STRESS & STRAIN

Unit 1

1.1 LOAD

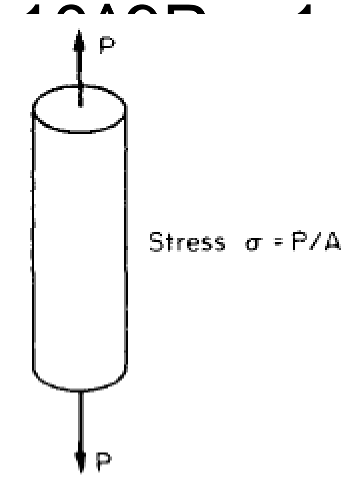
- Load is defined as the set of external forces acting on a mechanism or engineering structure which arise from service conditions in which the components work
- Common loads in engineering applications are tension and compression
- Tension:- Direct pull. Eg:Force present in lifting hoist
- Compression:- Direct push. Eg:- Force acting on the pillar of a building
- Sign convention followed: Tensile forces are positive and compressive negative

1.1.1TYPES OF LOAD

- There are a number of different ways in which load can be applied to a member. Typical loading types are:
- A) **Dead/ Static load**- Non fluctuating forces generally caused by gravity
- B) **Live load-** Load due to dynamic effect. Load exerted by a lorry on a bridge
- C) **Impact load or shock load**- Due to sudden blows
- D) **Fatigue or fluctuating or alternating loads**: Magnitude and sign of the forces changing with time

1.2 STRESS

- When a material is subjected to an external force, a resisting force is set up within the component, this internal resistance force per unit area is called stress. SI unit is N/m²(Pa). 1kPa=1000Pa, 1MPa=10^6 Pa, 1 Gpa
Terra Pascal=10^12 Pa

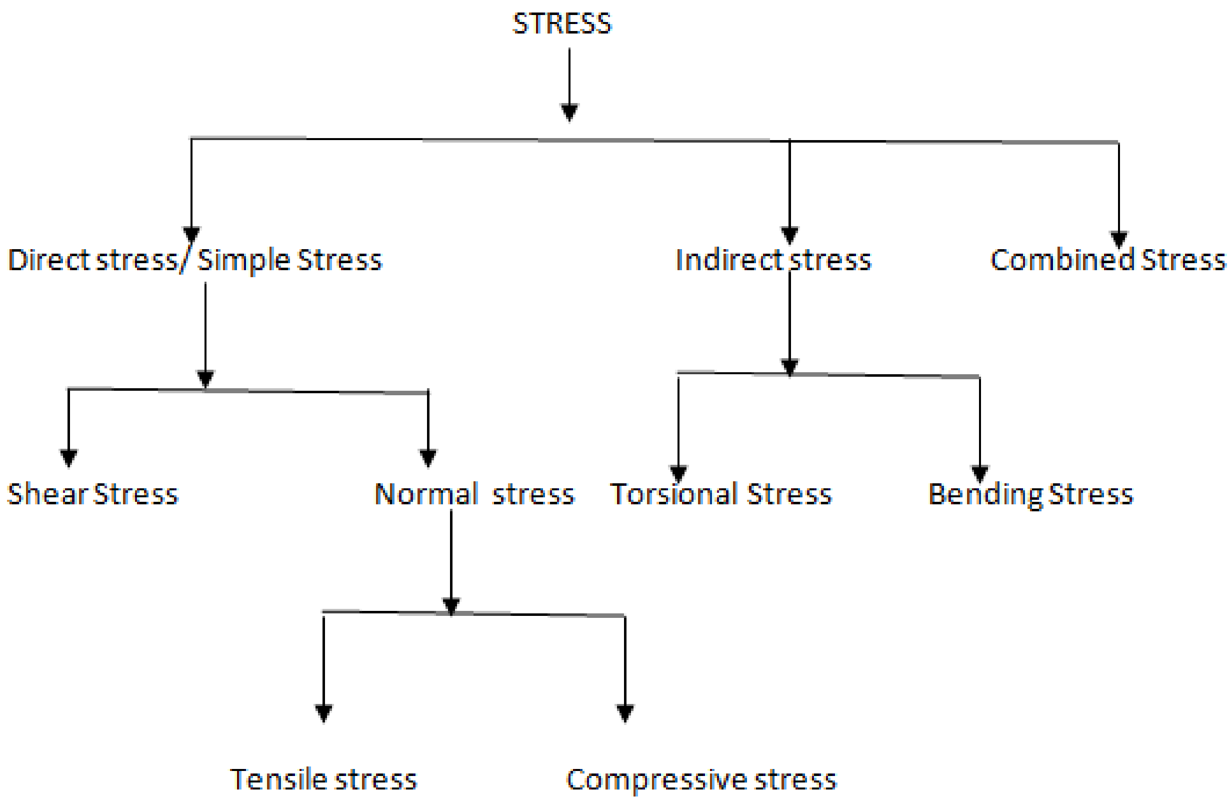


- In engineering applications, we use th
the original cross section area of the s
and it is known as conventional stress
Engineering stress

1.3 STRAIN

- When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to its original dimension is known as strain
- Strain is a dimensionless quantity
- Strain may be:- a) Tensile strain b) Compressive strain c) Volumetric strain d) Shear strain
- **Tensile strain-** Ratio of increase in length to original length of the body when it is subjected to a pull force
- **Compressive strain**- Ratio of decrease in length to original length of the body when it is subjected to a push force
- **Volumetric strain-** Ratio of change of volume of the body to the original volume
- **Shear strain-**Strain due to shear stress

1.4 TYPE OF STRESSES



1.4.1TYPES OF DIRECT STRESS

- Direct stress may be normal stress or shear stress
- **Normal stress (σ)** is the stress which acts in direction perpendicular to the area. Normal stress is further classified into tensile stress
- **Tensile stress** is the stress induced in a body, when it is subjected to two equal and opposite pulls (tensile forces) as a result of which there is a tendency in increase in length
- It acts normal to the area and pulls on the area

1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

- Consider a bar subjected to a tensile force P at its ends. Let

  A= Cross sectional area of the body

  L=Original length of the body

  dL= Increase in length of the body due to its pull P

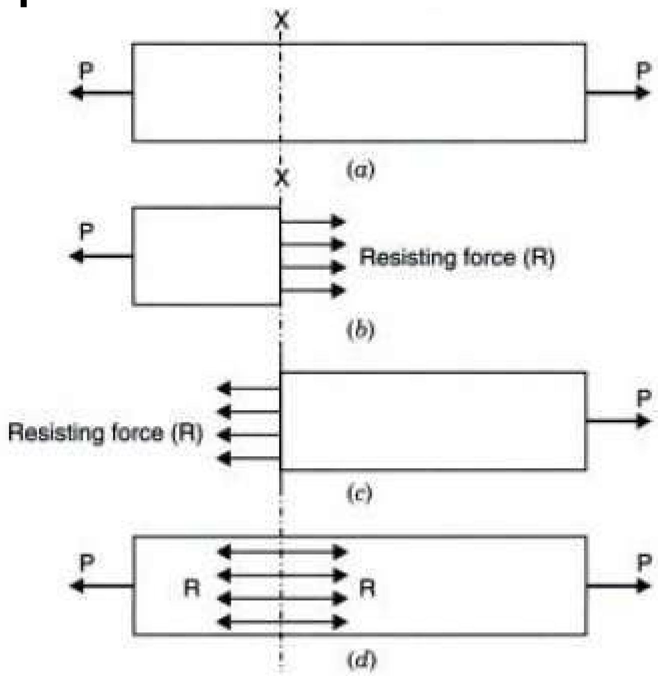
  ς= Stress induced in the body

  e= Tensile strain

Consider a section X-X which divides the body into two halves

1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

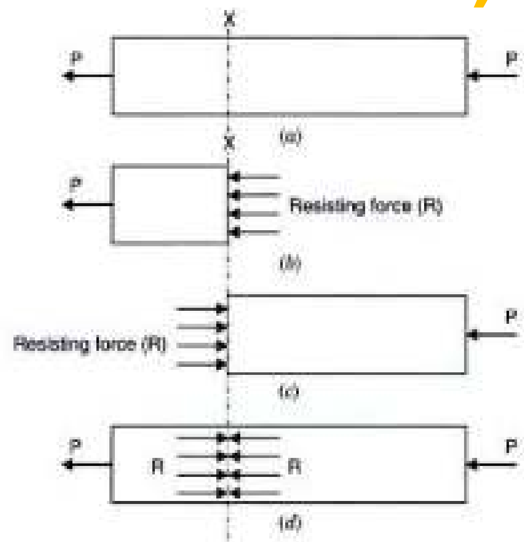
- The left part of the section x-x, will be in equilibrium if P=R (Resisting force). Similarly the right part of the section x-x will be in equilibrium if P=I



1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

- Tensile stress (ς)= Resisting force/ Cross sectional area= Applied force/Cross sectional area=P/A
- Tensile strain= Increase in length/Original length= dL/L
- Compressive stress:- Stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a tendency of decrease in length of the body
- It acts normal to the area and it pushes on the area
- In some cases the loading situation is such that the stress will vary across any given section. In such cases the stress at any given point is given by
- ς= Lt $\Delta A \rightarrow 0$ $\Delta P/\Delta A$= dP/dA= derivative of force w.r.t area

1.4.1 TYPES OF DIRECT STRESS (Compressive stress)



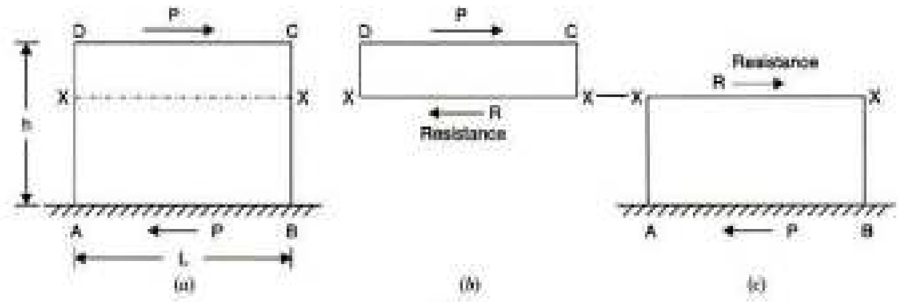
- **Compressive stress**=Resisting force/ cross sectional area= Applied force/ cross sectional area
- Compressive strain= Decrease in length/ Original length= -dL/L
- Sign convention for direct stress and strain:- Tensile stresses and strains are considered positive in sense producing an increase in length. Compressive stresses and strains are considered negative in sense producing decrease in length

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

- **Shear stress** :- Stress Induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across that section
- Consider a rectangular block of height h, length L and width unity. Let the bottom face AB of the block be fixed to the surface as shown. Let P be the tangential force applied along top face CD of the block. For the equilibrium of the block, the surface AB will offer a tangential reaction force R which is equal in magnitude and opposite in direction to the applied tangential force P

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

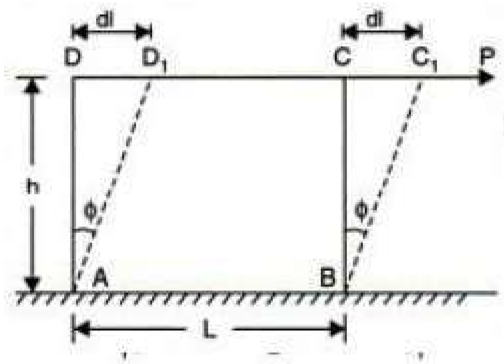
- Consider a section X-X cut parallel to the applied force which splits rectangle into two parts



- For the upper part to be in equilibrium; Applied force P=Resisting force R
- For the lower part to be in equilibrium; Applied force P=Resisting force R
- Hence, shear stress τ= Resisting force/Resisting area=P/L x 1=P/L
- Shear stress is tangential to the area on which it acts

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

- As the face AB is fixed, the rectangular section ABCD will be distorted to ABC1D1, such that new vertical face AD1 makes an angle φ with the initial face AD



(a)

- Angle φ is called shear strain. As φ is very small,
- φ=tan φ=DD1/AD=dl/h
- Hence shear strain=dl/h

1.5 ELASTICITY & ELASTIC LIMIT

- The property of a body by virtue of which it undergoes deformation when subjected to an external force and regains its original configuration (size and shape) upon the removal of the deforming external force is called elasticity.
- The stress corresponding to the limiting value of external force upto and within which the deformation disappears completely upon the removal of external force is called elastic limit
- A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed.
- If the external force is so large that the stress exceeds the elastic limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in the material

1.6 HOOKE'S LAW & ELASTIC MODULI

- Hooke's law states that: " When a body is loaded within elastic limit, the stress is proportional to strain developed" or "Within the elastic limit the ratio of stress applied to strain developed is a constant"
- The constant is known as Modulus of elasticity or Elastic modulus or Young's modulus
- Mathematically within elastic limit

  Stress/Strain=ς/e=E

  ς= P/A; e

  =ΔL/L E=PL/A

  ΔL

1.7 HOOKE’S LAW & ELASTIC MODULI

- Young's modulus (E) is generally assumed to be the same in tension or compression and for most of engineering applications has a high numerical value. Typically, $E=210 \times 10^9\ N/m^2$ (=210 GPa) for steel
- Modulus of rigidity, $G= \tau/\varphi$= Shear stress/ shear strain
- Factor of safety= Ultimate stress/Permissible stress
- In most engineering applications strains donot often exceed 0.003 so that the assumption that deformations are small in relation to orinal dimensions is generally valid

1.8 STRESS-STRAIN CURVE (TENSILE TEST)

- Standard tensile test involves subjecting a circular bar of uniform cross section to a gradually increasing tensile load until the failure occurs
- Tensile test is carried out to compare the strengths of various materials
- Change in length of a selected gauge length of bar is recorded by extensometers
- A graph is plotted with load vs extension or stress vs strain

1.8 STRESS-STRAIN CURVE (TENSILE TEST)

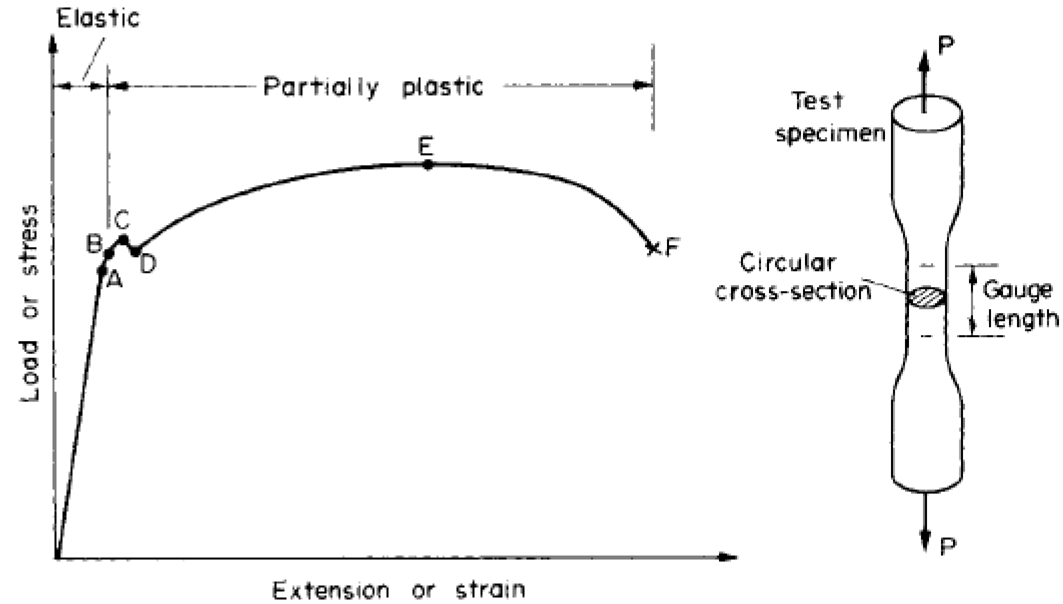


Fig. 1.3. Typical tensile test curve for mild steel.

1.8 STRESS-STRAIN CURVE (TENSILE TEST DIAGRAM)

- A→ Limit of proportionality; It is the point where the linear nature of the stress strain graph ceases
- B→ Elastic limit; It is the limiting point for the condition that material behaves elastically, but hooke's law does not apply . For most practical purposes it can be often assumed that limit of proportionality and elastic limits are the same
- Beyond the elastic limits, there will be some permanent deformation or permanent set when the load is removed
- C (Upper Yield point), D (Lower yield point)→ Points after which strain increases without correspondingly high increase in load or stress
- E→ Ultimate or maximum tensile stress; Point where the necking starts
- F→ Fracture point

RELATIONSHIPS BETWEEN STRESS & STRAIN

- **A) 1-Dimensional case** (due to pull or push or shear force)

  ς=Ee

- **B) 2-Dimensional case**
- Consider a body of length L, width B and height H. Let the body be subjected to an axial load. Due to this axial load, there is a deformation along the length of the body. This strain corresponding to this deformation is called longitudinal strain.
- Similarly there are deformations along directions perpendicular to line of application of fore. The strains corresponding to these deformations are called lateral strains

RELATIONSHIPS BETWEEN STRESS & STRAIN

δL = Increase in length,
δb = Decrease in breadth, and
δd = Decrease in depth.

Then longitudinal strain $= \dfrac{\delta L}{L}$

and lateral strain $= \dfrac{\delta b}{b}$ or $\dfrac{\delta d}{d}$

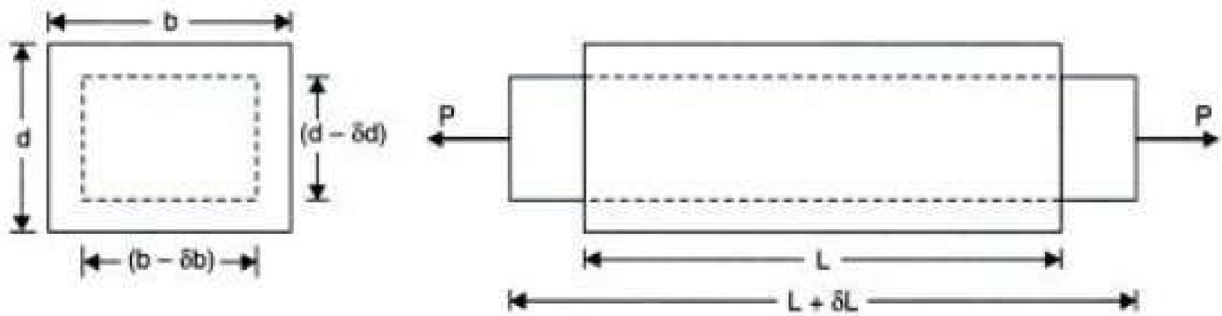


Fig. 1.3. Typical tensile test curve for mild steel.

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Longitudinal strain is always of opposite sign of that of lateral strain. Ie if the longitudinal strain is tensile, lateral strains are compressive and vice versa
- Every longitudinal strain is accompanied by lateral strains in orthogonal directions
- Ratio of lateral strain to longitudinal strain is called Poisson's ratio (μ); Mathematically,
- μ=-Lateral strain/Longitudinal strain
- Consider a rectangular figure ABCD subjected a stress in σx direction and in σ y direction

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to ςx= ς x/E

  Strain along x direction due to ς y=-μ x ςy/E

  Total strain in x direction ex= ς x/E - μ x

  ςy/E

  Similarly total strain in y direction, ey= ς y/E - μ x ςx/E

- In the above equation tensile stresses are considered as positive and compressive stresses as negative

- **C) 3 Dimensional case:-**

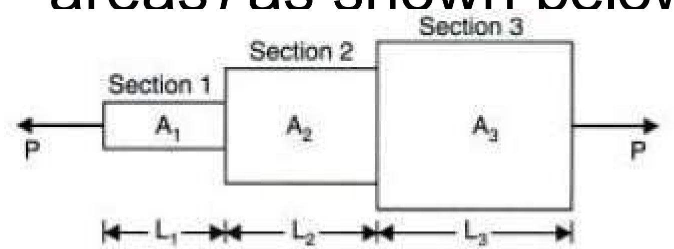
  Consider a 3 D body subjected to 3 orthogonal normal stresses in x,y and z directions

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to ςx= ς x/E
  Strain along x direction due to ς y=-μ x
  ςy/E Strain along x direction due to ς z=-
  μ x ςz/E
  Total strain in x direction ex= ς x/E - μ x (ςy/E + ςz/E)
  Similarly total strain in y direction, ey= ς y/E - μ x
  (ςxE + ςz/E)
  Similarly total strain in z direction, ez= ς z/E - μ x (ςxE + ςy/E)

1.10 ANALYSIS OF BARS OF VARYING CROSS SECTION

- Consider a bar of different lengths and of different diameters (and hence of different cross sectional areas) as shown below. Let this bar be subjected to



- The total change in length will be obtained by adding the changes in length of individual sections
- Total stress in section 1: $\varsigma 1 = E1 \times \Delta L1/L1$

  $\varsigma 1 \times L1/E1 = \Delta L1$

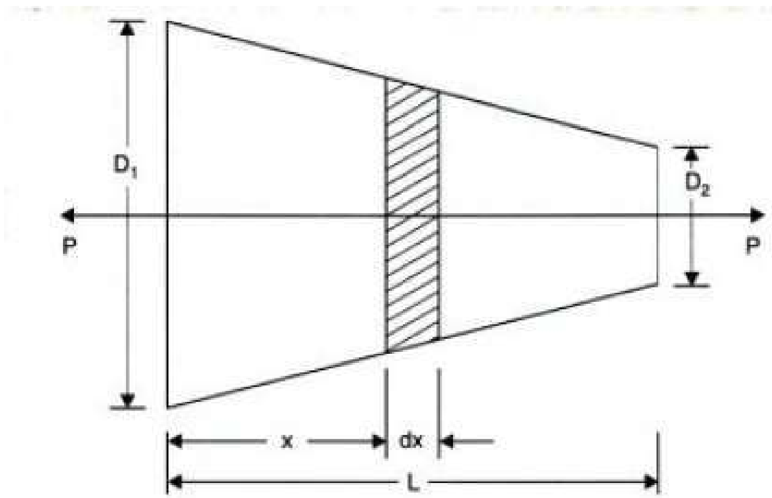
  $\varsigma 1 = P/A1$; Hence $\Delta L1 = PL1/A1E1$
- Similarly, $\Delta L2 = PL2/A2E2$; $\Delta L3 = PL3/A3E3$

1.10 ANALYSIS OF BARS OF VARYING CROSS SECTION

- Hence total elongation $\Delta L = P x \, (L1/A1E1 + L2/A2E2 + L3/A3E3)$
- If the Young's modulus of different sections are the same, $E1=E2=E3=E$; Hence $\Delta L = P/E x \, (L1/A1 + L2/A2 + L3/A3)$
- When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads
- While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of each section is calculated and the total deformation is equal to the algebraic sum of deformations of individual sections

1.11 ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider a bar uniformly tapering from a diameter D1 at one end to a diameter D2 at the other end
- Let
- P→ Axial load acting on the bar
- L→ Length of bar
- E→ Young's modulus of the material



1. 11 ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider an infinitesimal element of thickness dx, diameter Dx at a distance x from face with diameter D1.

  Deformation of the element $d(\Delta x) = P \times dx / (A_x \times E)$

  $A_x = \pi/4 \times D_x^2$; $D_x = D1 - (D1 - D2)/L \times x$

  Let $(D1-D2)/L = k$; Then $D_x = D1 - kx$

  $d(\Delta L_x) = 4 \times P \times dx / (\pi \times (D1-kx)^2 \times E)$

  Integrating from x=0 to x=L 4PL/(πED1D2)

$$\int_0^L d(\Delta x) = \int_0^L 4 \times P \times dx / (\pi \times (D1-kx)^2 \times E)$$

  Let $D1 - kx = \lambda$; then $dx = -(d\lambda/k)$

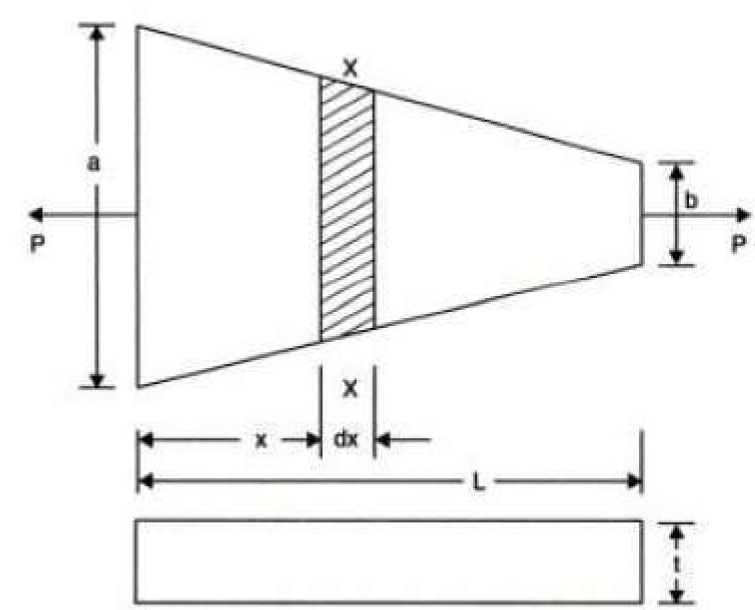
  When x=0, λ=D1; When x=L, λ=D2

$$\int_0^L d(\Delta L_x) = \int_{D1}^{D2} -4 \times P \times dx / (\pi \times \lambda^2 \, k \times E)$$

$$\Delta L_x = 4PL/(\pi E D1 D2)$$

1.12 ANALYSIS OF UNIFORMLY TAPERING RECTANGULAR BAR

A bar of constant thickness and uniformly tapering in width from one end to the other end is shown in Fig. 1.14.

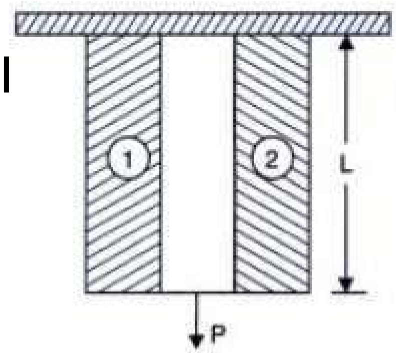


Let P = Axial load on the bar
L = Length of bar
a = Width at bigger end
b = Width at smaller end
E = Young's modulus
t = Thickness of bar

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}.$$

1.13 ANALYSIS OF BARS OF COMPOSITE SECTIONS

- A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for elongation and shortening when subjected to axial loads is called composite bar.
- Consider a composite bar as showı



- Let

P→ Applied load

L→ Length of bar

A1→ Area of cross section of Inner member

A2→ Cross sectional area of Outer member

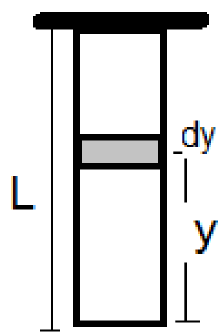
1.13 ANALYSIS OF BARS OF COMPOSITE SECTIONS

- Strain developed in the outer member= Strain developed in the inner member

  $\varsigma 1/E1 = \varsigma 2/E2$
- Total load (P)= Load in the inner member (P1) + Load in the outer member (P2)
- $\varsigma 1 \times A1 + \varsigma 2 \times A2 = P$
- Solving above two equations, we get the values of ς1, ς2 & e1 and e2

PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

- Consider a bar of length L, area of cross section A rigidly fixed at one end. Let ρ be the density of the material. Consider an infinitesimal element of thickness (ce y from the bottom of the bar.



- The force acting on the element considered= weight of the portion below it=$\rho A g y$

PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

- Tensile stress developed= Force acting on the element/Area of cross section= ρgy.

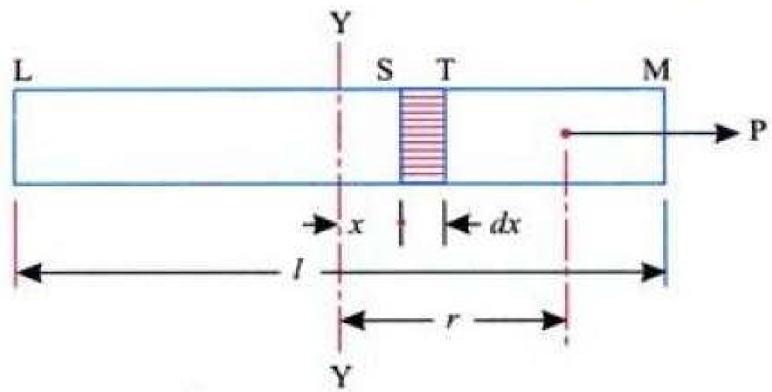
- From the above equation, it is clear that the maximum stress at the section where y=L, ie at the fixed end (ρgL) and minimum stress is at the free end(=0)

$$\Delta L_y = \int_0^L \rho g y \, dy / AE = \rho g L^2 / 2AE$$

- Elongation due to self weight

1.15 STRESS IN BAR DUE TO ROTATION

Consider a bar of length l rotating about the axis y at a constant angular velocity ω. Consider an infinitesimal element of thickness dx at a distance x from the axis of rotation.



