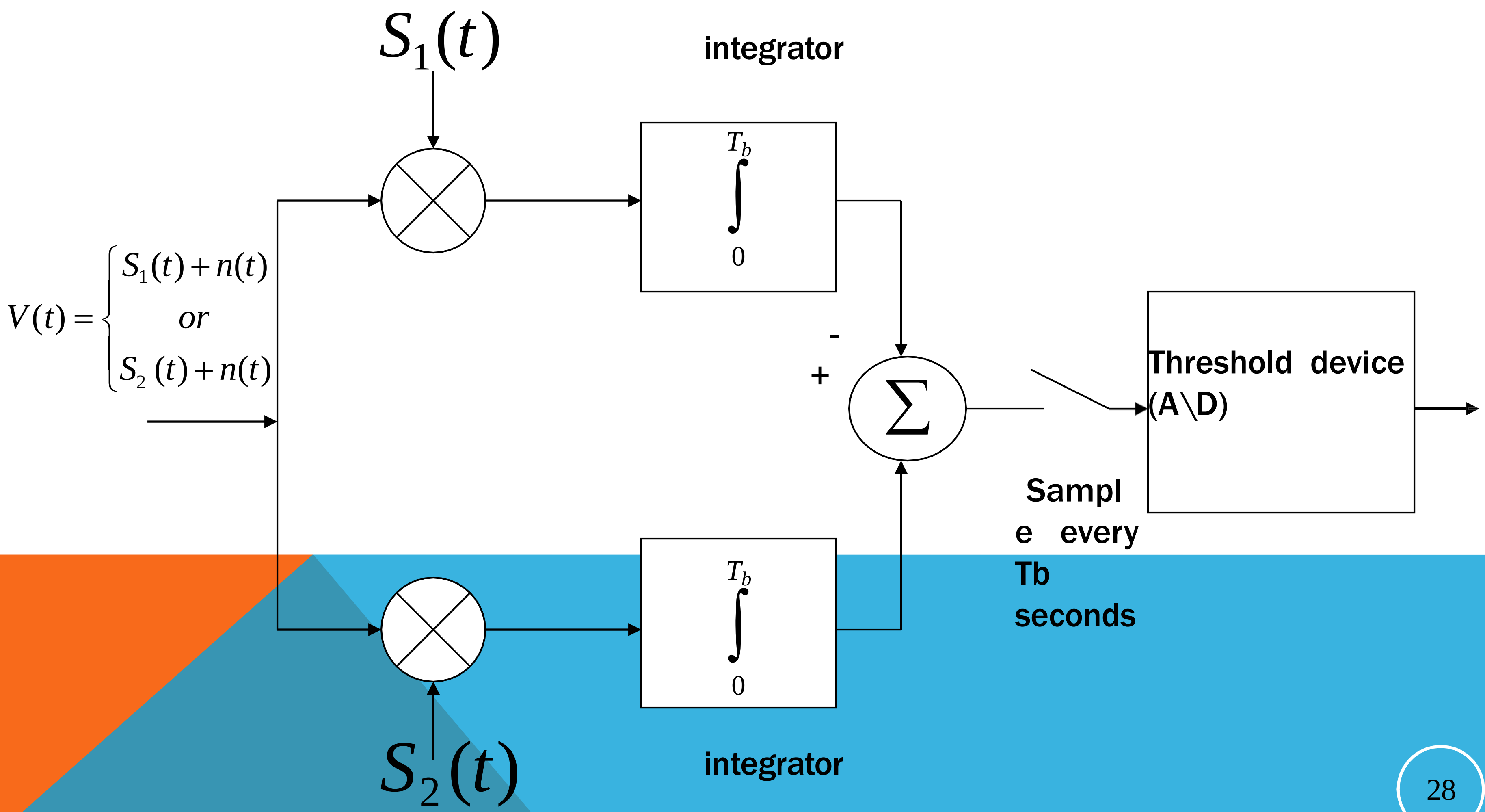
$$z(T) = h_{opt}(T) * r(T)$$

$$= \int_0^T r(\tau)\, s_i^*(\tau)\, d\tau = \langle r(t), s(t) \rangle$$

<u>Recall</u>: correlation operation is the projection of the received signal onto the signal space!

<u>Key idea</u>: Reject the noise (N) outside this space as irrelevant: => maximize S/N

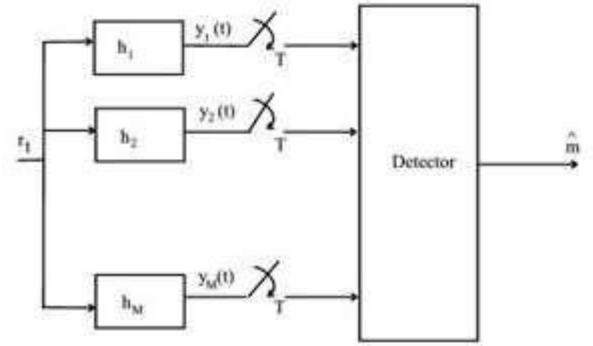
A Correlation Receiver



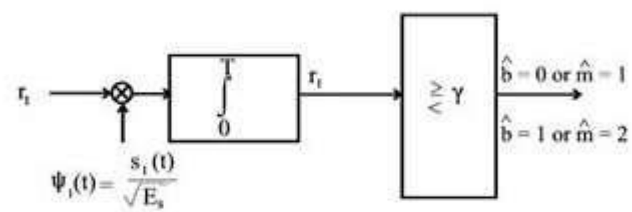
The equivalence of correlation and matched filter receivers

- So we can see that the detector part of the receiver may be implemented using either matched filters or correlators. The output of each correlator is equivalent to the output of a corresponding matched filter when sampled at t = T.

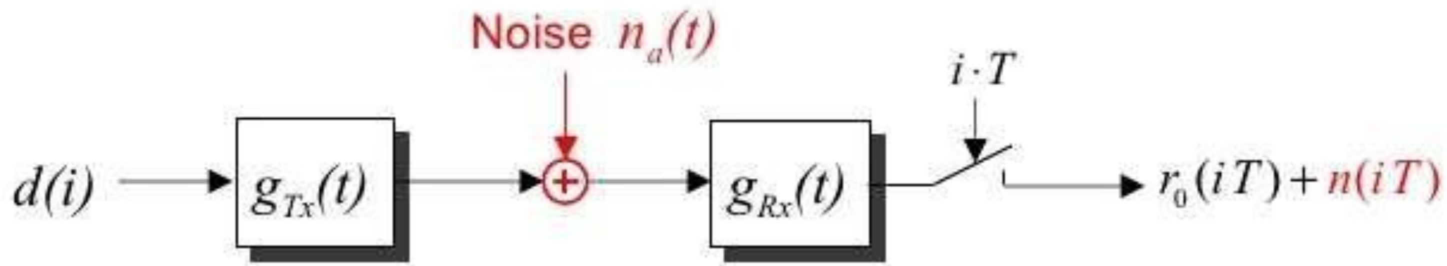
Matched filters



Correlators



Probability Of Error

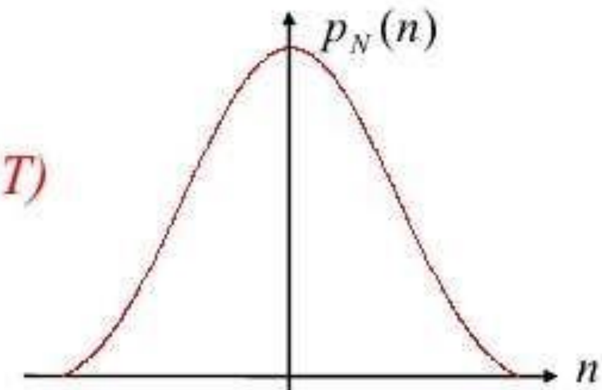


We assume:

- binary transmission with $d(i) \in \{d_0, d_1\}$
- transmission system fulfills 1st Nyquist criterion
- noise $n(iT)$, independent of data source

Probability density function (pdf) of $n(iT)$

Mean and variance

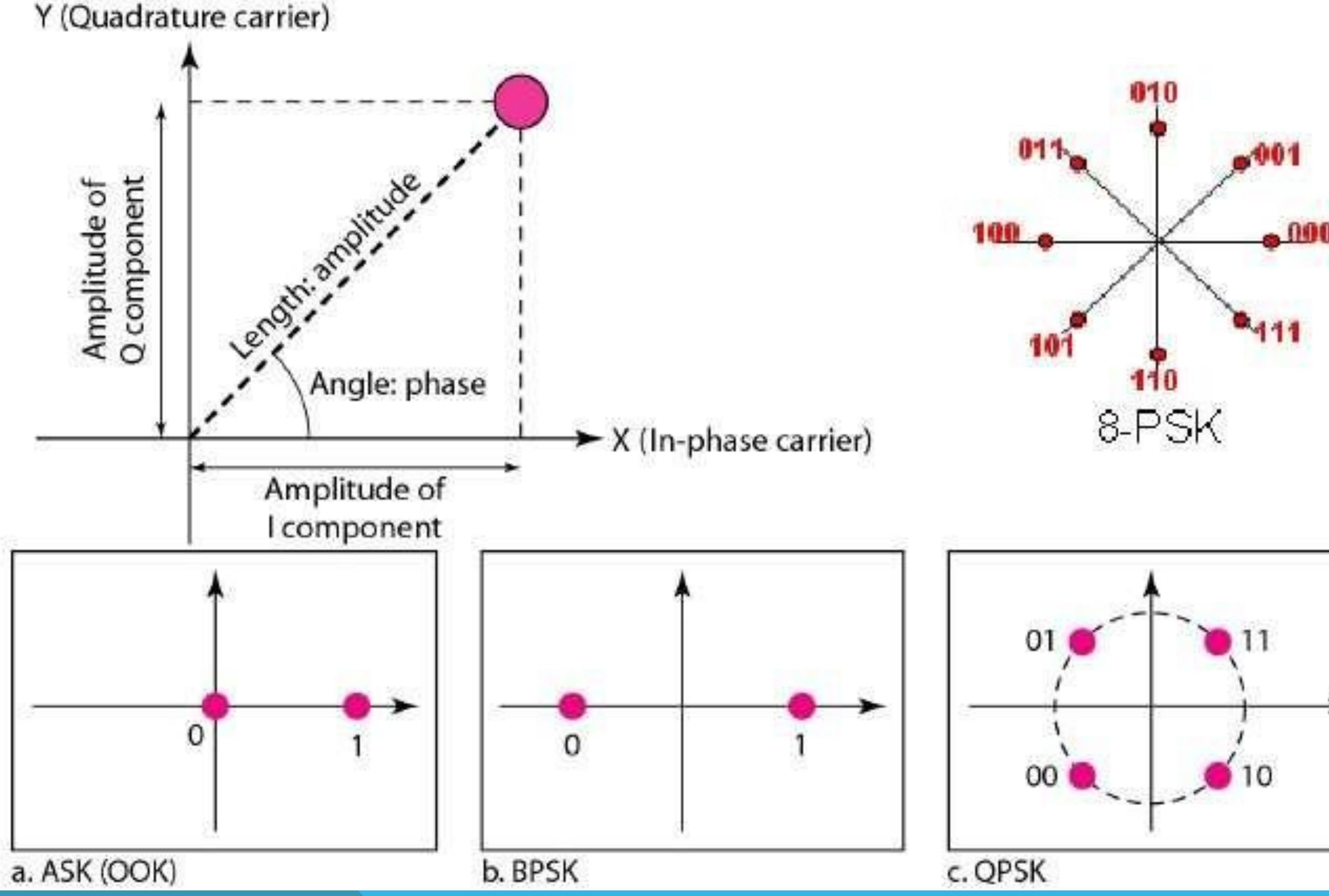


Signal Constellation Diagram

A constellation diagram is a representation of a signal modulated by a digital modulation scheme such as quadrature amplitude modulation or phase-shift keying.

It displays the signal as a two-dimensional xy-plane scatter diagram in the complex plane at symbol sampling instants.

Concept of a constellation diagram



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(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
P.Kothakota, Puthalapattu (M), Chittoor Dist – 517 112 AP, India

Department of Electronics and Communication Engineering

Digital Communication Systems

Unit-4
Passband Transmission Model

By
B SAROJA
Associate Professor Dept. of . ECE

Contents

Introduction, Pass band transmission model Coherent phase-shift keying – BPSK, QPSK Binary Frequency shift keying (BFSK)

Error probabilities of BPSK, QPSK, BFSK

Generation and detection of Coherent PSK, QPSK, & BFSK

Power spectra of above mentioned modulated signals

M-array PSK, M-array QAM

Non-coherent orthogonal modulation schemes -DPSK, BFSK,
Generation and detection of non-coherent BFSK, DPSK

Comparison of power bandwidth requirements for all the above

Introduction:
Baseband Vs Bandpass

Bandpass model of detection process is equivalent to baseband model because:

- The received bandpass waveform is first transformed to a baseband waveform.

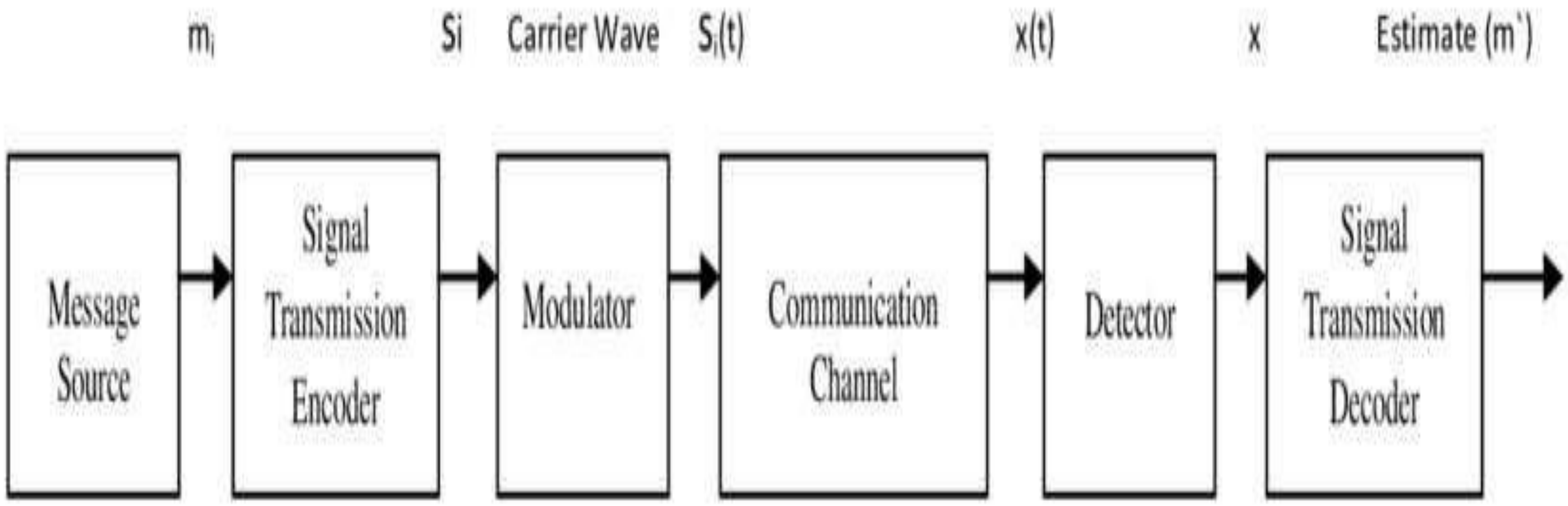
<u>Equivalence theorem:</u>

- Performing bandpass linear signal processing followed by heterodying the signal to the baseband, …

  … yields the same results as …

  … heterodying the bandpass signal to the baseband , followed by a baseband linear signal processing.

