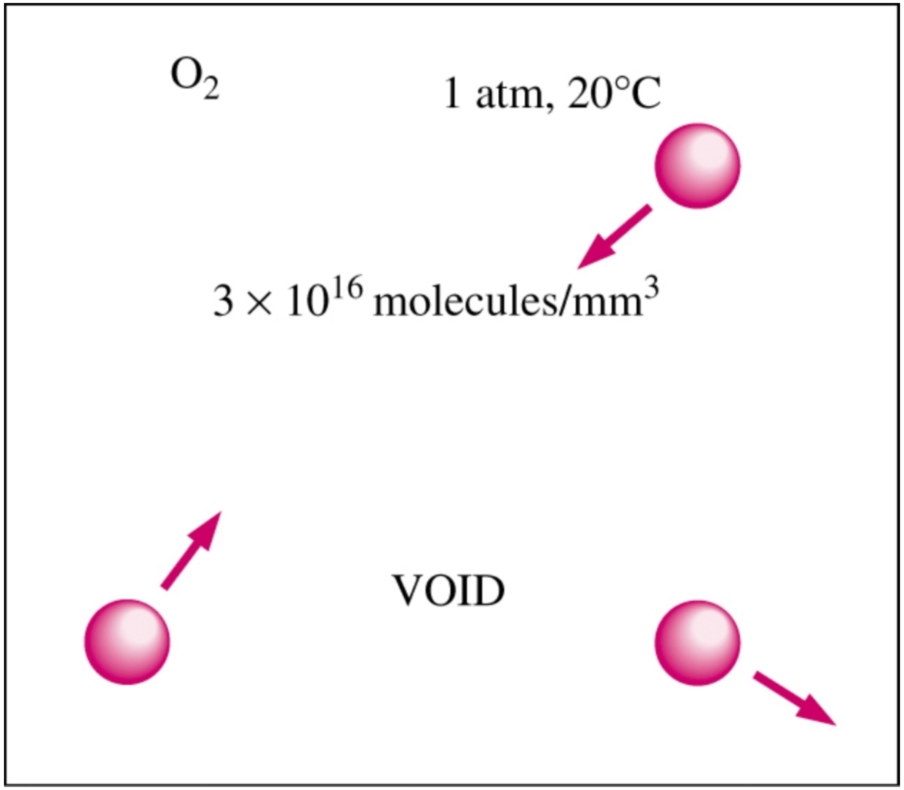
UNIT-I

FLUID PROPERTIES AND STATICS

Introduction

- Any characteristic of a system is called a **property.**
  - Familiar: pressure *P*, temperature *T,* volume *V*, and mass *m*.
  - Less familiar: viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, vapor pressure, surface tension.
- *Intensive* properties are independent of the mass of the system. Examples: temperature, pressure, and density.
- *Extensive* properties are those whose value depends on the size of the system. Examples: Total mass, total volume, and total momentum.
- Extensive properties per unit mass are called **specific properties**. Examples include specific volume $v = V/m$ and specific total energy $e=E/m.$

Continuum



- Atoms are widely spaced in the gas phase.
- However, we can disregard the atomic nature of a substance.
- View it as a continuous, homogeneous matter with no holes, that is, a **continuum**.
- This allows us to treat properties as smoothly varying quantities.
- Continuum is valid as long as size of the system is large in comparison to distance between molecules.

Density and Specific Gravity

- Density is defined as the *mass per unit volume* $\rho = m/V$. Density has units of kg/m^3
- Specific volume is defined as $v = 1/\rho = V/m$.
- For a gas, density depends on temperature and pressure.
- **Specific gravity**, or relative density is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C), i.e., $SG=\rho/\rho_{H_20}$. SG is a dimensionless quantity.
- The **specific weight** is defined as the weight per unit volume, i.e., $\gamma_s = \rho g$ where g is the gravitational acceleration. γ_s has units of N/m^3.

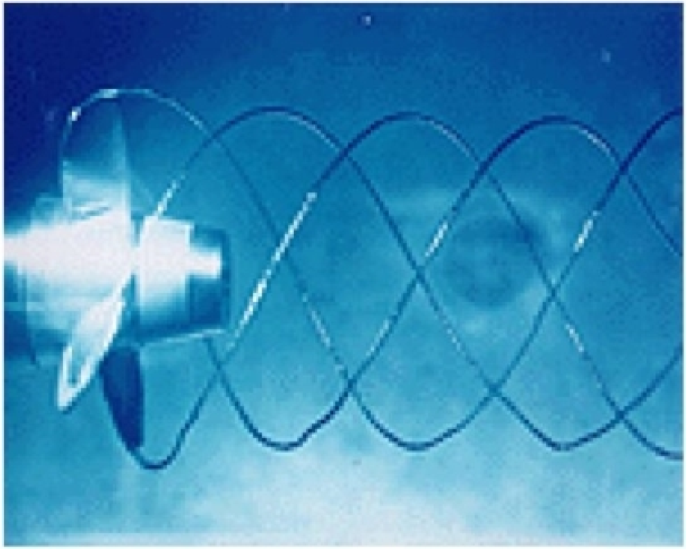
Density of Ideal Gases

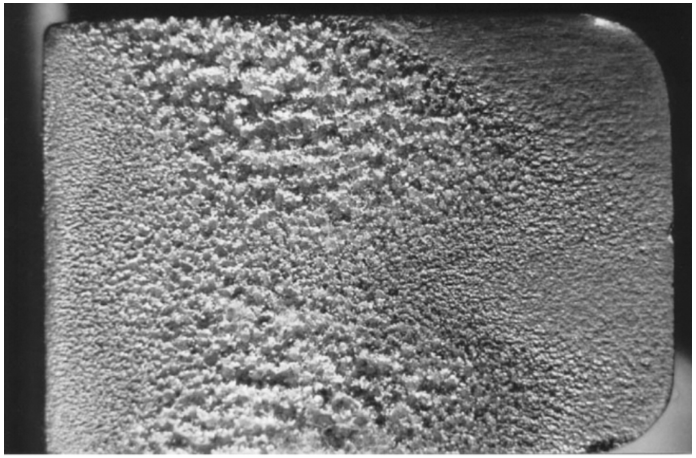
- **Equation of State**: equation for the relationship between pressure, temperature, and density.
- The simplest and best-known equation of state is the ideal-gas equation.

$$P\,v = R\,T \qquad \text{or} \qquad P = \rho\,R\,T$$

- Ideal-gas equation holds for most gases.
- However, dense gases such as water vapor and refrigerant vapor should not be treated as ideal gases. Tables should be consulted for their properties, e.g., Tables A-3E through A-6E in textbook.

Vapor Pressure and Cavitation





- **Vapor Pressure** P_v is defined as *the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature*
- If P drops below P_v, liquid is locally vaporized, creating cavities of vapor.
- Vapor cavities collapse when local P rises above P_v.
- Collapse of cavities is a violent process which can damage machinery.
- Cavitation is noisy, and can cause structural vibrations.

Energy and Specific Heats

- Total energy E is comprised of numerous forms: thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear.
- Units of energy are *joule* (*J*) or *British thermal unit* (BTU).
- Microscopic energy
  - Internal energy u is for a non-flowing fluid and is due to molecular activity.
  - Enthalpy h=u+Pv is for a flowing fluid and includes flow energy (Pv).
- Macroscopic energy
  - Kinetic energy $ke=V^2/2$
  - Potential energy $pe=gz$
- In the absence of electrical, magnetic, chemical, and nuclear energy, the total energy is $e_{flowing}=h+V^2/2+gz$.

Coefficient of Compressibility

- How does fluid volume change with *P* and *T*?
- Fluids expand as *T* ↑ or *P* ↓
- Fluids contract as *T* ↓ or *P* ↑
- Need fluid properties that relate volume changes to changes in *P* and *T*.
  - Coefficient of compressibility

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T$$

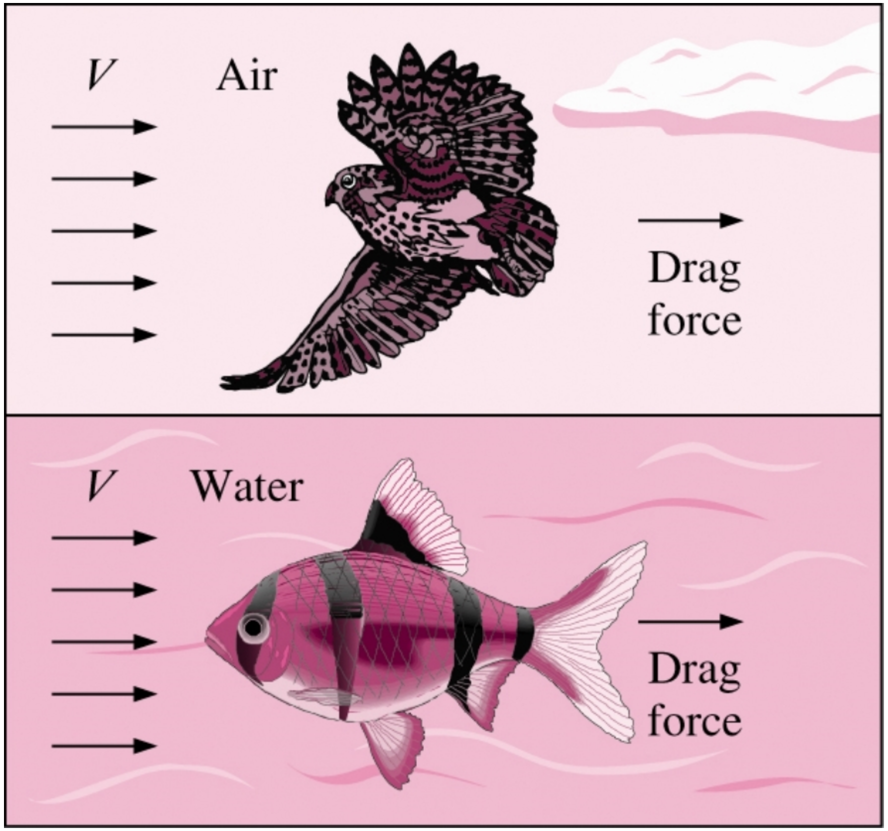
  - Coefficient of volume expansion

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

- Combined effects of *P* and *T* can be written as

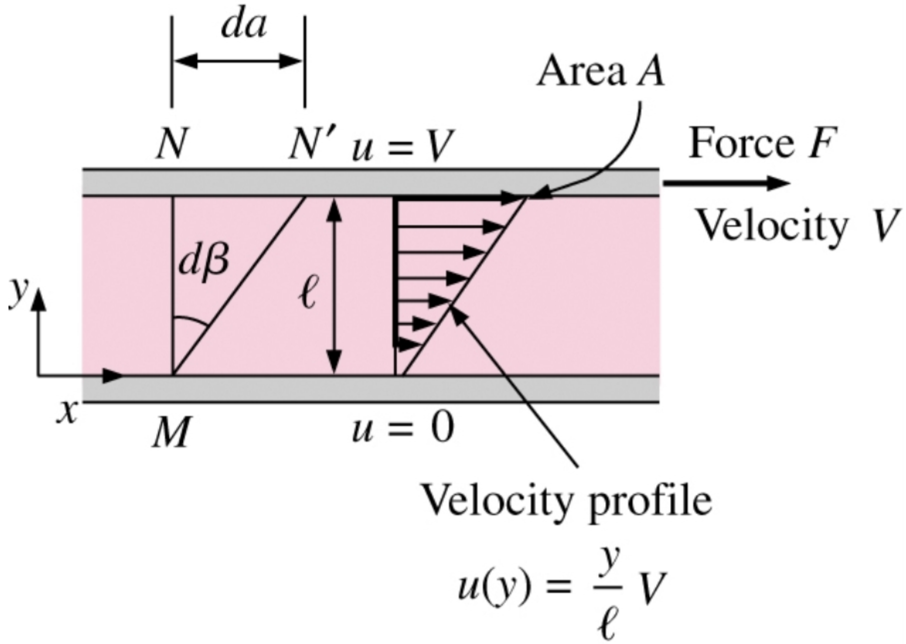
$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP$$