Tensile force on element ST= Centrifugal force on element TM

Centrifugal force on element TM= Mass of element TM x r x ω^2= {l/2 – (x+dx)} x A x ρ x r x ω^2

r= x + ½ x (l/2- (x+dx)

As dx is numerically very small, x + dx≈x

Hence tensile force on element ST=(l/2 – x) x A x{x + ½ x (l/2-x)} x ρ x ω^2

= A x ρ x ω^2 x (l^2/4 - x^2)/2

1.15 STRESS IN BAR DUE TO ROTATION

Tensile stress developed= Tensile force/cross sectional area= $A \times \rho \times \omega^2 \times (l^2/4 - x^2)/2A$

$\sigma rod = \rho \times \omega^2 \times (l^2/4 - x^2)/2$

$\sigma rod = 0$, when $x = l/2$

σrod =Maximum when $d(\sigma rod)/dx = 0$; ie when $x=0$

$\sigma rodmax == \rho \times \omega^2 \times l^2/8$

Extension of element= $\sigma rod \times dx/E$

Extension of entire bar= $\int_0^l \rho \times \omega^2 \times (l^2/4 - x^2) dx/2 = \rho \times \omega^2 \times l^3/12E$

Extension of entire bar= $\rho \times \omega^2 \times l^3/12E$

1.16 THERMAL STRESS

- Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are set up in a body, when the temperature of the body is raised or lowered and the body is restricted from expanding or contracting
- Consider a body which is heated to a certain temperature

  Let

  L= Original length of the body

  Δ T=Rise in temp

  E=Young's modulus

  α=Coefficient of linear expansion

  dL= Extension of rod due to rise of temp
- If the rod is free to expand, Thermal strain developed

  $et = \Delta L/L = \alpha \times \Delta T$

1.16 THERMAL STRESS

- The extension of the rod, $\Delta L = L \times \alpha \times \Delta T$
- If the body is restricted from expanding freely, Thermal stress developed is ςt/et=E
- $ςt = E \times \alpha \times \Delta T$
- Stress and strain when the support yields:-

  If the supports yield by an amount equal to δ, then the actual expansion is given by the

  difference between the thermal strain and δ

  Actual strain, $e = (L \times \alpha \times \Delta T - \delta)/L$

  Actual stress= Actual strain x E= $(L \times \alpha \times \Delta T - \delta)/L \times E$

UNIT II

SHEAR AND BENDING IN BEAMS

APPLIED AND REACTIVE FORCES

- **Forces that act on a Body can be divided into two Primary types: applied and reactive.**

- **In common Engineering usage, applied forces are forces that act directly on a structure like, dead, live load etc.)**

- **Reactive forces are forces generated by the action of one body on another and hence typically occur at connections or supports.**
- **The existence of reactive forces follows from Newton's third law, which state that to every action , there is an equal and opposite reaction.**

SUPPORTS

To bear or hold up (a load, mass, structure, part, etc.); serve as a foundation or base for any structure.

To sustain or withstand (weight, pressure, strain, etc.) without giving way

It is a aid or assistance to any structure by preserve its load

Supports are used to connect structures to the ground or other bodies in order to restrict (confine) their movements under the applied loads. The loads tend to move the structures, but supports prevent the movements by exerting opposing forces, or reactions, to neutralize the effects of loads thereby keeping the structures in equilibrium.

TYPES OF SUPPORTS

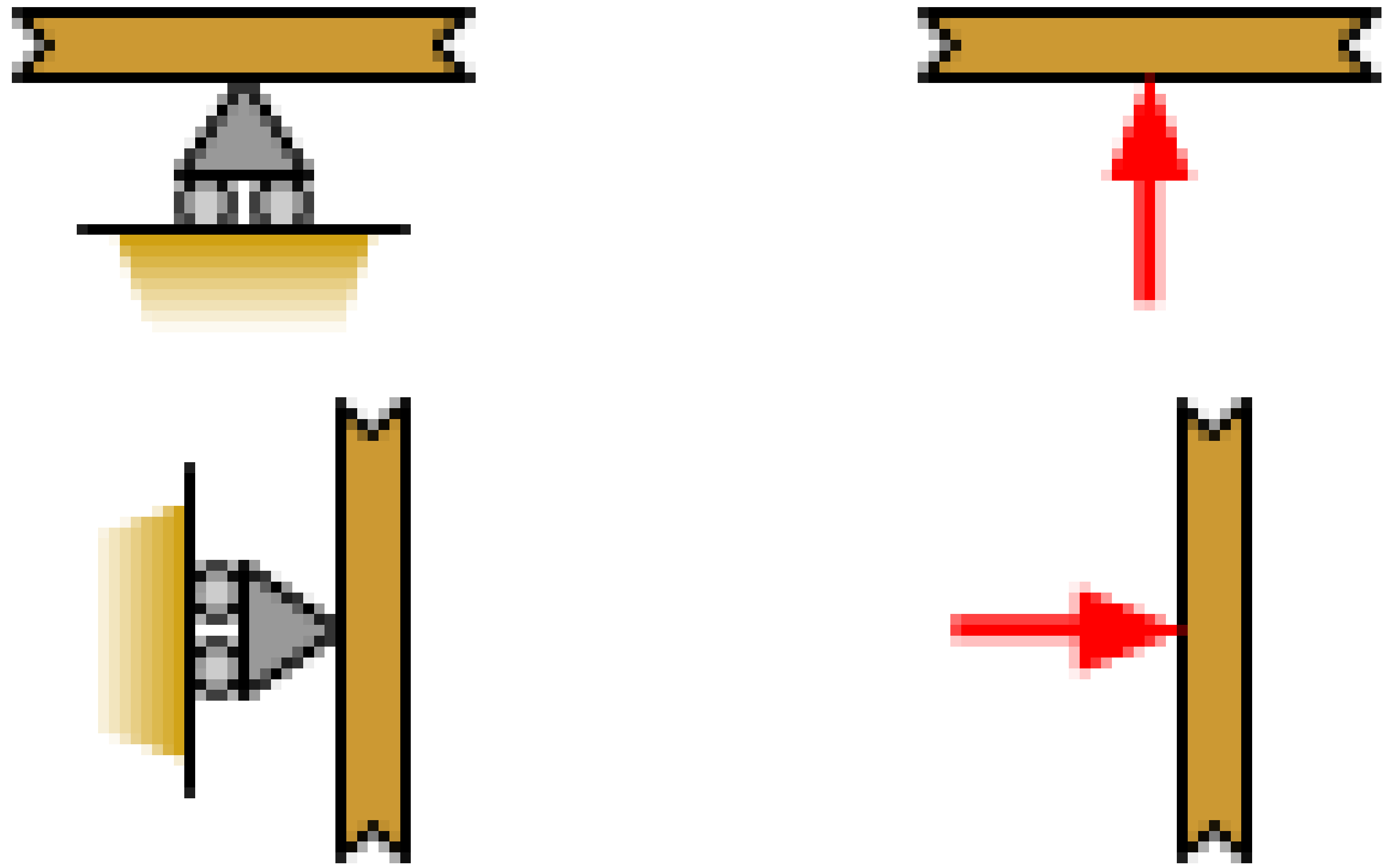
- Supports are grouped into three categories, depending on the number of reactions (1,2,or3) they exert on the structures.
- 1) Roller support
- 2) Hinge support
- 3) fixed support

ROLLER SUPPORT

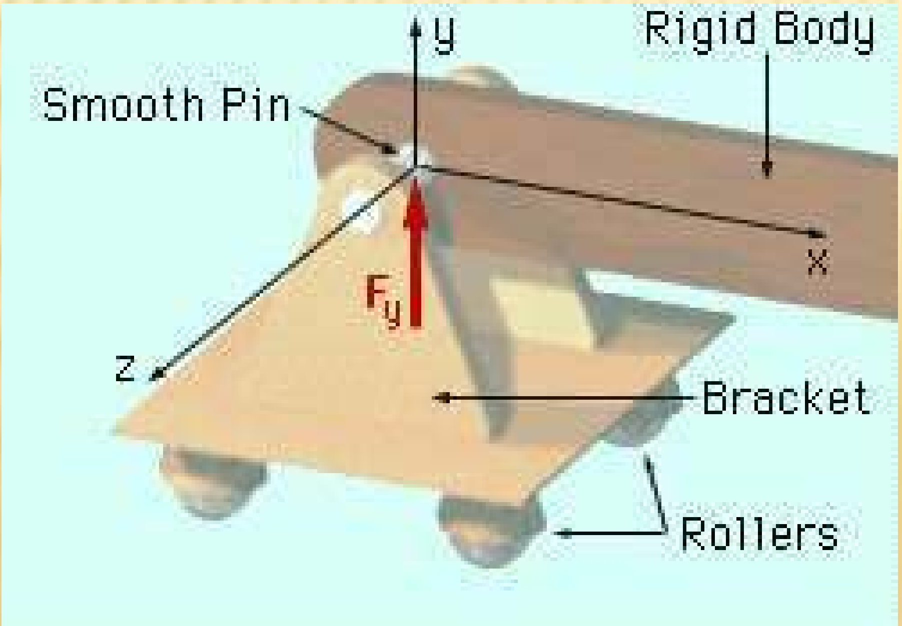
- Roller supports are free to rotate and translate along the surface upon which the roller rests.

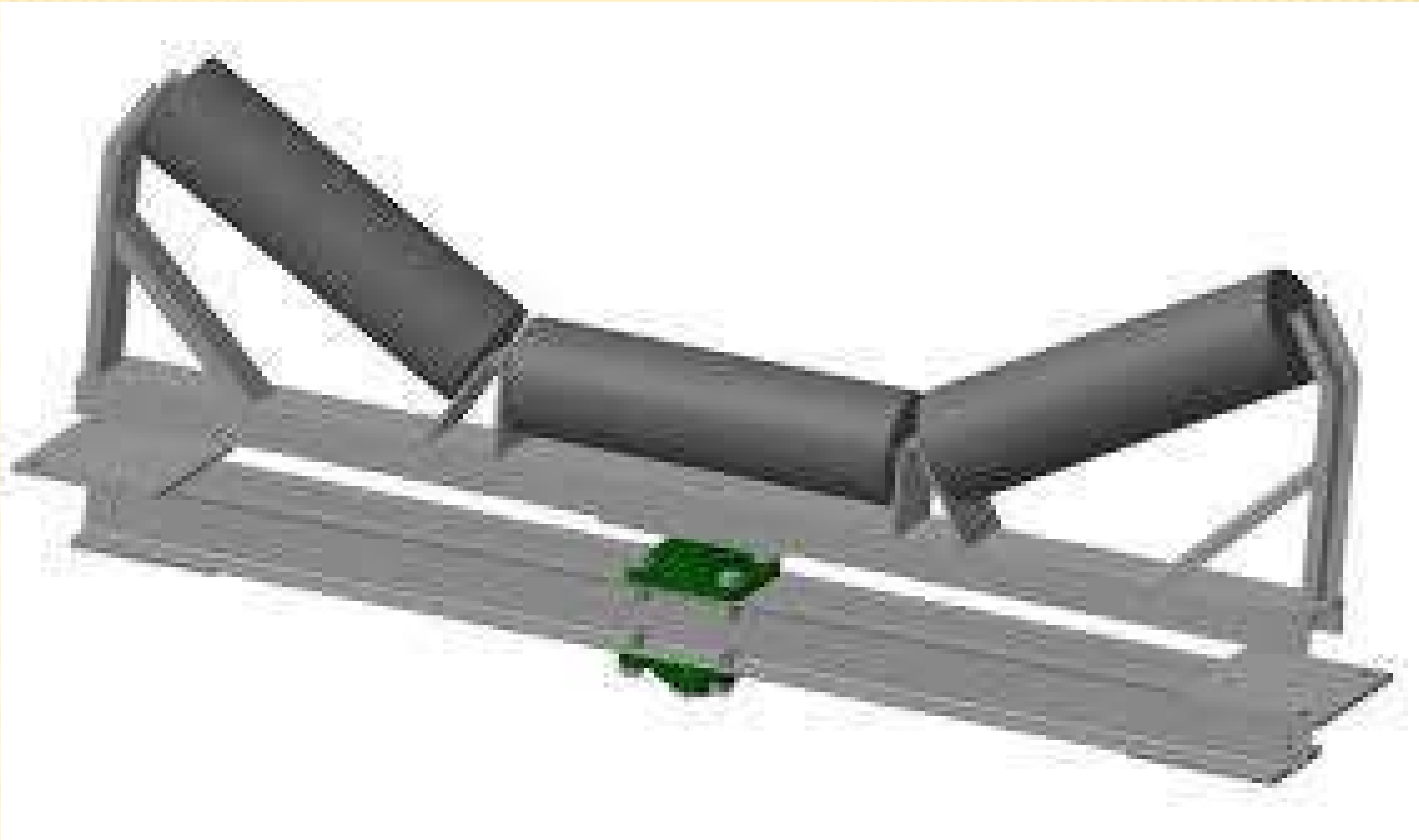
- The surface can be horizontal, vertical, or sloped at any angle.

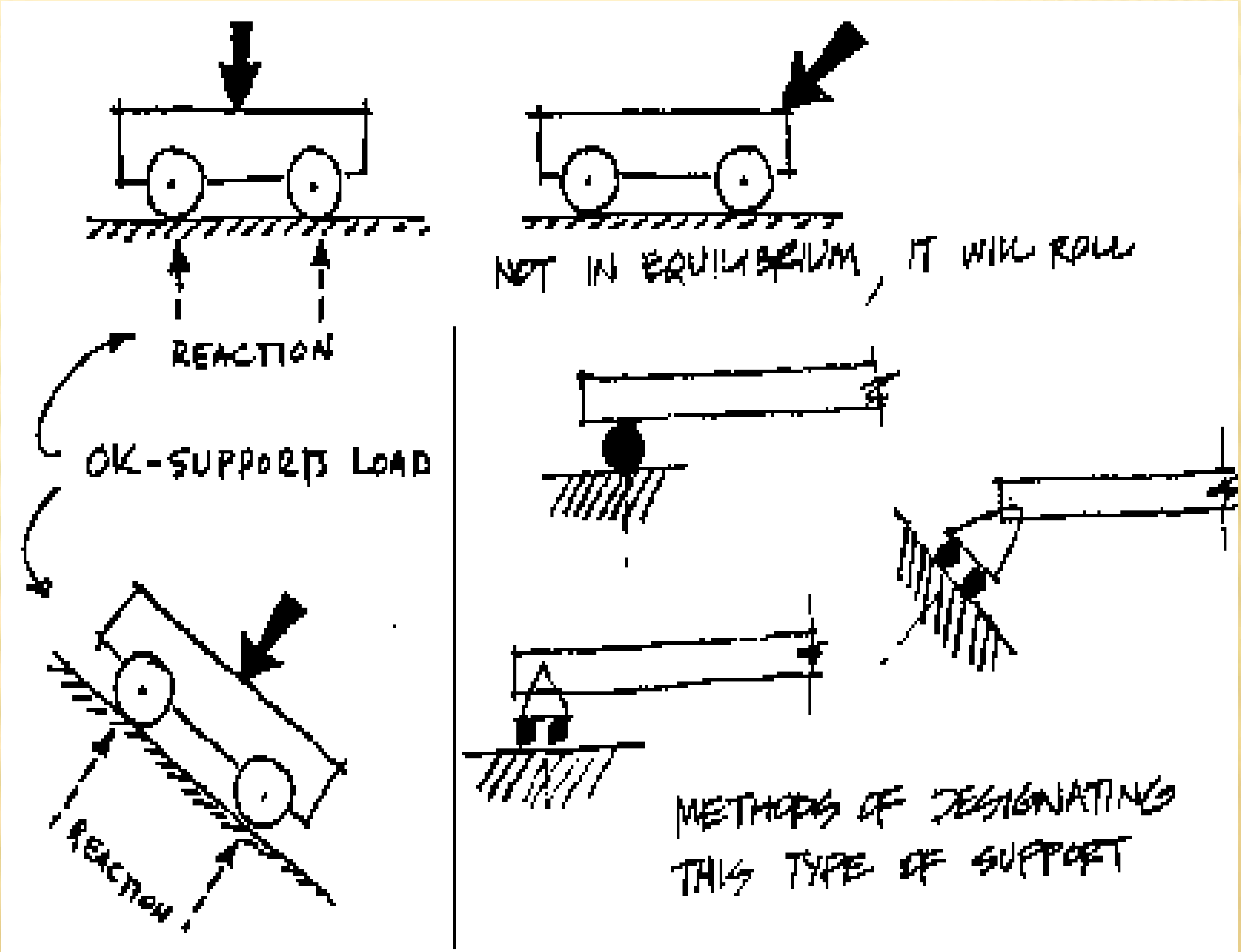
- The resulting reaction force is always a single force that is perpendicular to, and away from, the surface



Restrains the structure from moving in one or two perpendicular directions.

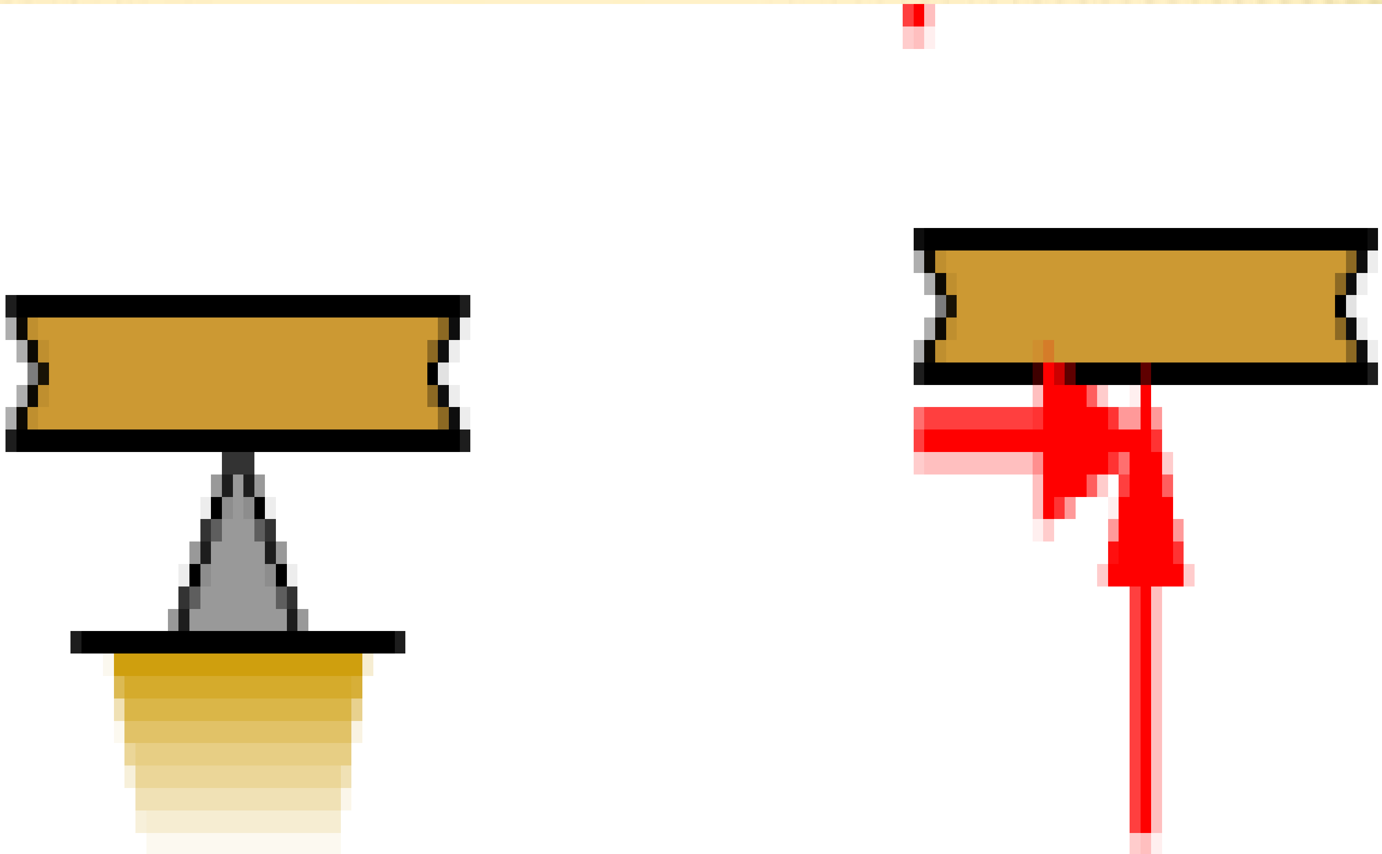

y
Rigid Body
Smooth Pin
x
F_y
z
Bracket
Rollers

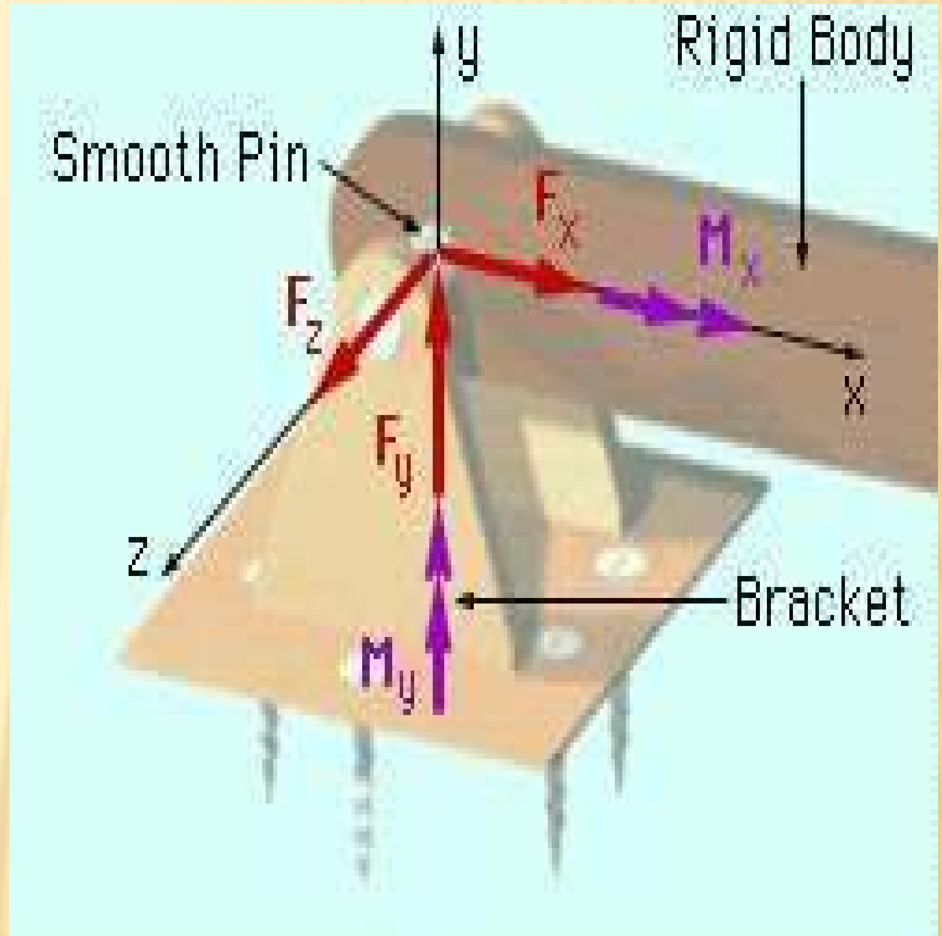



NOT IN EQUILIBRIUM, IT WILL ROLL
REACTION
OK-SUPPORTS LOAD
REACTION
METHODS OF DESIGNATING
THIS TYPE OF SUPPORT

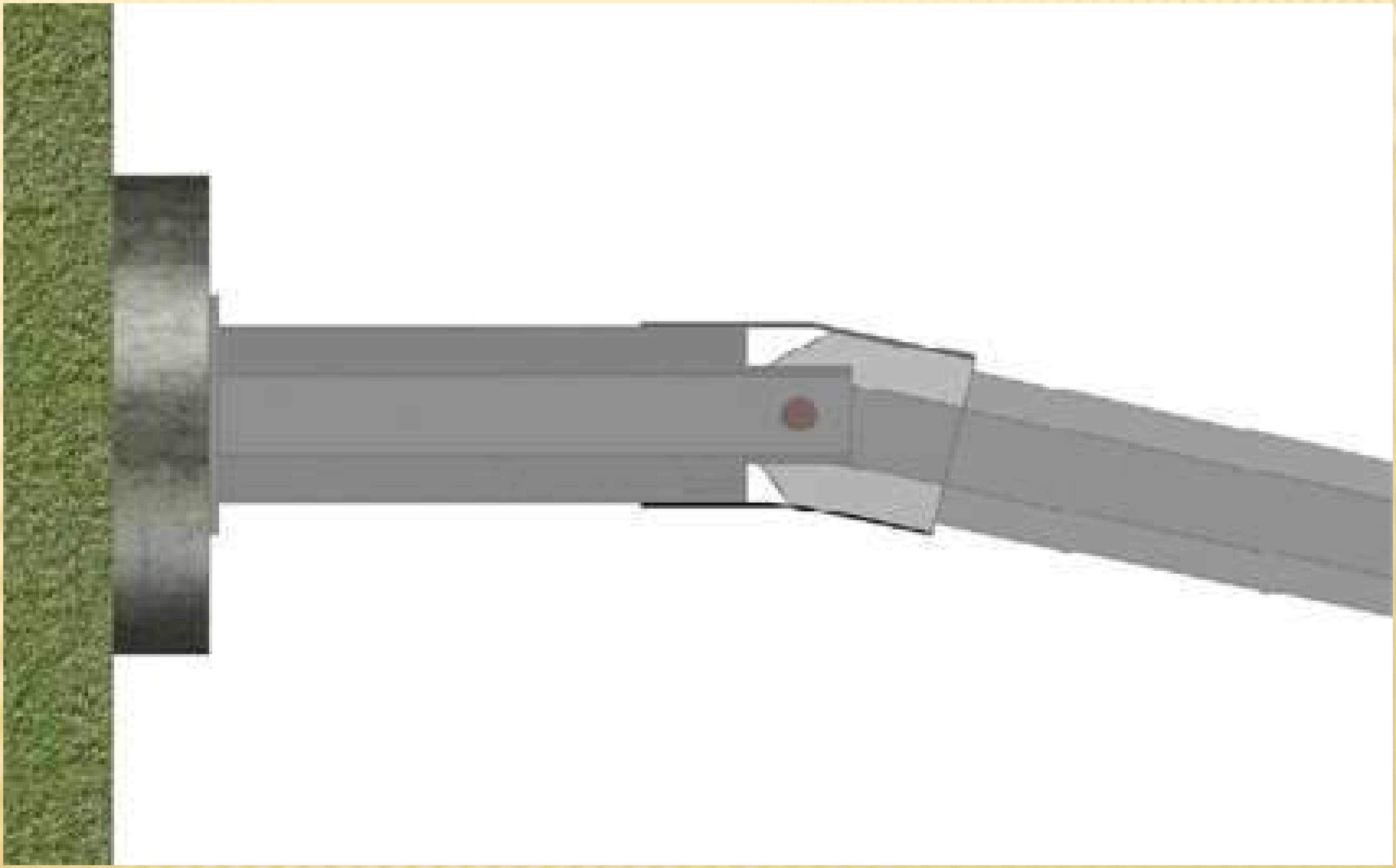
HINGE SUPPORT

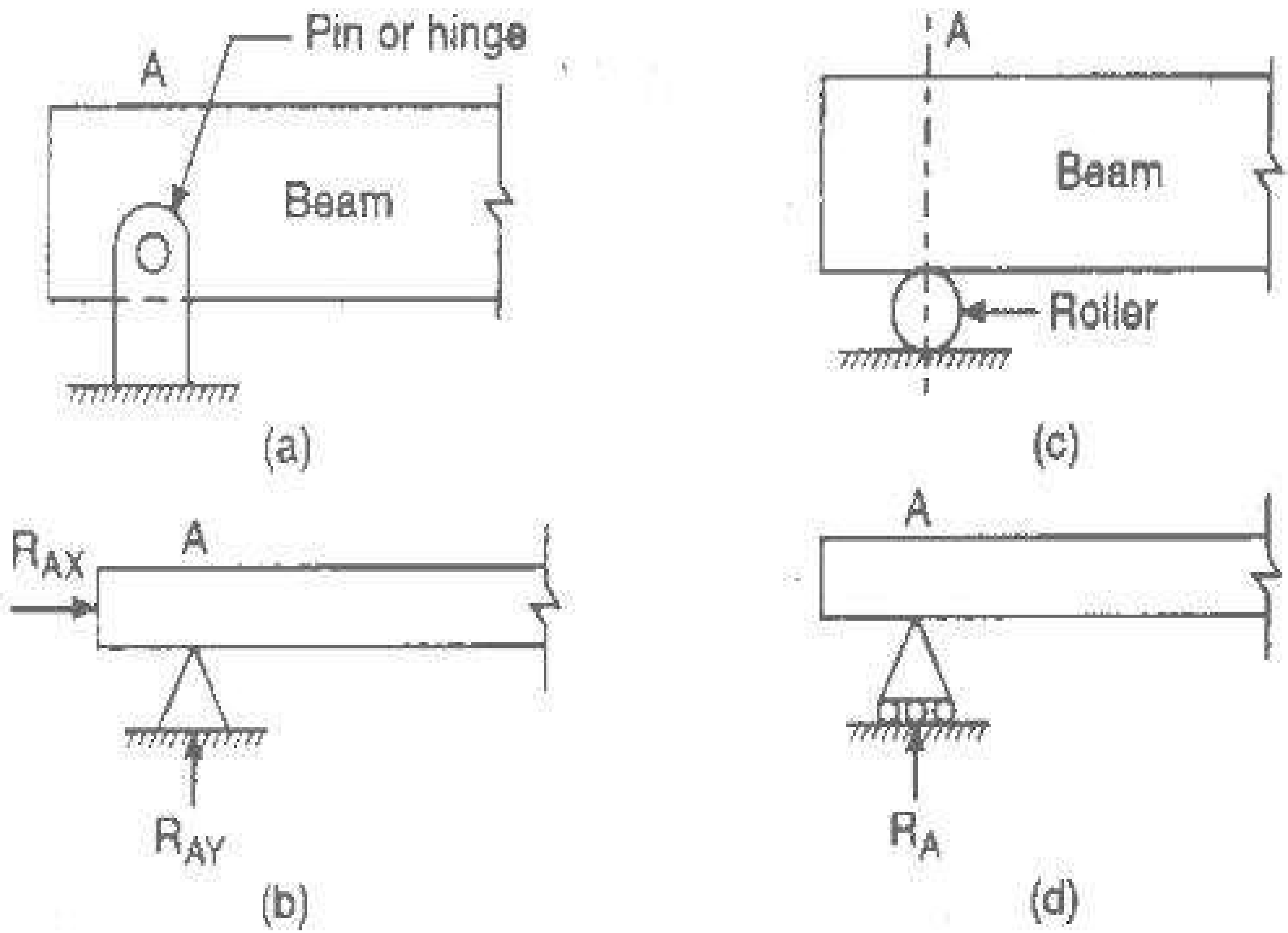
- A Hinge support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction
- Pin or hinge support is used when we need to prevent the structure from moving or restrain its translational degrees of freedom.
- A **hinge** is a type of bearing that connects two solid objects, typically allowing only a limited angle of rotation between them. Two objects connected by an ideal hinge rotate relative to each other about a fixed axis of rotation.