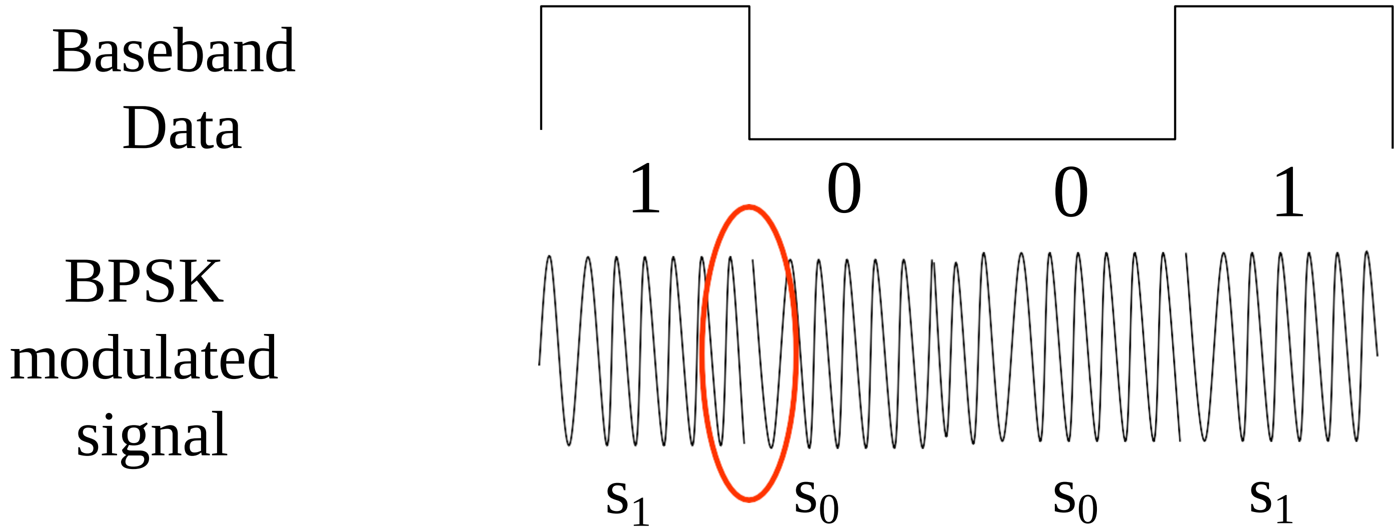
PASSBAND TRANSMISSION MODEL



Types Of Digital Modulation

- Amplitude Shift Keying (ASK)
  - The most basic (binary) form of ASK involves the process of switching the carrier either on or off, in correspondence to a sequence of digital pulses that constitute the information signal. One binary digit is represented by the presence of a carrier, the other binary digit is represented by the absence of a carrier. Frequency remains fixed
- Frequency Shift Keying (FSK)
  - The most basic (binary) form of FSK involves the process of varying the frequency of a carrier wave by choosing one of two frequencies (binary FSK) in correspondence to a sequence of digital pulses that constitute the information signal. Two binary digits are represented by two frequencies around the carrier frequency. Amplitude remains fixed
- Phase Shift Keying (PSK)
  - Another form of digital modulation technique which we will not discuss

BINARY PHASE SHIFT KEYING (PSK)

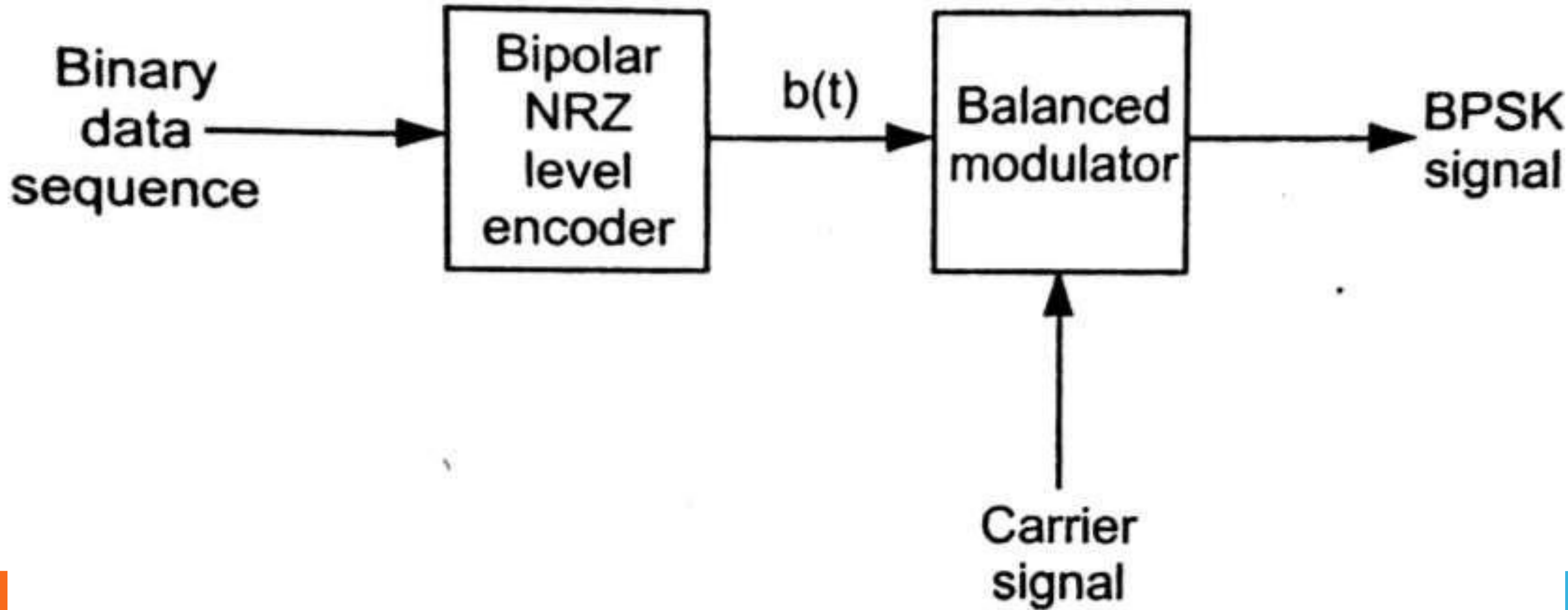


where $s_0 = -A\cos(\omega_c t)$ and $s_1 = A\cos(\omega_c t)$

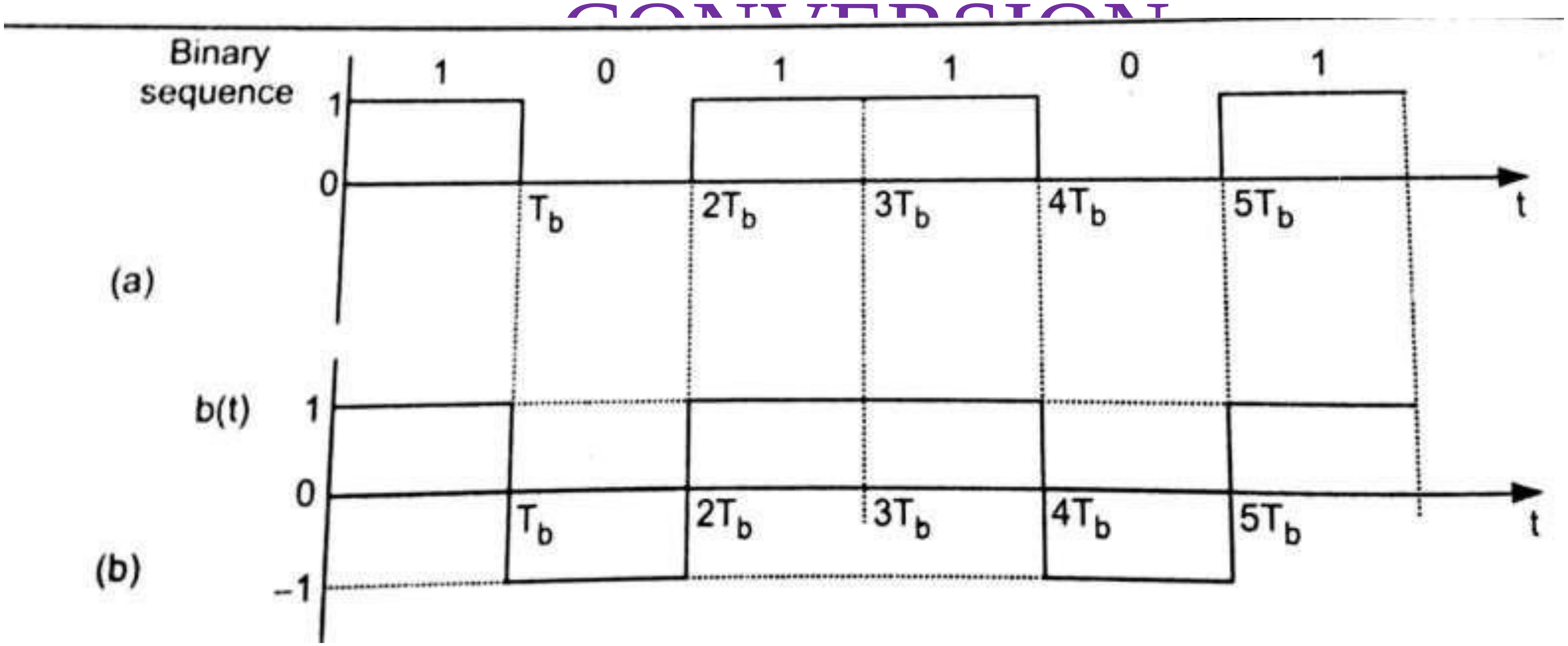
Major drawback – rapid amplitude change between symbols due to phase discontinuity, which requires infinite bandwidth. Binary Phase Shift Keying (BPSK) demonstrates better performance than ASK and BFSK

BPSK can be expanded to a M-ary scheme, employing multiple phases and amplitudes as different states