Viscosity



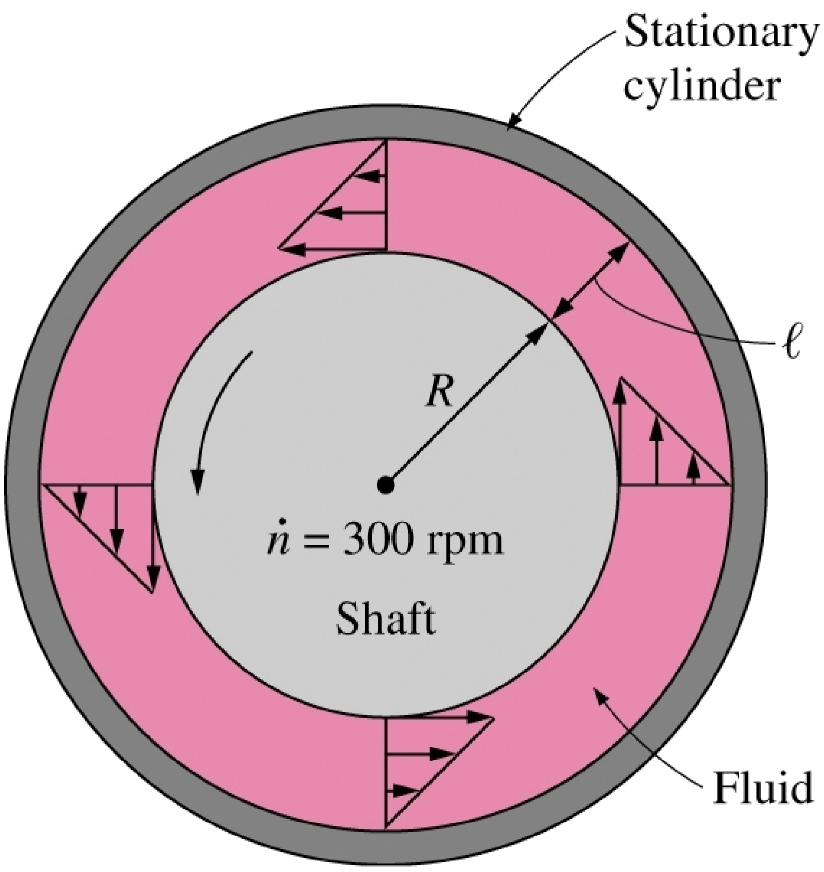
- **Viscosity** is a property that represents the internal resistance of a fluid to motion.
- The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity.

Viscosity



- To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates separated by a distance ℓ
- Definition of shear stress is $\tau = F/A$.
- Using the no-slip condition, $u(0) = 0$ and $u(\ell) = V$, the velocity profile and gradient are $u(y) = Vy/\ell$ and $du/dy = V/\ell$
- Shear stress for Newtonian fluid: $\tau = \mu du/dy$
- μ is the **dynamic viscosity** and has units of *kg/m·s*, *Pa·s*, or **poise**.

Viscometry

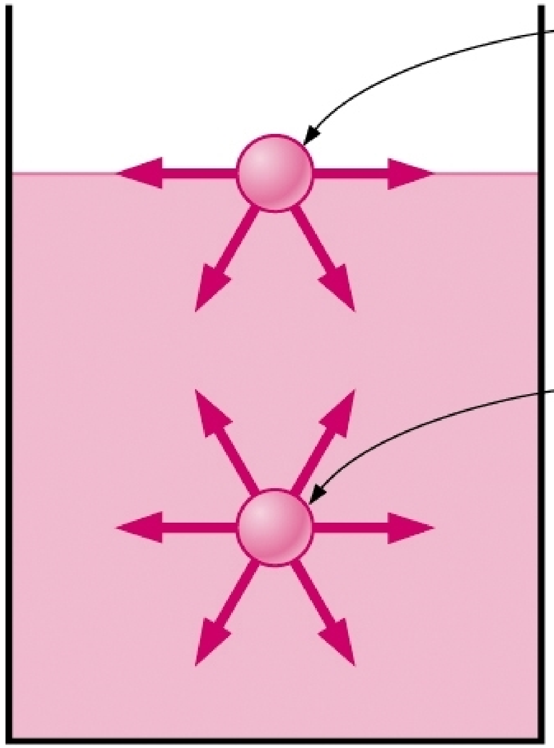


- How is viscosity measured? A rotating viscometer.
  - Two concentric cylinders with a fluid in the small gap ℓ.
  - Inner cylinder is rotating, outer one is fixed.
- Use definition of shear force:

$$F = \tau A = \mu A \frac{du}{dy}$$

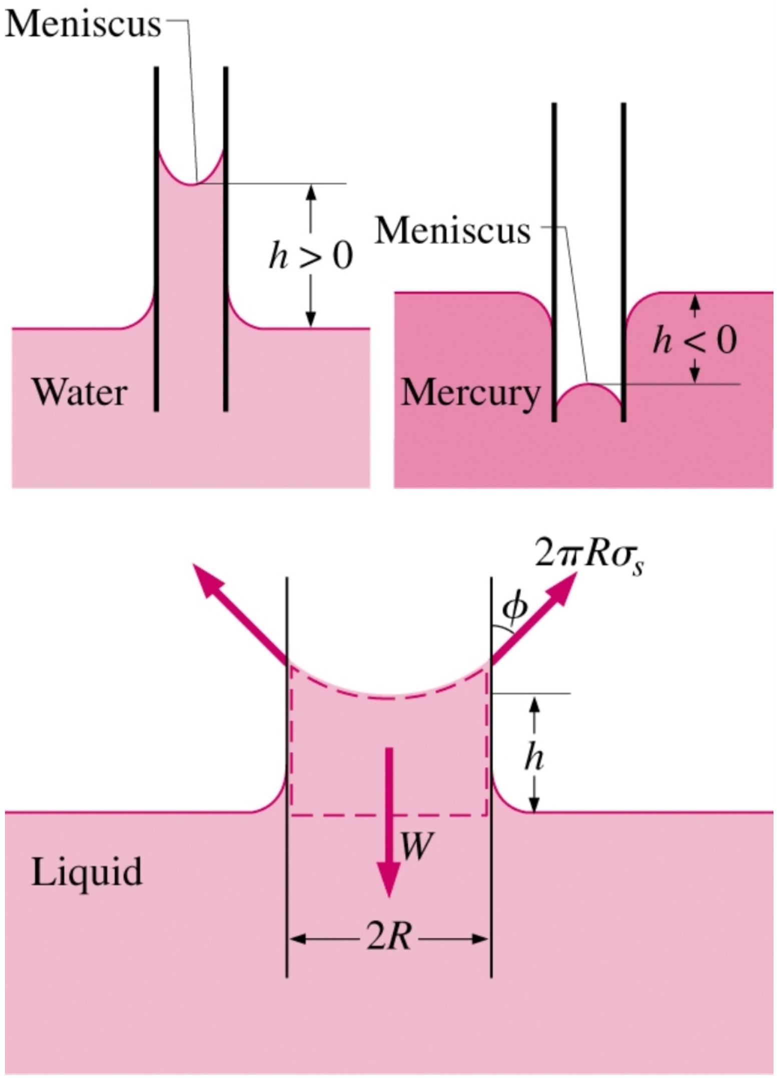
- If $\ell/R \ll 1$, then cylinders can be modeled as flat plates.
- Torque $T = FR$, and tangential velocity $V=\omega R$
- Wetted surface area $A=2\pi RL$.
- Measure T and ω to compute μ

Surface Tension



- Liquid droplets behave like small spherical balloons filled with liquid, and the surface of the liquid acts like a stretched elastic membrane under tension.
- The pulling force that causes this is
  - due to the attractive forces between molecules
  - called **surface tension** σ_s.
- Attractive force on surface molecule is not symmetric.
- Repulsive forces from interior molecules causes the liquid to minimize its surface area and attain a spherical shape.

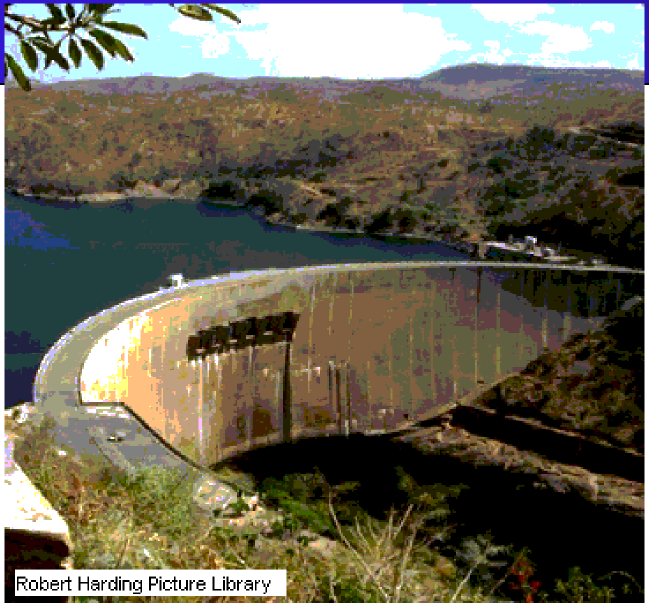
Capillary Effect



- **Capillary effect** is the rise or fall of a liquid in a small-diameter tube.
- The curved free surface in the tube is call the **meniscus**.
- Water meniscus curves up because water is a *wetting fluid.*
- Mercury meniscus curves down because mercury is a *nonwetting fluid*.
- Force balance can describe magnitude of capillary rise.