y
Rigid Body
Smooth Pin
F_x
M_x
F_z
x
F_y
z
Bracket
M_y




Pin or hinge
A
Beam
(a)
A
Beam
Roller
(c)
R_AX
A
R_AY
(b)
A
R_A
(d)

FIXED SUPPORT

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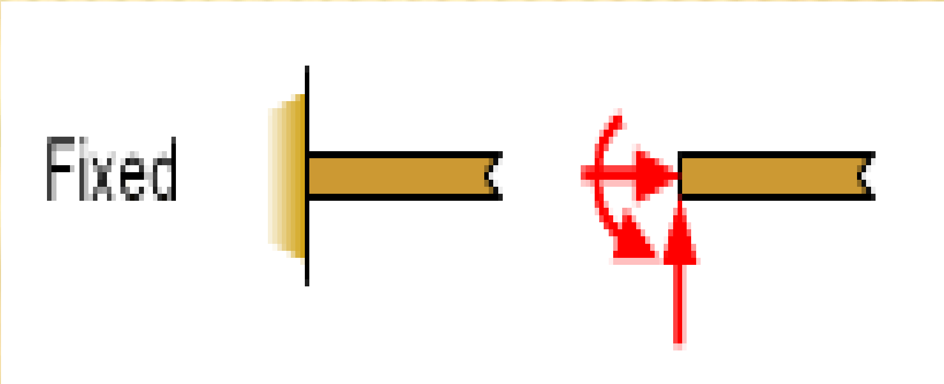
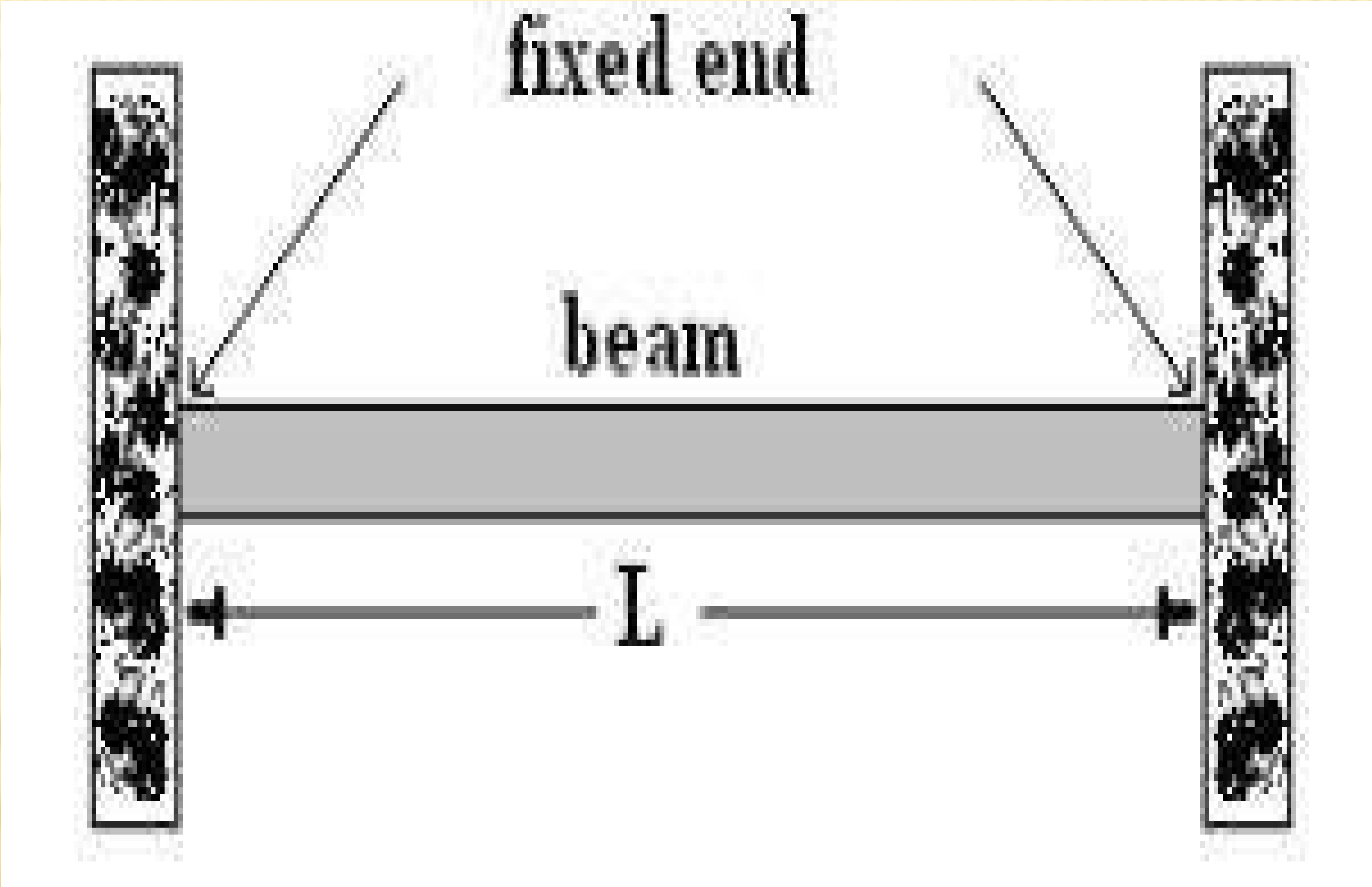



fixed end
beam
L

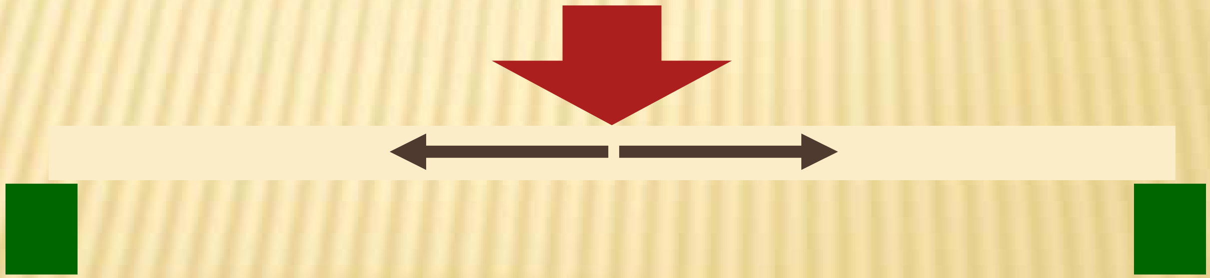
BEAM

- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse **shear force** and bending **moment**.
- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

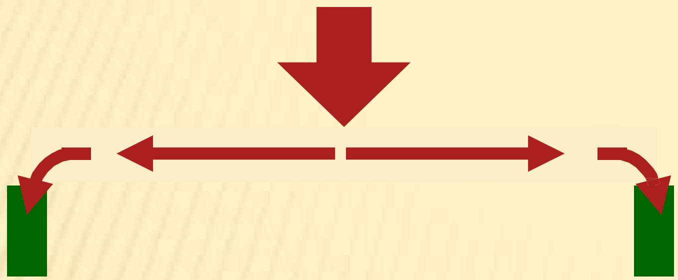
Beams

- Extremely common structural element

- In buildings majority of loads are vertical and majority of useable surfaces are horizontal



Beams



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action of beams involves combination of bending and shear

TYPES OF BEAMS

- The following are the important types of beams:
- 1. Cantilever
- 2. simply supported
- 3. overhanging
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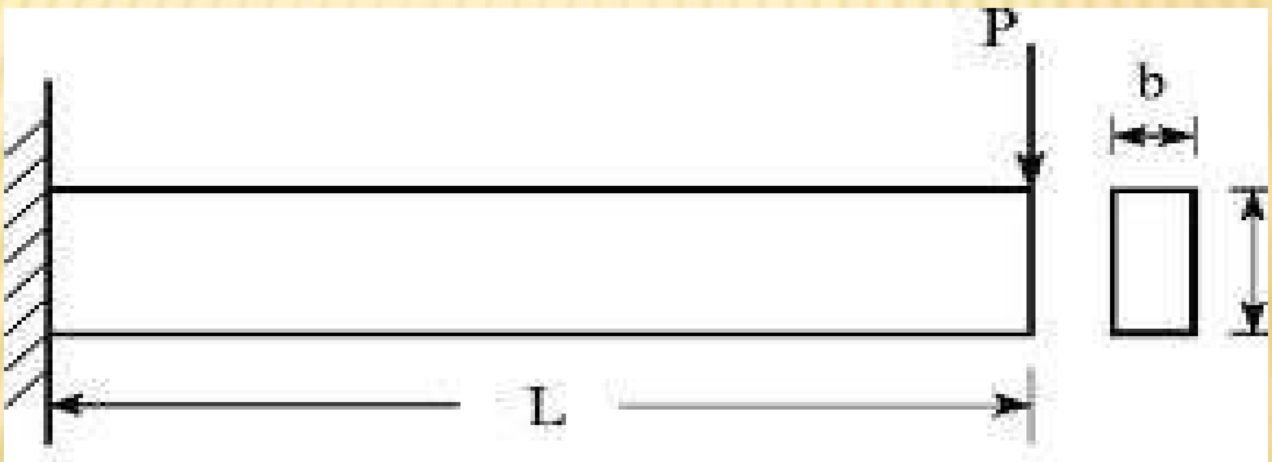


25

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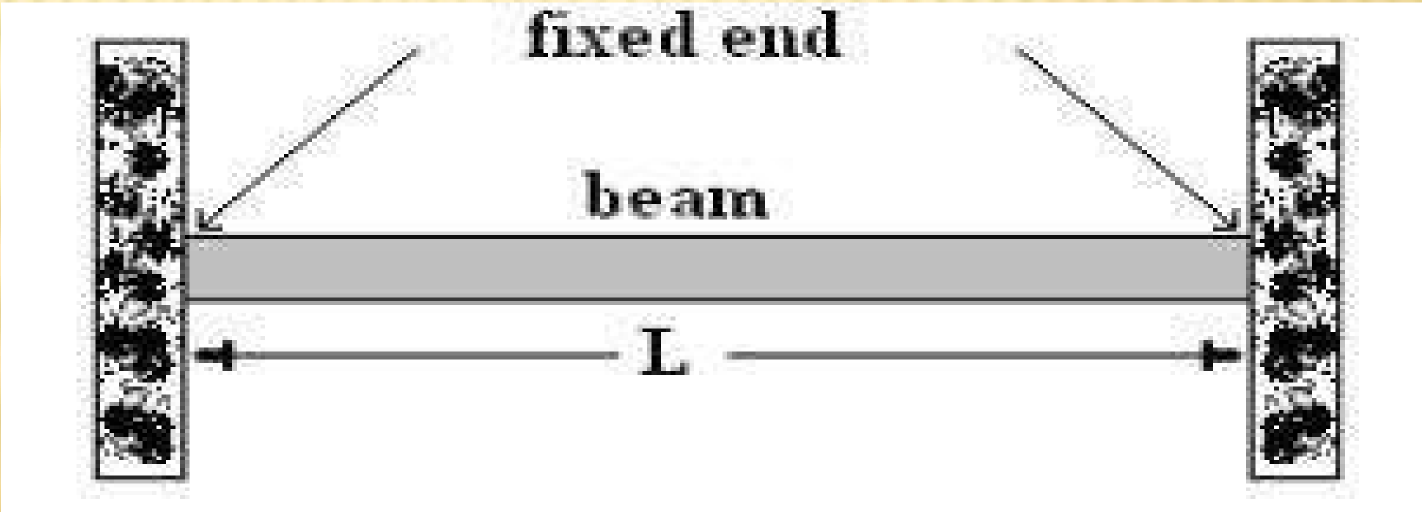

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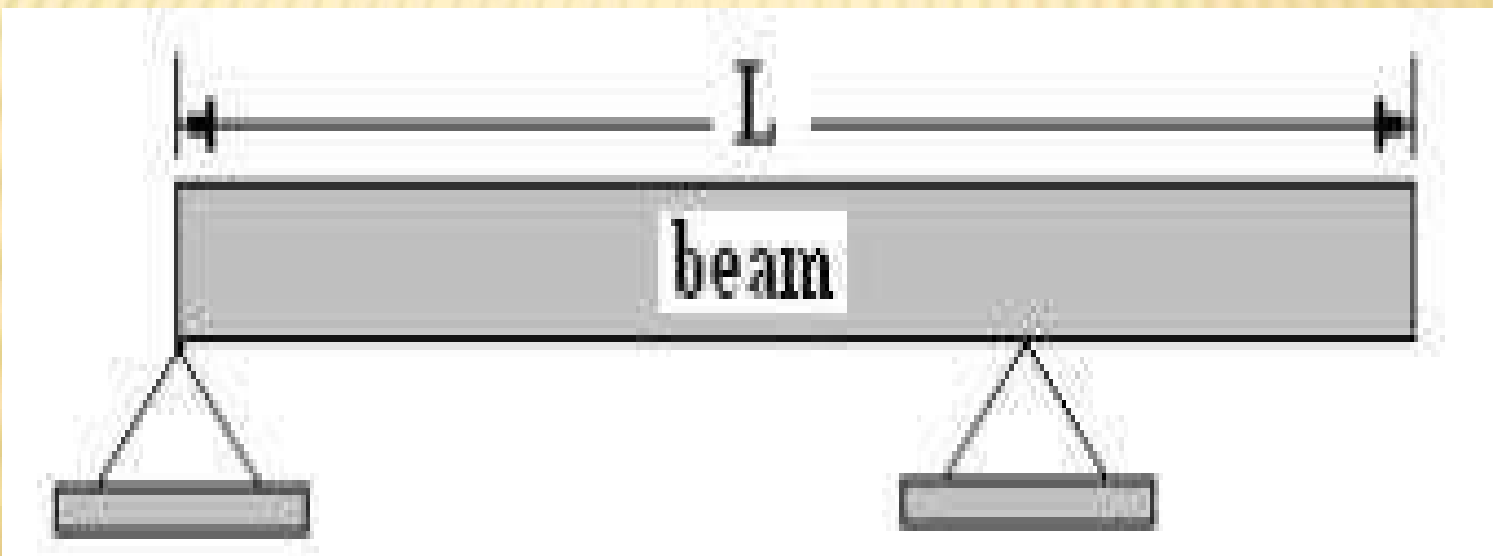

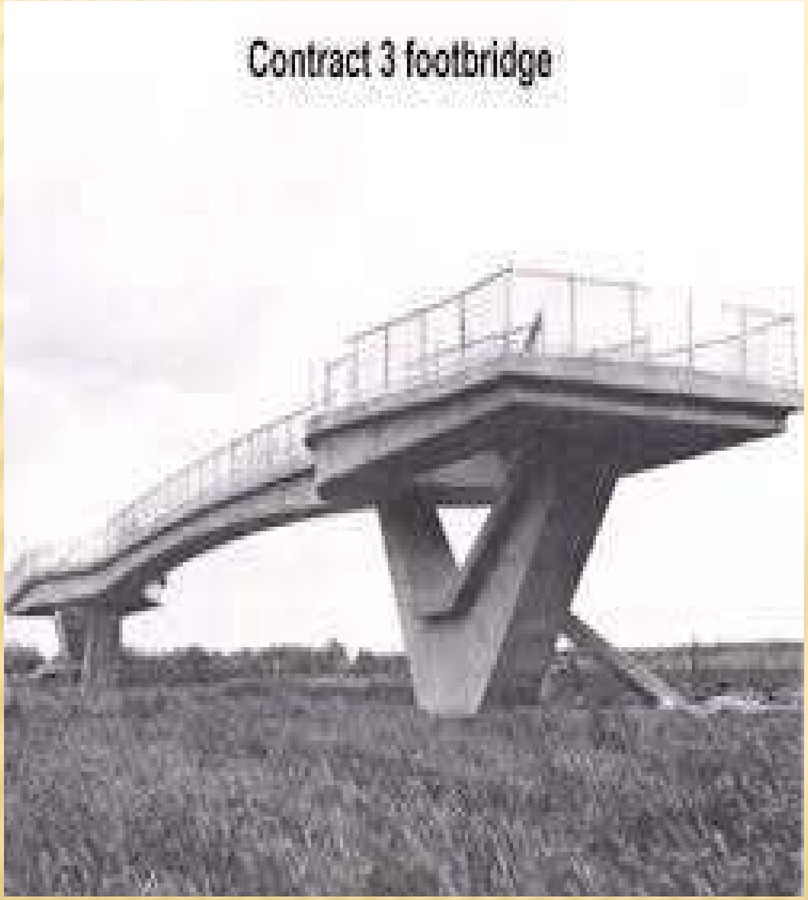
Contract 3 footbridge

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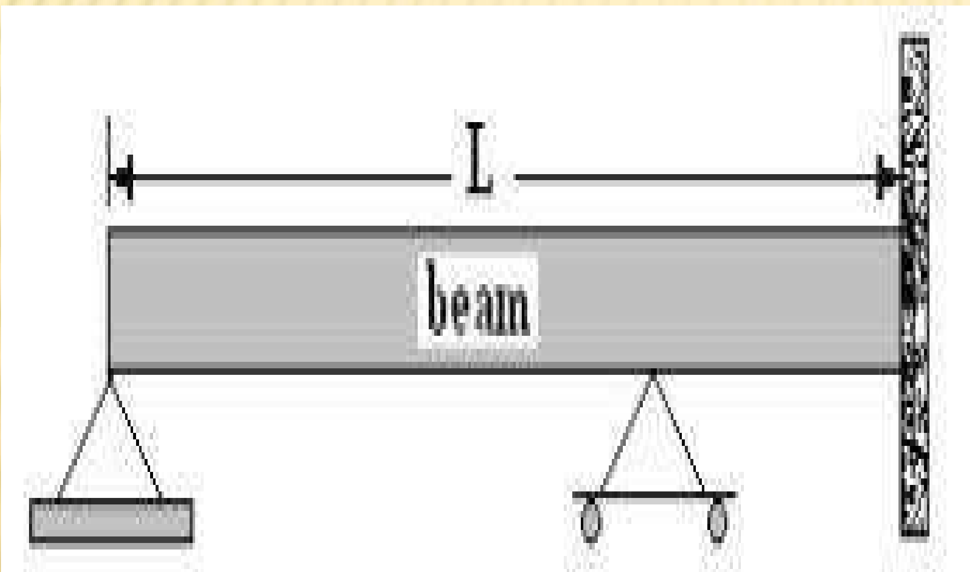

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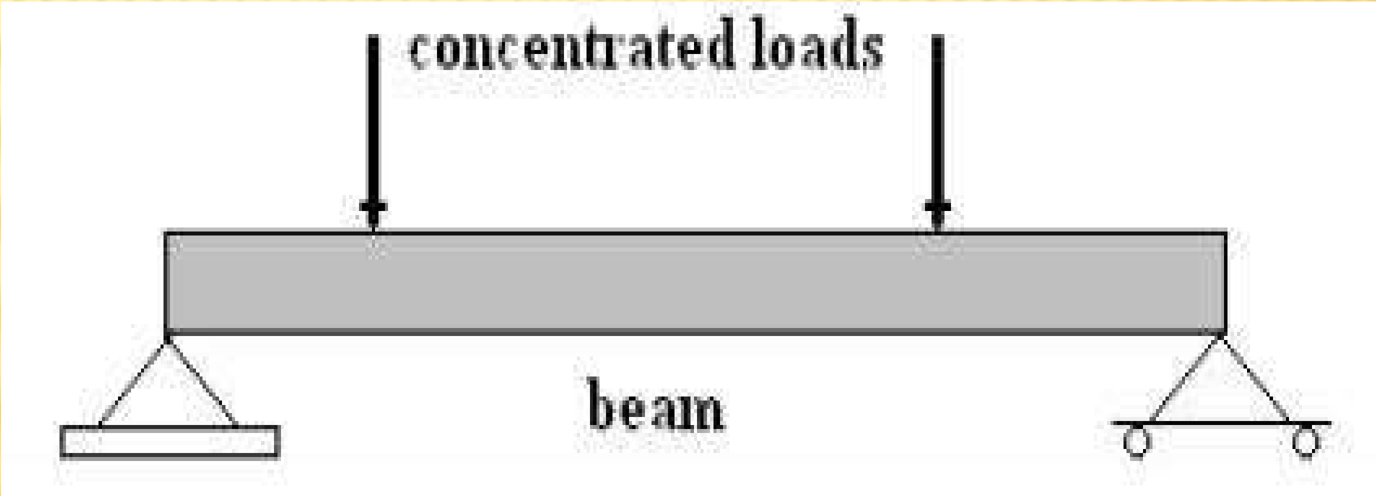
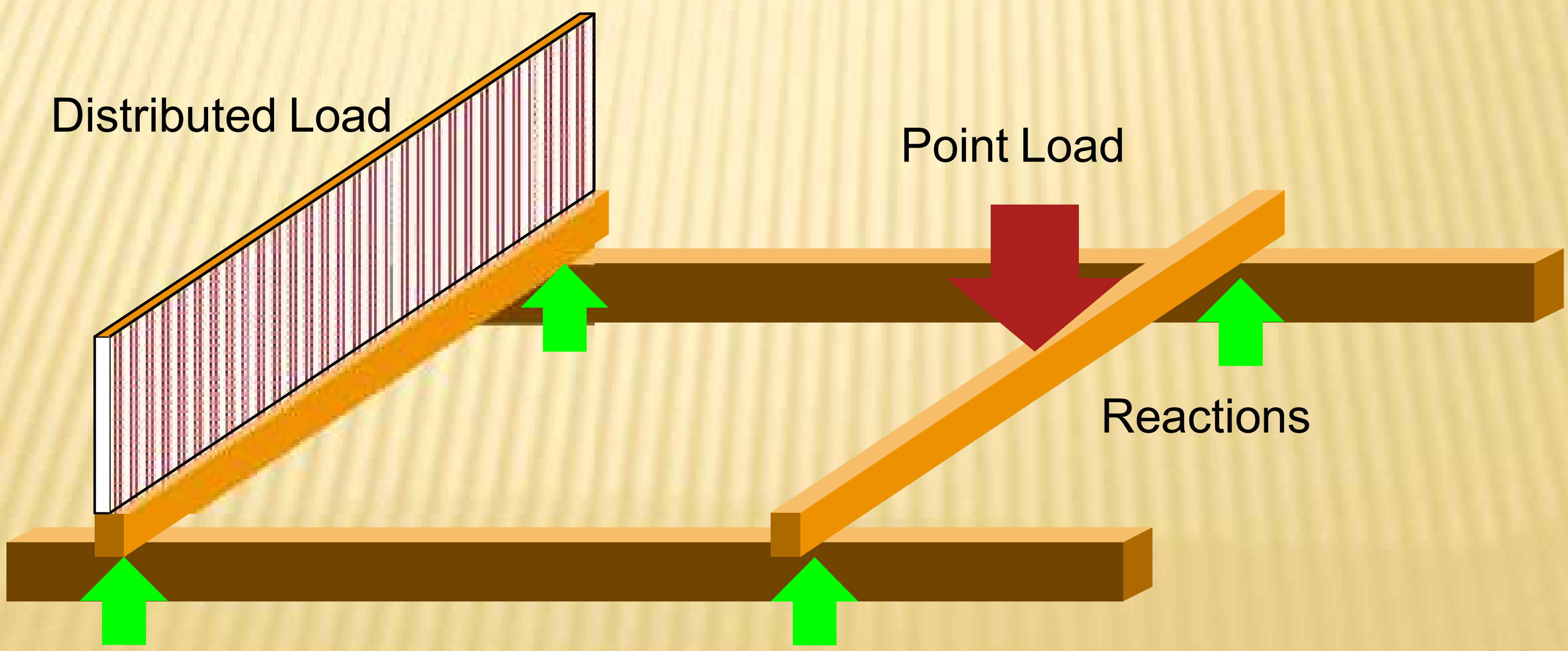
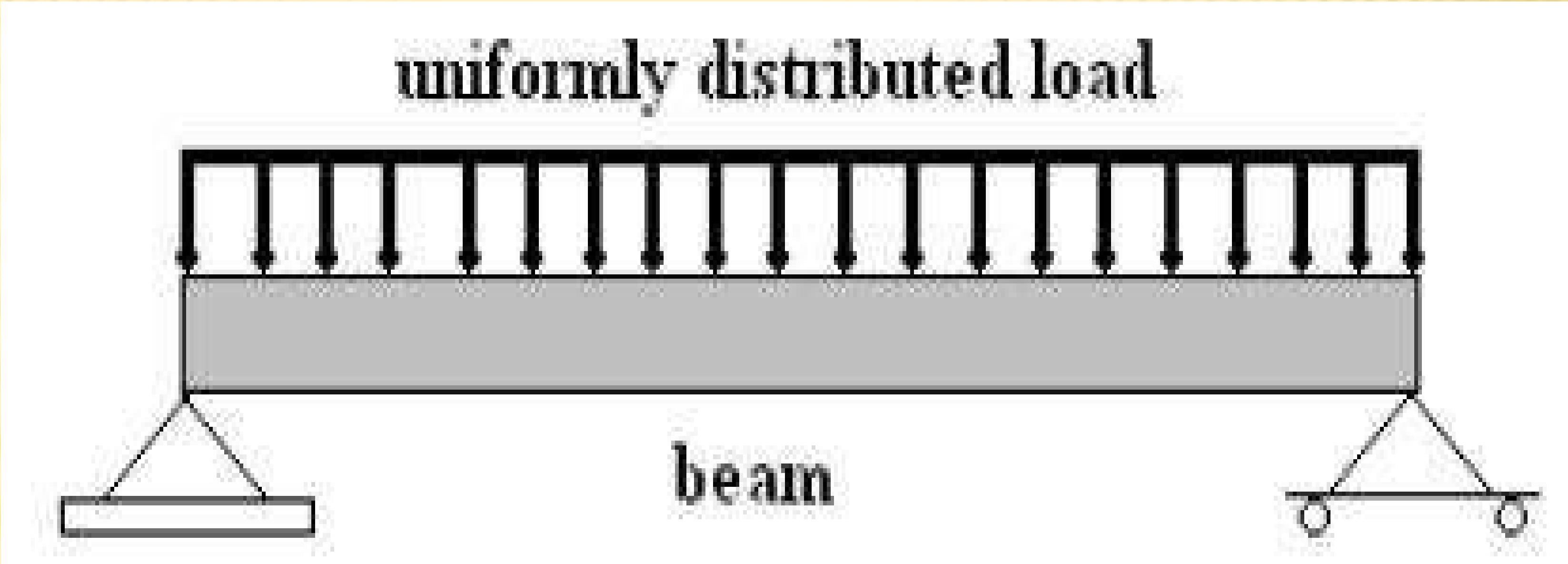
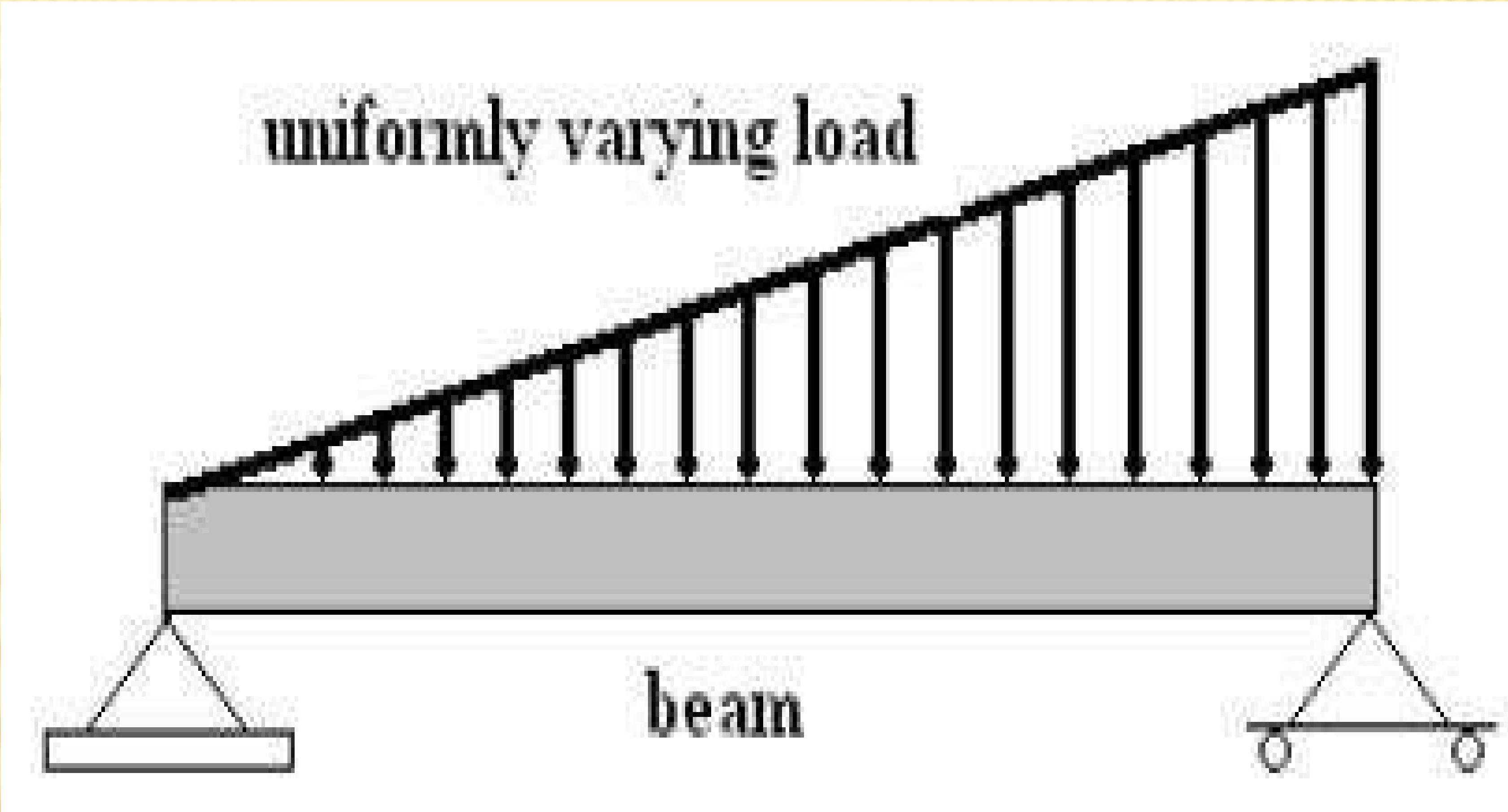