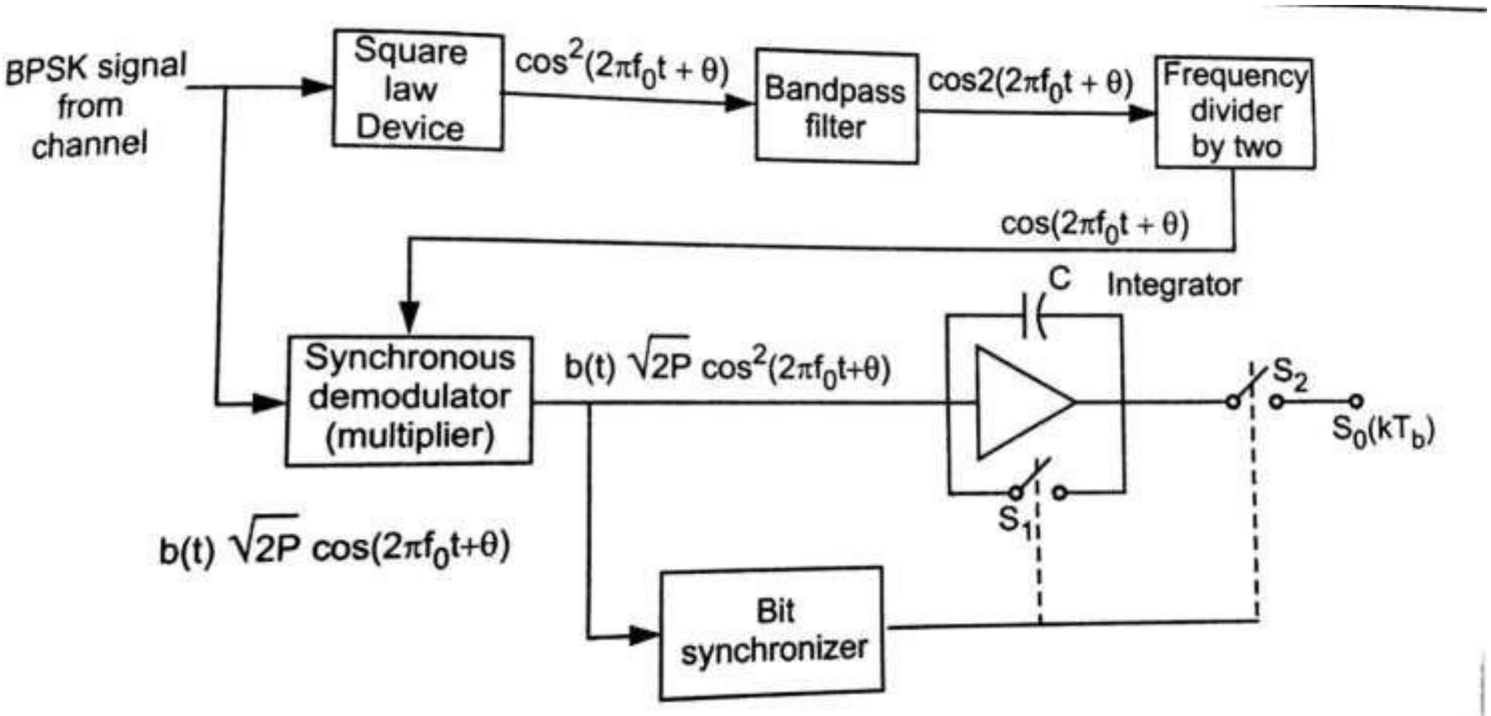
BPSK TRANSMITTER



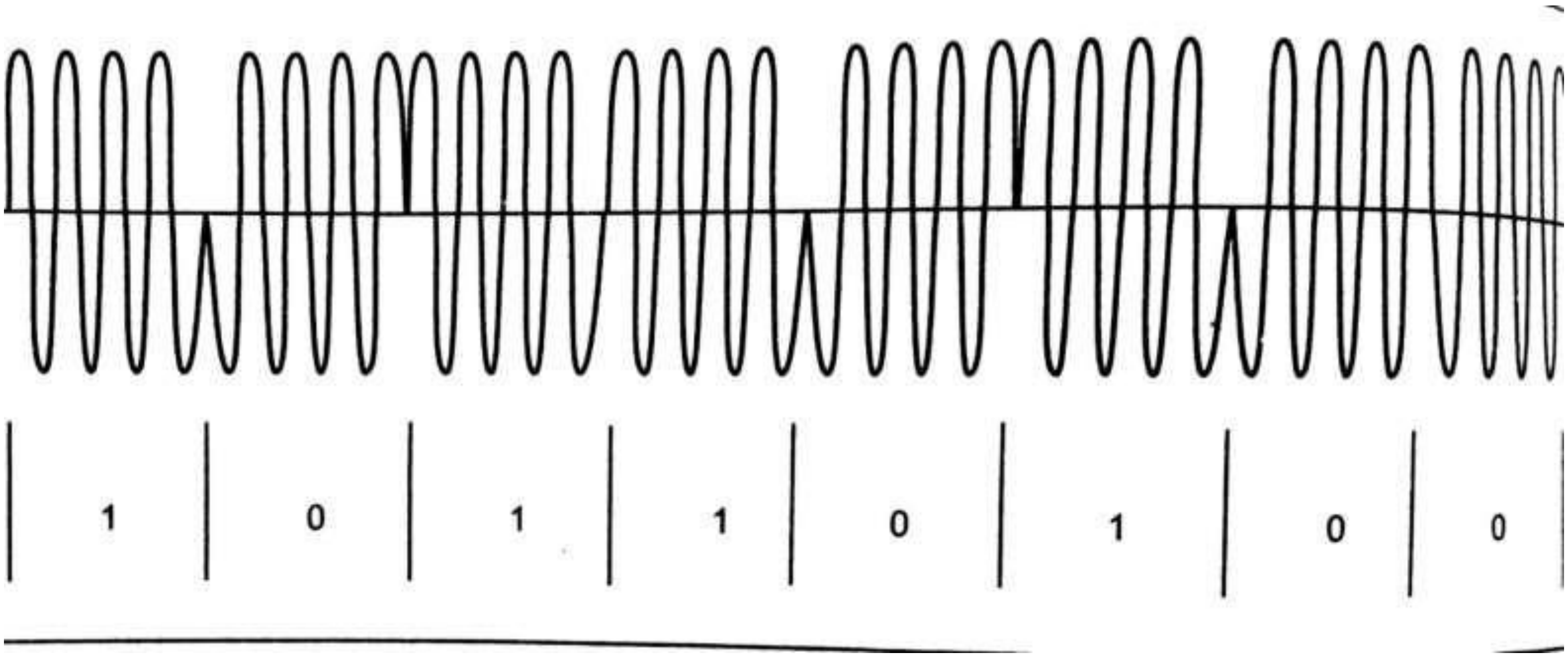
BINARY TO BIPOLAR CONVERSION



COHERENT BPSK RECEIVER



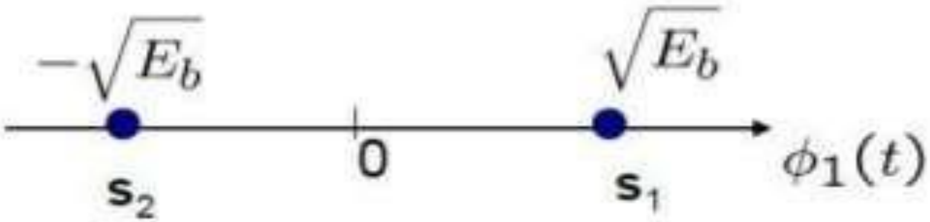
BPSK WAVEFORM



Probability of Error for BPSK

• Dependence of the error probability on the distance between two signal points. In general, is

$$P_e = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)$$



$$d_{12} = 2\sqrt{E_b}$$

• Since the signals $s_1(t)$ and $s_2(t)$ are equally likely to be transmitted, the average probability of error is

$$P_e = 0.5P(e|s_1) + 0.5P(e|s_2) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

QPSK

- **Quadrature Phase Shift Keying (QPSK) can be interpreted as two independent BPSK systems (one on the I-channel and one on Q-channel), and thus the same performance but twice the bandwidth (spectrum) efficiency.**
- **Quadrature Phase Shift Keying has twice the bandwidth efficiency of BPSK since 2 bits are transmitted in a single modulation symbol**
- **Quadrature Phase Shift Keying (QPSK) has twice the bandwidth efficiency of BPSK, since 2 bits are transmitted in a single modulation symbol.**

Sr.No.	Input successive bits		Symbol	Phase shift in carrier
$i = 1$	1 (1V)	0 (−1V)	S_1	$\pi / 4$
$i = 2$	0 (−1V)	0 (−1V)	S_2	$3\pi / 4$
$i = 3$	0 (−1V)	1 (1V)	S_3	$5\pi / 4$
$i = 4$	1 (1V)	1 (1V)	S_4	$7\pi / 4$

Symbol and corresponding phase shifts in QPSK

QPSK

QPSK → Quadrature Phase Shift Keying

- **Four** different phase states in **one** symbol period
- **Two** bits of information in each symbol

Phase: 0 $\pi/2$ π $3\pi/2$ → possible phase values

Symbol: 00 01 11 10

Note that we choose binary representations so an error between two adjacent points in the constellation only results in a single bit error

- For example, decoding a phase to be π instead of $\pi/2$ will result in a "11" when it should have been "01", only one bit in error.

$$s_{QPSK} = \left\{ \sqrt{\frac{2E_s}{T_S}} \cos(2\pi f_c t) \cos[(i-1)\frac{\pi}{2}] - \sqrt{\frac{2E_s}{T_S}} \sin(2\pi f_c t) \sin[(i-1)\frac{\pi}{2}] \right\}$$

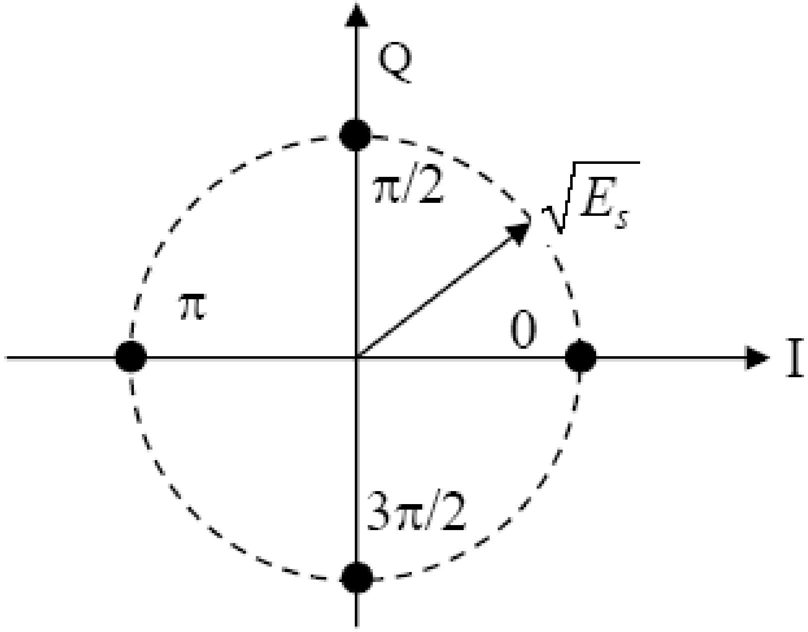
Then we can make the following definition:

$$s_{QPSK} = \left\{ \sqrt{E_s} \cos[(i-1)\frac{\pi}{2}]\phi_I(t) - \sqrt{E_s} \sin[(i-1)]\frac{\pi}{2}\phi_Q(t) \right\} \text{ for } i = 1, 2, 3, 4$$

where $\phi_I(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$ and $\phi_Q(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$

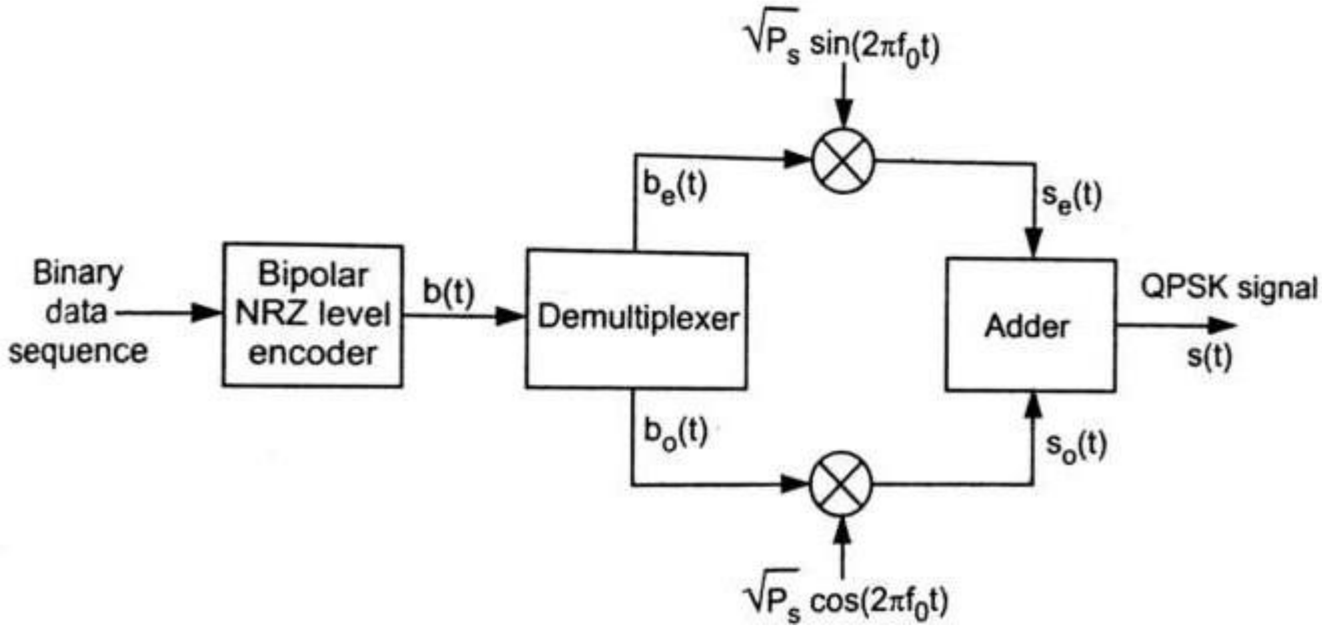
$$E_s = \text{signal energy per symbol}$$

$$= \int_{\substack{\text{over one}\\ \text{symbol time}}} power\ dt = \int_0^{T_s} s^2(t)\, dt$$



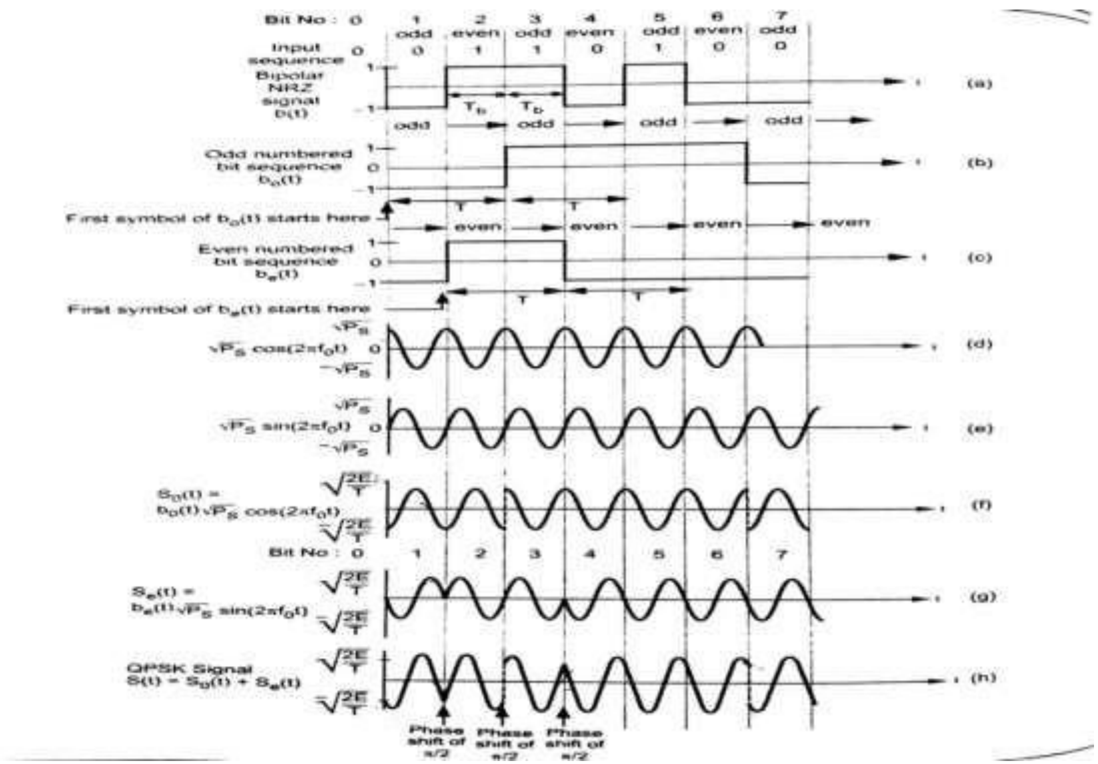
- Now we ha
- $E_s = 2\ E_b$
- I = in-phas
- Q = quadrature component that is $s_Q(t)$.