Dams

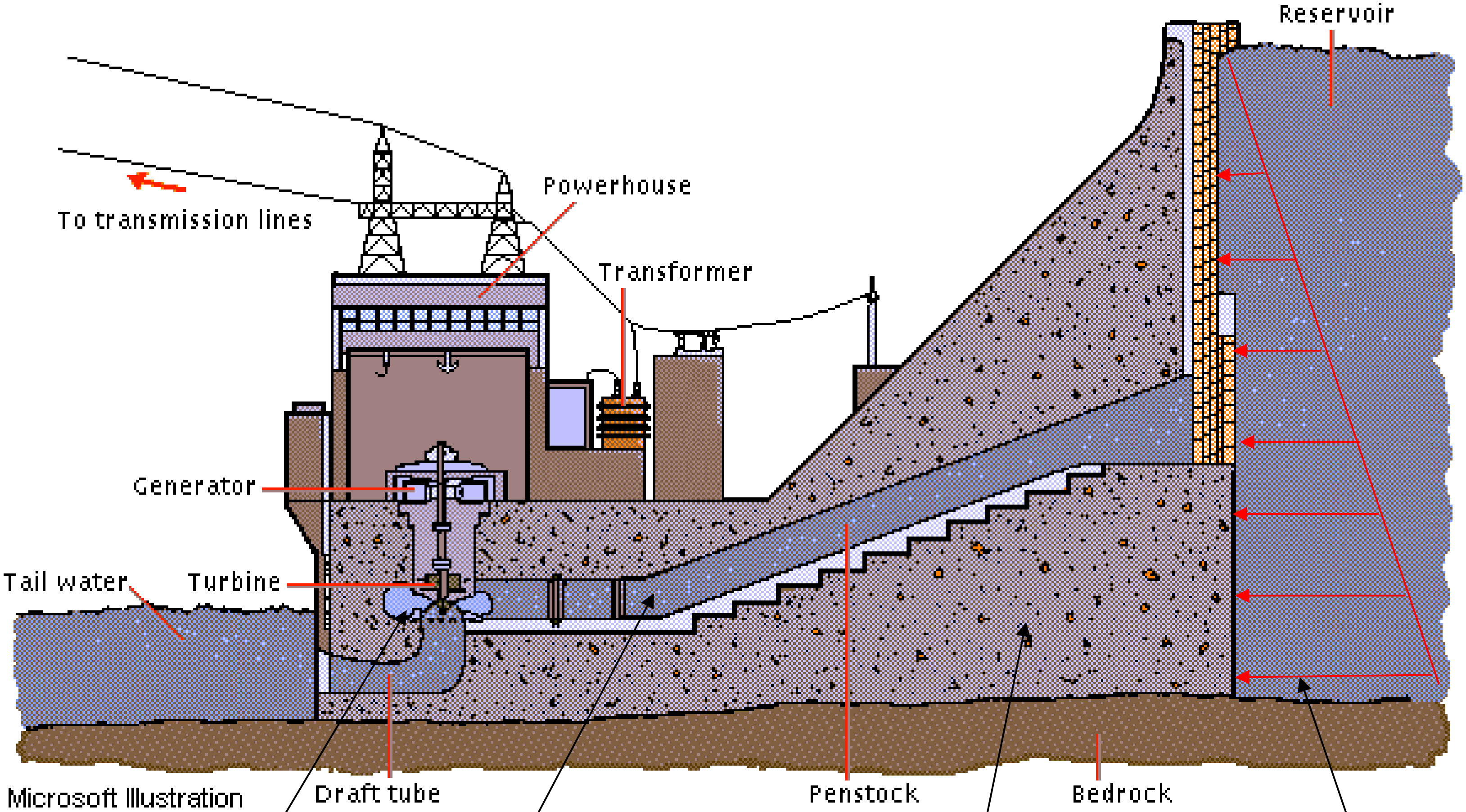
Arch Dam



Arch & Gravity Dam

Gravity Dam

Dams (cont.)



Turbine

Energy conversion

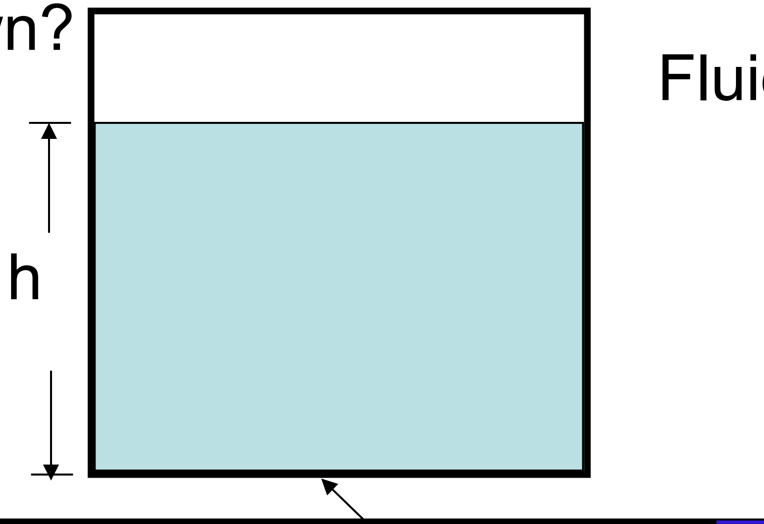
Hydrostatic uplift

Hydrostatic forces

Fluid Statics

When a surface is submerged in a fluid at rest, hydrostatic forces develop on the surface due to the fluid pressure. These forces must be perpendicular to the surface since there is no shear action present. These forces can be determined by integrating the static pressure distribution over the area it is acting on.

Example: What is the force acting on the bottom of the tank shown?



Fluid with density ρ

Dam Design

Design concern: (**Hydrostatic Uplift**) Hydrostatic pressure above the heel (upstream edge) of the dam may cause seepage with resultant uplift beneath the dam base (depends largely on the supporting material of the dam). This reduces the dams stability to sliding and overturning by effectively reducing the weight of the dam structure. (Question: What prevents the dam from sliding?)

Determine the minimum compressive stresses in the base of a concrete gravity dam as given below. It is important that this value should be greater than zero because (1) concrete has poor tensile strength. Damage might occur near the heel of the dam. (2) The lifting of the dam structure will accelerate the seeping rate of the water underneath the dam and further increase hydrostatic uplift and generate more instability.

Catastrophic breakdown can occur if this factor is not considered: for example, it is partially responsible for the total collapse of the St. Francis Dam in California, 1928.

Dam Design

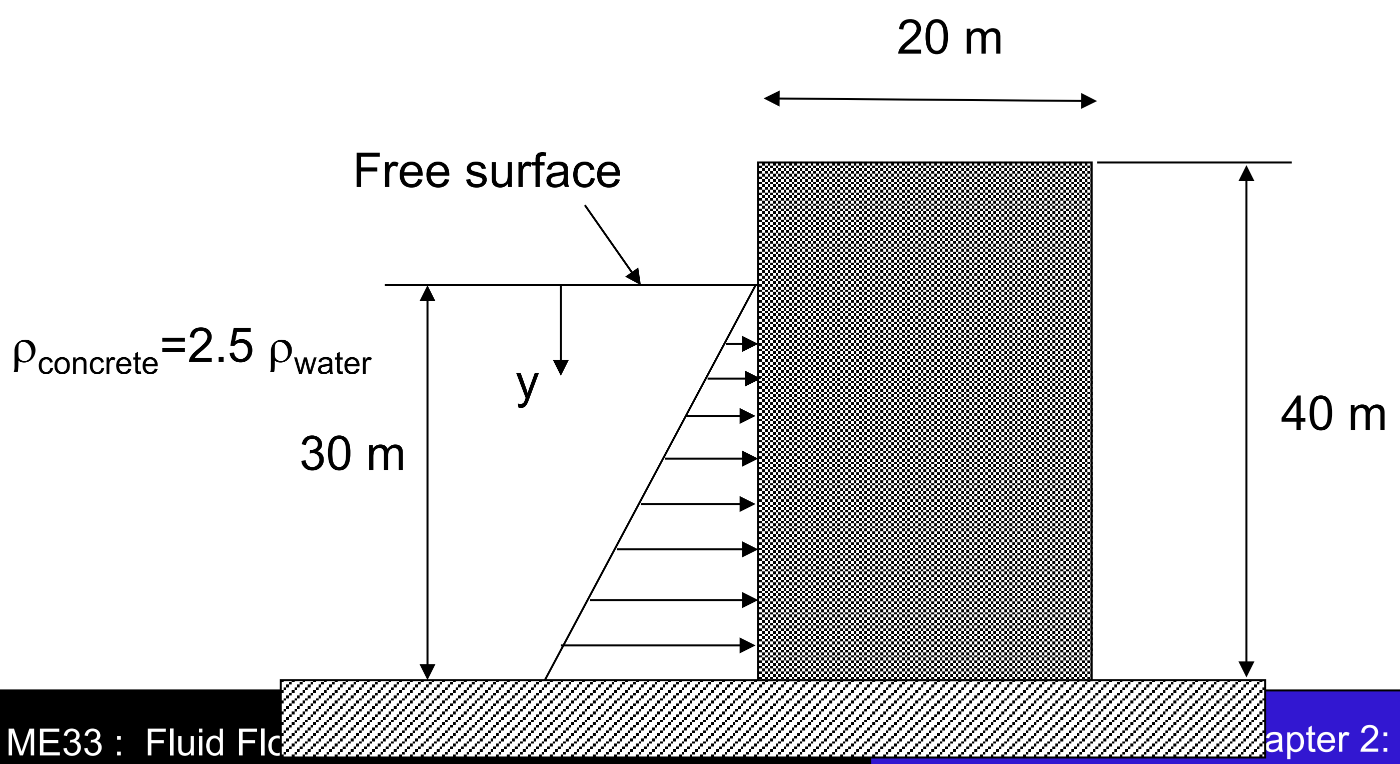
First, calculate the weight of the dam (per unit width):

$W = \rho V g = (2.5)(1000)(20)(40)(1)(9.8) = 19.6 \times 10^6$ (N)

The static pressure at a depth of y: $P(y) = \rho_w g y$

The total resultant force acting on the dam by the water pressure is:

$$R = \int P(y)dy = \int_0^{h=30} \rho_w g y dy = \rho_w g \left(\frac{h^2}{2} \right) = (1000)(9.8)(1/2)(30)^2 = 4.4 \times 10^6 (N)$$



Example (cont.)

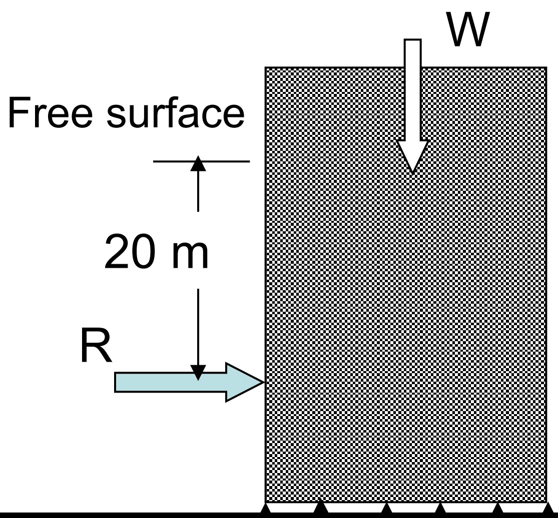
The resultant force, R, is acting at a depth $\bar{h}$ below the free surface so that

$$R\bar{h}=\int P(y)ydy=\int(\rho_w gy)ydy=\rho_w g\int_0^{h=30} y^2dy=\rho_w g\frac{h^3}{3},\quad \bar{h}=\frac{\rho_w g\frac{h^3}{3}}{R}=\frac{2h}{3}=20(m)$$

Assume the load distribution under the dam is linear (it might not be linear if the soil distribution is not uniform)

Therefore, the stress distribution can be written as

$$\sigma(x)=\sigma_{min}+\frac{\sigma_{max}-\sigma_{min}}{20}x$$



In order to reach equilibrium, both the sum of forces and the sum of moments have to balance to zero

$$\sum F_x=0,\ R=F_{dam,x}\ \text{(frictional force and the air drag force)}$$

$$\sum F_y=0,\ W=F_{dam,y}=\int_0^{20}\sigma(x)dx=10(\sigma_{max}+\sigma_{min})$$

$$1.96\times10^6(N)=\sigma_{max}+\sigma_{min}$$

Example (cont.)

The sum of moments has to be zero also: Taking moment w.r.t. the heel of the dam

$$\sum \mathrm{M_O} = 0, \quad -R(10) - W(10) + \int_0^{20} \sigma(x) x dx = 0$$

$$(10)(4.4\times10^6 + 19.6\times10^6) = \sigma_{\max}\int_0^{20} x dx + \frac{\sigma_{\max}-\sigma_{\min}}{20}\int_0^{20} x^2 dx$$

$$240\times10^6 = 133.3\sigma_{\max} + 66.7\sigma_{\min}$$

Solve: $\sigma_{\max} = 1.64\times10^6 (N)$, $\sigma_{\min} = 0.32\times10^6 (N)$

The minimum compressive stress is significantly lower than the maximum stress

The hydrostatic lift under the dam (as a result of the buoyancy induced by water seeping under the dam structure) can induce as high as one half of the maximum hydrostatic head at the heel of the dam and gradually decrease to zero at the other end.