Loads on Beams

- Point loads, from concentrated loads or other beams
- Distributed loads, from anything continuous




uniformly distributed load
beam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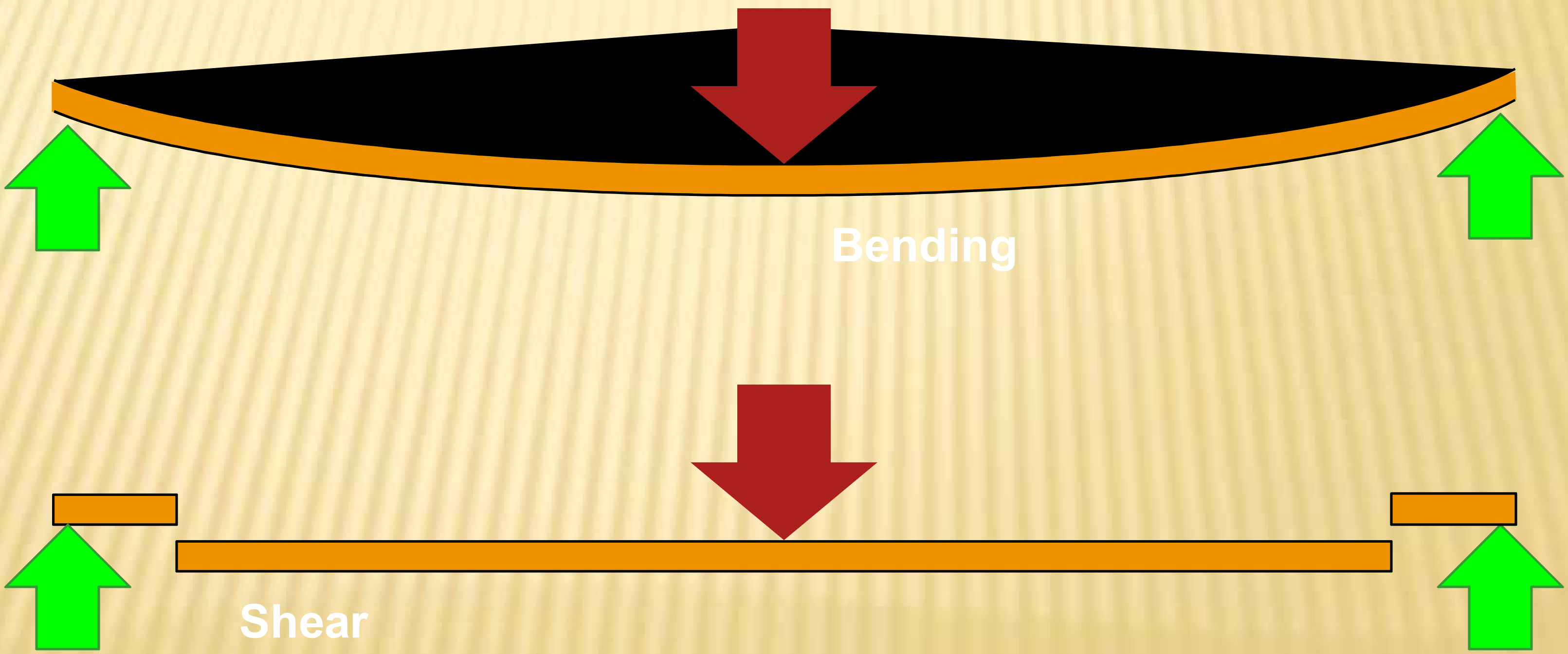
What the Loads Do

- The loads (& reactions) bend the beam, and try to shear through it



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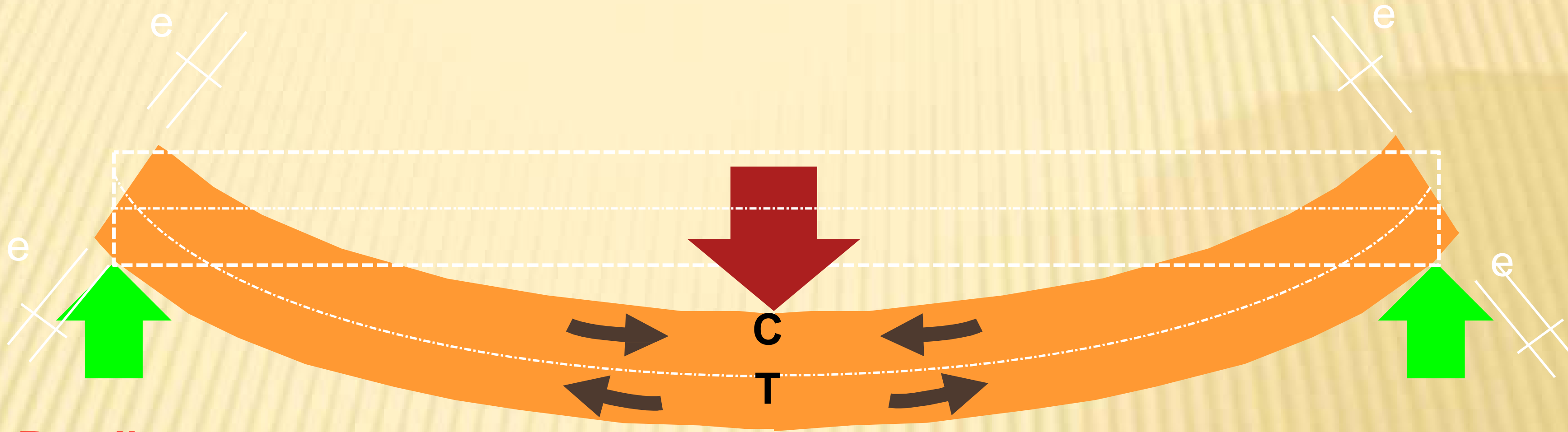
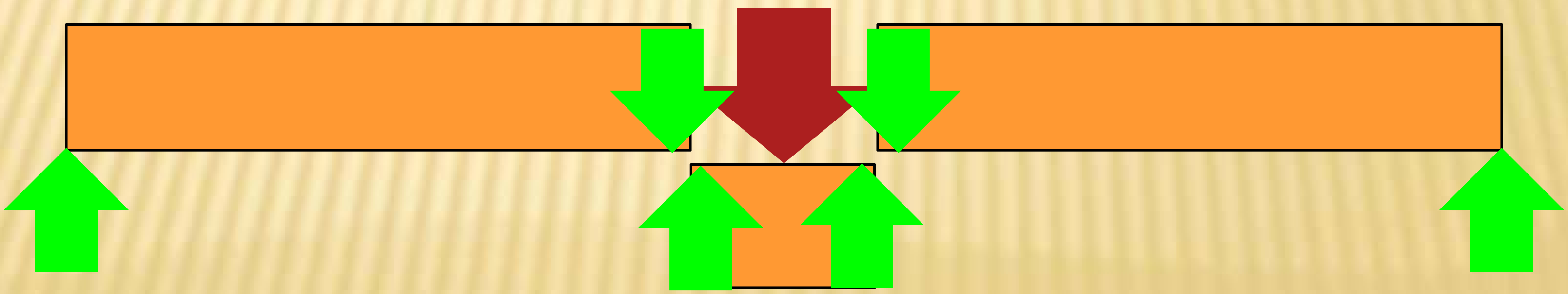


Bending



Shear

Designing Beams

- in architectural structures, bending moment more important
  - importance increases as span increases
- short span structures with heavy loads, shear dominant
  - e.g. pin connecting engine parts

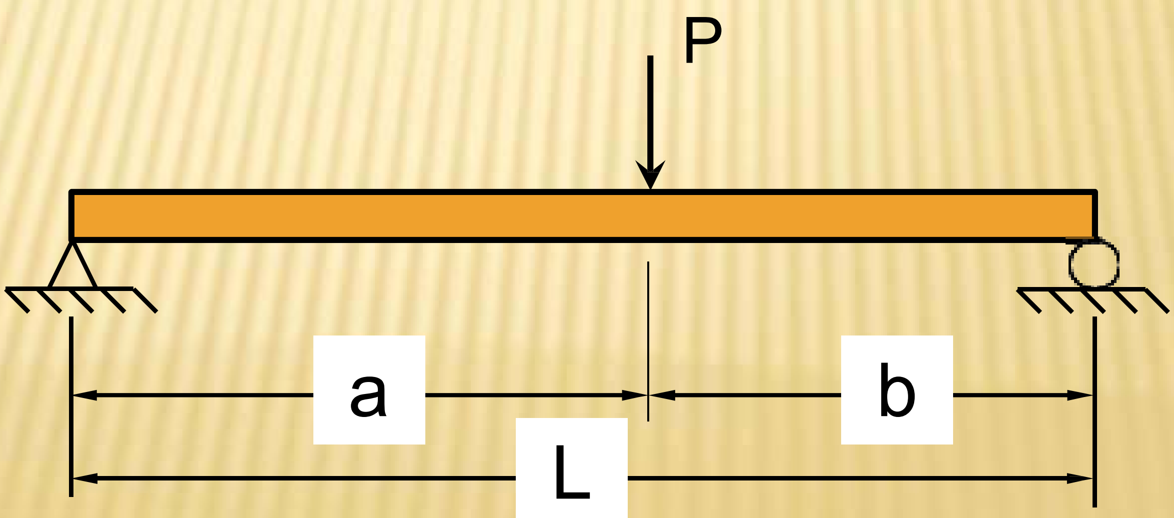
beams in building designed for bending checked for shear

How we calculate the Effects

- First, find ALL the forces (loads and reactions)
- Make the beam into a free body (cut it out and artificially support it)
- Find the reactions, using the conditions of equilibrium

INTERNAL REACTIONS IN BEAMS

- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - normal force,
  - shear force,
  - bending moment.



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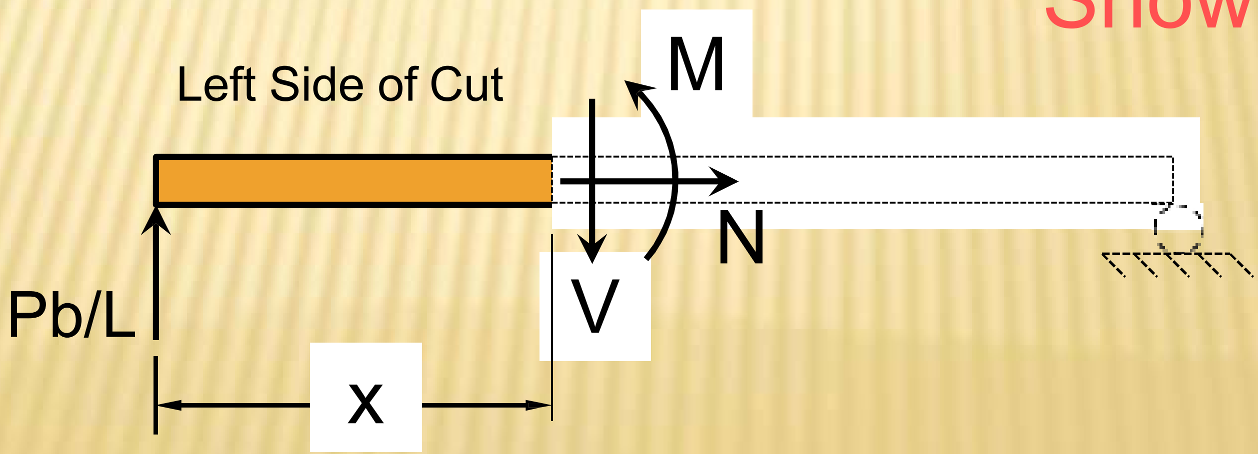

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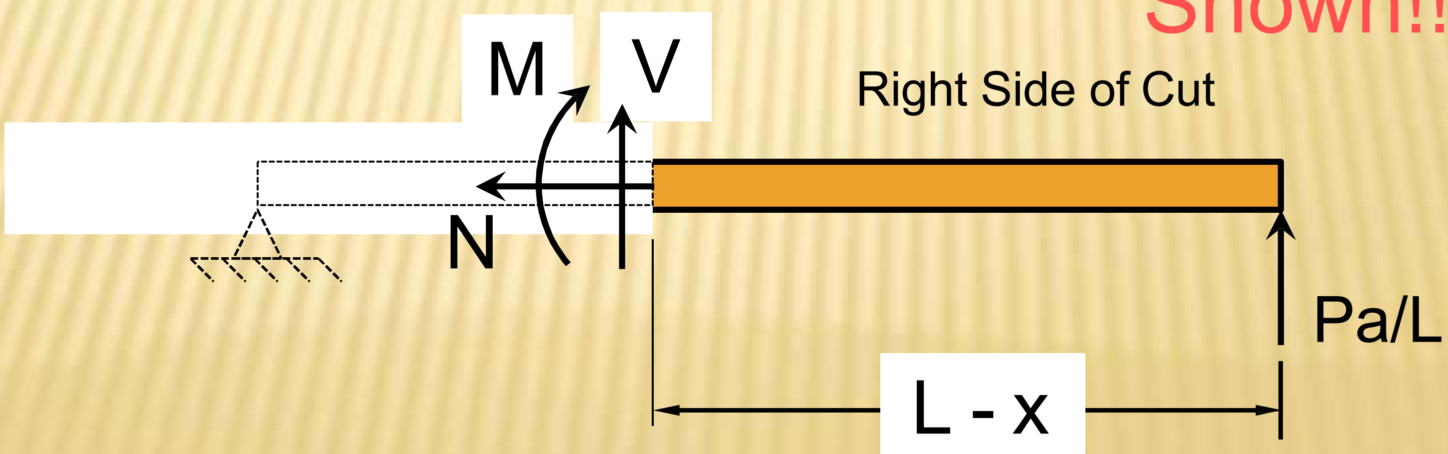
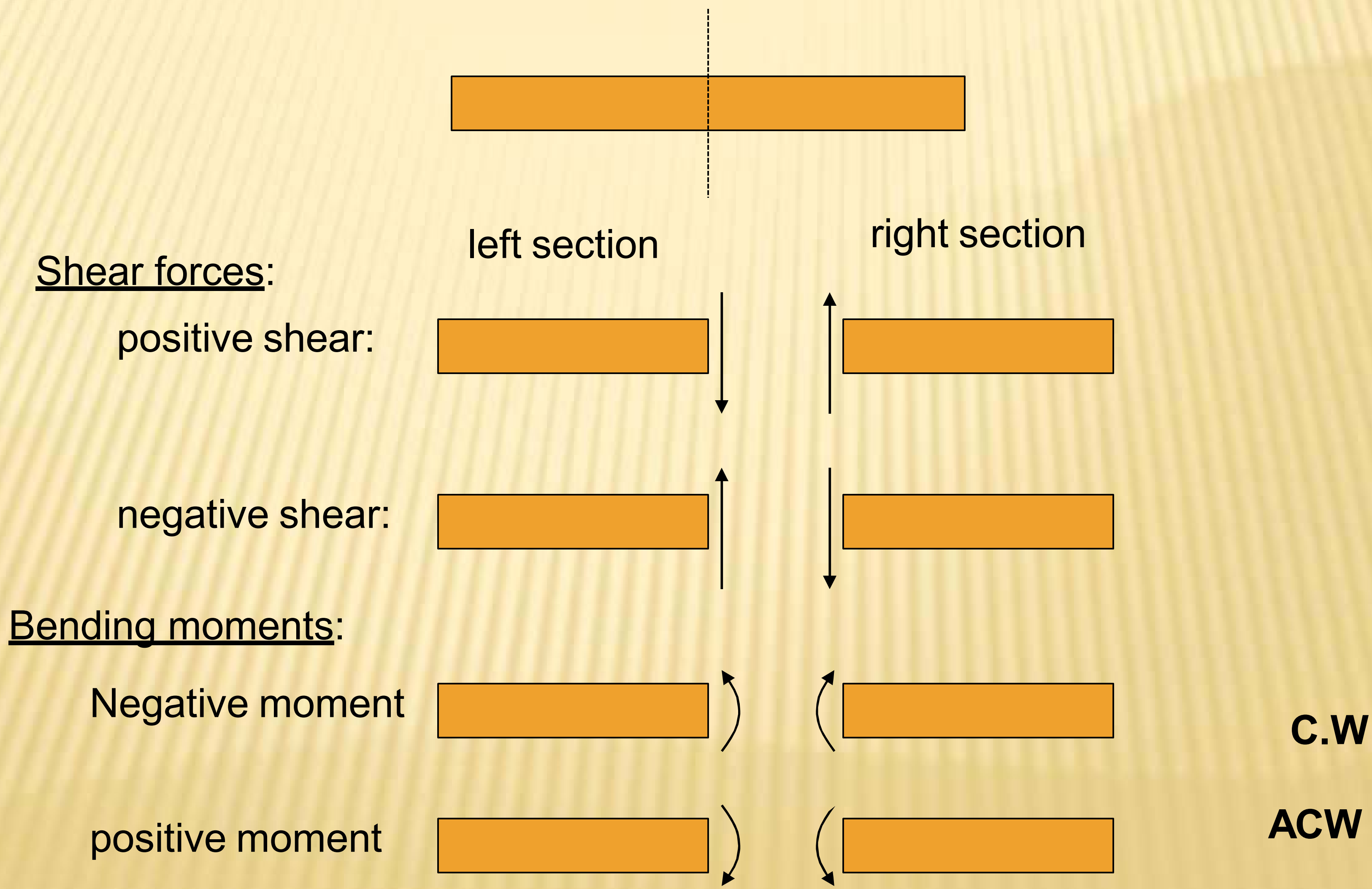
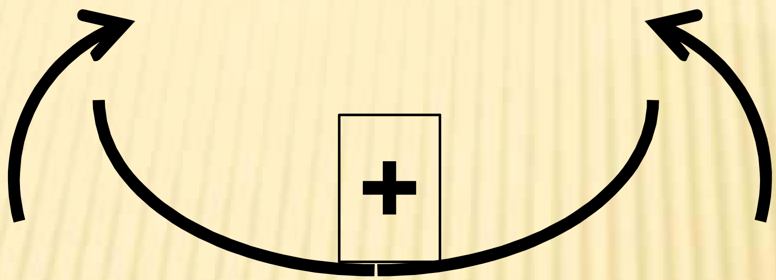


SHEAR FORCES, BENDING MOMENTS - SIGN CONVENTIONS

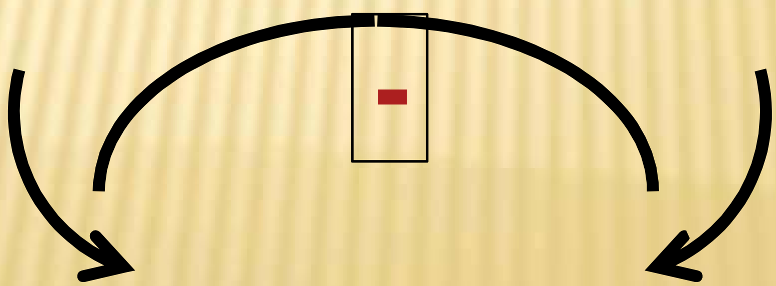


Sign Conventions
Bending Moment Diagrams (cont.)

Sagging bending moment is POSITIVE (happy)

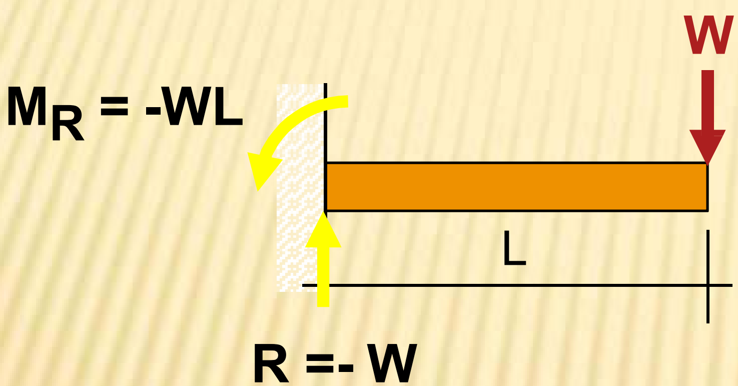


Hogging bending moment is NEGATIVE (sad)



Cantilever Beam Point Load at End

- Consider cantilever beam with point load on end



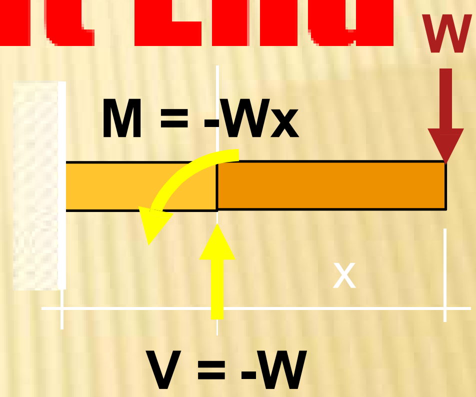
vertical reaction, R = -W
and moment reaction M_R = - WL

- **Use the free body idea to isolate part of the beam**
- **Add in forces required for equilibrium**

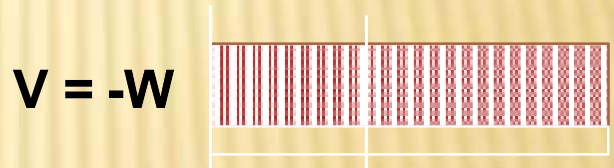
Cantilever Beam
Point Load at End

Take section anywhere at distance, x from end

Add in forces, V = -W and moment M = - Wx

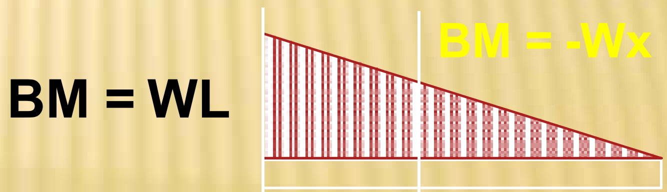


Shear V =- W constant along length



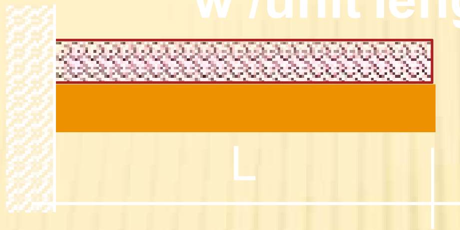
Shear Force Diagram

Bending Moment BM = -W.x
when x = L BM = -WL
when x = 0 BM = 0

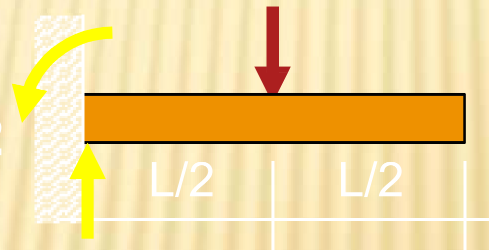


Bending Moment Diagram

Cantilever Beam
Uniformly Distributed Load



For maximum shear V and bending moment BM



vertical reaction, $R = W = wL$

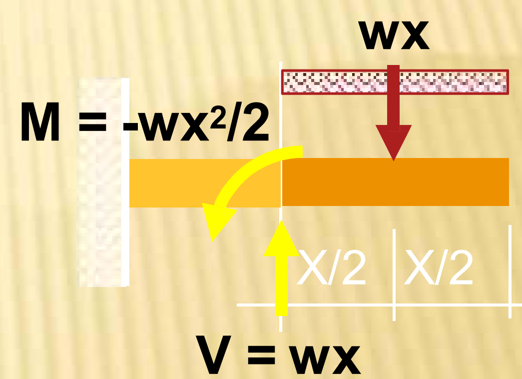
and moment reaction $M_R = -WL/2 = -wL^2/2$

Example 2 - Cantilever Beam Uniformly Distributed Load (cont.)

For distributed V and BM

Take section anywhere at distance, x from end

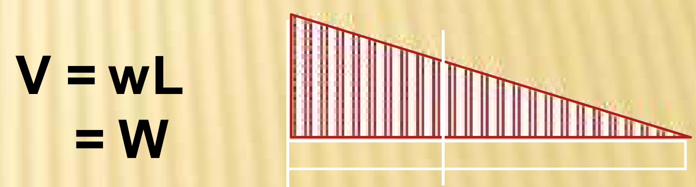
Add in forces, V = w.x and moment M = - wx.x/2



Shear $V = wx$

when x = L $V = W = wL$

when x = 0 $V = 0$



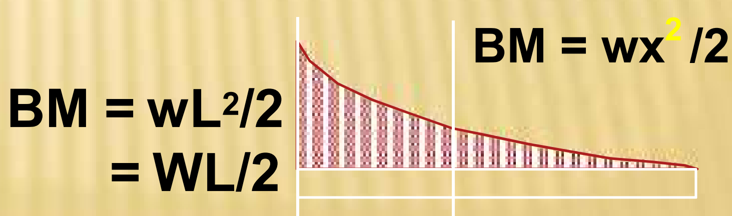
Shear Force Diagram

Bending Moment $BM = w.x^2/2$

when x = L $BM = wL^2/2 = WL/2$

when x = 0 $BM = 0$

(parabolic)



Bending Moment Diagram

Fig. 6.22 shows a cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.

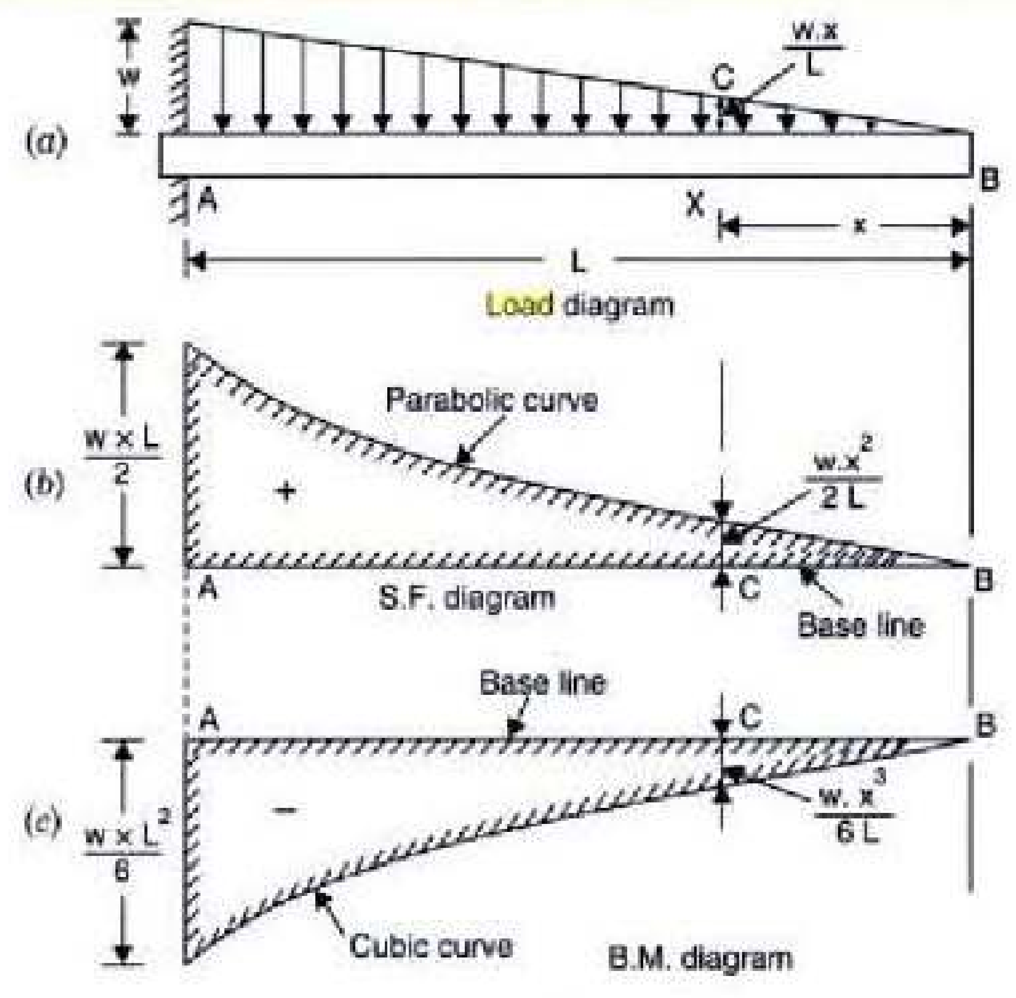


Fig. 6.22

Take a section X at a distance x from the free end B.

Let $\quad F_x$ = Shear force at the section X, and

ce.

DEFLECTION

Unit - III

APPLIED AND REACTIVE FORCES

- Forces that act on a Body can be divided into two Primary types: applied and reactive.

- In common Engineering usage, applied forces are forces that act directly on a structure like, dead, live load etc.)

- Reactive forces are forces generated by the action of one body on another and hence typically occur at connections or supports.
- The existence of reactive forces follows from Newton's third law, which state that to every action , there is an equal and opposite reaction.

SUPPORTS

To bear or hold up (a load, mass, structure, part, etc.); serve as a foundation or base for any structure.

To sustain or withstand (weight, pressure, strain, etc.) without giving way

It is a aid or assistance to any structure by preserve its load

Supports are used to connect structures to the ground or other bodies in order to restrict (confine) their movements under the applied loads. The loads tend to move the structures, but supports prevent the movements by exerting opposing forces, or reactions, to neutralize the effects of loads thereby keeping the structures in equilibrium.