QPSK Transmitter

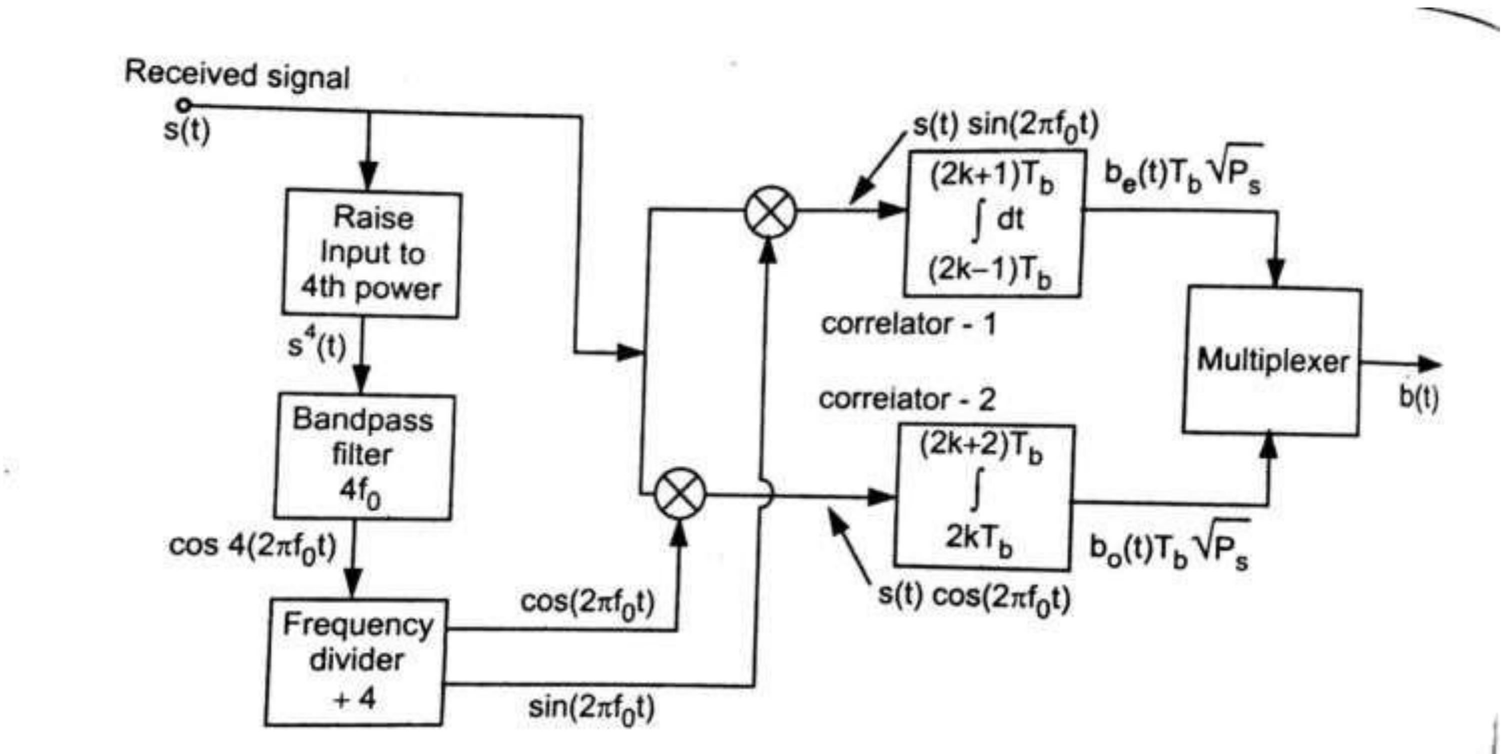


AN OFFSET QPSK TRANSMITTER

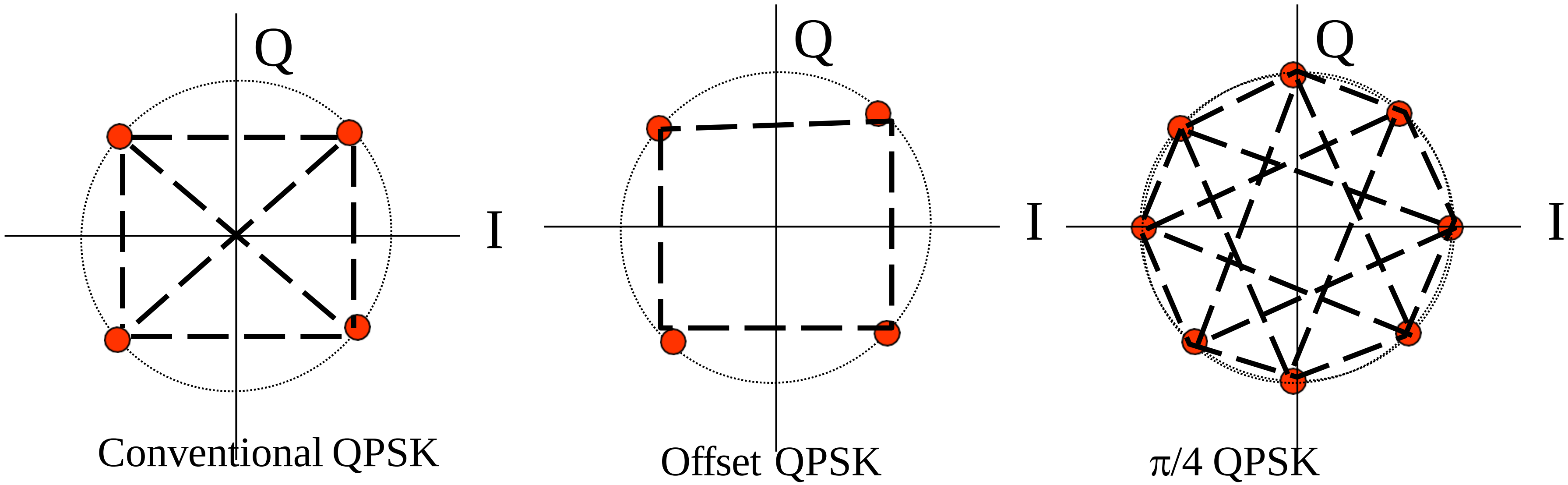
QPSK Waveforms



QPSK Receiver

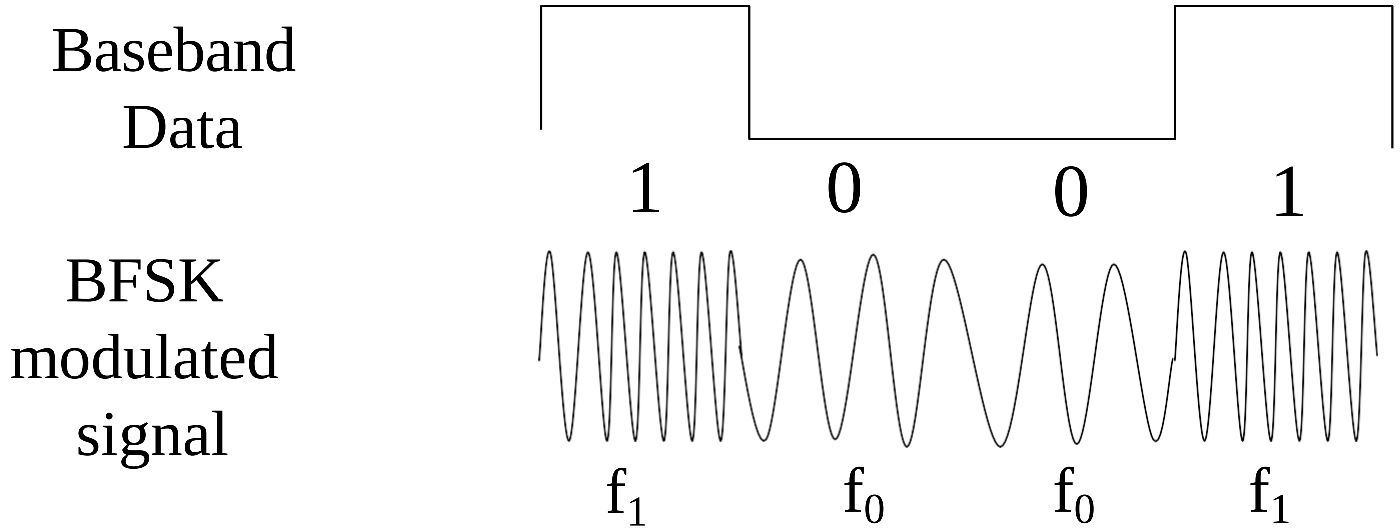


Types of QPSK



- **Conventional QPSK has transitions through zero (i.e. 180⁰ phase transition). Highly linear amplifiers required.**
- **In Offset QPSK, the phase transitions are limited to 90⁰, the transitions on the I and Q channels are staggered.**
- **In π/4 QPSK the set of constellation points are toggled each symbol, so transitions through zero cannot occur. This scheme produces the lowest envelope variations.**
- **All QPSK schemes require linear power amplifiers**

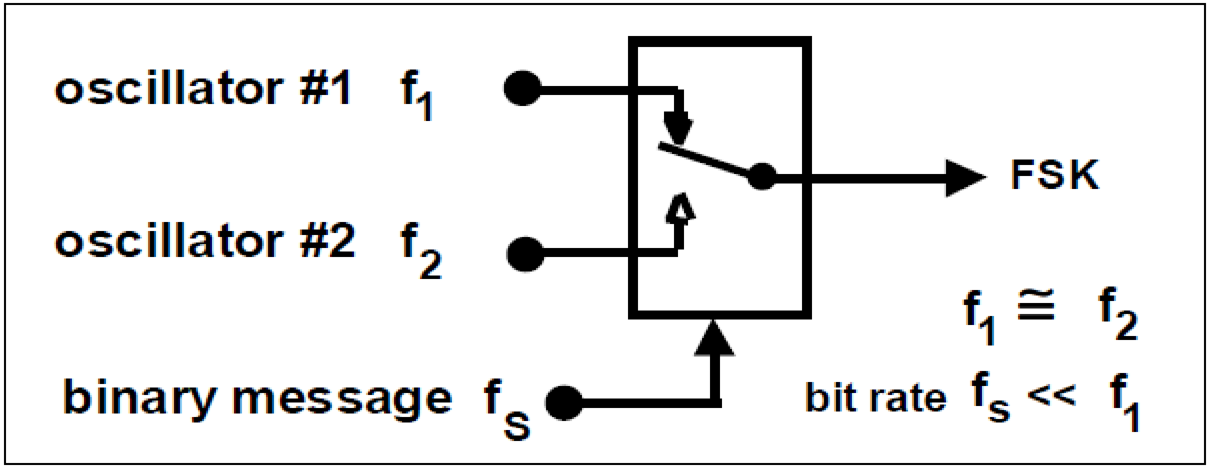
Frequency Shift Keying (FSK)

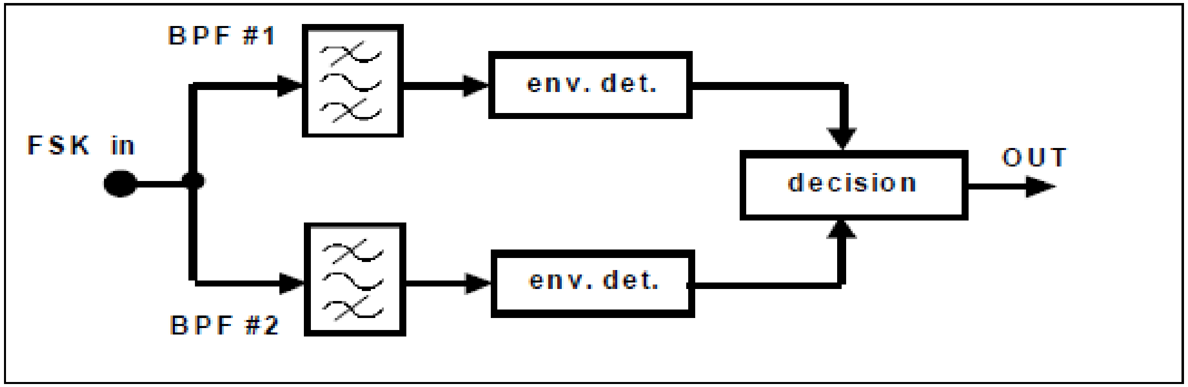


where $f_0 = A\cos(\omega_c - \Delta\omega)t$ and $f_1 = A\cos(\omega_c + \Delta\omega)t$

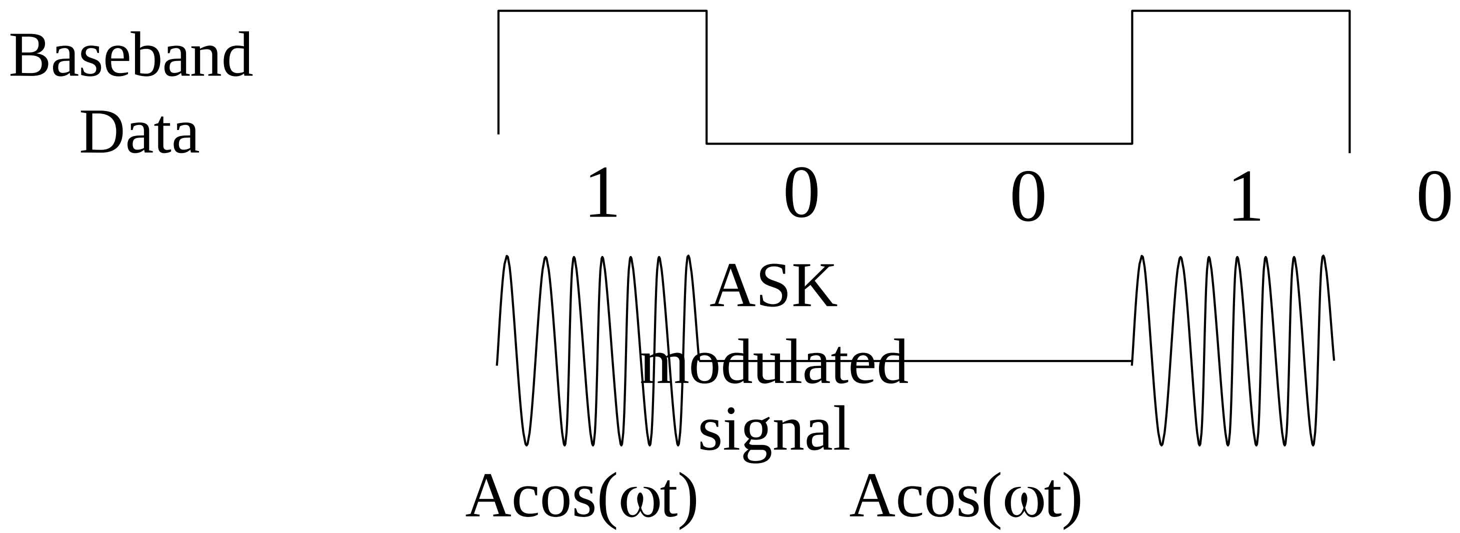
Example: The ITU-T V.21 modem standard uses FSK
FSK can be expanded to a M-ary scheme, employing multiple frequencies as different states

Generation & Detection of FSK





Amplitude Shift Keying (ASK)



Pulse shaping can be employed to remove spectral spreading
ASK demonstrates poor performance, as it is heavily affected by noise,
fading, and interference

Error Probabilities of PSK,QPSK,BFSK

- Error Probability of Binary PSK is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Error Probability of QPSK is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Error Probability of coherent binary FSK is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Power Spectral Density (PSD)

- In practical, pulse shaping should be considered for a precise bandwidth measurement and considered in the spectral efficiency calculations.

- Power spectral density (PSD) describes the distribution of signal power in the frequency domain. If the baseband equivalent of the transmitted signal sequence is given as

$$g(t) = \sum_{k=-\infty}^{\infty} a_k \, p(t - kT_s)$$

a_k : Baseband modulation symbol

T_s : Signal interval $\quad p(t)$: Pulse shape

then the PSD of $g(t)$ is given as

$$\Phi_g(f) = \frac{1}{T_s} |P(f)|^2 \Phi_a(f)$$

where

$$P(f) = F\{p(t)\}$$

$$\Phi_a(f) = \sum_{n=-\infty}^{\infty} R_a(n) e^{-j2\pi f n T_s}$$

$$R_a(n) = \frac{1}{2} E\left[a_k a_{k+n}^*\right]$$

M-ARY Phase Shift Keying (MPSK)

In M-ary PSK, the carrier phase takes on one of the M possible values, namely $\theta_i = 2 * (i - 1)\pi / M$

where i = 1, 2, 3, …..M.