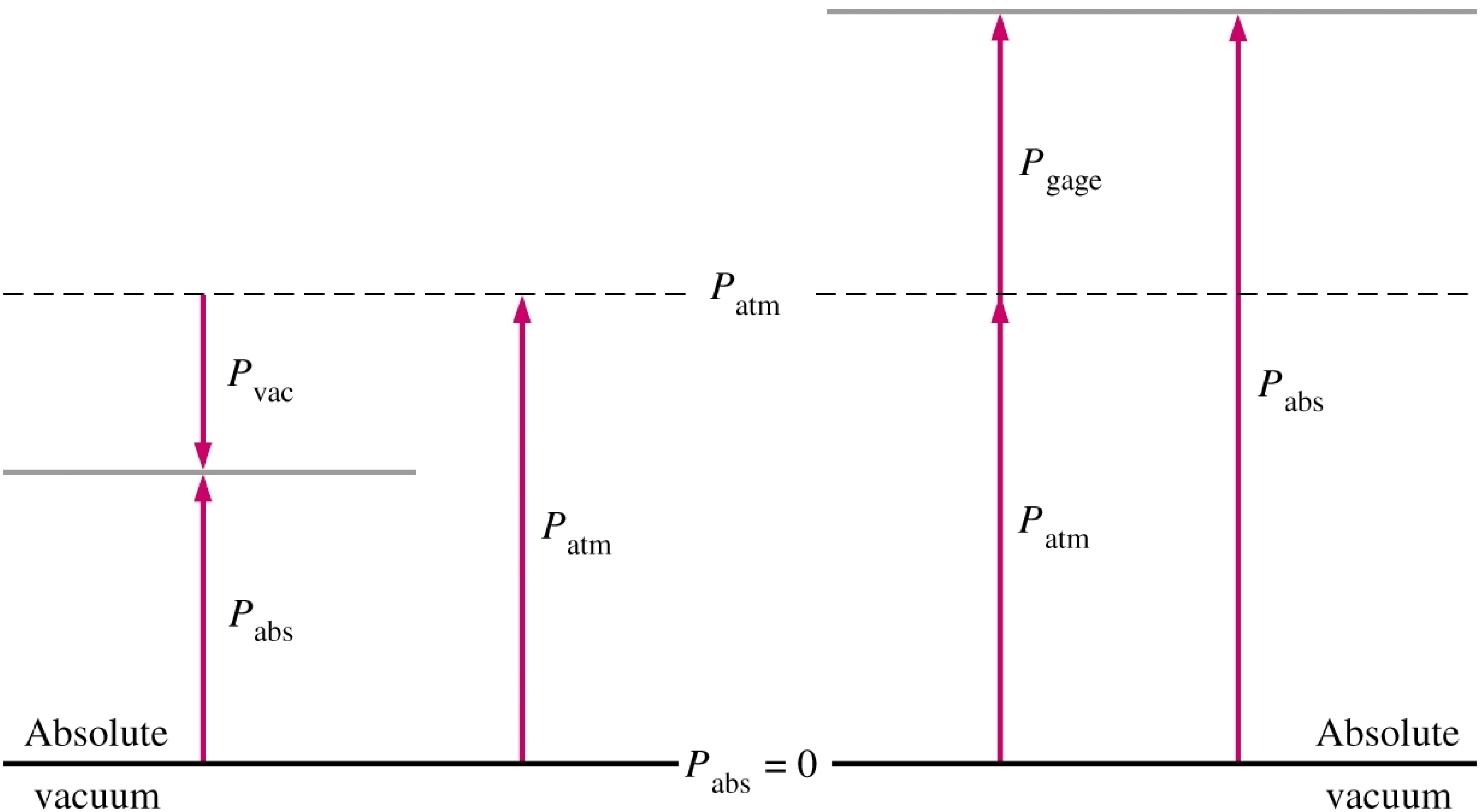
That is $\sigma_{\text{lift}} = \frac{1}{2}(\rho_w g h) = (0.5)(1000)(9.8)(30) = 0.147\times10^6 (N)$

Therefore, the effective compressive stress will only be $0.173(=0.32\text{-}0.147)\times10^6 (N)$.

Absolute, gage, and vacuum pressures

- Actual pressure at a give point is called the **absolute pressure**.
- Most pressure-measuring devices are calibrated to read zero in the atmosphere, and therefore indicate **gage pressure**, $P_{gage}=P_{abs} - P_{atm}$.
- Pressure below atmospheric pressure are called **vacuum pressure**, $P_{vac}=P_{atm} - P_{abs}$.

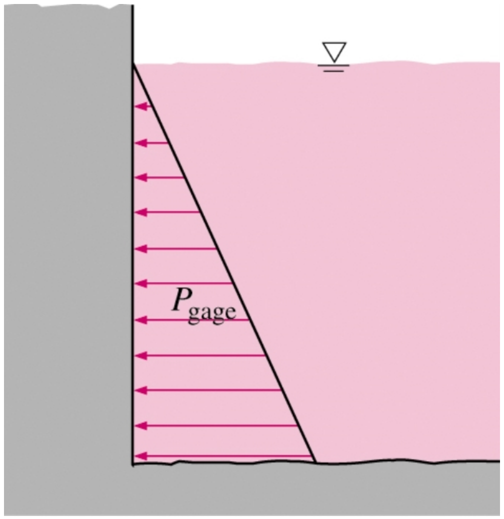
Absolute, gage, and vacuum pressures

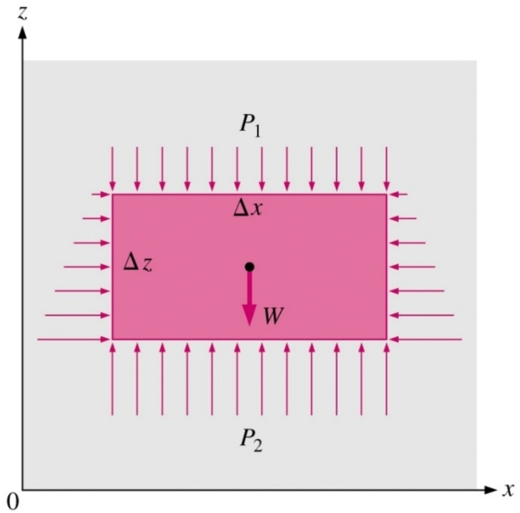


Pressure at a Point

- Pressure at any point in a fluid is the same in all directions.
- Pressure has a magnitude, but not a specific direction, and thus it is a scalar quantity.

Variation of Pressure with Depth





- In the presence of a gravitational field, pressure increases with depth because more fluid rests on deeper layers.
- To obtain a relation for the variation of pressure with depth, consider rectangular element
  - Force balance in *z*-direction gives

$$\sum F_z = ma_z = 0$$

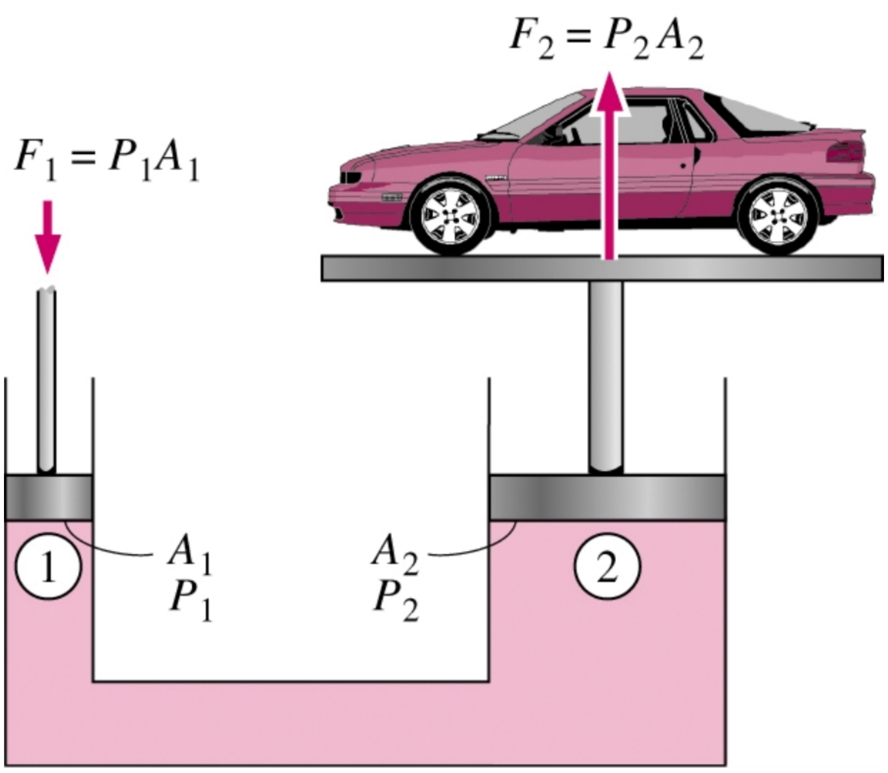
$$P_2 \Delta x - P_1 \Delta x - \rho g \Delta x \Delta z = 0$$

  - Dividing by Δx and rearranging gives

$$\Delta P = P_2 - P_1 = \rho g \Delta z = \gamma_s \Delta z$$

Scuba Diving and Hydrostatic Pressure

Pascal's Law

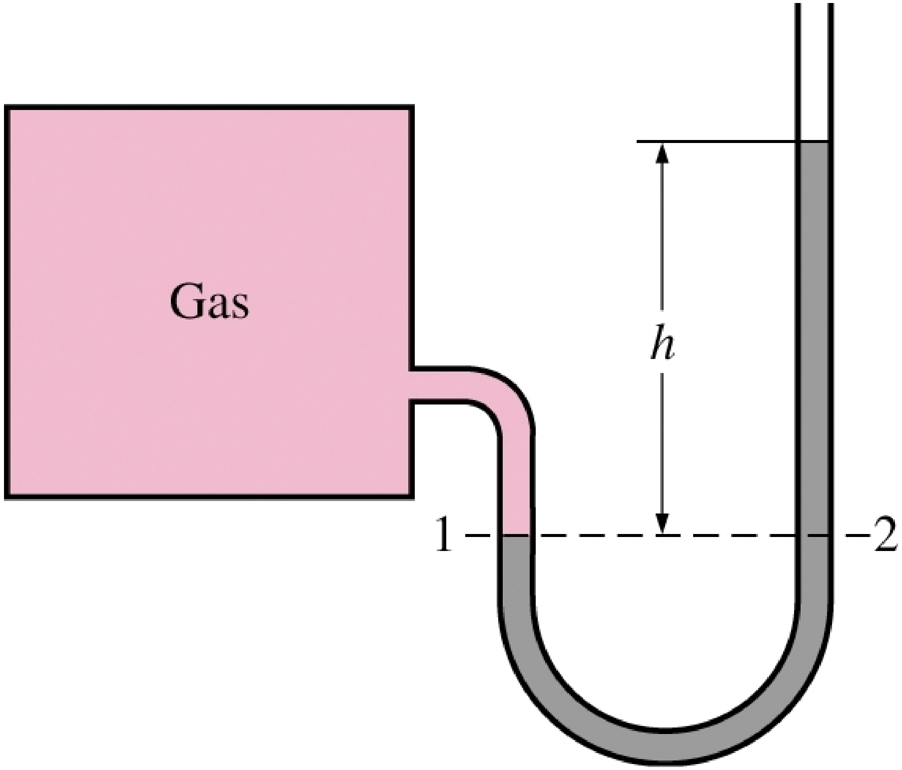


- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A_2/A_1 is called *ideal mechanical advantage*