TYPES OF SUPPORTS

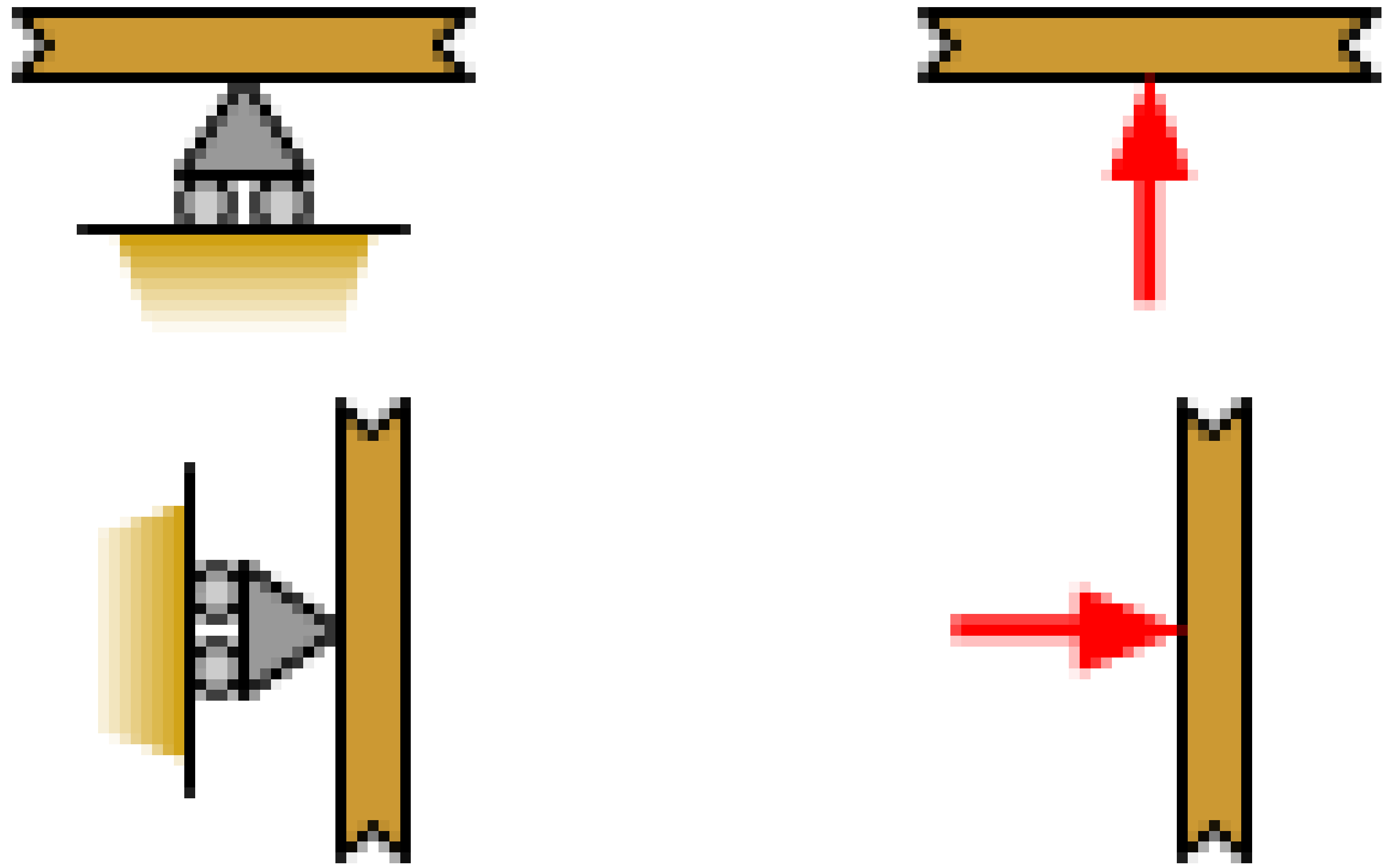
- Supports are grouped into three categories, depending on the number of reactions (1,2,or3) they exert on the structures.
- 1) Roller support
- 2) Hinge support
- 3) fixed support

ROLLER SUPPORT

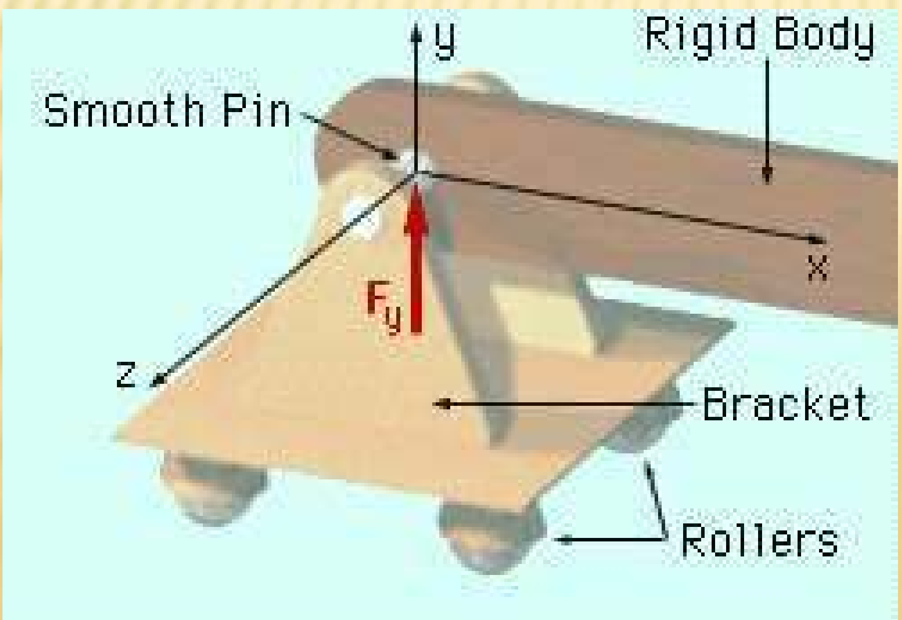
- Roller supports are free to rotate and translate along the surface upon which the roller rests.

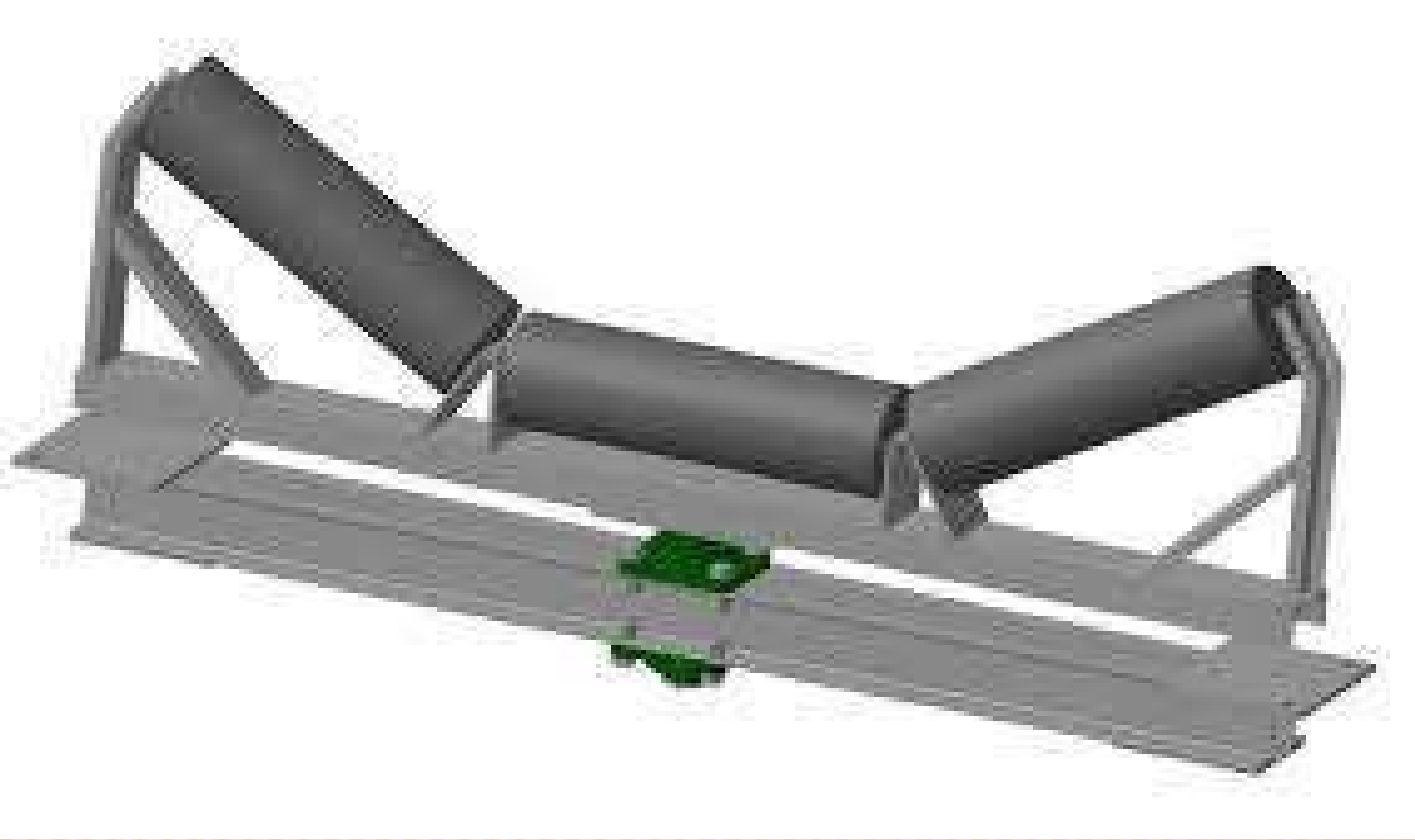
- The surface can be horizontal, vertical, or sloped at any angle.

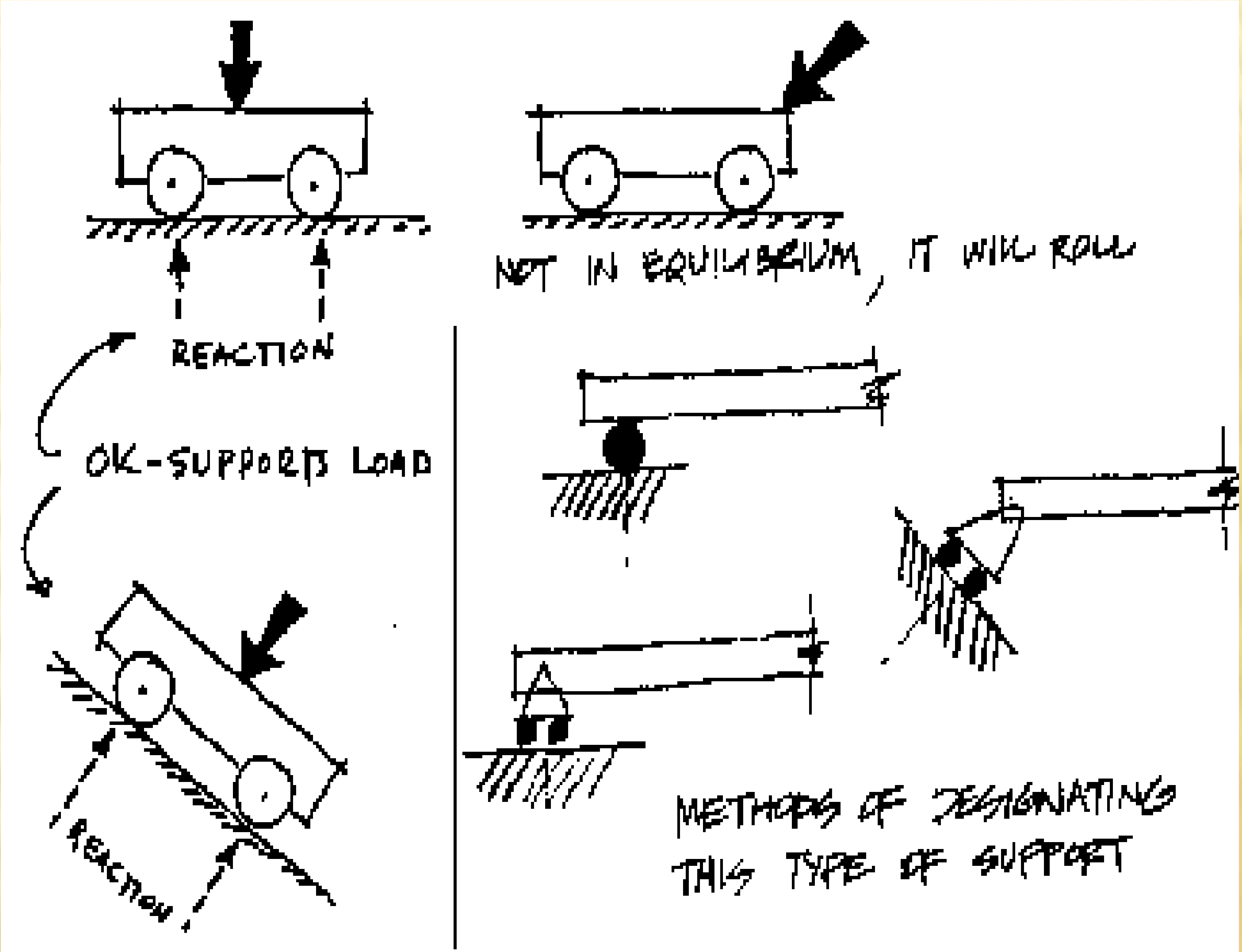
- The resulting reaction force is always a single force that is perpendicular to, and away from, the surface



Restrains the structure from moving in one or two perpendicular directions.

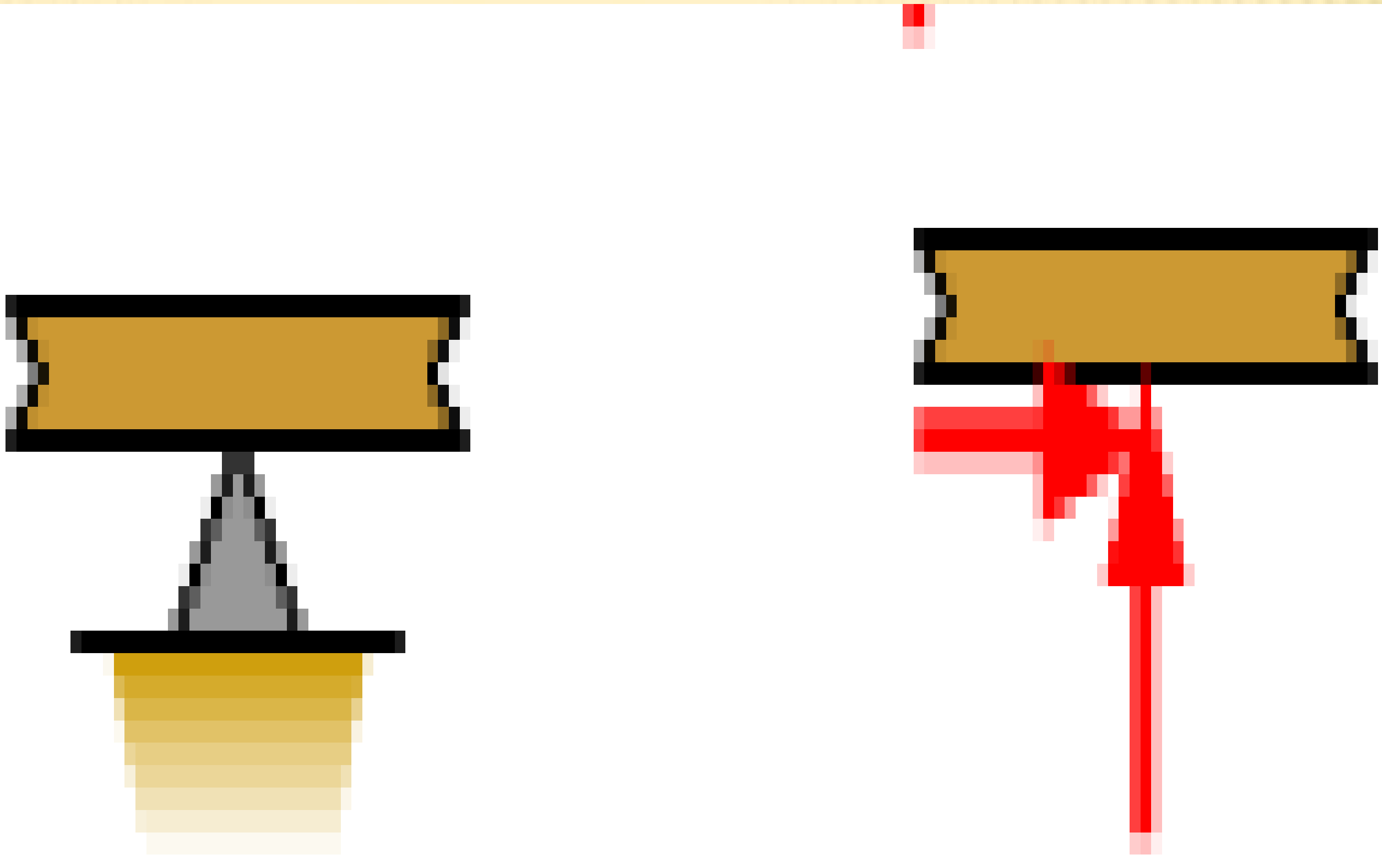

y
Rigid Body
Smooth Pin
x
F_y
z
Bracket
Rollers

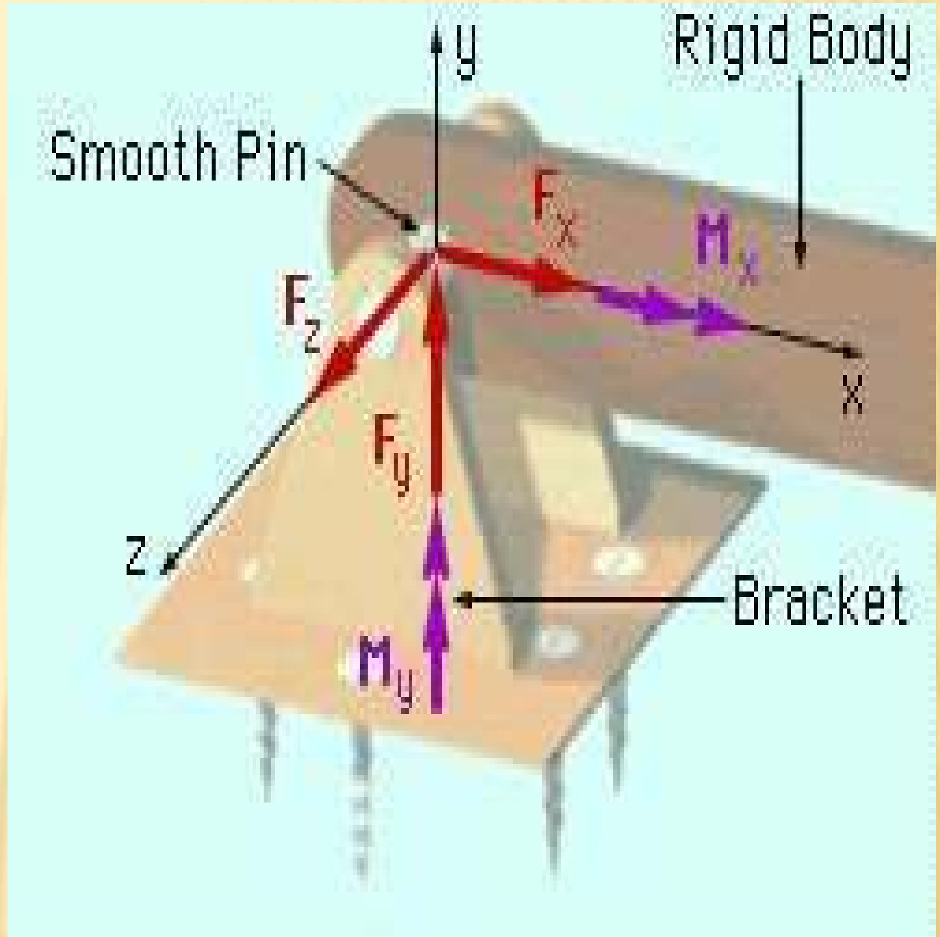



NOT IN EQUILIBRIUM, IT WILL ROLL
REACTION
OK-SUPPORTS LOAD
REACTION
METHODS OF DESIGNATING THIS TYPE OF SUPPORT

HINGE SUPPORT

- A Hinge support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction
- Pin or hinge support is used when we need to prevent the structure from moving or restrain its translational degrees of freedom.
- A **hinge** is a type of bearing that connects two solid objects, typically allowing only a limited angle of rotation between them. Two objects connected by an ideal hinge rotate relative to each other about a fixed axis of rotation.




y
Rigid Body
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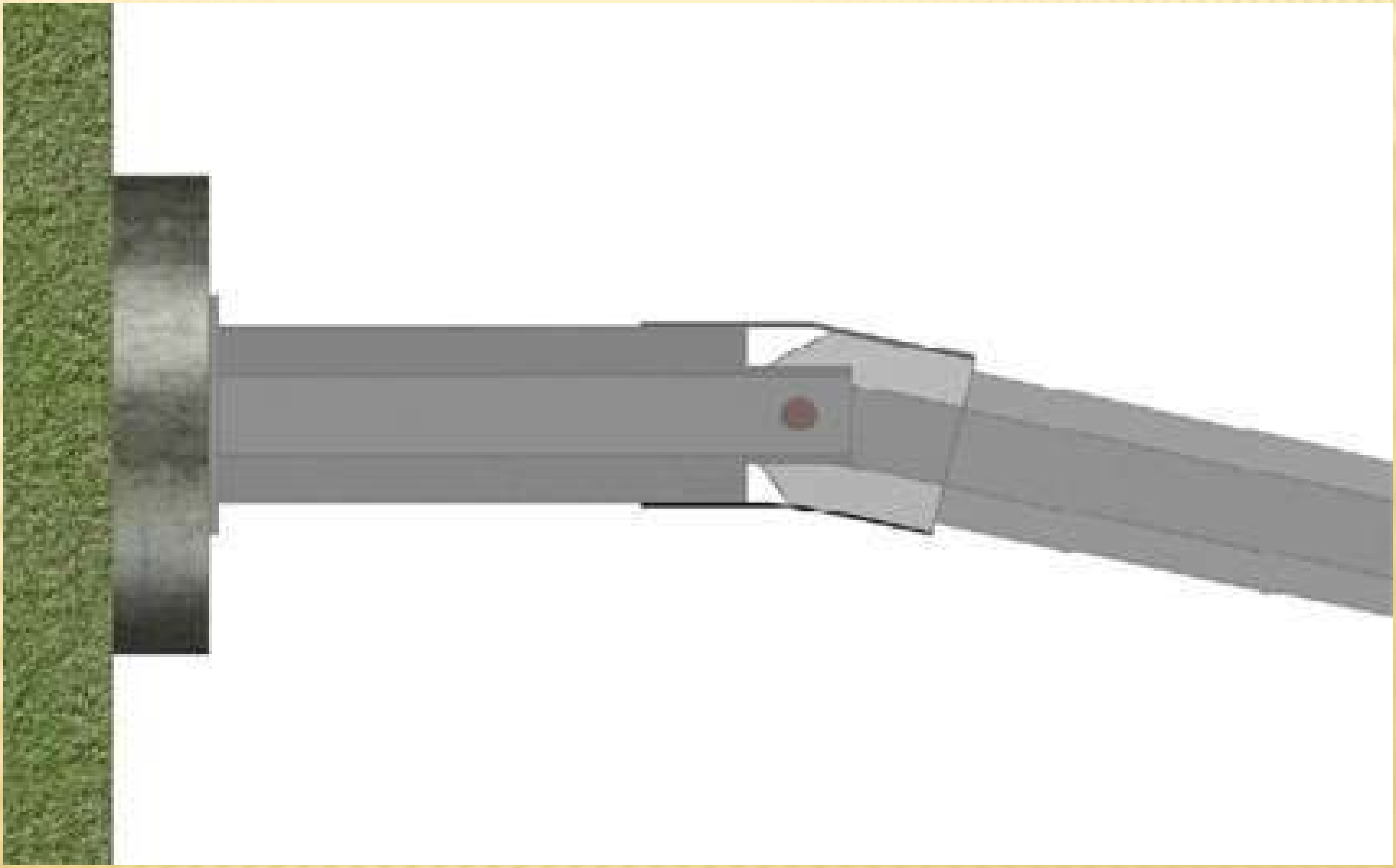
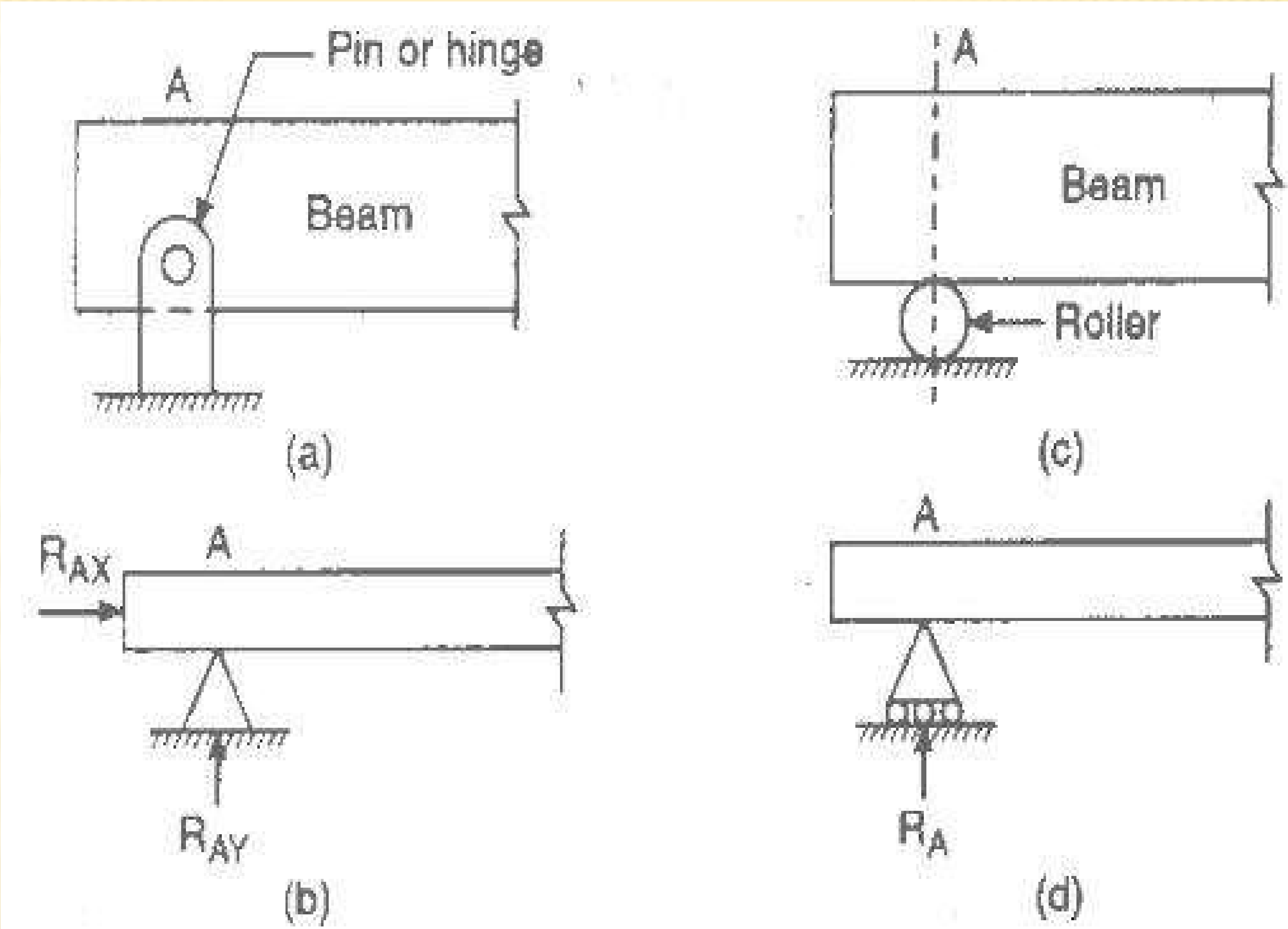
F_x
M_x
F_z
x
F_y
z
Bracket
M_y





Pin or hinge
A
Beam
(a)
A
Beam
Roller
(c)
R_{AX}
A
R_{AY}
(b)
A
R_A
(d)

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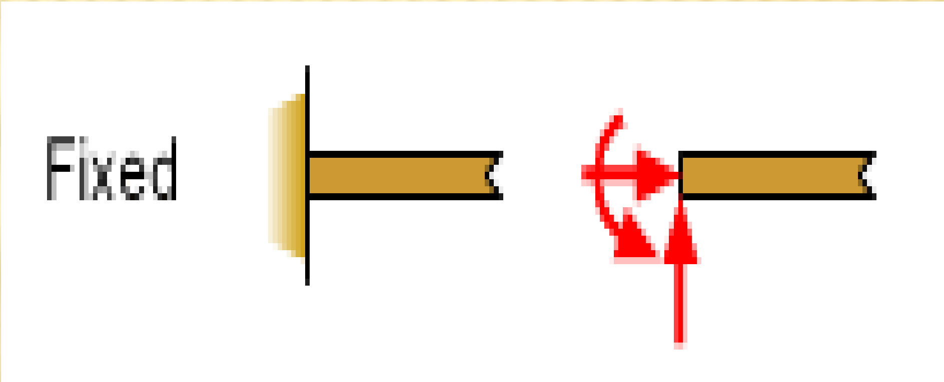