The modulated waveform can be expressed as

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t + \boxed{\frac{2\pi}{M}(i-1)}\right), 0 \le t \le T_s \quad i = 1, 2, ..., M$$

where E_s is energy per symbol = $(\log_2 M)\, E_b$

T_s is symbol period = $(\log_2 M)\, T_b$.

The above equation in the Quadrature form is

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[(i-1)\frac{2\pi}{M}\right]\cos(2\pi f_c t) \qquad i = 1, 2, \ldots, M$$
$$-\sqrt{\frac{2E_s}{T_s}}\sin\left[(i-1)\frac{2\pi}{M}\right]\sin(2\pi f_c t)$$

By choosing orthogonal basis signals

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),$$

$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

defined over the interval $0 \le t \le T_s$

M-ary signal set can be expressed as

$$S_{\text{M-PSK}}(t) = \{\sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t) - \sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t)\}$$

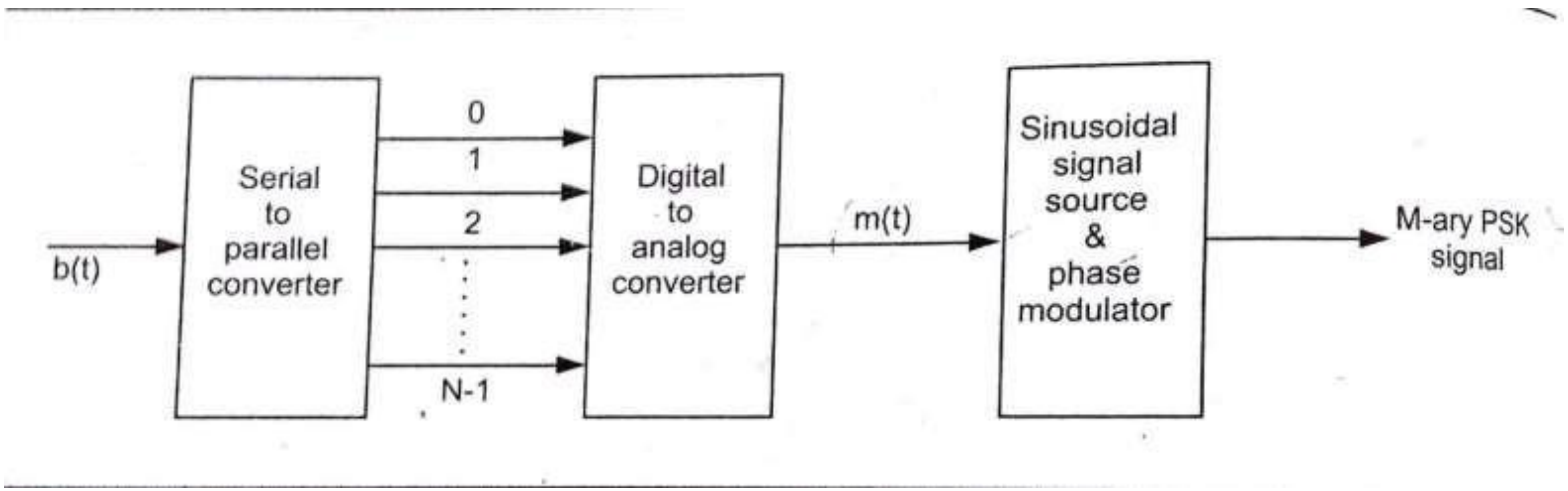
$$i = 1, 2, \ldots\ldots, M$$

- Since there are only two basis signals, the constellation of M-ary PSK is two dimensional.

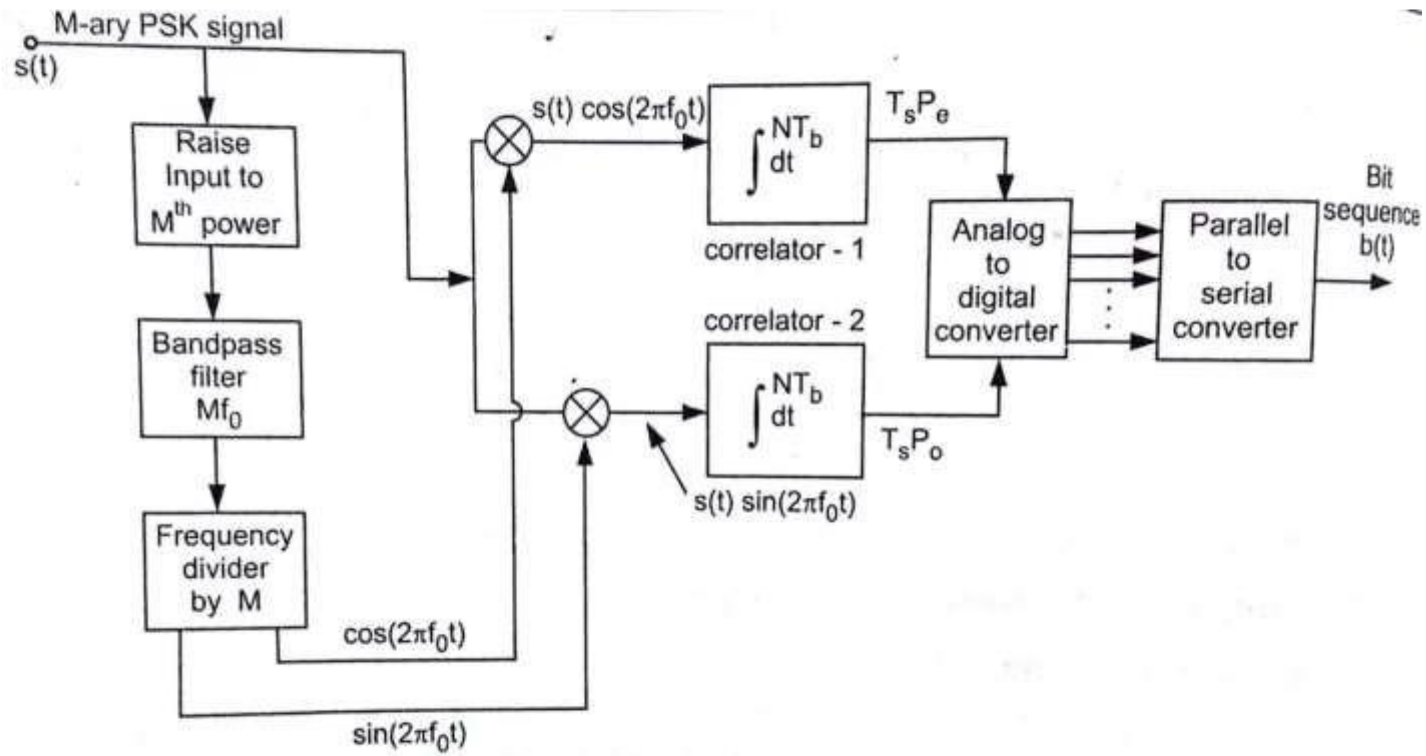
- The M-ary message points are equally spaced on a circle of radius $\sqrt{E_s}$, centered at the origin.

- The constellation diagram of an 8-ary PSK signal set is shown in fig.

M-ARY PSK Transmitter



Coherent M-ARY PSK Receiver



M-ARY Quadrature Amplitude Modulation (QAM)

It's a Hybrid modulation

As we allow the amplitude to also vary with the phase, a new modulation scheme called quadrature amplitude modulation (QAM) is obtained.

The constellation diagram of 16-ary QAM consists of a square lattice of signal points.

Combines amplitude and phase modulation

One symbol is used to represent n bits using one symbol

BER increases with n,

The general form of an M-ary QAM signal can be defined as

$$S_i(t) = \sqrt{\frac{2E_{min}}{T_s}}a_i\cos(2\pi f_c t) + \sqrt{\frac{2E_{min}}{T_s}}b_i\sin(2\pi f_c t)$$

$$0 \le t \le T \qquad i = 1, 2, \ldots, M$$

where

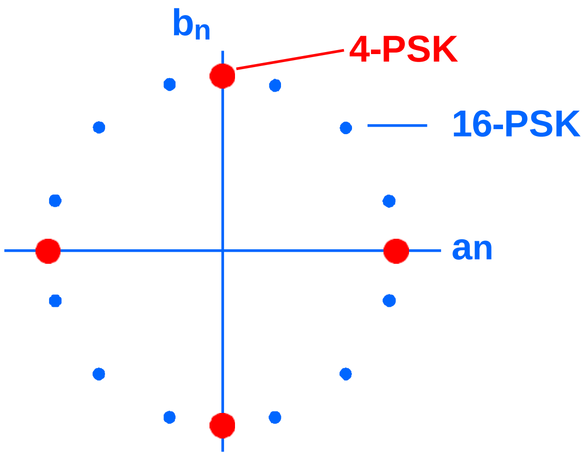
E_{min} is the energy of the signal with the lowest amplitude and

a_i and b_i are a pair of independent integers chosen according to the location of the particular signal point.

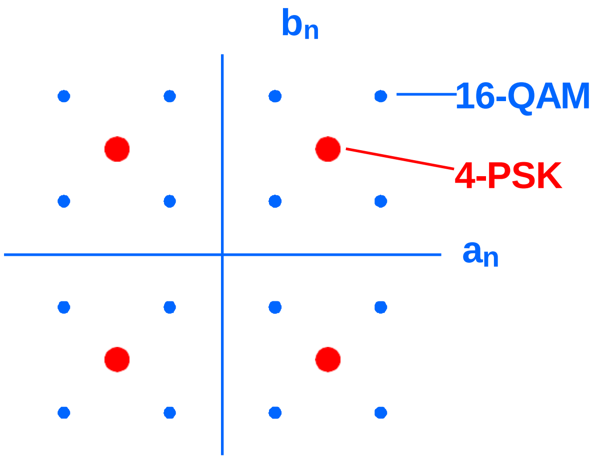
⑩ In M-ary QAM energy per symbol and also distance between possible symbol states is not a constant.

M-PSK AND M-QAM

M-PSK (Circular Constellations)



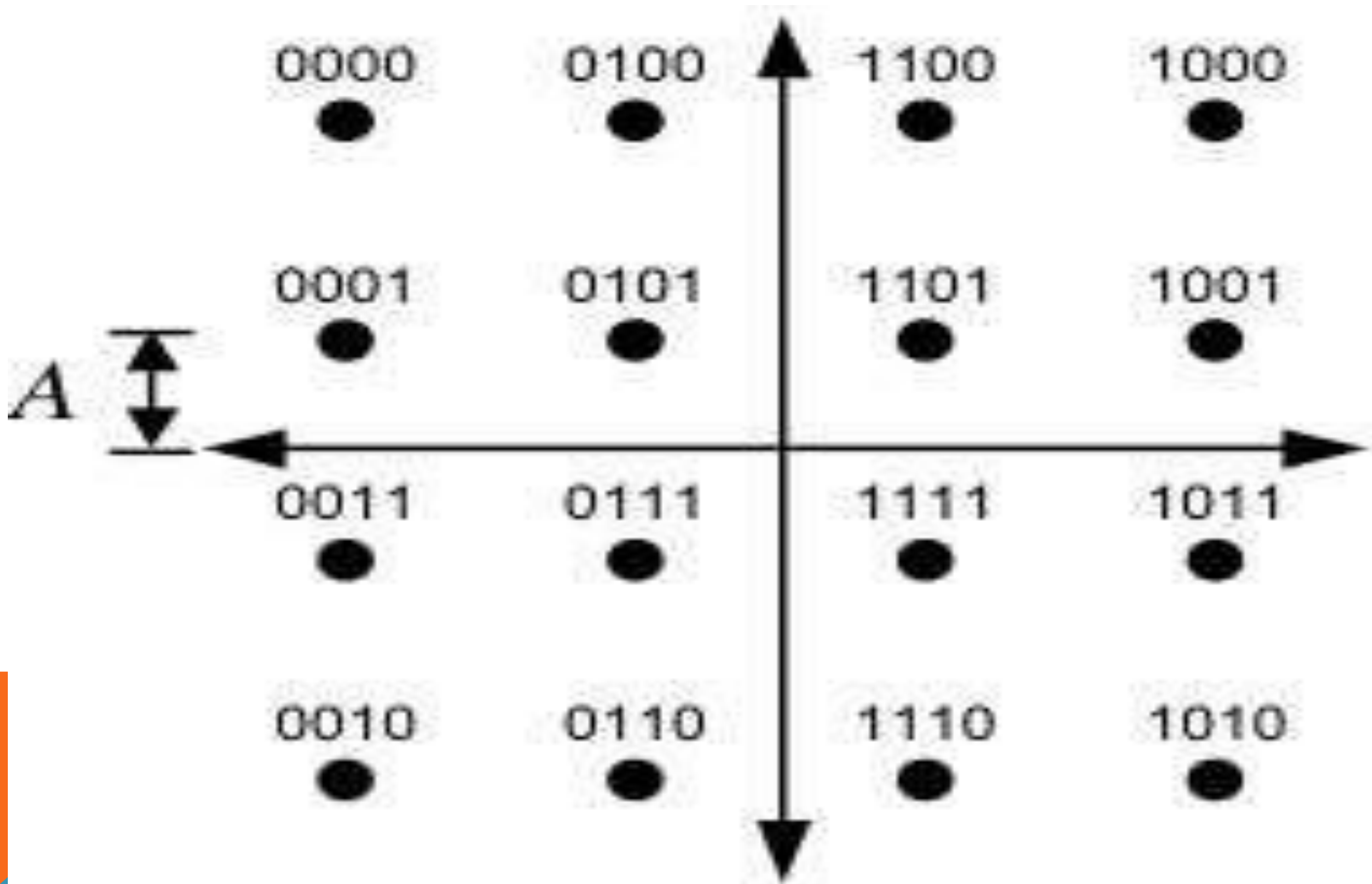
M-QAM (Square Constellations)



Tradeoffs

- **Higher-order modulations (M large) are *more spectrally efficient* but *less power efficient (i.e. BER higher)*.**
- **M-QAM is more spectrally efficient than M-PSK but also more sensitive to system nonlinearities.**