The Manometer

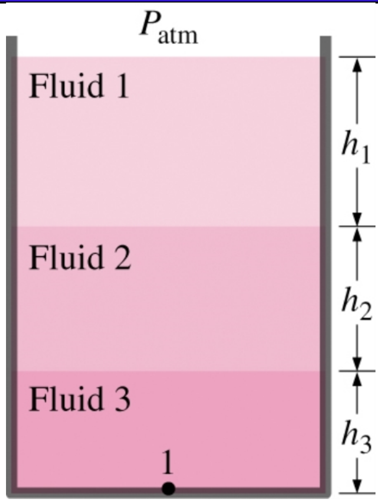


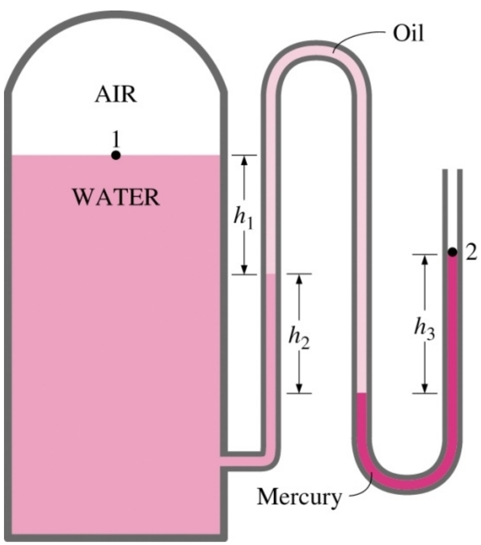
$$P_1 = P_2$$

$$P_2 = P_{atm} + \rho g h$$

- An elevation change of Δz in a fluid at rest corresponds to $\Delta P/\rho g$.
- A device based on this is called a **manometer.**
- A manometer consists of a U-tube containing one or more fluids such as mercury, water, alcohol, or oil.
- Heavy fluids such as mercury are used if large pressure differences are anticipated.

Mutlifluid Manometer





- For multi-fluid systems
  - Pressure change across a fluid column of height *h* is $\Delta P = \rho g h$.
  - Pressure increases downward, and decreases upward.
  - Two points at the same elevation in a continuous fluid are at the same pressure.
  - Pressure can be determined by adding and subtracting $\rho g h$ terms.

$$P_2 + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

Unit-II :

FLUID KINEMATICS AND DYNAMICS

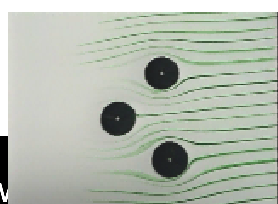
Fluid Flow Concepts and Reynolds Transport Theorem

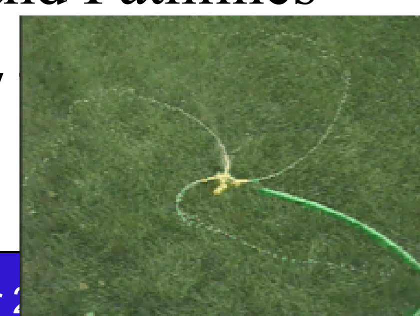
- Descriptions of:
  - fluid motion
  - fluid flows
  - temporal and spatial classifications
- Analysis Approaches
  - Lagrangian vs. Eulerian
- Moving from a system to a control volume

Descriptions of Fluid Motion

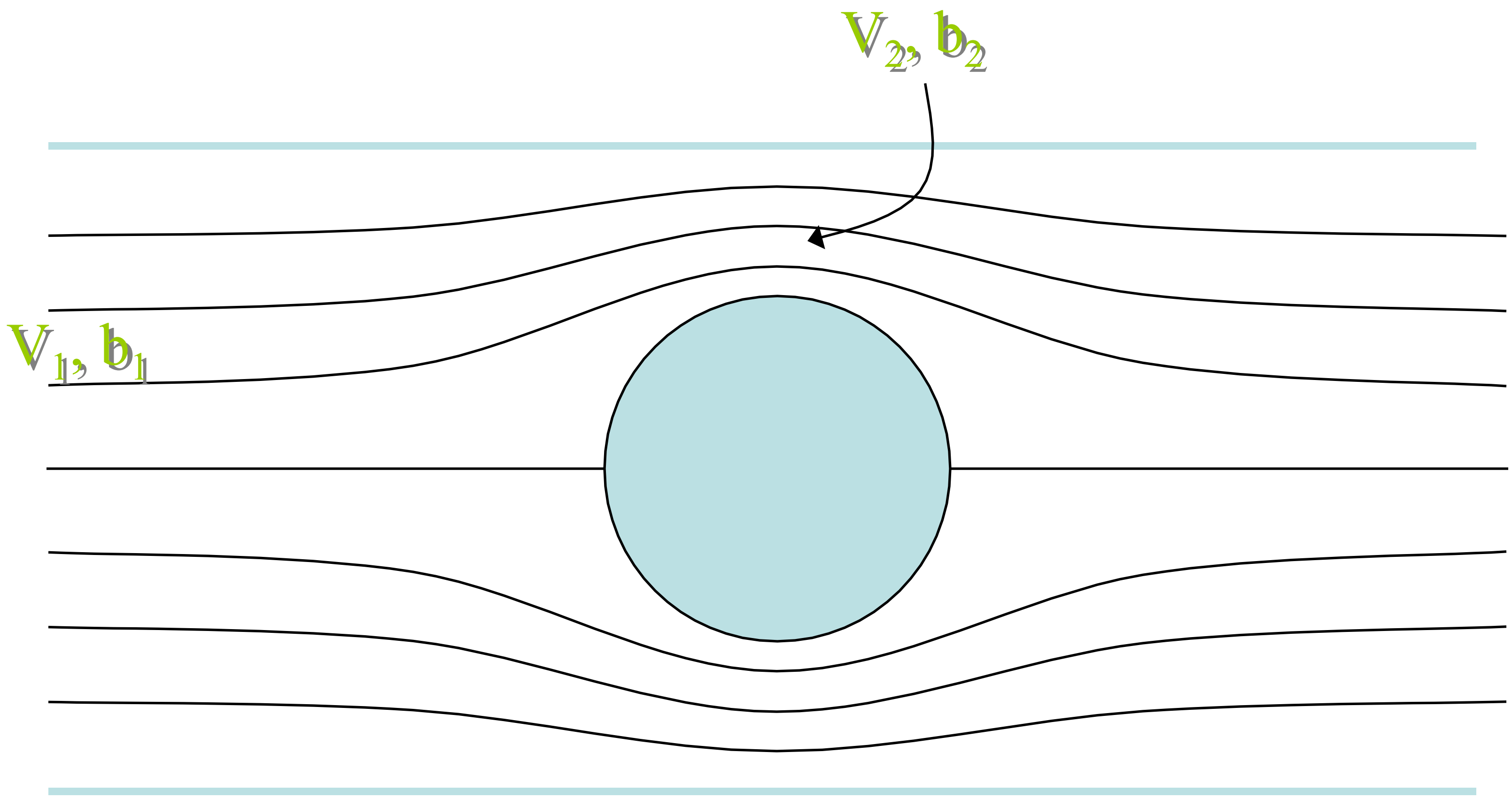
- streamline
  - Defined instantaneously
  - has the direction of the velocity vector at each point
  - no flow across the streamline
  - steady flow streamlines are fixed in space
  - unsteady flow streamlines move
- pathline
  - Defined as particle moves (over time)
  - path of a particle
  - same as streamline for steady flow
- streakline
  - tracer injected continuously into a flow
  - same as pathline and streamline for steady

Draw Streamlines and Pathlines





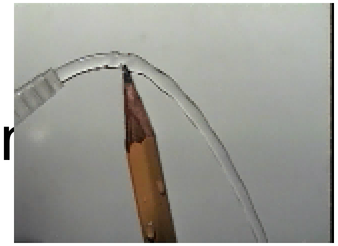
Streamlines



[Ideal flow machine]

Descriptors of Fluid Flows

- Laminar flow
  - fluid moves along smooth paths
  - viscosity damps any tendency to swirl or r

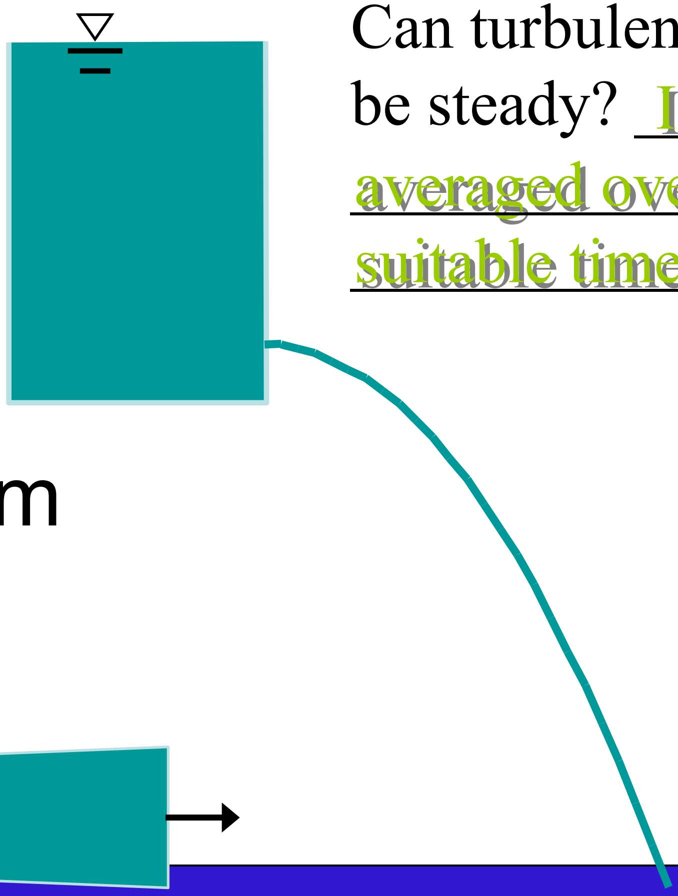


- Turbulent flow
  - fluid moves in very irregular paths
  - efficient mixing
  - velocity at a point fluctuates

Transition to turbulence movie

Temporal/Spatial Classifications

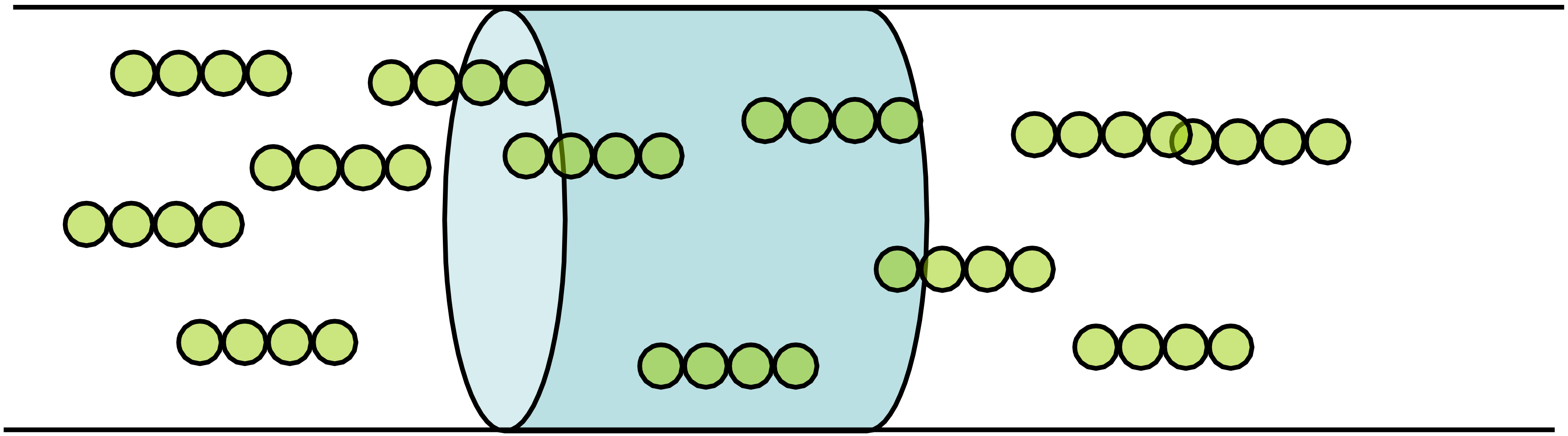
- Steady - unsteady
  - Changing in time



Can turbulent flow be steady? If averaged over a suitable time

- Uniform - nonuniform
  - Changing in space

Control Volume Conservation Equation



$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \, d\mathcal{V} + \int_{cs} \rho b \, \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$$

$$0 = -1 + (-0 + 1)$$

$$0 = 1 + (-1 + 0)$$

$$0 = 0 + (-0 + 0)$$

Fluid Kinematics

What is fluid kinematics?