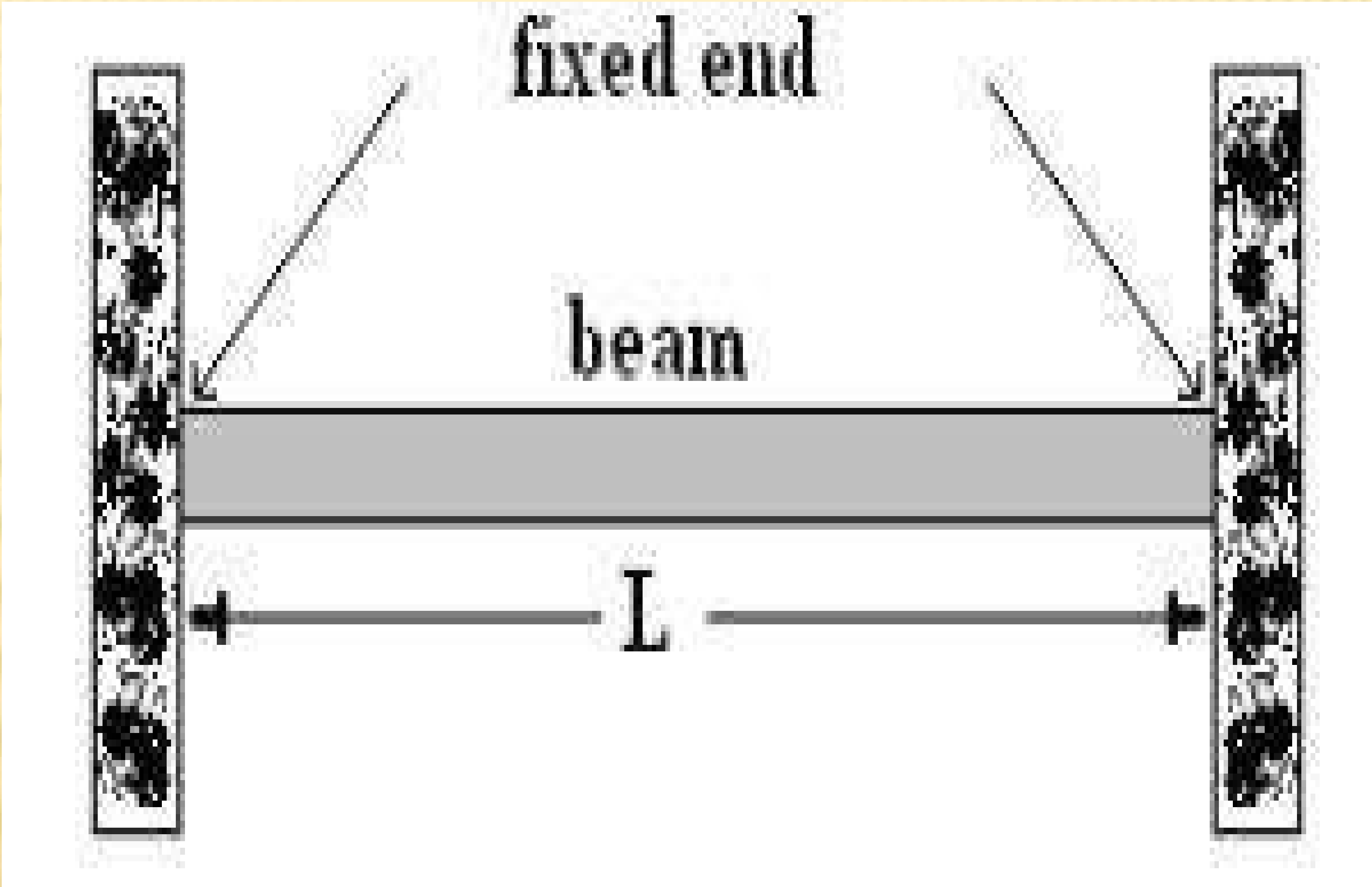
fixed end
beam
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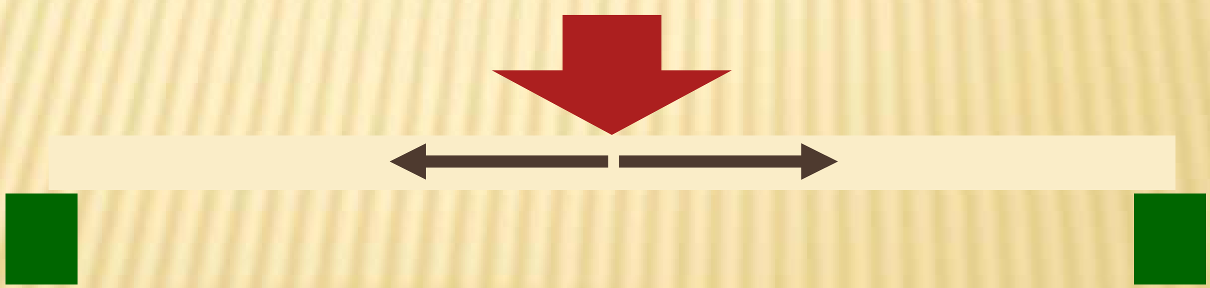

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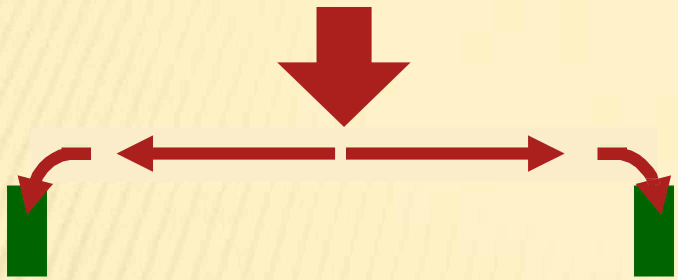

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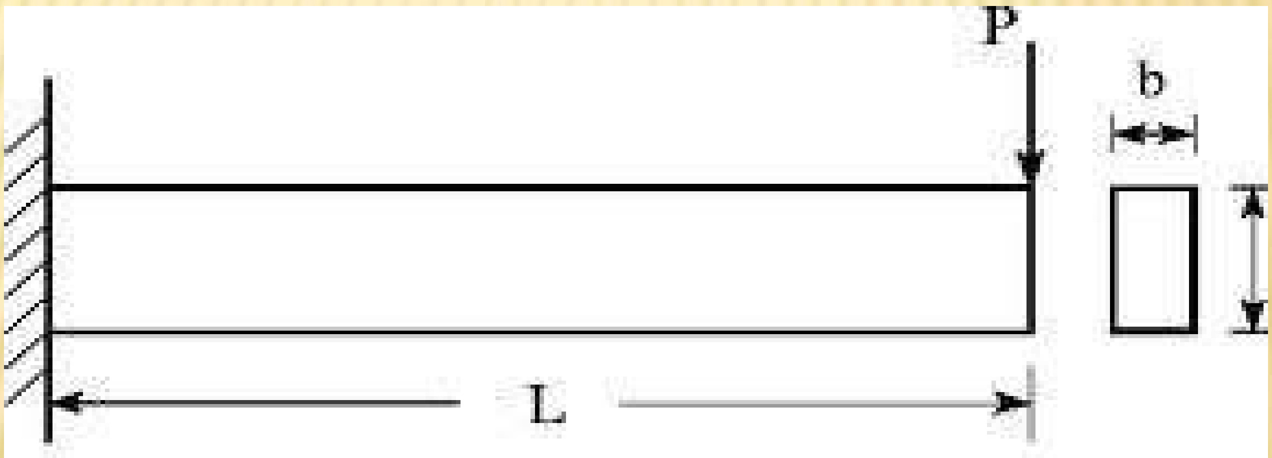


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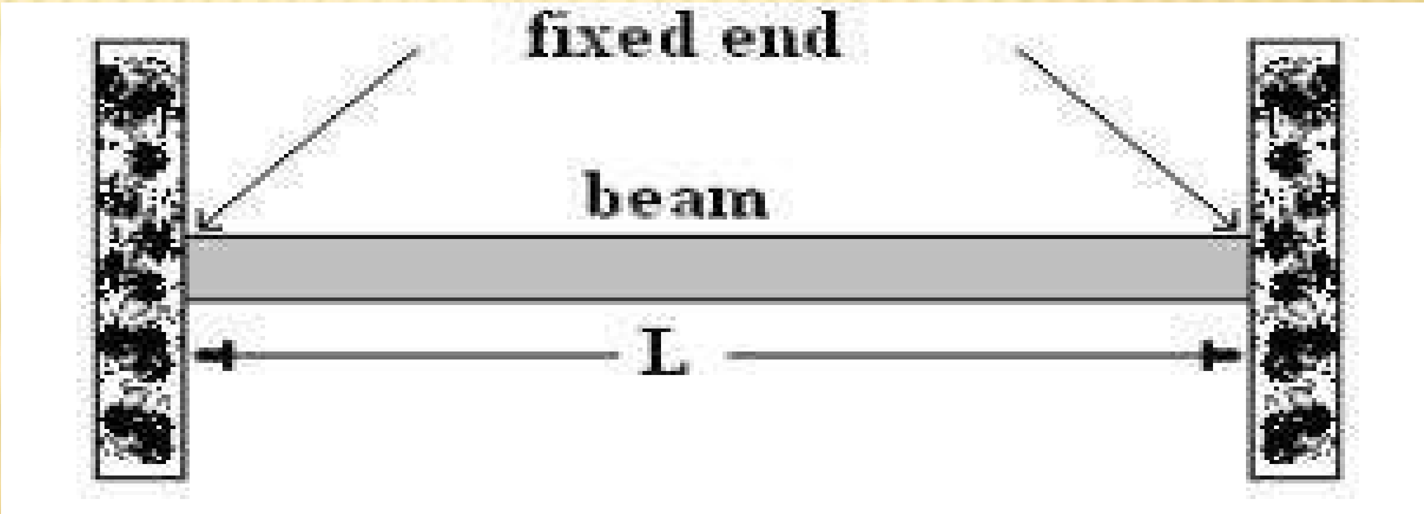

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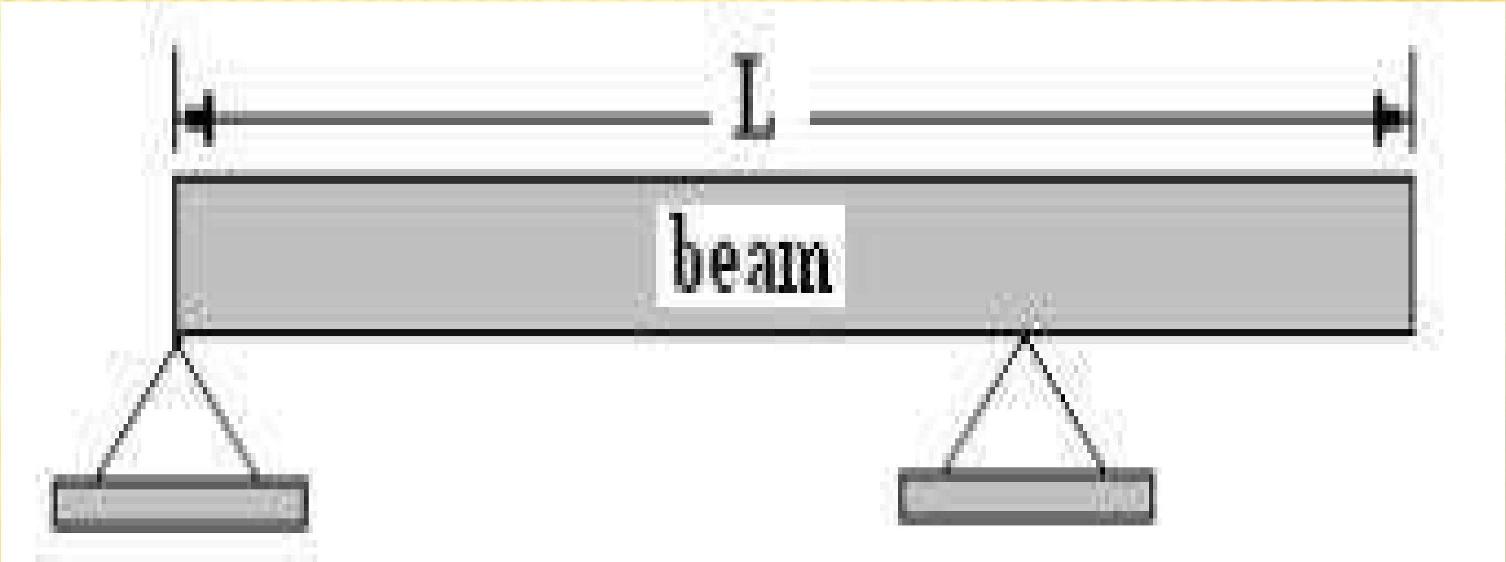

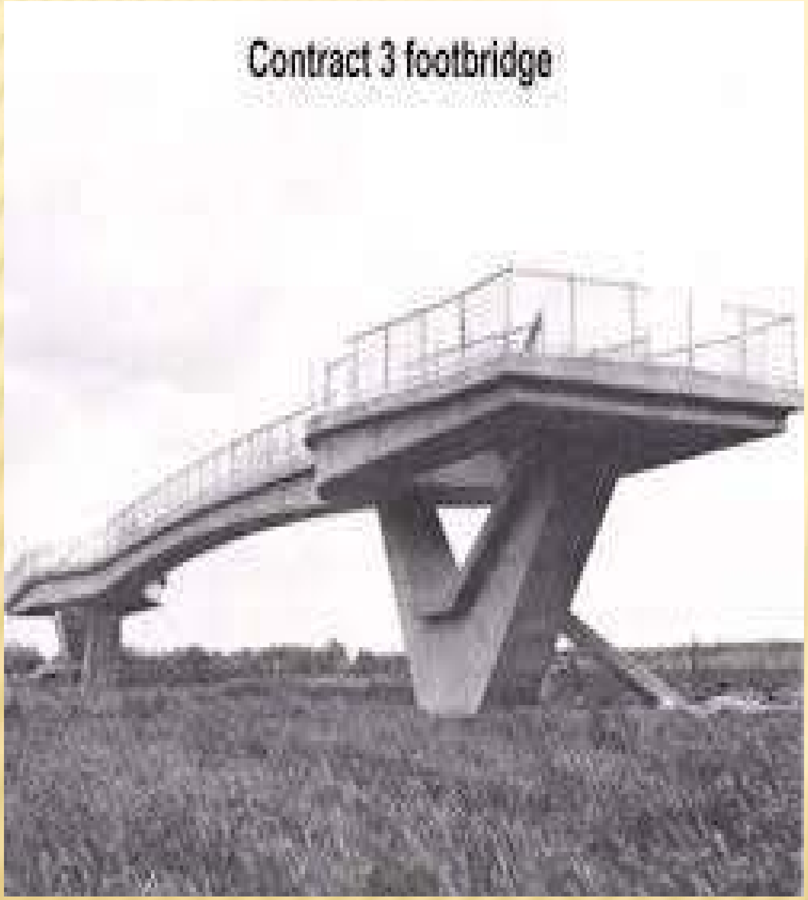
Contract 3 footbridge

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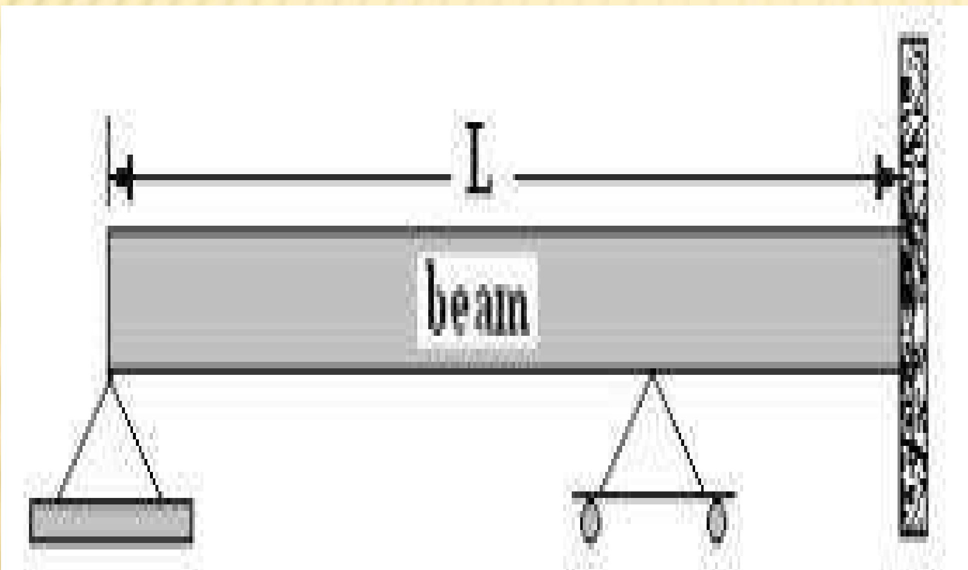

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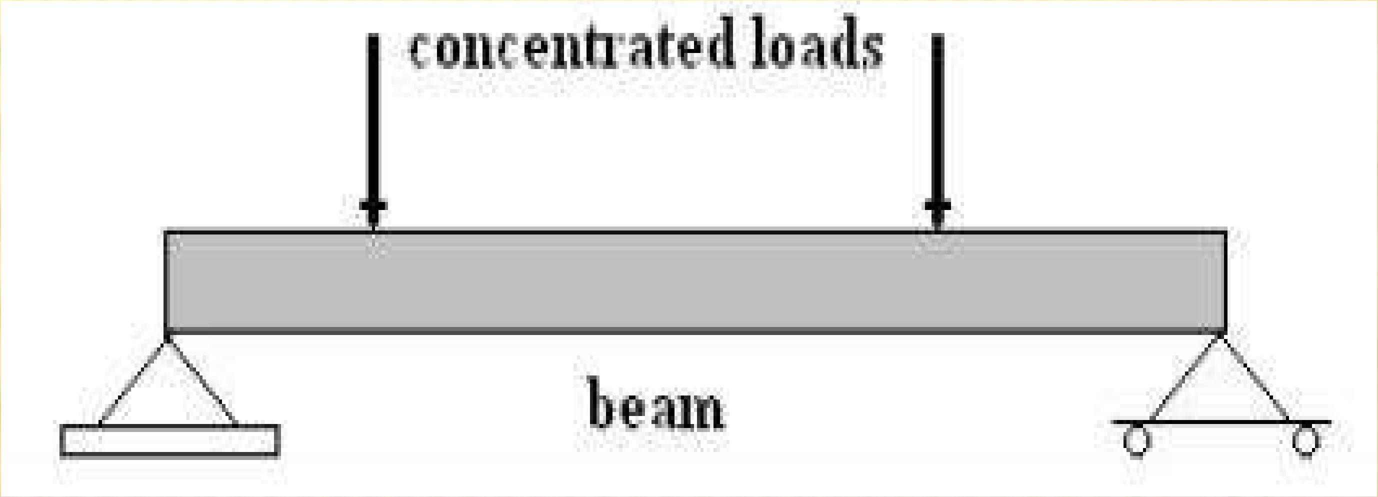

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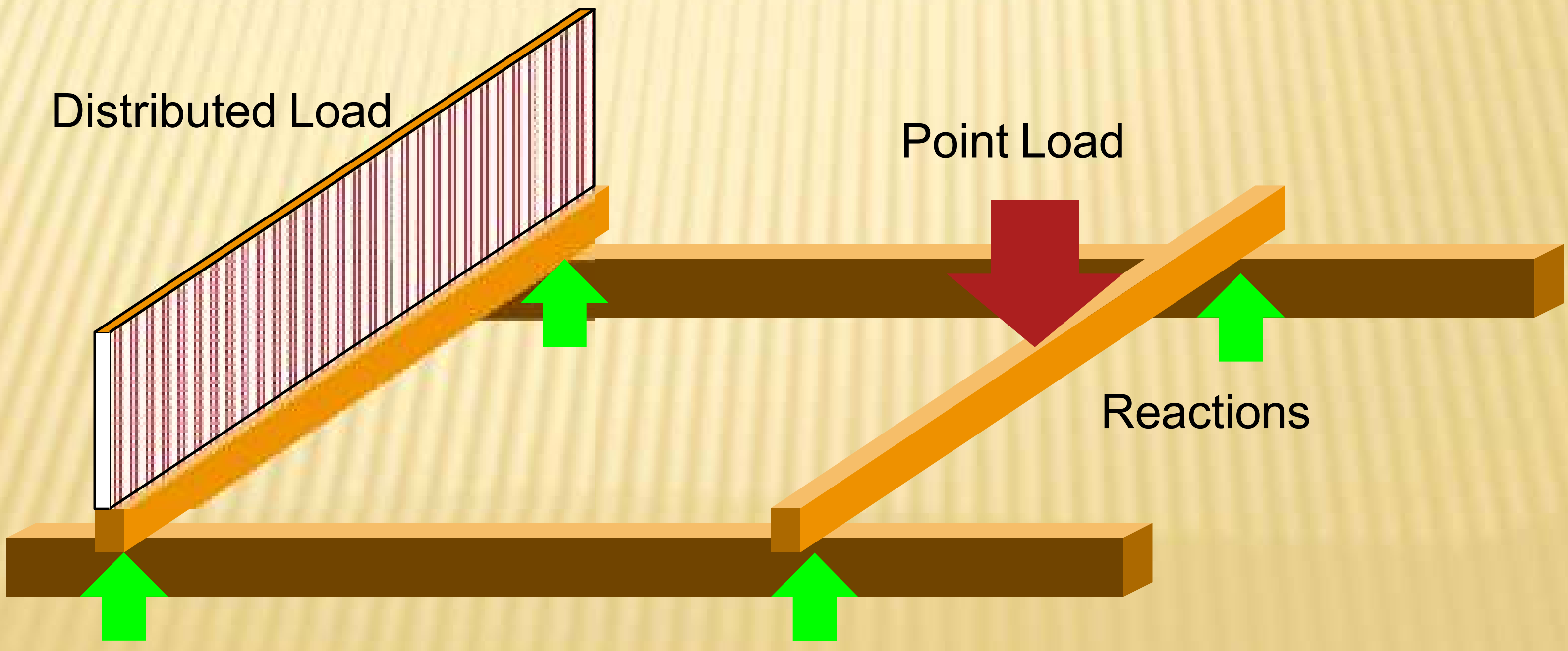
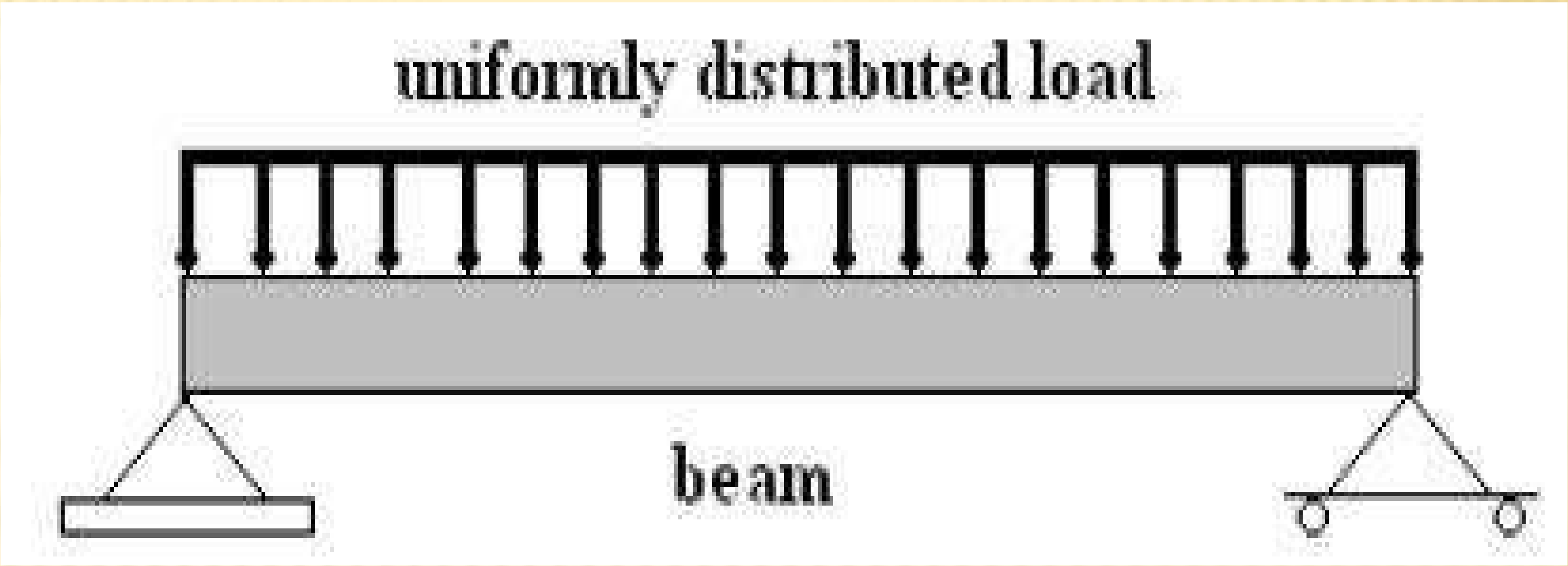
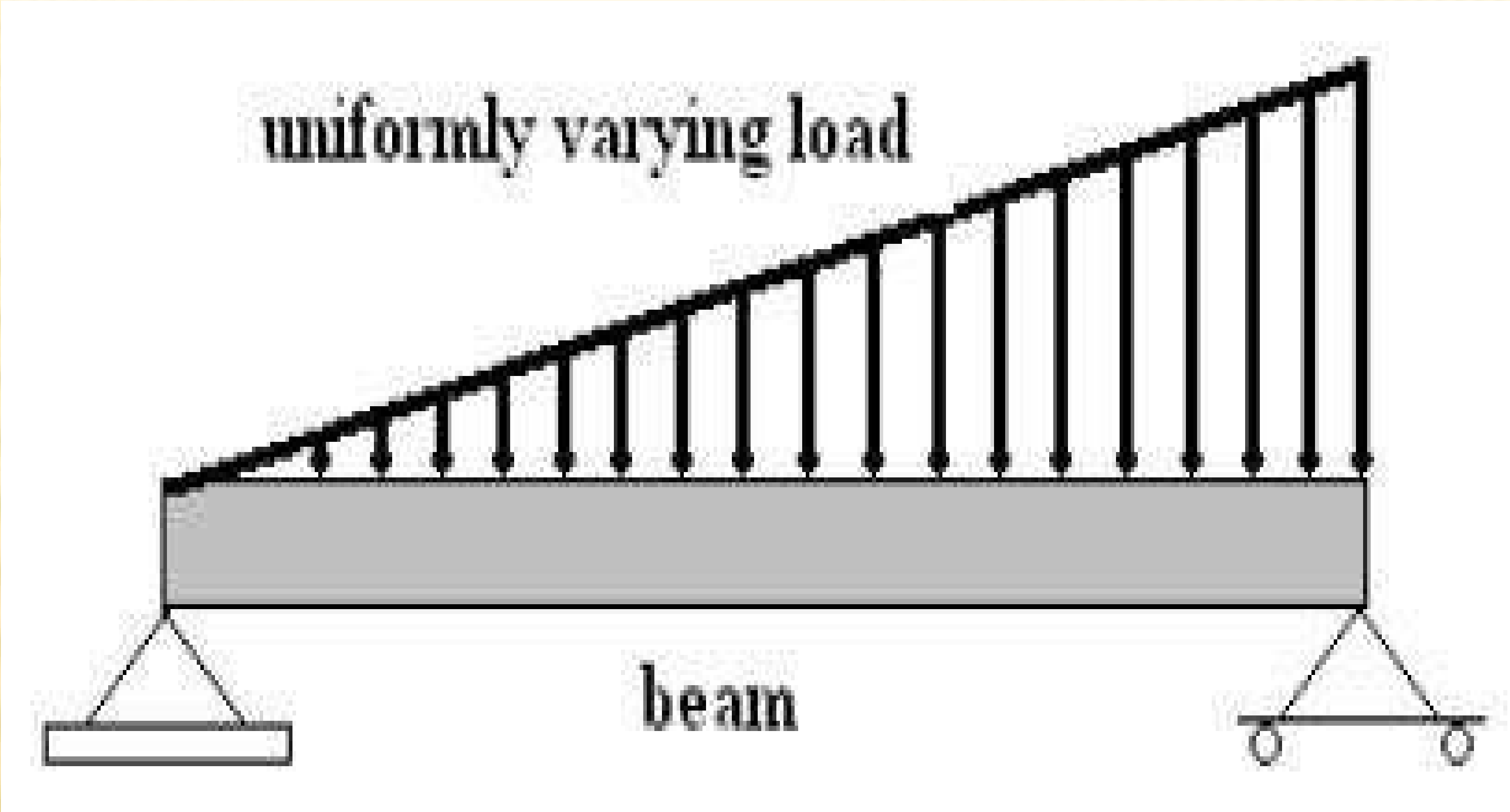



uniformly distributed load
beam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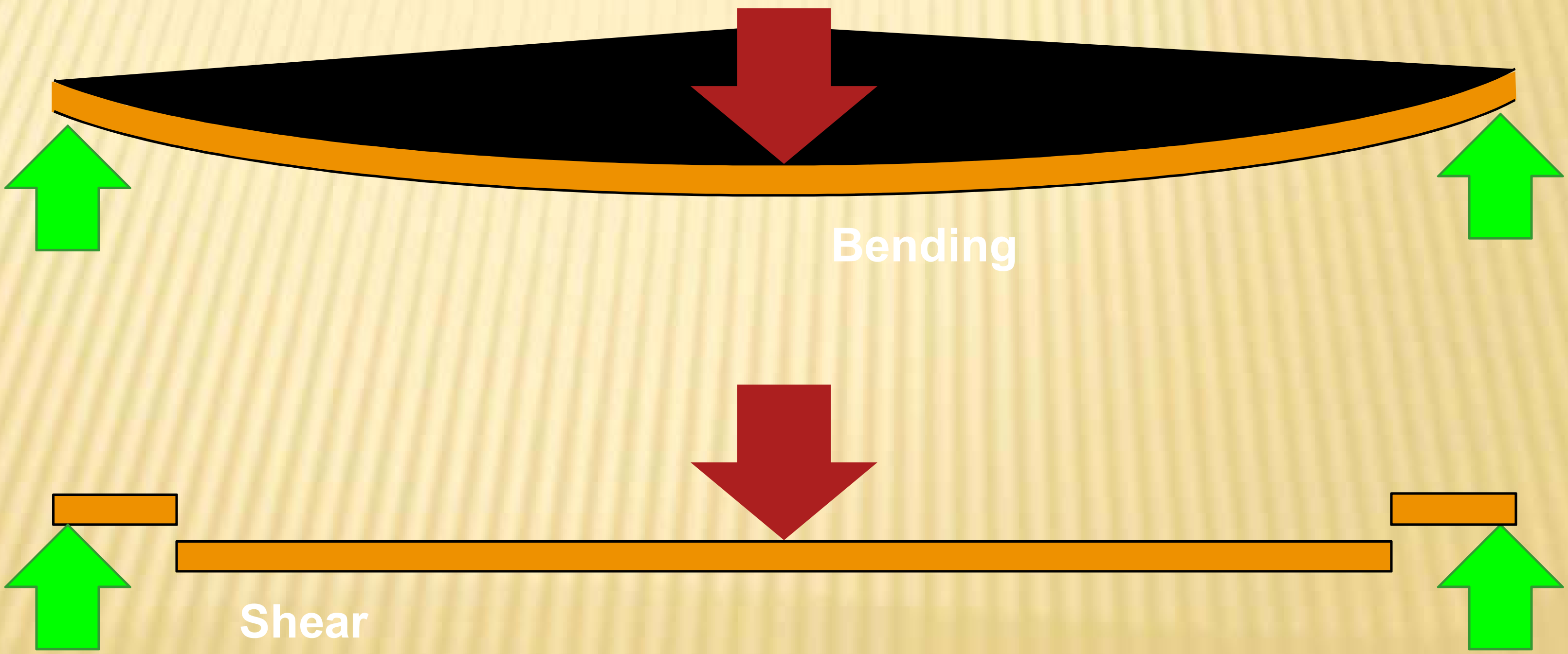
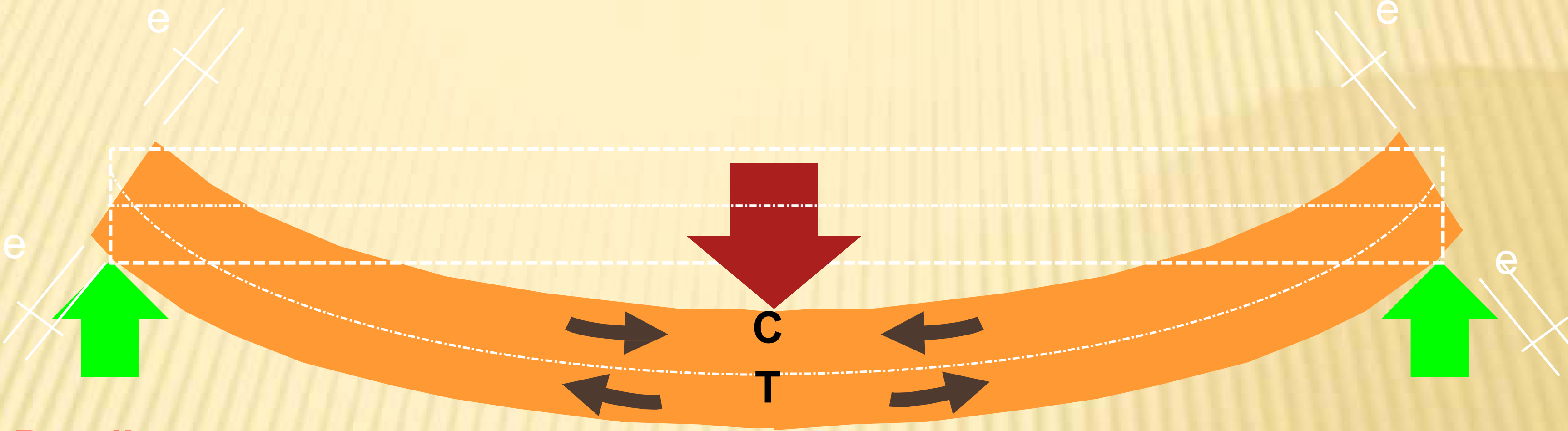
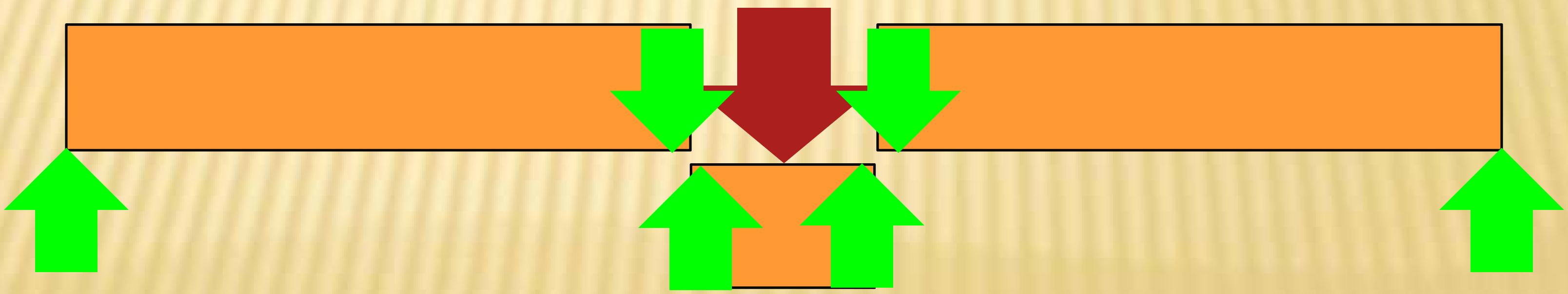