QAM Constellation Diagram



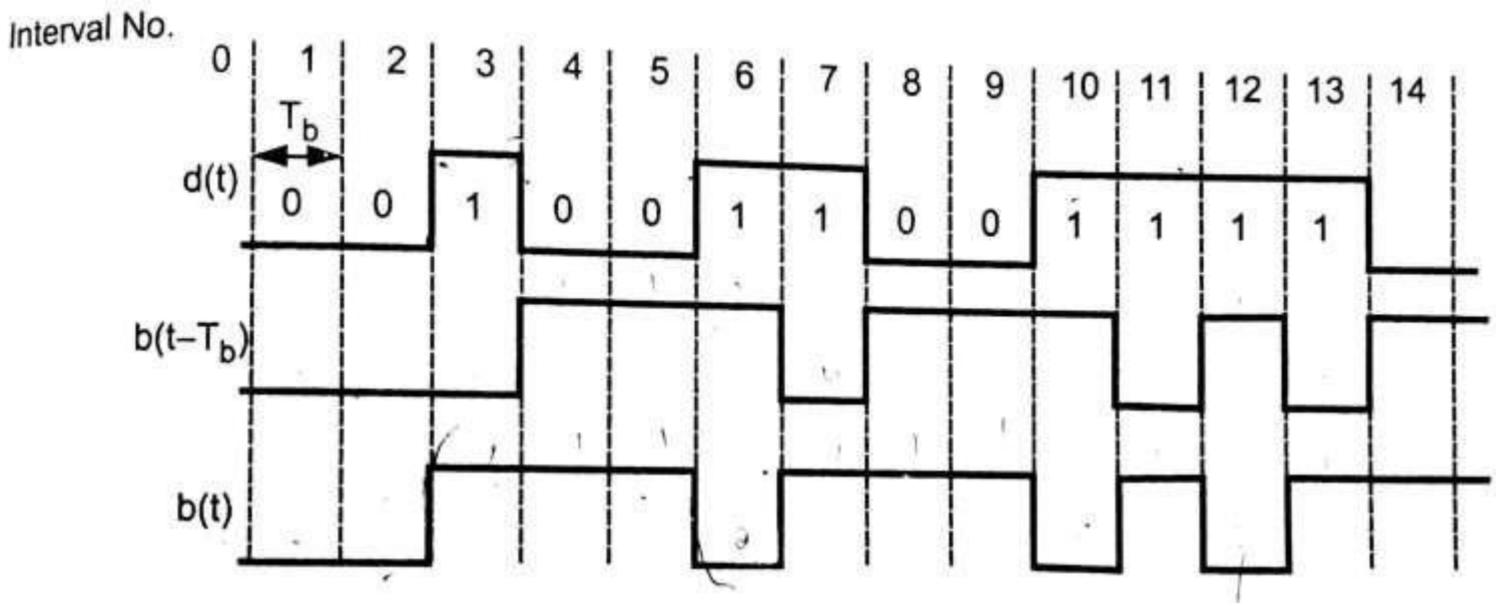
Differential Phase Shift Keying (DPSK)

- DPSK is a non coherent form of phase shift keying which avoids the need for a coherent reference signal at the receiver.

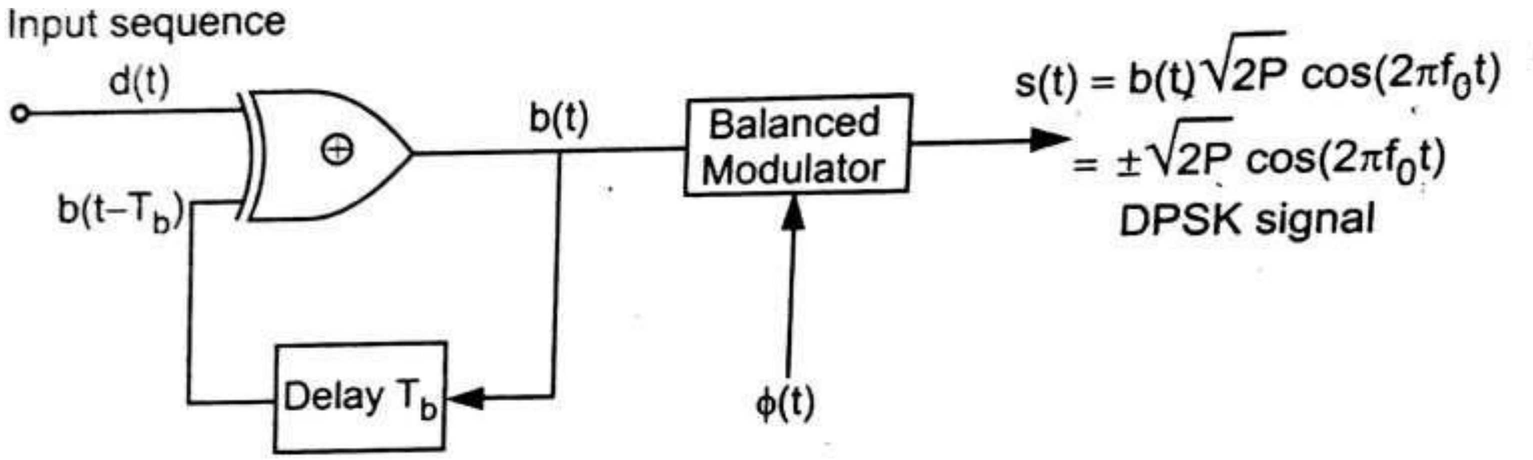
Advantage:

- Non coherent receivers are easy and cheap to build, hence widely used in wireless communications.
- DPSK eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter:

DPSK Waveforms



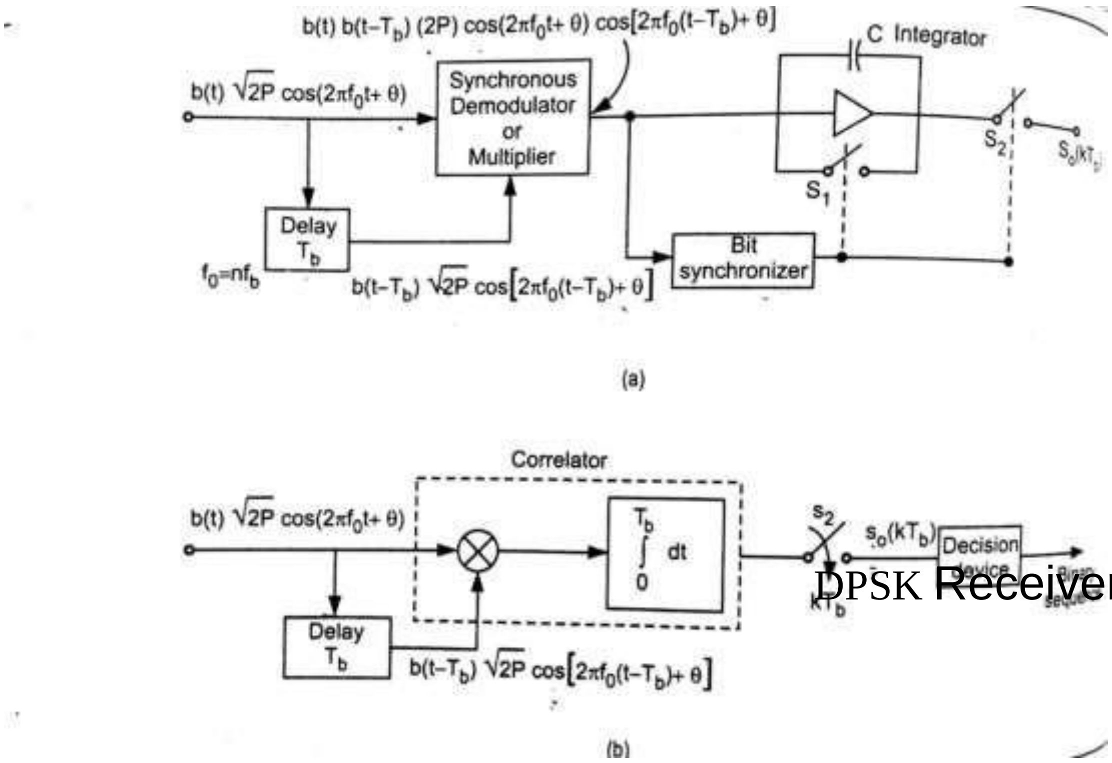
Transmitter/Generator of DPSK Signal



Non-coherent Detection

- Non-coherent detection
  - does not require carrier phase recovery (uses differentially encoded mod. or energy detectors) and hence, has less complexity at the price of higher error rate.
- No need in a reference in phase with the received carrier
- Differentially coherent detection
  - Differential PSK (DPSK)
    - *The information bits and previous symbol, determine the phase of the current symbol.*
- Energy detection
  - Non-coherent detection for orthogonal signals (e.g. M-FSK)
    - *Carrier-phase offset causes partial correlation between I and Q braches for each candidate signal.*
    - *The received energy corresponding to each candidate signal is used for detection.*

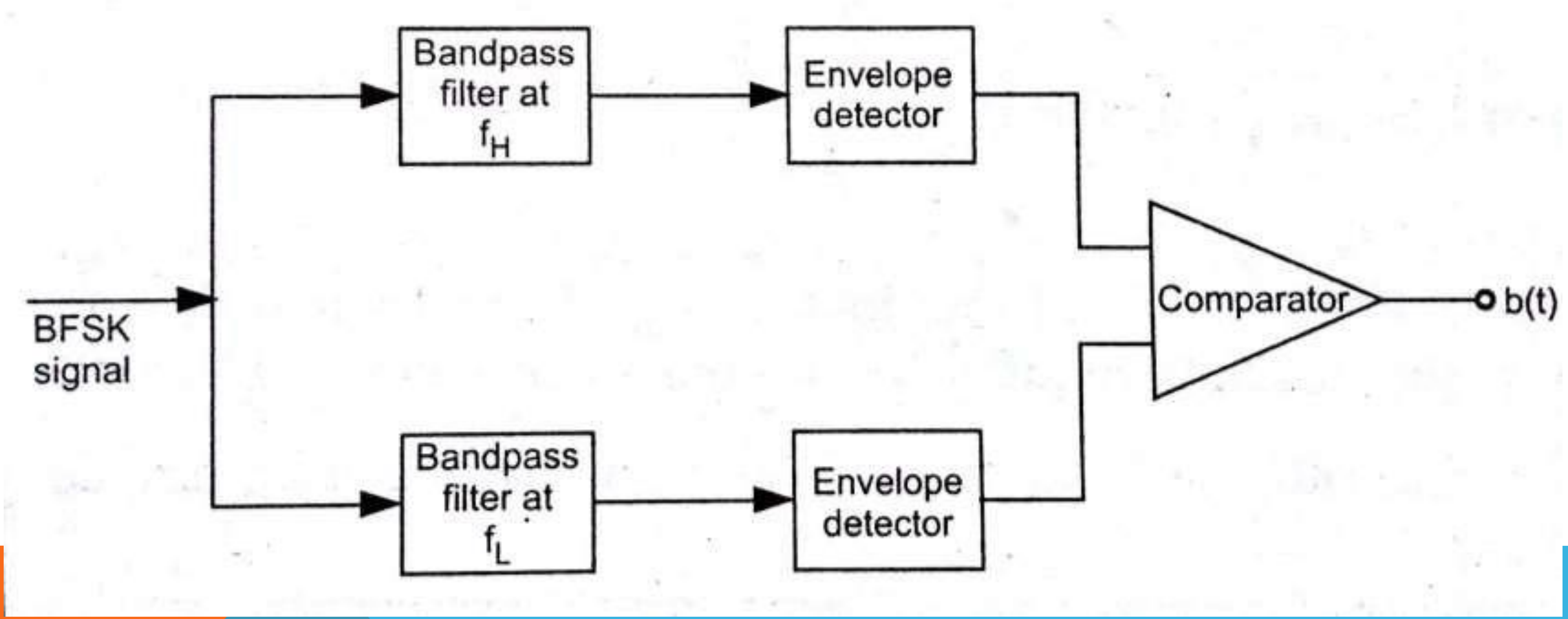
Non-coherent DPSK Receiver



DPSK Receiver

Dpsk receiver using correlator

Non-coherent BFSK Receiver



Comparisons Between Modulation Techniques

Sr. No.	Parameter	BPSK	DPSK	QPSK	M-ary PSK	QASK	BFSK	M-ary FSK	MSK	ASK
1	Modulation of	Phase	Phase	Phase	Phase	Amplitude and phase	frequency	frequency	frequency	amplitude
2	Equation of the transmitted signal $s(t)$	$s(t) = b(t)\sqrt{2P_s}\cos(2\pi f_0 t)$	$s(t) = b(t)\sqrt{2P_s}\cos(2\pi f_0 t)$ $b(t)$ differentially coded	$s(t) = \sqrt{2P_s}\cos[2\pi f_0 t + (2m+1)\frac{\pi}{4}]$ $m = 0, 1, 2, 3$	$s(t) = \sqrt{2P_s}\cos(2\pi f_0 t + \phi_m)$ $\phi_m = (2m+1)\frac{\pi}{M}$ $m = 0, 1, 2, \ldots, M-1$	$s(t) = k_1\sqrt{0.2P_s}\cos(2\pi f_0 t) + k_2\sqrt{0.2P_s}\sin(2\pi f_0 t)$ $k_1, k_2 = \pm 1$ or ± 3 for $M = 16$	$s(t) = \sqrt{2P_s}\cos[2(\pi f_0 + d(t)\Omega)t]$ Ω is frequency shift	$s(t) = \sqrt{2P_s}\cos(2\pi f_i t)$ $i = 1, 2, \ldots M$	$s(t) = b_0(t)\sqrt{2P_s}\sin 2\pi\left[f_0 + b_e(t)\,b_0(t)\frac{f_b}{4}\right]t$ $b_e(t), b_o(t)$ = odd/even sequence	$s(t) = 2\sqrt{2P_s}\cos(2\pi f_0 t)$ for symbol '1' = 0 for symbol '0'
3	Bits per symbol	One	One	Two	N	N	One	N	Two	One
4	Number of possible symbols $M = 2^N$	Two	Two	Four	$M = 2^N$	$M = 2^N$	Two	$M = 2^N$	Four	Two
5	Detection method	Coherent	Non-Coherent	Coherent	Coherent	Coherent	Non-coherent	Non-coherent	Coherent	Coherent
6	Minimum Euclidean distance	$2\sqrt{E_b}$		$2\sqrt{E_b}$	$2\sqrt{E_s}\sin\frac{\pi}{M}$	$\sqrt{0.4E_s}$ for $M = 16$	$\sqrt{2E_b}$	$\sqrt{2NE_b}$	$2\sqrt{E_b}$	$\sqrt{E_b}$
7	Minimum bandwidth (BW)	$2f_b$	f_b	f_b	$\frac{2f_b}{N}$	$\frac{2f_b}{N}$	$4f_b$	$\frac{2^{N+1}}{N}f_b$	$1.5f_b$	$2f_b$
8	Symbol duration (T_s)	T_b	$2T_b$	$2T_b$	NT_b	NT_b	T_b	NT_b	$2T_b$	T_b