- Fluid kinematics is the study on fluid motion in space and time without considering the force which causes the fluid motion.
- According to the continuum hypothesis the local velocity of fluid is the velocity of an infinitesimally small fluid particle/element at a given instant t. It is generally a continuous function in space and time.

• How small an how large should be a fluid particle/element in frame of the continuum concept?

• The characteristic length of the fluid system of interest >> The characteristic length of a fluid particle/element >> The characteristic spacing between the molecules contained in the volume of the fluid particle/element :

$$L >> d >> \lambda ; \lambda / L = Kn \; (Knudsen \;\; No.)$$

For air at sea-level conditions,

- molecules in a volume of 3×10^7 $(10^{-3}\; mm)^3$

- (λ : mean free path) $15 \;^\circ C \;\; and \;\; 10.133 \times 10^4 \;\; Pa$ $\lambda = 10^{-6} \; mm$

The continuum concept is valid!

4.1. Velocity Field

- Eulerian Flow Description
- Lagrangian Flow Description
- Streamline
- Pathline
- Streakline

4.1.1. In the Eulerian Method

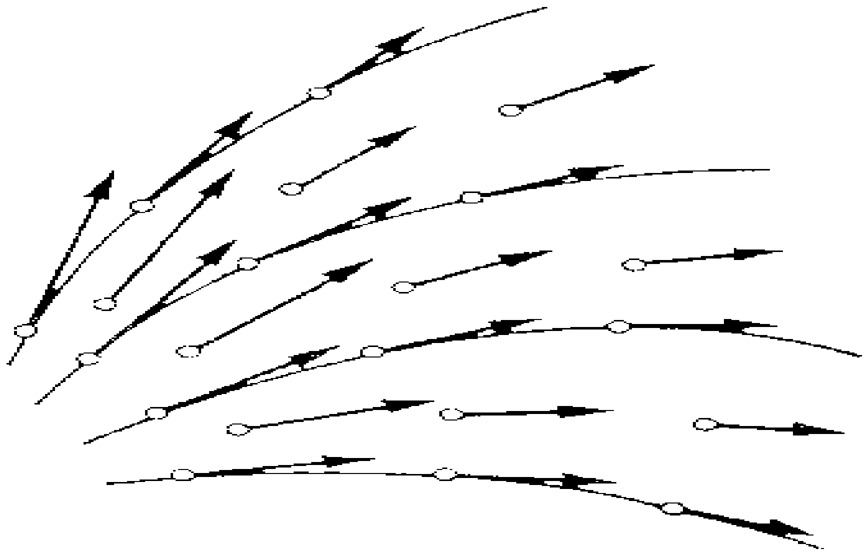
- The flow quantities, like $\vec{u}$, p, ρ, T, are described as a function of space and time without referring to any individual identity of the fluid particle : $\vec{u}(\vec{r}, t)$

4.1.2. Streamline

- A line in the fluid whose tangent is parallel to $\vec{u}$ at a given instant t.
- The family of streamlines at time t are solutions of

$$\frac{dx}{u_x(\vec{r},t)} = \frac{dy}{u_y(\vec{r},t)} = \frac{dz}{u_z(\vec{r},t)}$$

- Where $\vec{u}_x,\ \vec{u}_y,\ and\ \vec{u}_z$ are velocity components in the respective direction



▲ **Fig. 4.1**

- Steady flow : the streamlines are fixed in space for all time.
- Unsteady flow : the streamlines are changing from instant to instant.

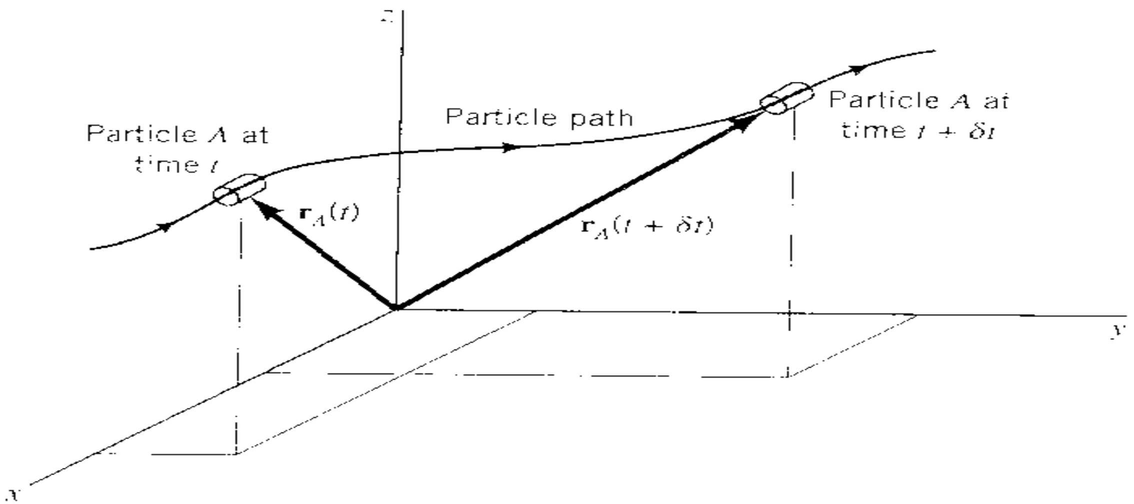
4.1.3. Flow Dimensionality

- Most of the real flow are
  3-dimensional and unsteady : $\vec{u}(x, y, z, t)$
- For many situations simplifications can be made :
  2-dimensional unsteady and steady flow
  $\vec{u}(x, y, t)$; $\vec{u}(x, y)$
  1-dimensional unsteady and steady flow
  $\vec{u}(x, t)$; $\vec{u}(x)$

4.1.4. In the Lagrangian Method

- The flow quantities are described for each individually identifiable fluid particle moving through flow field of interest. The position of the individual fluid particle is a function of time :

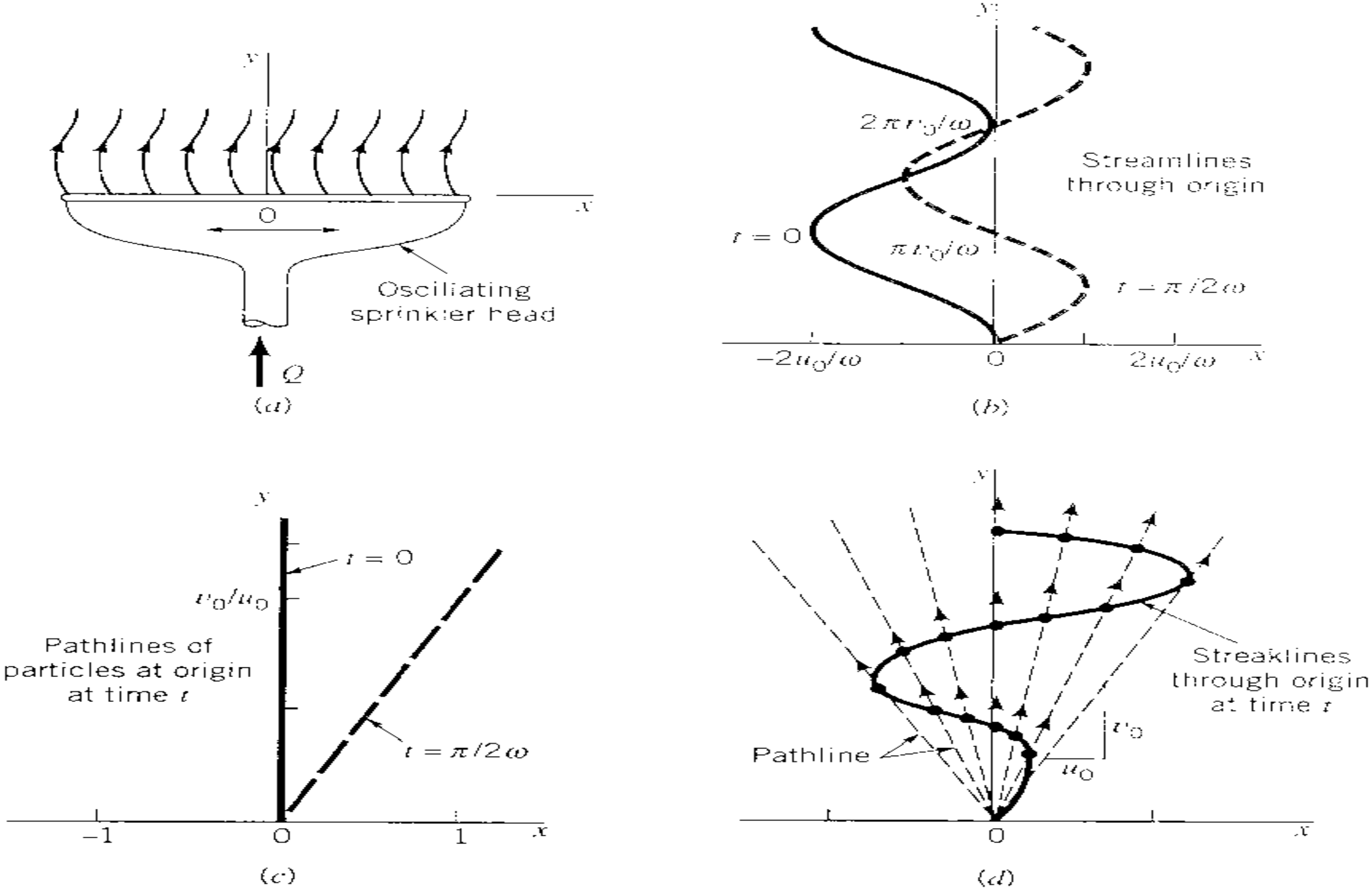
$$\vec{V}\,(\vec{r}(t))$$



▲ Fig. 4.2

4.1.5. Pathline

- A line traced by an individual fluid particle: $\vec{r}(t)$
- For a steady flow the pathlines are identical with the streamlines.



Oscillating sprinkler head
(a)
Streamlines through origin
$2\pi v_0/\omega$
$\pi v_0/\omega$
$t = 0$
$t = \pi/2\omega$
$-2u_0/\omega$
$2u_0/\omega$
(b)
v_0/u_0
Pathlines of particles at origin at time t
$t = 0$
$t = \pi/2\omega$
(c)
Streaklines through origin at time t
Pathline
v_0
u_0
(d)

▲ **Fig. 4.3**

4.1.6. Streakline

- A streakline consists of all fluid particles in a flow that have previously passed through a common point. Such a line can be produced by continuously injecting marked fluid (smoke in air, or dye in water) at a given location.