What the Loads Do





Designing Beams

- in architectural structures, bending moment more important
  - importance increases as span increases
- short span structures with heavy loads, shear dominant
  - e.g. pin connecting engine parts

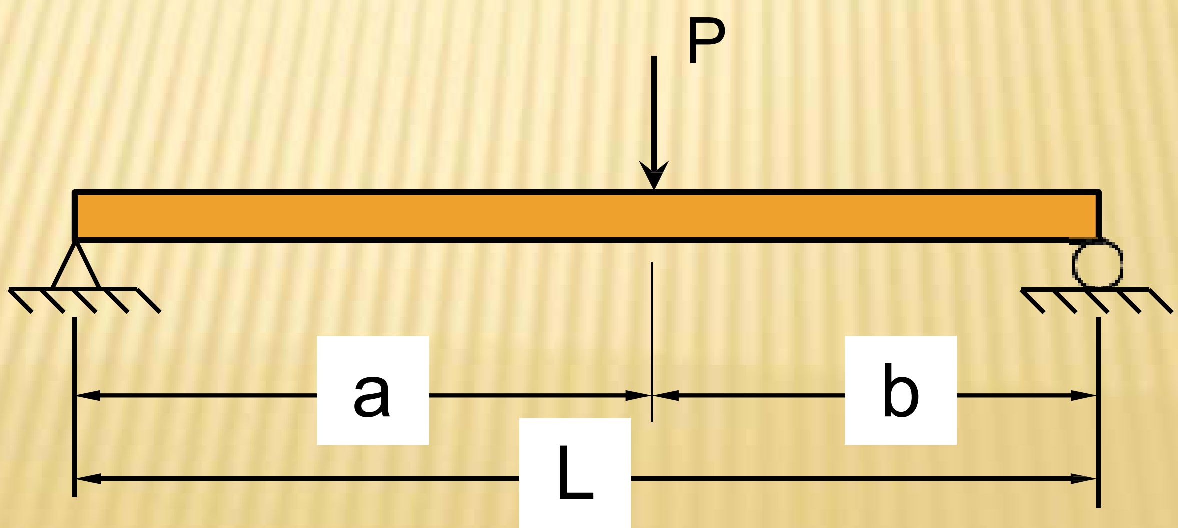
beams in building designed for bending checked for shear

How we calculate the Effects

- First, find ALL the forces (loads and reactions)
- Make the beam into a free body (cut it out and artificially support it)
- Find the reactions, using the conditions of equilibrium

INTERNAL REACTIONS IN BEAMS

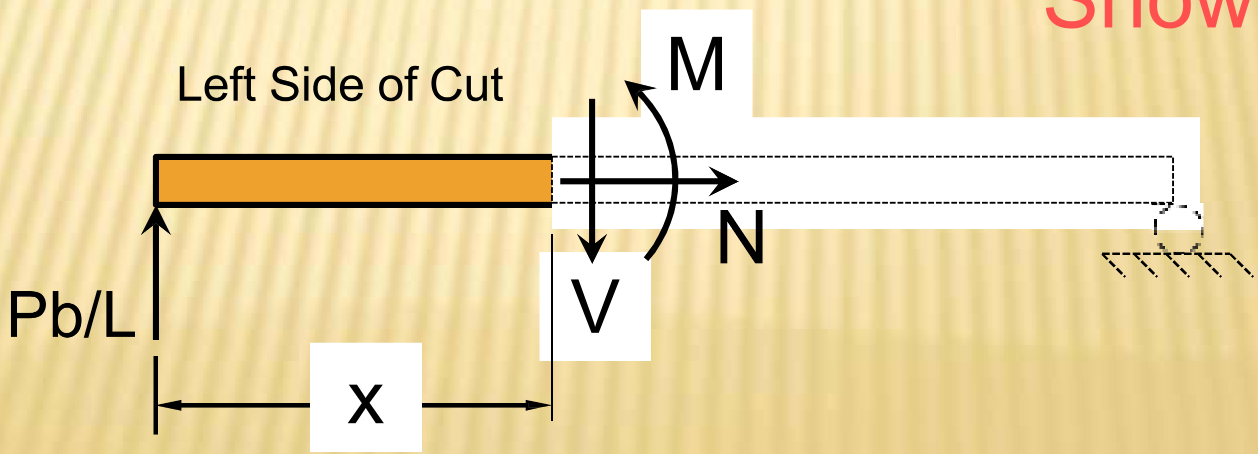
- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - normal force,
  - shear force,
  - bending moment.



INTERNAL REACTIONS IN BEAMS

- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - normal force,
  - shear force,
  - bending moment.

Positive Directions Shown!!!



INTERNAL REACTIONS IN BEAMS

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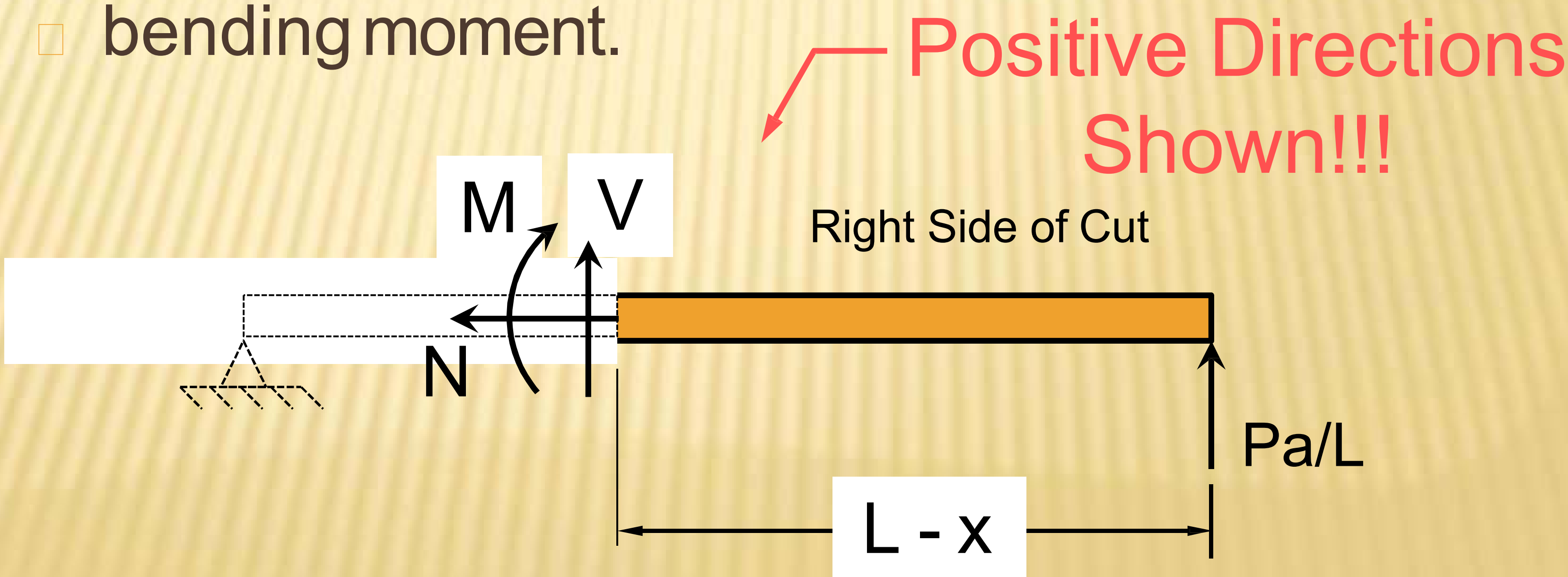
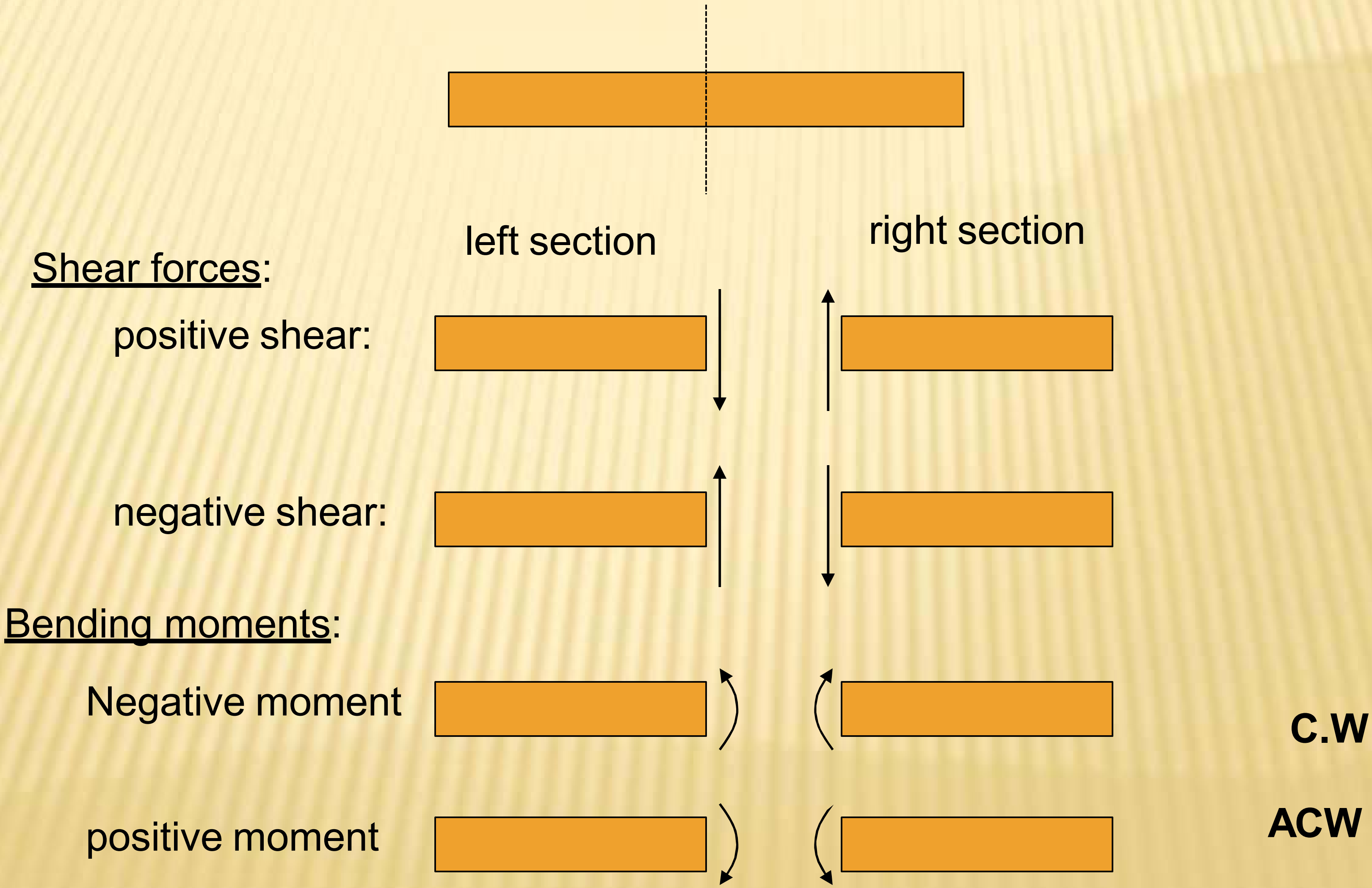
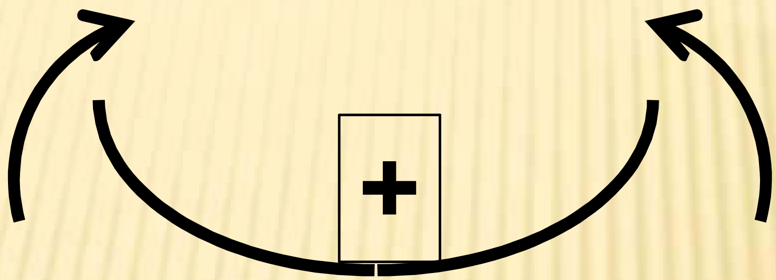


SHEAR FORCES, BENDING MOMENTS - SIGN CONVENTIONS

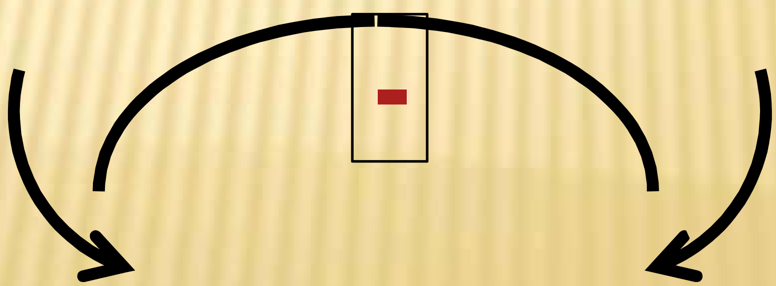


Sign Conventions
Bending Moment Diagrams (cont.)

Sagging bending moment is POSITIVE (happy)

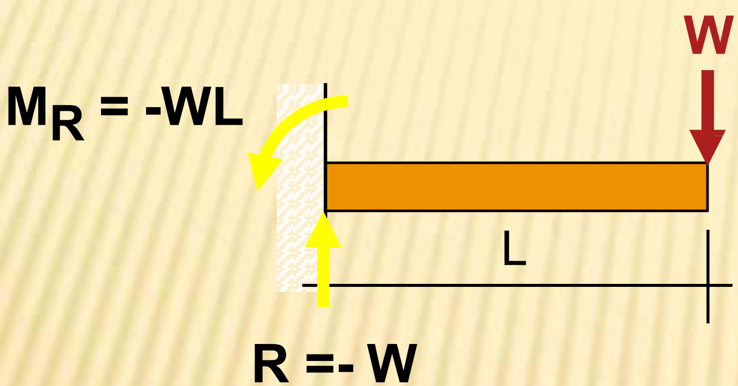


Hogging bending moment is NEGATIVE (sad)



Cantilever Beam Point Load at End

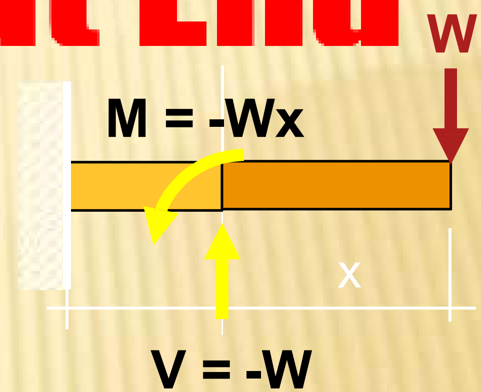
- Consider cantilever beam with point load on end



vertical reaction, $R = -W$
and moment reaction $M_R = -WL$

- **Use the free body idea to isolate part of the beam**
- **Add in forces required for equilibrium**

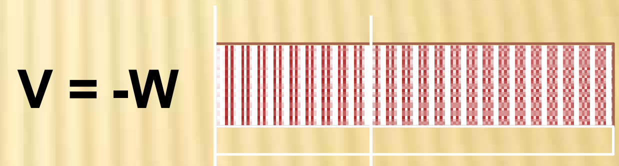
Cantilever Beam
Point Load at End



Take section anywhere at distance, x from end

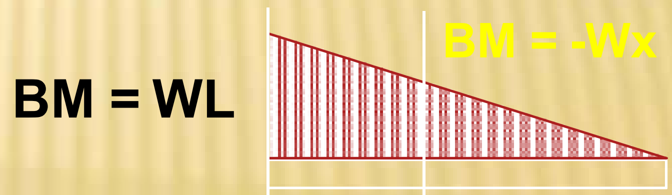
Add in forces, V = -W and moment M = - Wx

Shear V =- W constant along length



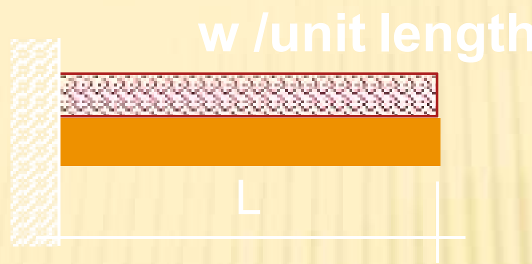
Shear Force Diagram

Bending Moment BM = -W.x
when x = L BM = -WL
when x = 0 BM = 0

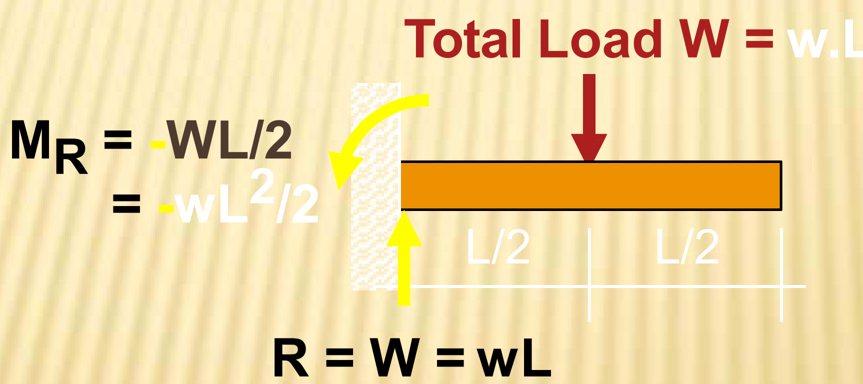


Bending Moment Diagram

Cantilever Beam
Uniformly Distributed Load



For maximum shear V and bending moment BM



vertical reaction, $R = W = wL$

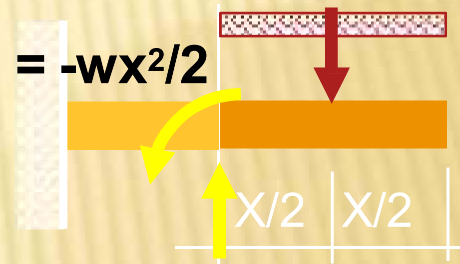
and moment reaction $M_R = -WL/2 = -wL^2/2$

Example 2 - Cantilever Beam Uniformly Distributed Load (cont.)

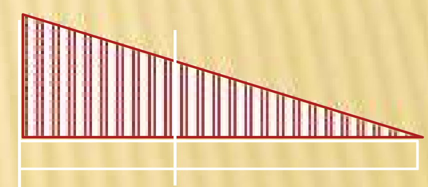
For distributed V and BM

Take section anywhere at distance, x from end

Add in forces, V = w.x and moment M = - wx.x/2

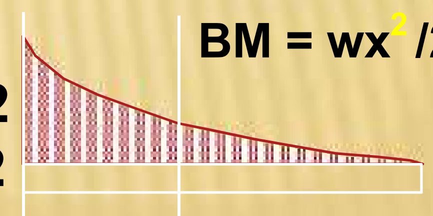


Shear V = wx
when x = L V = W = wL
when x = 0 V = 0



Shear Force Diagram

Bending Moment BM = w.x²/2
when x = L BM = wL²/2 = WL/2
when x = 0 BM = 0
(parabolic)



Bending Moment Diagram

Fig. 6.22 shows a cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.

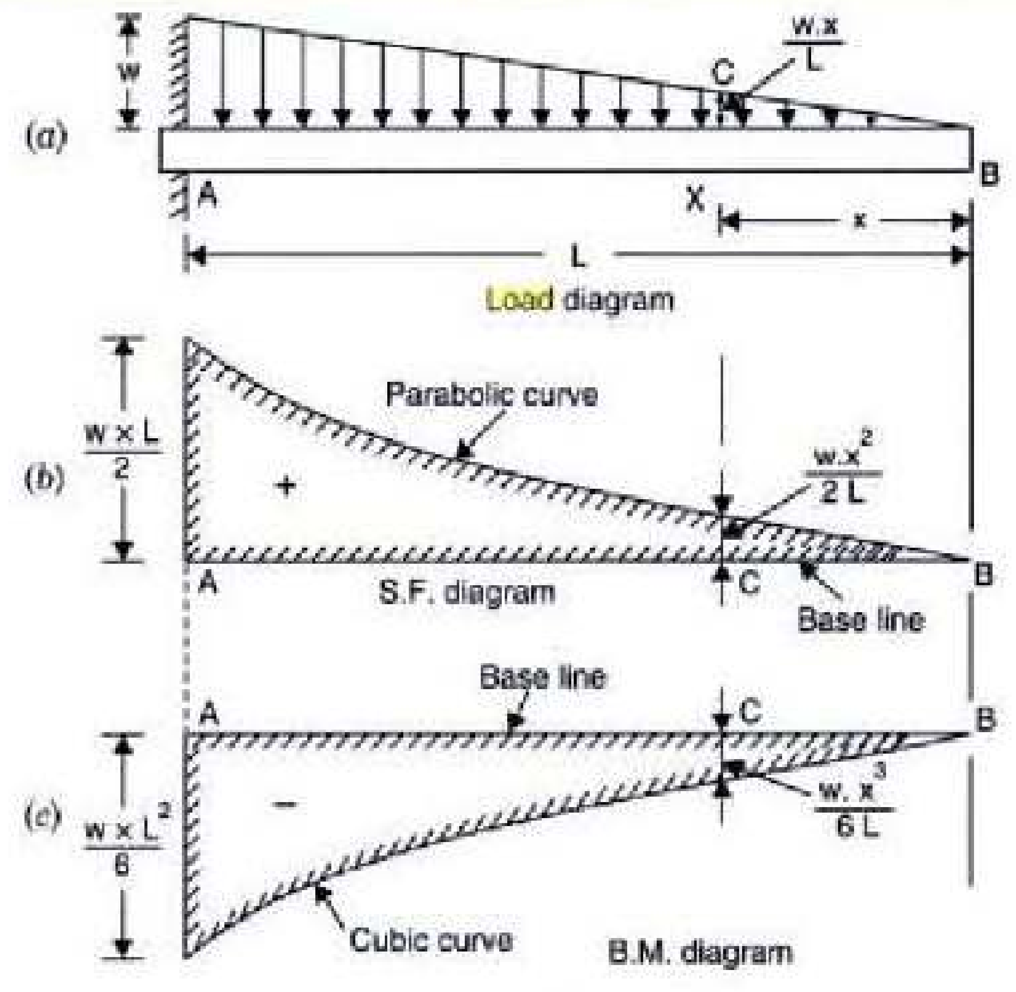


Fig. 6.22

Take a section X at a distance x from the free end B.

Let $\quad F_x$ = Shear force at the section X, and

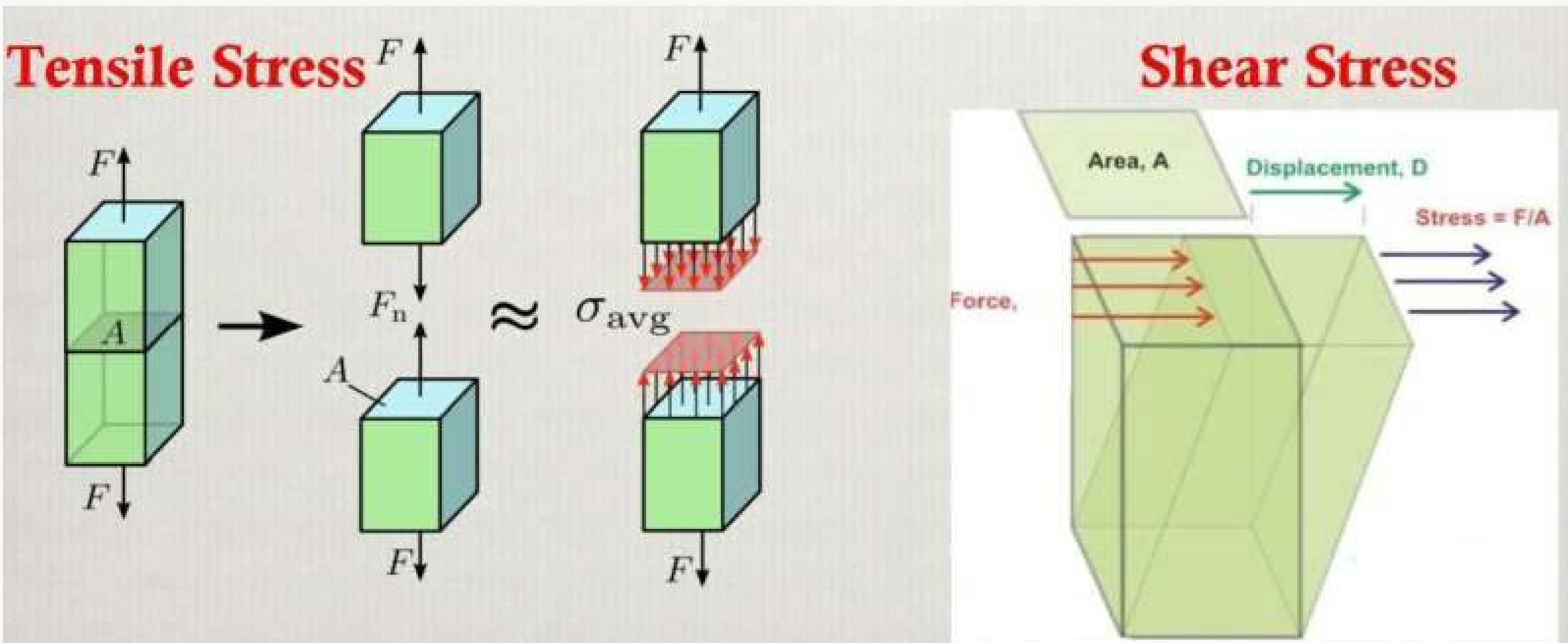
ce.

Principal stresses and strains

Unit -4

Stresses and strains

- In last lecture we looked at stresses were acting in a plane that was at right angles/parallel to the action of force.