VEMU INSTITUTE OF TECHNOLOGY
(Approved by AICTE, New Delhi & Affiliated to JNTUA, Ananthapuramu)
P.Kothakota, Puthalapattu (M), Chittoor Dist – 517 112 AP, India

Department of Electronics and Communication Engineering

Digital Communication Systems

Unit-5
Channel Coding

By
B SAROJA
Associate Professor
Dept. of . ECE

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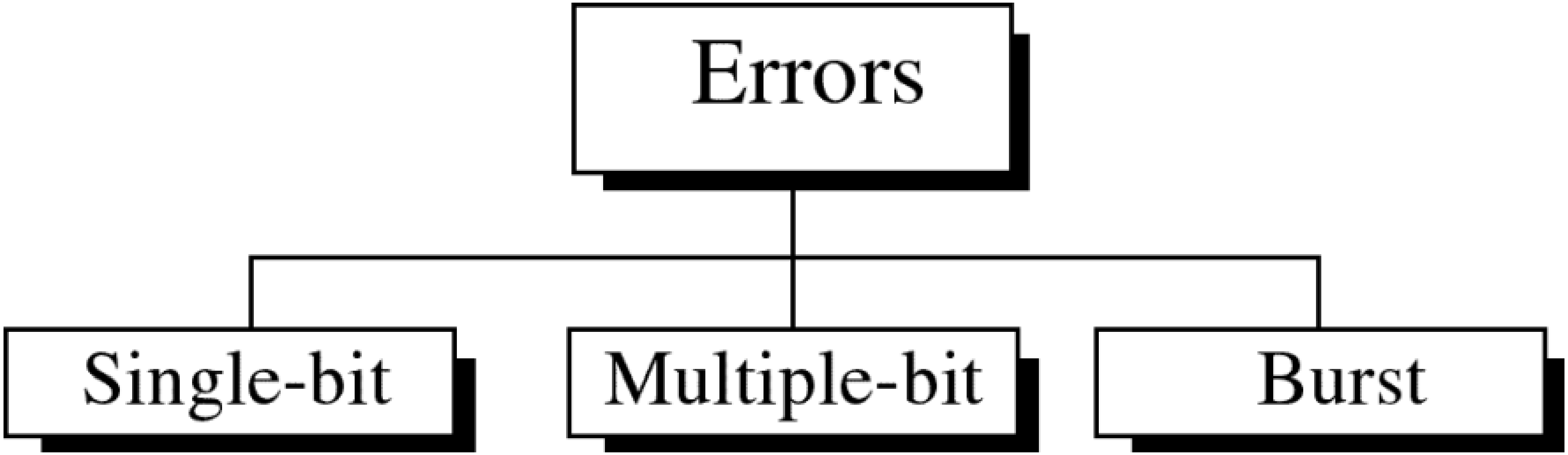
Introduction : Types of Errors



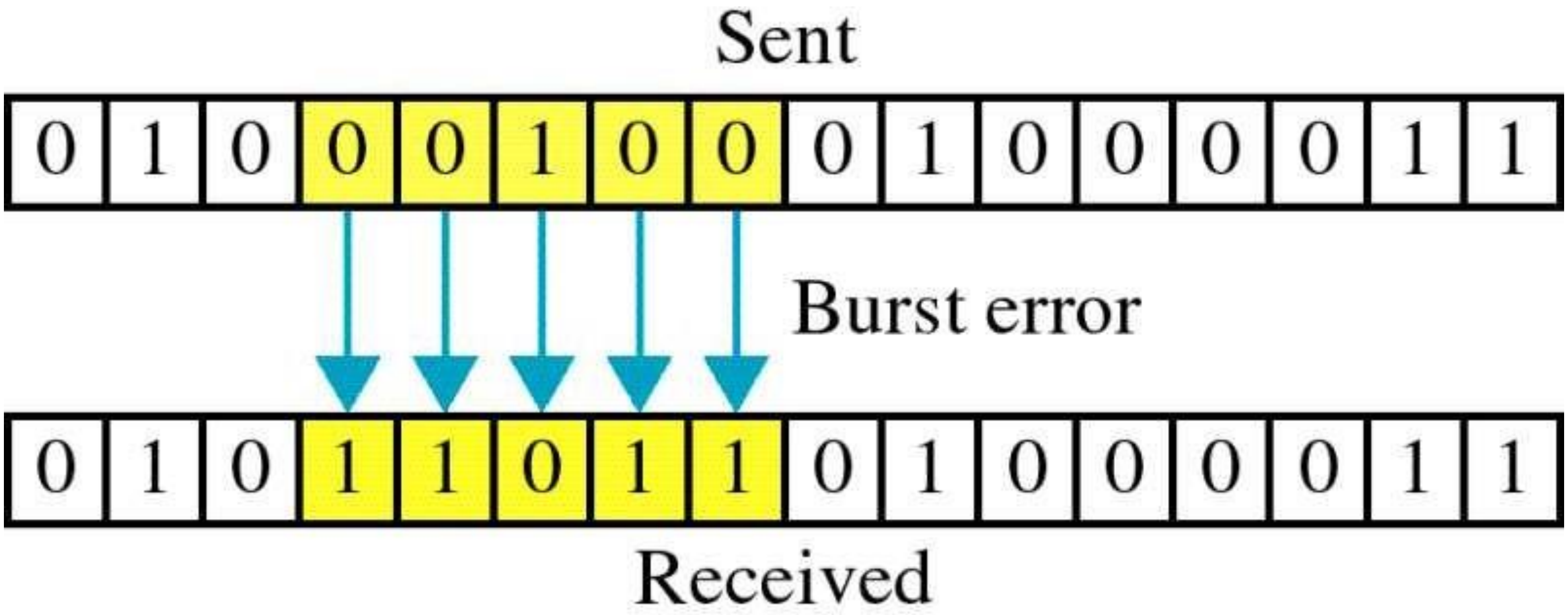
Single bit errors are the least likely type of errors in serial data transmission because the noise must have a very short duration which is very rare. However this kind of errors can happen in parallel transmission.

Example:

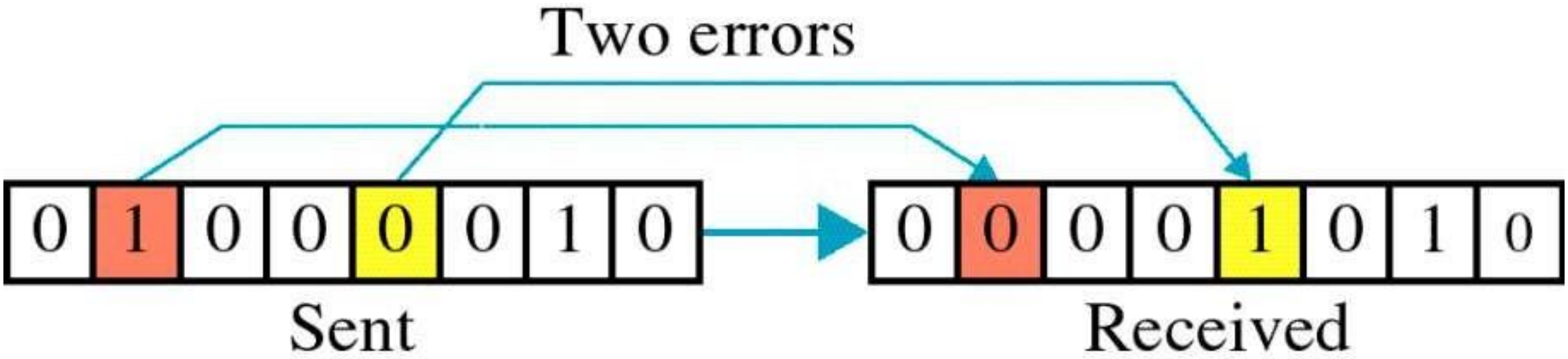
Example:

- ★ **If data is sent at 1Mbps then each bit lasts only 1/1,000,000 sec. or 1 μs.**
- ★ **For a single-bit error to occur, the noise must have a duration of only 1 μs, which is very rare.**

Burst Error



Sent
0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 1
Burst error
0 1 0 1 1 0 1 1 0 1 0 0 0 0 1 1
Received


Two errors
0
1
0
0
0
0
1
0
Sent
0
0
0
0
1
0
1
0
Received

The term burst error means that two or more bits in the data unit have changed from 1 to 0 or from 0 to 1.

Burst errors does not necessarily mean that the errors occur in consecutive bits, the length of the burst is measured from the first corrupted bit to the last corrupted bit. Some bits in between may not have been corrupted.

- ★ **Burst error is most likely to happen in serial transmission since the duration of noise is**

normally longer than the duration of a bit.

- ★ **The number of bits affected depends on the data rate and duration of noise.**

Example:

Example:

- **If data is sent at rate = 1Kbps then a noise of 1/100 sec can affect 10 bits.(1/100*1000)**

- **If same data is sent at rate = 1Mbps then a noise of 1/100 sec can affect 10,000 bits.($1/100*10^6$)**

Error Detection

Error detection means to decide whether the received data is correct or not without having a copy of the original message.

Error detection uses the concept of redundancy, which means adding extra bits for detecting
errors at the destination.

Error Correction

It can be handled in two ways:

1) **receiver can have the sender retransmit the entire data unit.**
2) **The receiver can use an error-correcting code, which automatically corrects certain errors.**

Single-bit Error Correction

To correct an error, the receiver reverses the value of the altered bit. To do so, it must know which bit is in error.

Number of redundancy bits needed Let

data bits = m

Redundancy bits = r

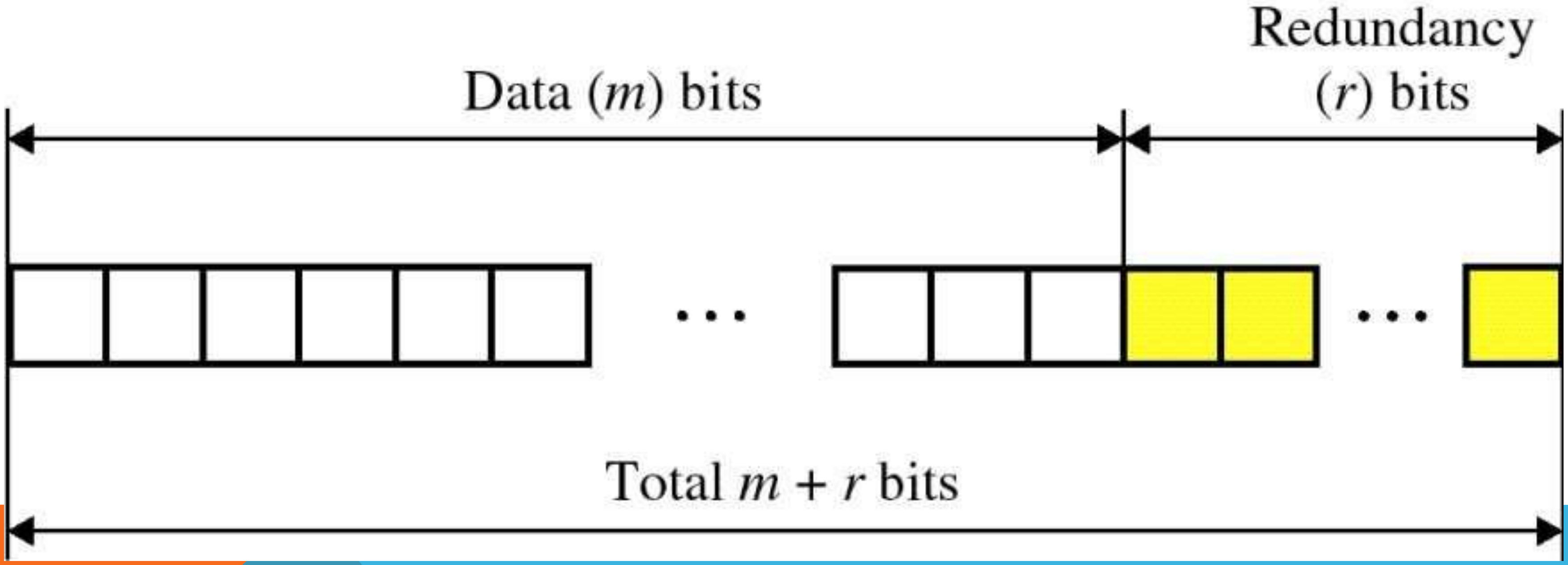
$\therefore$ Total message sent = $m+r$

The value of r must satisfy the following relation:

$$2^r \geq m+r+1$$

$$2^r \geq m+r+1$$

Error Correction



Repetetion

Retransmission is a very simple concept. Whenever one party sends something to the other party, it retains a copy of the data it sent until the recipient has acknowledged that it received it. In a variety of circumstances the sender automatically retransmits the data using the retained copy.

Parity Check Codes

information bits transmitted =

k bits actually transmitted = n

= k+1 Code Rate R = k/n =

k/(k+1)

Error detecting capability = 1

Error correcting capability = 0

Parity Codes – Example 1

Even parity

(i) d=(10110) so,

c=(101101)

(ii) **d=(11011) so,**

c=(110110)

Parity Codes – Example 2

Coding table for (4,3) even parity code

Dataword			Codeword			
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	1	0	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	0	1	1	0	1	0
1	1	0	1	1	0	0
1	1	1	1	1	1	1

Parity Codes

To decode

- Calculate sum of received bits in block (mod 2)
- If sum is 0 (1) for even (odd) parity then the dataword is the first *k* bits of the received codeword
- Otherwise error

Code can <u>detect</u> single errors

But cannot <u>correct</u> error since the error could be in any bit

For example, if the received dataword is (100000) the transmitted dataword could have been (000000) or (110000) with the error being in the first or second place respectively

Note error could also lie in other positions including the parity bit.

Interleaving

Interleaving is a process or methodology to make a system more efficient, fast and reliable by arranging data in a non contiguous manner. There are many uses for interleaving at the system level, including:

Storage: As hard disks and other storage devices are used to store user and system data, there is always a need to arrange the stored data in an appropriate way.

Error Correction: Errors in data communication and memory can be corrected through interleaving.

Code Vectors

In practice, we have a message (consisting of words, numbers, or symbols) that we wish to transmit. We begin by encoding each "word" of the message as a binary vector.

A binary code is a set of binary vectors (of the same length) called code vectors.

The process of converting a message into code vectors is called encoding, and the reverse process is called decoding.

Hamming Distance

Hamming distance is a metric for comparing two binary data strings. While comparing two binary strings of equal length, Hamming distance is the number of bit positions in which the two bits are different.

The Hamming distance between two strings, a and b is denoted as d(a,b).