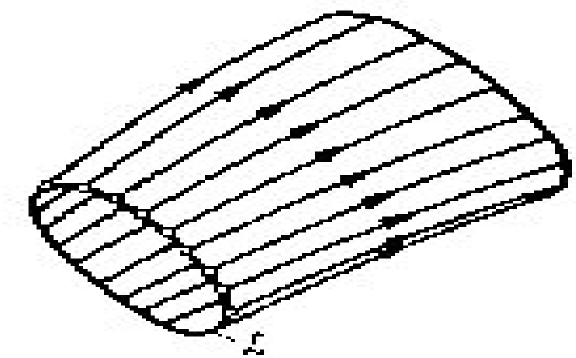
- For steady flow : The streamline, the pathline, and the streakline are the same.

4.2. Stream-tube and Continuity Equation

- Stream-tube
- Continuity Equation of a Steady Flow

4.2.1. Stream-tube

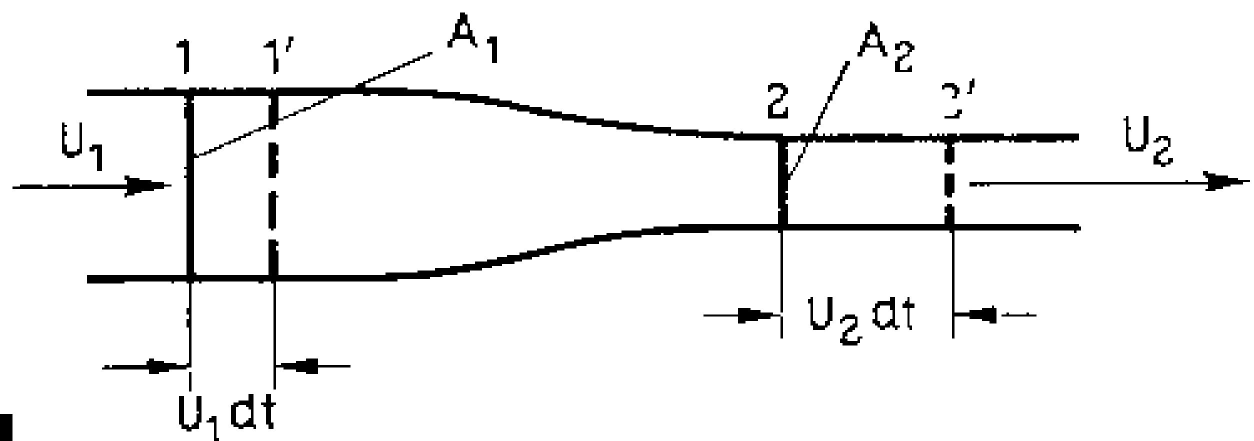
- is the surface formed instantaneously by all the streamlines that pass through a given closed curve in the fluid.



▲ **Fig. 4.4**

4.2.2. Continuity Equation of a Steady Flow

- For a steady flow the stream-tube formed by a closed curved fixed in space is also fixed in space, and no fluid can penetrate through the stream-tube surface, like a duct wall.



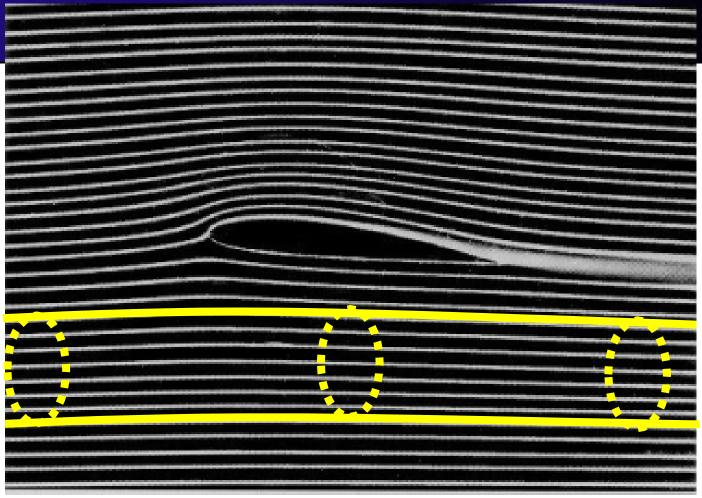
▲ Fig. 4.5

Fluid Motion

- Density and Pressure
- Hydrostatic Equilibrium and Pascal’s Law
- Archimedes' Principle and Buoyancy
- Fluid Dynamics
- Conservation of Mass: Continuity Equation
- Conservation of Energy: Bernoulli’s Equation
- Applications of Fluid Dynamics

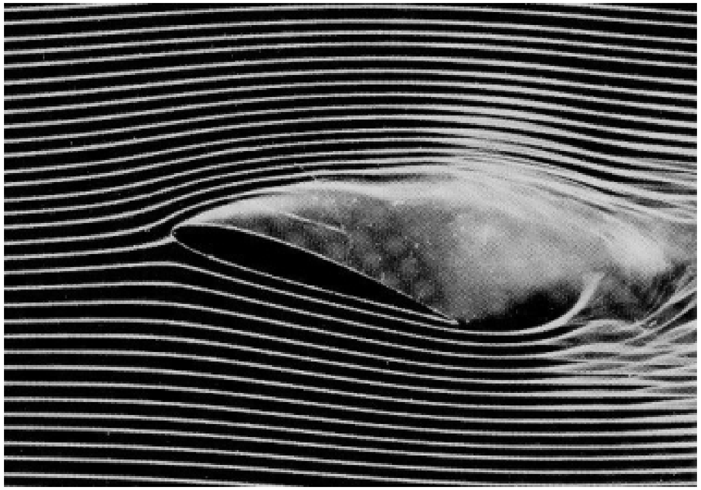
Fluid Dynamics

Laminar (steady) flow is where each particle in the fluid moves along a smooth path, and the paths do not cross.



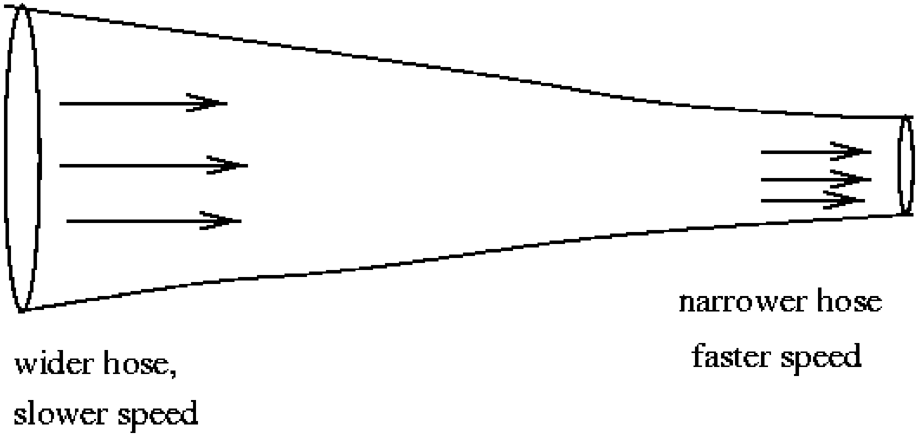
Streamlines spacing measures velocity and the flow is always tangential, for steady flow don't cross. A set of streamlines act as a pipe for an **incompressible** fluid

Non-viscous flow – no internal friction (water OK, honey not)



Turbulent flow above a critical speed, the paths become irregular, with whirlpools and paths crossing. Chaotic and **not considered here.**

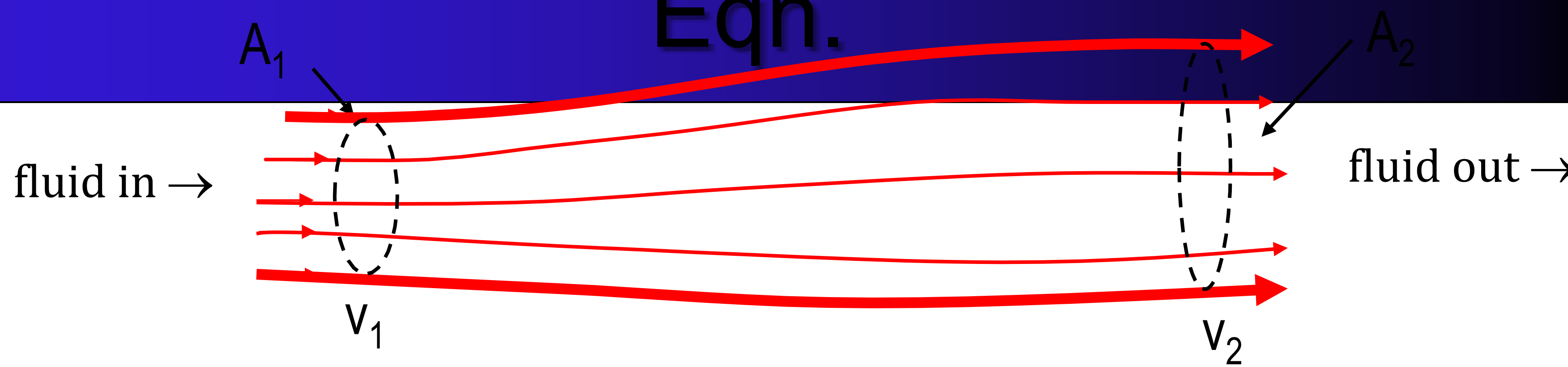
Conservation of Mass: The Continuity Eqn.



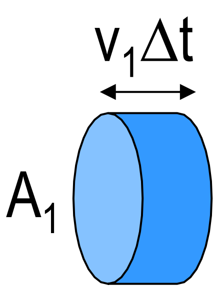
"The water all has to go somewhere"

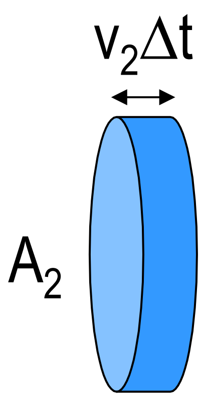
The rate a fluid enters a pipe must equal the rate the fluid leaves the pipe. i.e. There can be **no sources or sinks** of fluid.

Conservation of Mass: The Continuity Eqn.



Q. How much fluid flows across each area in a time Δt:





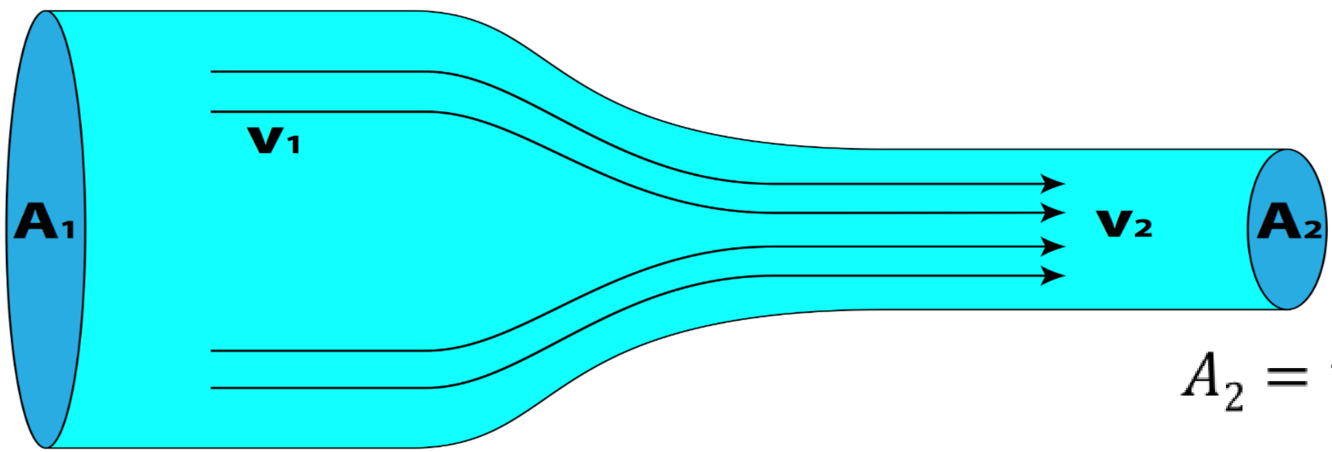
$$\Delta m = \rho V_1 = \rho A_1 v_1 \Delta t$$

$$\Delta m = \rho V_2 = \rho A_2 v_2 \Delta t$$

$$flow\ rate: \frac{\Delta m}{\Delta t} = \rho A v \qquad continuity\ eqn: \quad A_1 v_1 = A_2 v_2$$

Conservation of Mass: The Continuity Eqn.