Compressive load **Failure in shear**

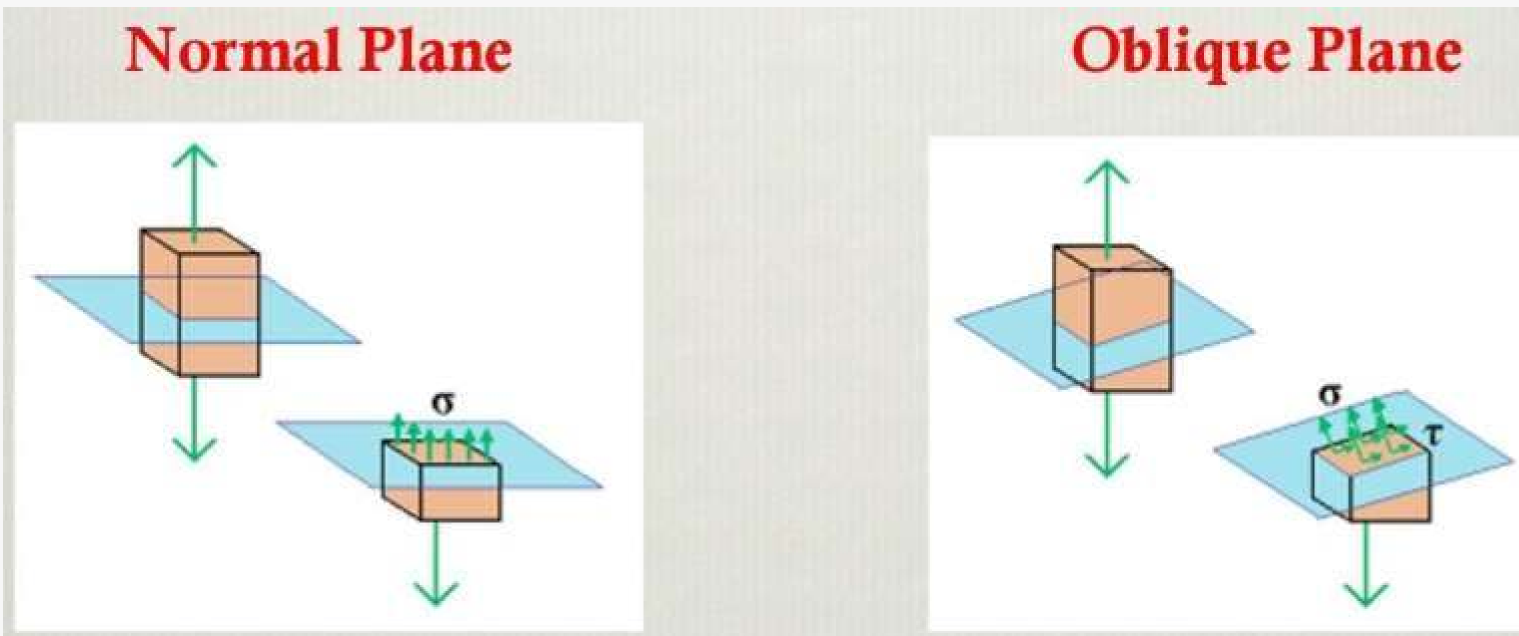
Stresses are acting normal to the surface yet the material failed in a different plane

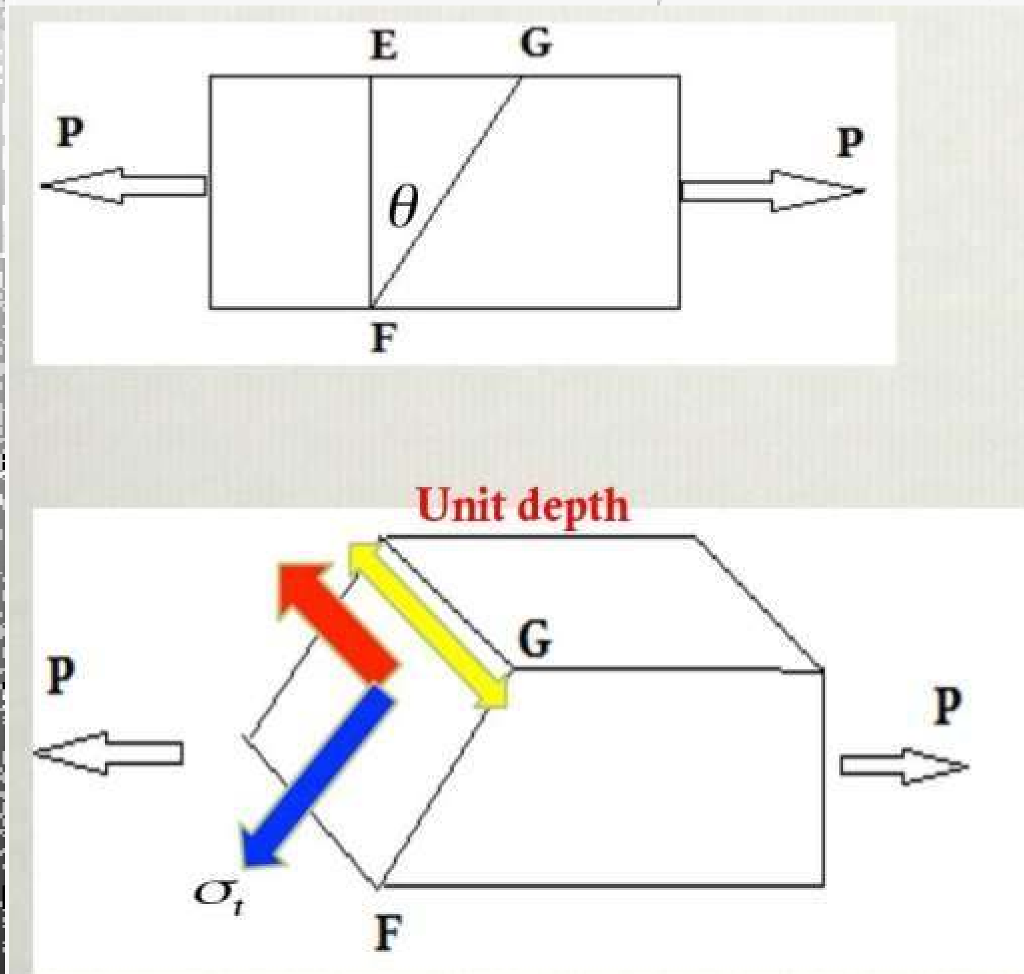
Principal stresses and strains

- What are principal stresses.
- Planes that have no shear stress are called as principal planes.
- Principal planes carry only normal stresses

Stresses in oblique plane

- In real life stresses does not act in normal direction but rather in inclined planes.





$\sigma = \frac{P}{A}$

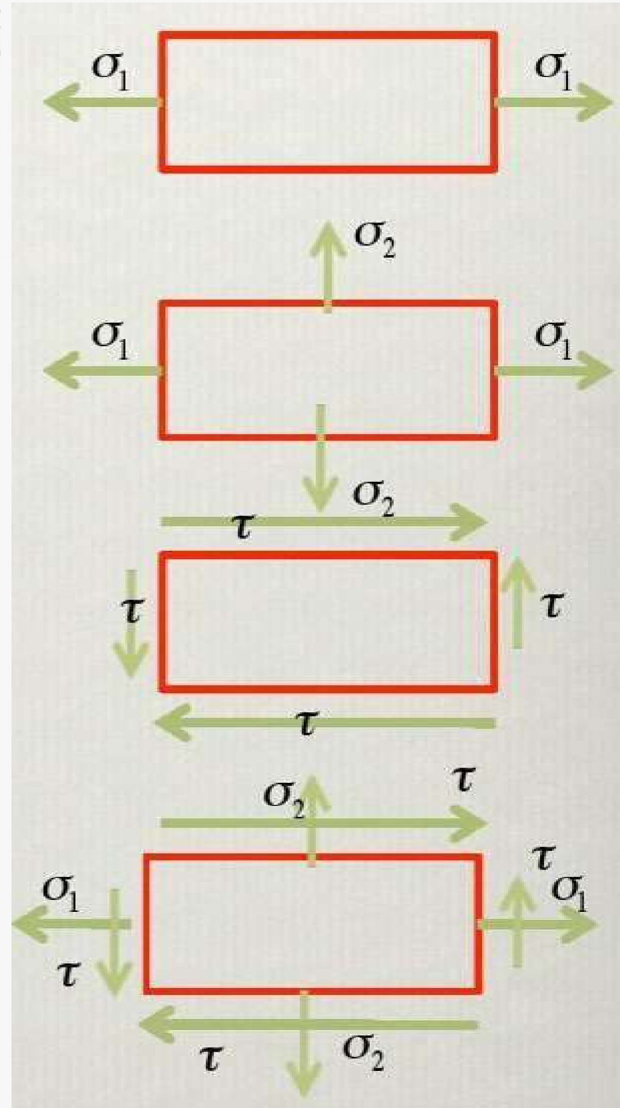
P = Axial forces

A = cross sectional area

$\sigma_n = \sigma\cos^2\theta$

$\sigma_t = \frac{\sigma}{2}\sin 2\theta$

- Member subjected to direct stress in one plane
- Member subjected to direct stress in two mutually perpendicular plane.
- Member subjected to simple shear stress.
- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress.



- Member subjected to direct stress in two mutually
  perpendicular directions + simple shear stress

$$\sigma n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

- Member subjected to direct stress in two mutually
  perpendicular directions + simple shear stress
- POSITION OF PRINCIPAL PLANES
- Shear stress should be zero

$$\sigma_t = \frac{\sigma_{1-\sigma 2}}{2} \sin 2\theta - \tau \cos 2\theta = 0$$

$$\tan 2\theta = 2T/(\sigma_1 - \sigma_2)$$

- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress .

Major principal Stress= $\frac{\sigma_1+\sigma_2}{2} \pm \frac{\sigma_1-\sigma_2}{2} + T$

Minor principal Stress = $\frac{\sigma_1+\sigma_2}{2} \pm \frac{\sigma_1-\sigma_2}{2} + T$

- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress

- MAX SHEAR STRESS

$$\frac{d}{d\theta}(\sigma_t) = 0$$

$$\frac{d}{d\theta}[tan 2\theta \sin 2\theta - \tau \cos 2\theta] = 0$$

$$\tan 2\theta = \frac{\sigma_1 - \sigma_2}{2T}$$

- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress

- MAX SHEAR STRESS

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$$

$$\tan 2\theta = \frac{\sigma_1 - \sigma_2}{2T}$$

$$\sigma_{t(max)} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4T^2}$$

- Member subjected to direct stress in one plane
- Member subjected to direct stress in two mutually
  perpendicular plane
- Member subjected to simple shear stress.
- Member subjected to direct stress in two mutually
  perpendicular directions + simple shear stress

❖ Member subjected to direct stress in one plane

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Stress in one direction and no shear stress $\sigma_2 = 0, \tau = 0$

$$\sigma_n = \frac{\sigma_1}{2} + \frac{\sigma_1}{2}\cos 2\theta = \sigma_1 \cos^2\theta$$

$$\sigma_t = \frac{\sigma_1}{2} \sin 2\theta$$

- Member subjected to direct stress in two mutually perpendicular plane

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Stress in two direction and no shear stress $\tau=0$

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta$$

- Member subjected to simple shear stress.

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

No stress in axial direction but only shear stress $\sigma_1=\sigma_2=0$

$$\sigma_n = \tau \sin 2\theta$$

$$\sigma_t = -\tau \cos 2\theta$$

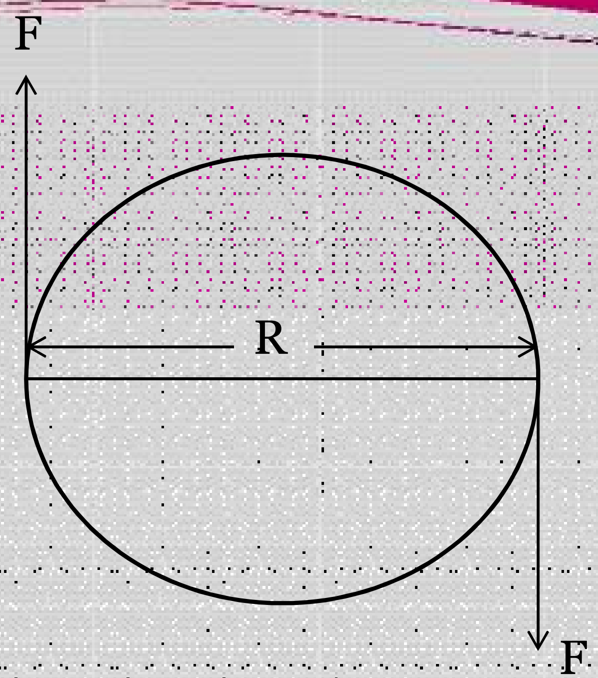
TORSION OF SHAFTS AND SPRING

Unit 5

TORSION OF CIRCULAR SHAFT

TORQUE OR TURNING MOMENT OR TWISTING MOMENT :-

- In factories and workshops, shafts is used to transmit energy from one end to other end.
- To transmit the energy, a turning force is applied either to the rim of a pulley, keyed to the shafts, or to any other suitable point at some distance from the axis of the shaft.
- The moment of couple acting on the shaft is called torque or turning moment or twisting moment.



Torque = turning force x diameter of shaft

T = F x 2R

where :

T=Torque

F=Turning force

S=Radius of the shaft

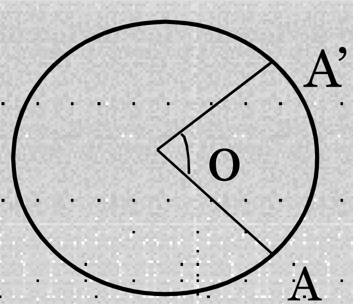
Unit of Torque(T) is N.mm or kN.mm

ANGLE OF TWIST (θ)

- When the shaft is subjected to Torque (T), point A on the surface of the shaft comes to A' position. The angle AOA' at the centre of the shaft is called the angle of twist.

- $\angle AOA' = \theta$ = Angle of twist

- Angle of twist is measured in radians.



SHEAR STRESS IN SHAFT:(τ)

- When a shaft is subjected to equals and opposite end couples, whose axes coincide with the axis of the shaft, the shaft is said to be in pure torsion and at any point in the section of the shaft stress will be induced.

- That stress is called shear stress in shaft.

STRENGTH OF SHAFTS

Maximum torque or power the shaft can transmit from one pulley to another, is called strength of shaft.

(a) For solid circular shafts:

Maximum torque (T)is given by :

$$T = \frac{\pi}{16} \times \tau \times D^3$$

where, D=dia. of the shaft

τ=shear stress in the shaft

(B) for hollow circular shaft

maximum torque (t) is given by.

$$T = \frac{\pi}{16} \times \tau \times \frac{D^4 - d^4}{D}$$

Where, d= outer dia of shaft
d=inner dia of shaft.

ASSUMPTION IN THE THEORY OF TORSION:

- The following assumptions are made while finding out shear stress in a circular shaft subjected to torsion.

1) The material of shaft is uniform throughout the length.
2) The twist along the shaft is uniform.
3) The shaft is of uniform circular section throughout the length.
4) Cross section of the shaft, which are plane before twist remain plain after twist.
5) All radii which are straight before twist remain straight after twist.

POLLAR MOMENT OF INERTIA : (J)

- The moment of inertia of a plane area, with respect to an axis perpendicular to the plane of the figure is called polar moment of inertia.
- As per the perpendicular axis theorem.

$$I_{ZZ} = I_{XX} + I_{YY} = J$$

$$= \frac{\pi}{64} \times D^4 + \frac{\pi}{64} \times D^4$$

$$J = \frac{\pi}{32} \times D^4$$

THEORY OF TORSION AND TORSION EQUATION

- Consider a shaft fixed at one end subjected to torque at the other end.

  Let T= Torque
  l= length of the shaft
  R=Radius of the shaft

- As a result of torque every cross-section of the shaft will be subjected to shear stress.
- Line CA on the surface of the shaft will be deform to CA' and OA to OA', as shown in figure.
- Let, ∠ACA'=shear strain
- ∠AOA'=angle of twist

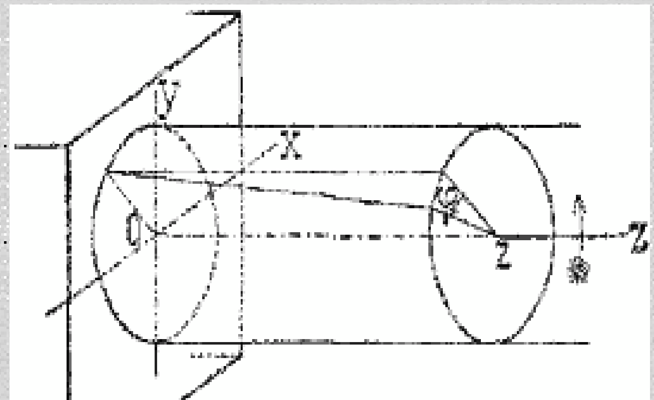


Fig. 1.

TORSION RIGIDITY

- Let twisting moment Produce a twist radians in length L.

$$\frac{T}{J}=\frac{C\theta}{L}$$

- for given shaft the twist is therefore proportional to the twisting moment T.
- In a beam the bending moment produce deflection, in the same manner a torque produces a twist in shaft .
- The quantity CJstands for the torque required to produce a twist of 1 radian per unit of the shaft.
- The quantity CJcorresponding to a similar EI, in expression for deflection of beams, EI is known as flexure rigidity.

EXAMPLE FOR SHAFT

EXAMPLE 1:-

Calculate diameter of shaft to transmit 10 KW at a speed of 15 Hz. The maximum shear stress should not exceed 60 mpa.

P = 10 kw

N = 15 Hz = 15 cycles/sec

= 15 x 60 rpm

= 900 rpm

τ= 60 Mpa

$$P = \frac{2\pi N T}{60}$$

$$10 \times 10^3 = \frac{2\pi \times 900 \times T}{60}$$

T = 106.10 N.m

now,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$106.10 \times 10^3 = \frac{\pi}{16} \times 60 \times D^3$$

$$D^3 = 9006.0$$

D = 20.80 mm

EXAMPLE 2:-

A haft of 60 mm diameter rotates with 180 rpm. If permissible shear stress is 100 Mpa, find torque and power in KW.

Solution:-

D = 60 mm

N = 180rpm

T = 100 Mpa

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 100 \times 60^3$$

T = 4240 N.mm

Now,

$$P = \frac{2\pi N T}{60}$$

$$P = \frac{2\pi \times 180 \times 4240}{60}$$

$$= 79922.11 \text{watt}$$

$$\boxed{P = 79.92\ \text{Kw}}$$

EXAMPLE NO:- 3

External and internal diameter of a propeller shaft are 400mm and 200mm respectively. Find maximum shear stress developed in the cross section when a twisting moment of 50kn.M is applied. Take modulus of rigidity $C = 0.8 \times 10^3 \text{ N/mm}^2$ .if span of shaft is 4m,also find twisting angle of shaft.

Solution:

D = 400mm

D = 200mm

T = 50 KN.m

L = 4m

$$T = \frac{\pi}{16} \times \tau \times \frac{D^4 - d^4}{D}$$

$$50 \times 10^6 = \frac{\pi}{16} \times \tau \times \frac{400^4 - 200^4}{400}$$

$$\tau = 4.24 \text{ N/mm}^2$$

Now,

$$\frac{\tau}{\text{R}} = \frac{C\theta}{l}$$

$$R = \frac{D}{2} = \frac{400}{2} = 200mm$$

$$\therefore \boxed{\theta = 0.00106 \text{ radians}}$$

EXAMPLE-4 :-

Calculate the diameter of the shaft required to transmit 45 kw at 120 rpm. The maximum torque is likely to exceed the mean by 30% for a maximum permissible shear stress of 55 N/mm^2 .Calculate also the angle of twist for a length of 2m.

Solution :

P = 45 Kw

N = 120rpm

$\tau = 55\ N/mm^2$

$$P = \frac{2\pi \times N \times T}{60}$$

$$45 \times 10^3 = \frac{2\pi \times 120 \times T}{60}$$

$\therefore T = 3580.98$ N.m

$$T_{max} = 1.30 \times T_{min}$$

$= 1.30 \times 3580.98$

$= 4655.28$ N.m

$= 465528$ N.mm

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$4655.28 \times 10^3 = \frac{\pi}{16} \times 55 \times D^3$$

D = 75.54 mm

Now using the relation

$$\frac{\tau}{R} = \frac{C\theta}{l}$$

$$\frac{80\times10^{3}\times\theta}{2000} = \frac{55}{37.77}$$

$$\theta = 0.0364 \text{ radians}$$

EXAMPLE 5 :-

A shaft has to transmit 105 kw power at 160 rpm. If the shear stress is not to exceed $65 N/mm^2$ & the twist in a length of 3.5 m must not to exceed 1 degree. Find suitable diameter. Take $T = 6266.72 \times 10^3$ N.mm

P = 105 kw

N = 160 rpm

$\tau = 65 N/mm^2$

L = 3500 mm

$G = 8 \times 10^4 N/mm^2$

Now,

$$P = \frac{2\pi N T}{60}$$

$$105 \times 10^3 = \frac{2\pi \times 160 \times T}{60}$$

$T = 6266.72 \times 10^3$ N.mm

1) For strength

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$6266.72 \times 10^3 = \frac{\pi}{16} \times 65 \times D^3$$

$$\boxed{D = 78.89 \text{ mm}}$$

2) For stiffness

$$\frac{T}{J} = \frac{C\,\theta}{l}$$

$$\frac{6266.72\times10^{3}}{J} = \frac{8\times10^{4}\times0.0174}{3500}$$

$$\therefore\ J = 15.756\times10^{6}\,mm^{4}$$

Now,

$$J = \frac{\pi}{32}\times D^{4}$$

$$15.756\times10^{6} = \frac{\pi}{32}\times D^{4}$$

$$D = 112.55\ mm$$

EXAMOLE 6 :-

A solid shaft ABC is fixed at A and free at C and torque of 900 N.m is applied at B. The length of BC is 2m and that of BC is 1m. The diameter of AB is 40mm and that of BC is 20mm. If the shaft is made up of same material, find the angle of twisty in radius at the free end C.

Solution :

$d_1 = 40mm$

$d_2 = 20mm$

$l_1 = 2000mm$

$l_2 = 1000mm$

$T_B = 900 \times 10^3 \ N.mm$

As per given data,

twist will occur in the shaft AB and there will be zero twist in shaft BC.

The torque $T = 600 \times 10^3$ N.mm will act only on part AB.

We know that,

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\frac{900\times10^3}{0.251\times10^6} = \frac{80\times10^3\times\theta}{2000}$$

$\boxed{\theta = 0.0896 \text{ radians}}$ twist at part B.

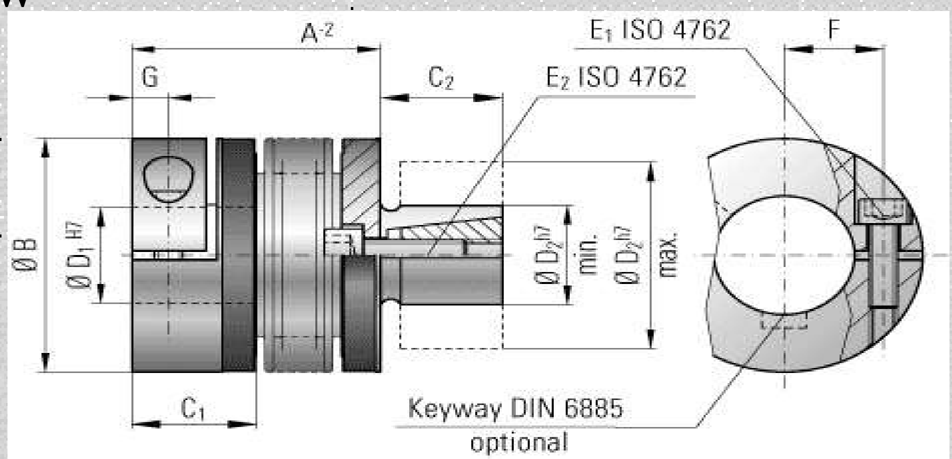
$$\theta_B = \theta_C = 0.0896 \text{radians}$$

$$J = \frac{\pi}{32}\times40^4$$

$$= 0.251\times10^6 \ mm^4$$

SHAFT COUPLING

When length of shaft required is very large, due to non availability of a single shaft of required length, it becomes necessary to connect two shafts together. This is usually done by means of flanged coupling as shown below



- The flange of two shafts are joined together by bolts nuts or rivets and the torque is then transferred from one shaft to another through the couplings.

- As the torque is transferred through the bolts, will be subjected to shear stress. As the diameter of bolts is small, as compared to the diameter of the flange therefore shear stress is assumed to be uniform in the bolts.

1) Design of bolts :

We know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

Where, τ= shear stress in shaft

d = diameter of shaft

Now,

Torque rested by one bolt,

= (area x shear stress) x radius of bolt circle

Where

$$= \frac{\pi}{4} \times db^2 \times \tau_b \times R$$

$$= \frac{\pi}{4} \times db^2 \times \tau_b \times \frac{D}{2}$$

$$= \frac{\pi}{4} \times db^2 \times \tau_b \times D$$

$\therefore$ total torque resisted by n bolts

$$= n \times \tau \times db^2 \times \tau_b \times D \quad \text{-------------} \ (2)$$

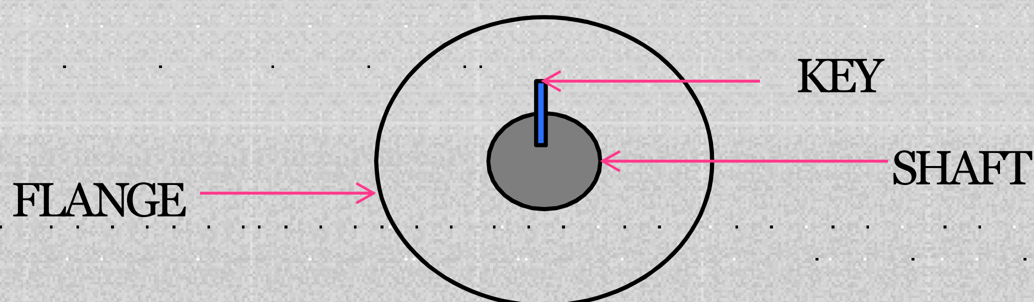
From equation (1) and (2)

$$\boxed{\frac{\pi}{16} \times \tau \times d^3 = n \times \frac{\pi}{8} \times db^2 \times \tau_b \times D}$$

DESIGN OF KEYS

A flange is attached to the shaft by means of a key. A rectangle notch is cut on the circumference of the shaft and a similar notch is cut on the inner side of the flange. The flange is then placed over the shaft in such a way that the two notches from a rectangular hole. A rectangular key is then inserted in to the hole and the flange is said to be keyed to the shaft.

Torque is transmitted by the shaft to flange through the key. Key is subjected to the shear stress.



EXAMPLE OF KEYS

EXAMPLE -7:-

A flanged coupling connecting two lengths of solid circular shaft has 6 nos of 20 mm diameter bolts equally spaced along a pitch circle of 240 mm diameter. Determine the shaft if the average shear stress in the bolts is to be the same as maximum shear stress in the shaft.

Solution :

n = 6 Nos

d = 20 mm

D = 240 mm

$$\tau_b = \tau$$

We know that,

Torque transmitted by shaft = Torque resisted by bolt

Now,

$$\frac{\pi}{16} \times \tau \times d^3 = n \times \frac{\tau}{8} \times db^2 \times \tau_b \times D$$

$$\therefore \frac{\pi}{16} \times \tau \times d^3 = 6 \times \frac{\pi}{8} \times 20^2 \times \tau \times 240$$

$$\therefore \boxed{d = 104.82 \text{mm dia of shafty}}$$

EXAMPLE -8 :-

The shaft each of 100 mm diameter are to be connected to the end by a bolted coupling. If the maximum shear stress in the shaft is 80 Mpa and in the bolts is 70 Mpa , find the number of 20 mm diameter bolts required for the coupling. Take diameter of bolt circle as 200 mm.

Solution :

$$d = 100mm$$

$$\tau = 80N/mm^2$$

$$\tau_b = 70N/mm^2$$

$$d_b = 20mm$$

$$D = 200mm$$

We know that,

Torque transmitted by the shaft = torque rested by the bolt

Now,

$$\frac{\pi}{16} \times \tau \times d^3 = n \times \frac{\pi}{8} \times db^2 \times \tau_b \times D$$

$$\frac{\pi}{16} \times 80 \times 100^3 = n \times \frac{\pi}{8} \times 20^2 \times 70 \times 200$$

$$\therefore n = 7.143 \text{Nos.}$$

Say $n = 8$ Nos Bolt

EXAMPLE OF DESIGN OF SHAFT

EXAMPLE – 9 :-

A shaft 100 in diameter is transmitted torque of 6000 N.m by means of key 200 mm long and 25 mm wide. Find the stress developed in shaft.

d = 100 mm

T = 6000 N.m

L = 200 mm

τ = shear stress in shaft

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{6000 \times 10^3}{9.817 \times 10^6} = \frac{\tau}{50}$$

$$\boxed{\tau = 30.56 N/mm^2}$$

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times 100^4$$

$$J = 9.817 \times 10^6$$

$$R = \frac{100}{2} = 50mm$$

Example -10 :-

Two shaft of diameter 50 mm are joined by a rigid flange coupling and transmit a torque in such a way that the shear stress in shaft does not exceed $100 N/mm^2$. If six bolt are used to join the flange and the bolt circle is 150 mm in diameter. Determine the diameter of the bolt if the permitted shear stress in the bolt is $\tau_b = 80 N/.mm^2$.

Solution :

d= 50mm

$\tau = 100 N/mm^2$

$\tau_b = 80 N/mm^2$

n= 6 Nos

D=150 mm

We know that,

Torque transmitted by the shaft = Torque resisted by bolts.

Now,

$$\frac{\pi}{16} \times \tau \times d^3 = n \times \frac{\tau}{8} \times db^2 \times \tau_b \times D$$

$$\frac{\pi}{16} \times 100 \times 50^3 = 6 \times \frac{\pi}{8} \times db^2 \times 80 \times 150$$

$\boxed{d_b = 9.32\text{mm}}$.....dia of bolt.

TORSION VEDIO

THANK YOU