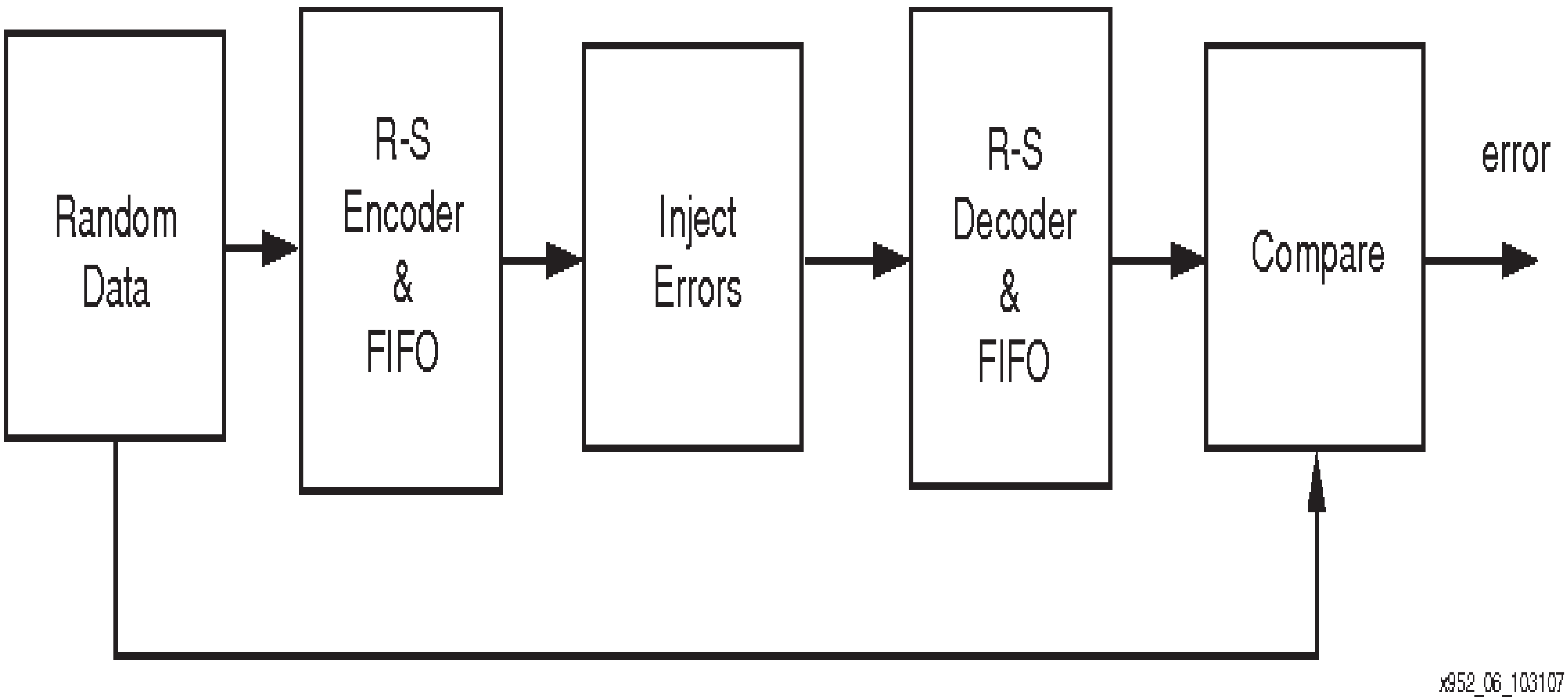
It is used for error detection or error correction when data is transmitted over computer networks. It is also using in coding theory for comparing equal length data words.

Example :

Suppose there are two strings 1101 1001 and 1001 1101.

11011001 $\oplus$ 10011101 = 01000100. Since, this contains two 1s, the Hamming distance, d(11011001, 10011101) = 2.

FEC System


Random Data
R-S Encoder & FIFO
Inject Errors
R-S Decoder & FIFO
Compare
error
x952_06_103107

Forward error correction (FEC) or channel coding is a technique used for controlling errors in data transmission over unreliable or noisy communication channels.

The central idea is the sender encodes the message in a redundant way by using an error-correcting code (ECC).

FEC gives the receiver the ability to correct errors without needing a reverse channel to request retransmission of data, but at the cost of a fixed, higher forward channel bandwidth.

FEC is therefore applied in situations where retransmissions

are costly or impossible, such as one-way communication

links and when transmitting to multiple receivers in multicast.

Automatic Repeat Request (ARQ)

Automatic Repeat reQuest (ARQ), also known as Automatic Repeat Query, is an error-control method for data transmission that uses acknowledgements and timeouts to achieve reliable data transmission over an unreliable service.

If the sender does not receive an acknowledgment before the timeout, it usually re-transmits the frame/packet until the sender receives an acknowledgment or exceeds a predefined number of re-transmissions.

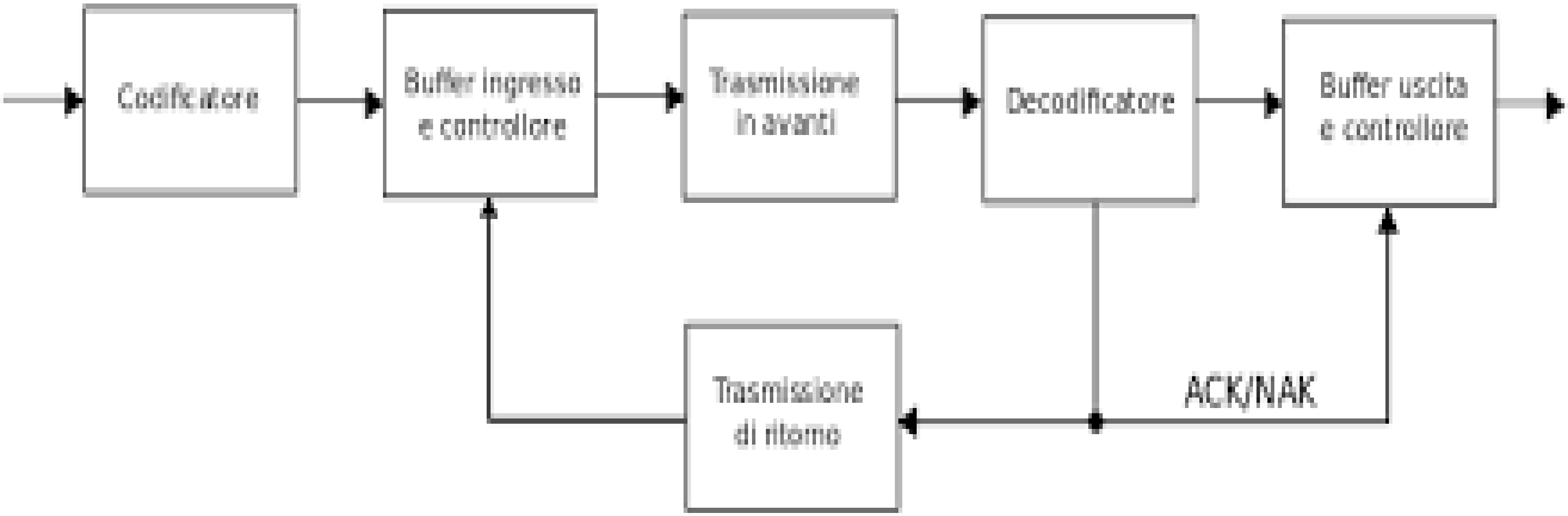
The types of ARQ protocols include Stop-and-wait ARQ

Go-Back-N ARQ

Selective Repeat ARQ

All three protocols usually use some form of sliding window protocol to tell the transmitter to determine which (if any) packets need to be retransmitted.

ARQ System



Codificatore
Buffer ingresso e controllore
Trasmissione in avanti
Decodificatore
Buffer uscita e controllore
Trasmissione di ritorno
ACK/NAK

Block Codes

Data is grouped Into Blocks Of Length *k* bits (dataword)

Each dataword is coded into blocks of length *n* bits (codeword), where in general *n*>*k*

This is known as an (*n*,*k*) block code

A vector notation is used for the datawords and codewords,

- Dataword d = $(d_1\, d_2 \ldots. d_k)$
- Codeword c = $(c_1\, c_2 \ldots\ldots. c_n)$

The redundancy introduced by the code is quantified by the code rate,

Code rate = k/n

i.e., the higher the redundancy, the lower the code rate

Block Code - Example

Data word length *k* = 4 Codeword length

***n* = 7**

This is a (7,4) block code with code rate

= 4/7 For example, d = (1101), c =

(1101001)

Linear Block Codes:Matrix Representation

parity bits n-k (=1 for Parity Check) Message m = $\{m_1\ m_2 \ldots m_k\}$ Transmitted Codeword c = $\{c_1\ c_2 \ldots\ c_n\}$

A generator matrix G_{kxn}

$$c = mG$$

Linear Block Codes

Linearity

$$c_1 = m_1 G,$$
$$c_2 = m_2 G$$

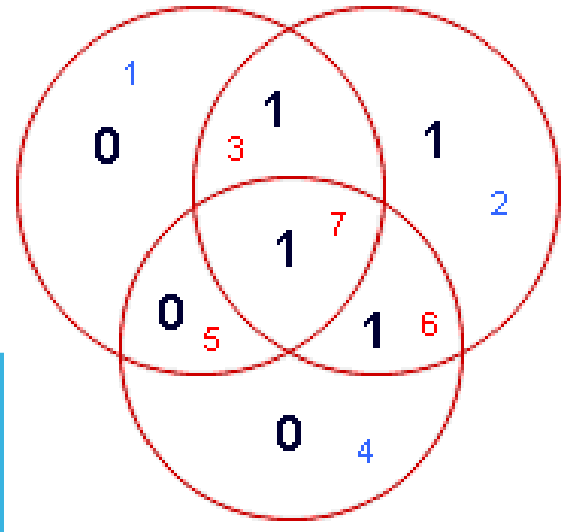
$$c_1 \oplus c_2 = (m_1 \oplus m_2)G$$

Example : 4/ 7 Hamming Code

- k = 4, n = 7
- 4 message bits at (3,5,6,7)
- 3 parity bits at (1,2,4)

Error correcting capability =1

Error detecting capability = 2



Linear Block Codes

If there are *k* data bits, all that is required is to hold *k* linearly independent code words, i.e., a set of *k* code words none of which can be produced by linear combinations of 2 or more code words in the set.

The easiest way to find *k* linearly independent code words is to choose those which have '1' in just one of the first *k* positions and '0' in the other *k*-1 of the first *k* positions.

Linear Block Codes

For example for a (7,4) code, only four codewords are required, e.g.,

1	0	0	0	1	1	0
0	1	0	0	1	0	1
0	0	1	0	0	1	1
0	0	0	1	1	1	1

- So, to obtain the codeword for dataword 1011, the first, third and fourth codewords in the list are added together, giving 1011010
- This process will now be described in more detail

An (*n*,*k*) block code has code vectors

d=($d_1 d_2 \ldots d_k$) and c=($c_1 c_2 \ldots c_n$)

The block coding process can be written as

c=dG

where G is the <u>Generator Matrix</u>

$$G = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{k1} & a_{k2} & \cdots & a_{kn} \end{bmatrix} = \begin{bmatrix} \mathrm{a}_1 \\ \mathrm{a}_2 \\ . \\ \mathrm{a}_k \end{bmatrix}$$

Thus,

$$c = \sum_{i=1}^{k} d_i a_i$$

- a_i must be linearly independent, i.e., Since codewords are given by summations of the a_i vectors, then to avoid 2 datawords having the same codeword the a_i vectors must be linearly independent.
- Sum (mod 2) of any 2 codewords is also a codeword, i.e.,

Since for datawords d_1 and d_2 we have;

$$d_3 = d_1 + d_2$$

So,

$$c_3 = \sum_{i=1}^{k} d_{3i} a_i = \sum_{i=1}^{k} (d_{1i} + d_{2i}) a_i = \sum_{i=1}^{k} d_{1i} a_i + \sum_{i=1}^{k} d_{2i} a_i$$

$$c_3 = c_1 + c_2$$

0 is always a codeword, i.e.,

Since all zeros is a dataword then,

$$c = \sum_{i=1}^{k} 0 \; a_i = 0$$

Decoding Linear Codes

One possibility is a ROM look-up table

In this case received codeword is used as an address

Example – Even single parity check code;

Address	Data
000000	0
000001	1
000010	1
000011	0
………	.

Data output is the error flag, i.e., 0 – codeword ok, If no error, dataword is first *k* bits of codeword

For an error correcting code the ROM can also store datawords.

Convolutional Codes

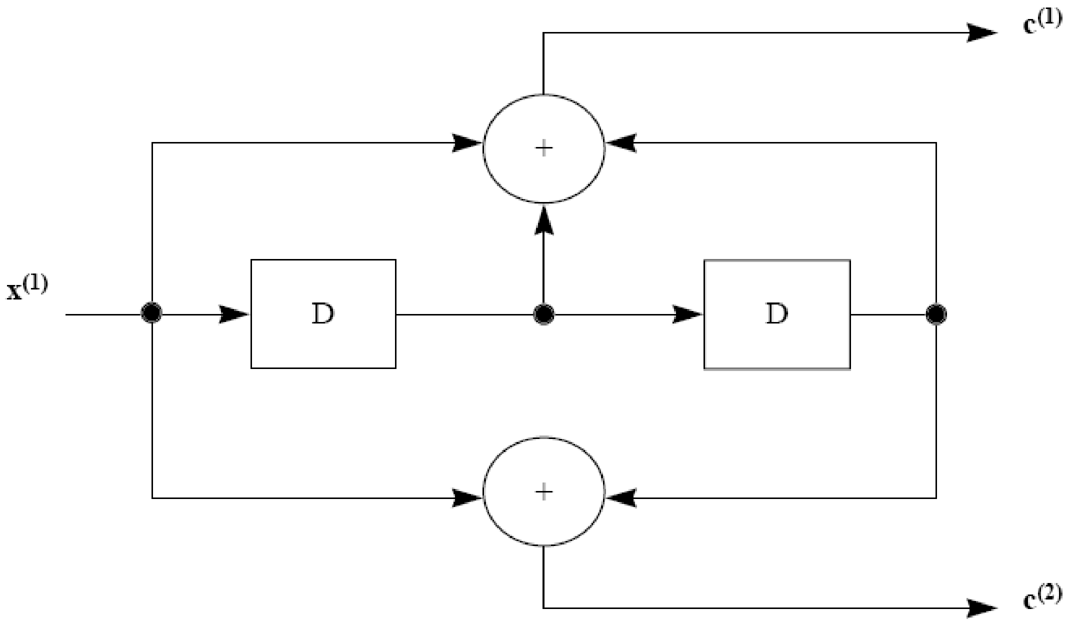
Block codes require a

buffer Example

k = 1

n = 2

Rate R = ½



Convolutional Codes:Decoding

Encoder consists of shift registers forming a finite state machine
Decoding is also simple – Viterbi Decoder which works by tracking
these states First used by NASA in the voyager space programme
Extensively used in coding speech data in mobile phones