Q. A river is 40m wide, 2.2m deep and flows at 4.5 m/s. It passes through a 3.7-m wide gorge, where the flow rate increases to 6.0 m/s. How deep is the gorge?



$A_2 = w_2 d_2$

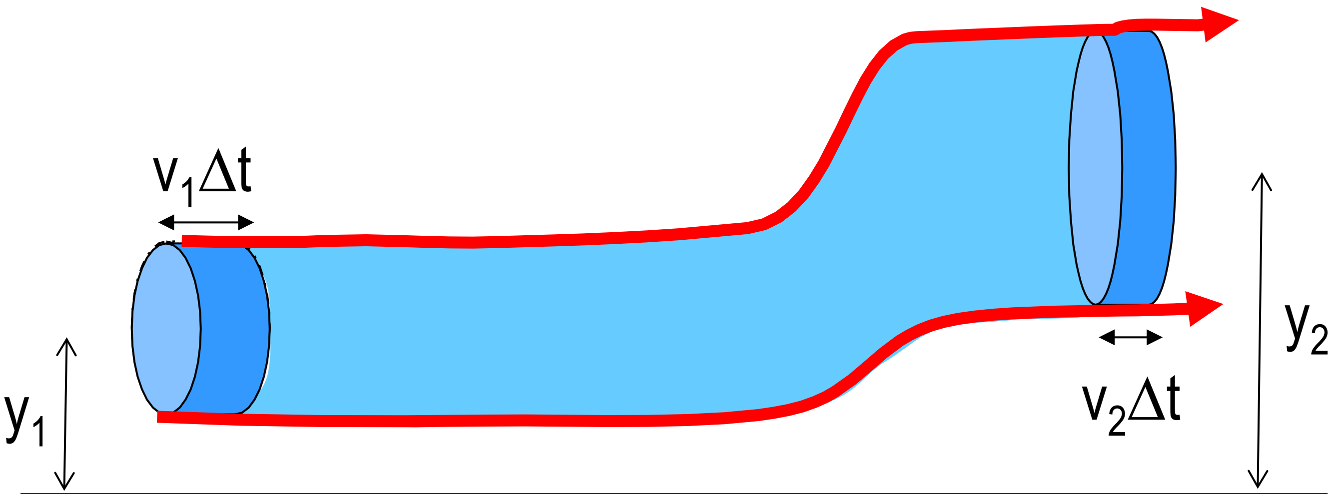
$A_1 = w_1 d_1$

$$Continuity\ equation: A_1 v_1 = A_2 v_2 \rightarrow w_1 d_1 v_1 = w_2 d_2 v_2$$

$$d = \frac{w_1 d_1 v_1}{\ } = \frac{40 \times 2.2 \times 4.5}{3.7 \times 6.0} = 18\ m$$

Conservation of Energy: Bernoulli's Eqn.

What happens to the energy density of the fluid if I raise the ends ?



Energy per unit volume

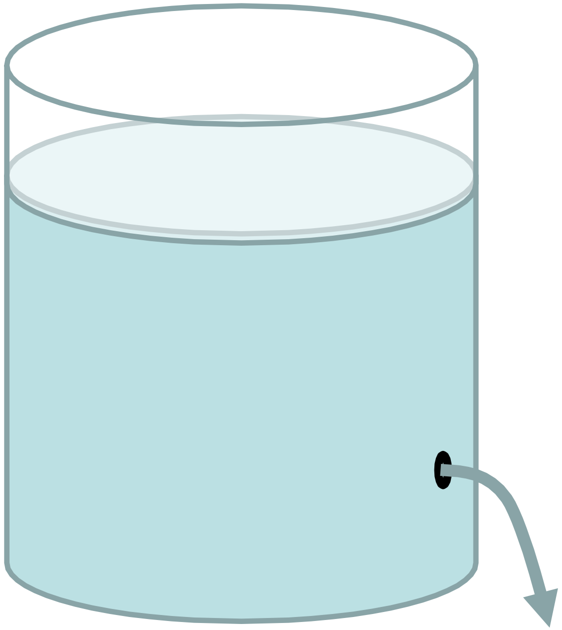
$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = const$$

Total energy per unit volume is constant at **any** point in fluid.

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = const$$

Conservation of Energy: Bernoulli's Eqn.

Q. Find the velocity of water leaving a tank through a hole in the side 1 metre below the water level.



$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

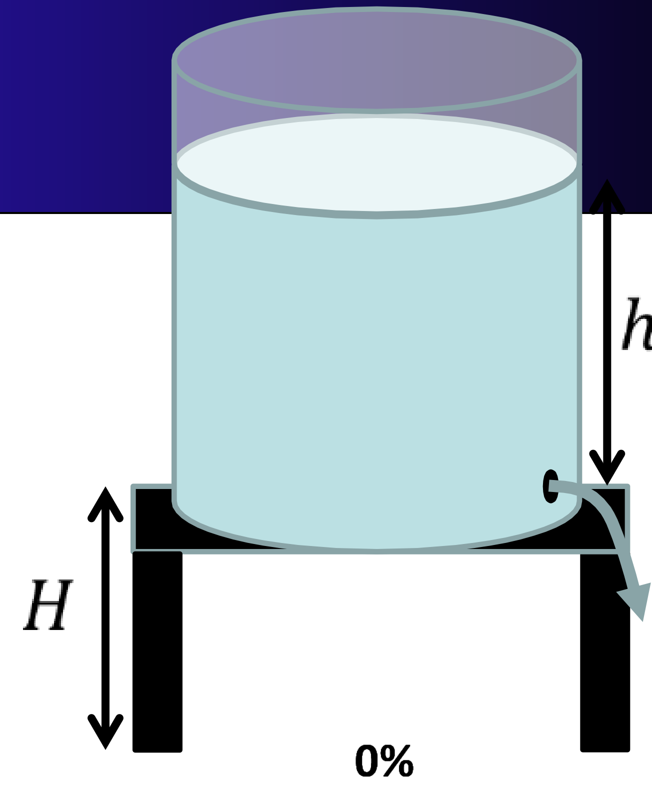
$$At\ the\ top:\ P = 1\ atm, v = 0, y = 1\ m$$

$$At\ the\ bottom:\ P = 1\ atm, v = ?, y = 0\ m$$

$$\not{P} + \rho g y = \not{P} + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gy} = \sqrt{2 \times 9.8 \times 1} = 4.4\ m/s$$

Which of the following can be done to increase the **flow rate** out of the water tank ?



1. Raise the tank ($\uparrow H$)
2. Reduce the hole size
3. Lower the water level ($\downarrow h$)
4. Raise the water level ($\uparrow h$)
5. None of the above

Summary: fluid dynamics

Continuity equation: mass is conserved!

$$\rho \times v \times A = constant$$

For liquids:

$$\rho = constant \rightarrow v \times A = constant$$

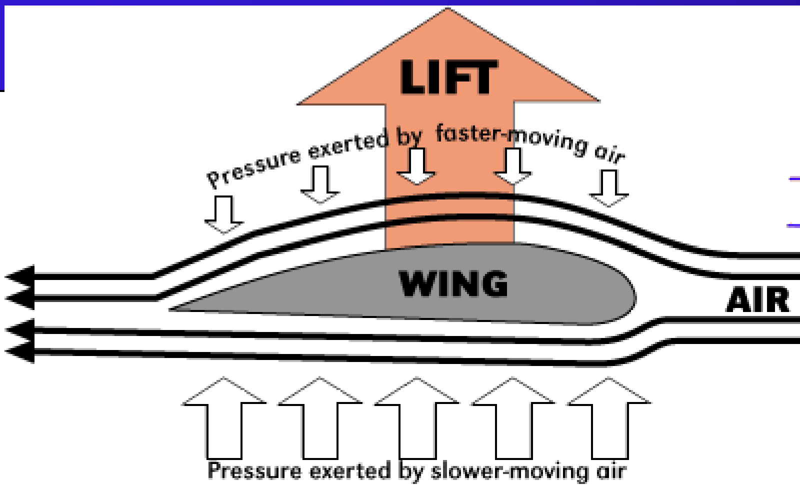
$$(Density\ \rho, velocity\ v, pipe\ area\ A)$$

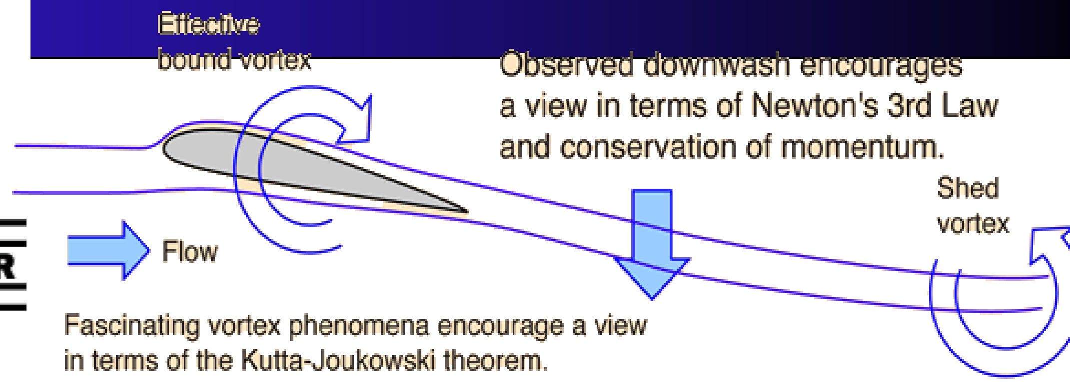
Bernoulli's equation: energy is conserved!

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

$$(Pressure\ P, density\ \rho, velocity\ v, height\ y)$$

Bernoulli's Effect and Lift





Newton's 3rd law
(air pushed downwards)

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

Lift on a wing is often explained in textbooks by Bernoulli's Principle: the air over the top of the wing moves faster than air over the bottom of the wing because it has further to move (?) so the pressure upwards on the bottom of the wing is smaller than the downwards pressure on the top of the wing.

Is that convincing? So why can a plane fly upside down?

Chapter 15 Fluid Motion Summary

- Density and Pressure describe bulk fluid behaviour
- Pressure in a fluid is the same for points at the same height
- In hydrostatic equilibrium, pressure increases with depth due to gravity
- The buoyant force is the weight of the displaced fluid
- Fluid flow conserves mass (continuity eq.) and energy (Bernoulli's equation)
- A constriction in flow is accompanied by a velocity **and** pressure change.
- Reread, Review and Reinforce concepts and techniques of Chapter 15

Examples 15.1 , 15.2 Calculating Pressure and Pascals Law
Examples 15.3 , 15.4 Buoyancy Forces: Working Underwater + Tip of Iceberg
Examples 15.5 Continuity Equation: Ausable Chasm
Examples 15.6, 15.7 Bernoulli's Equation – Draining a Tank and Venturi Flow

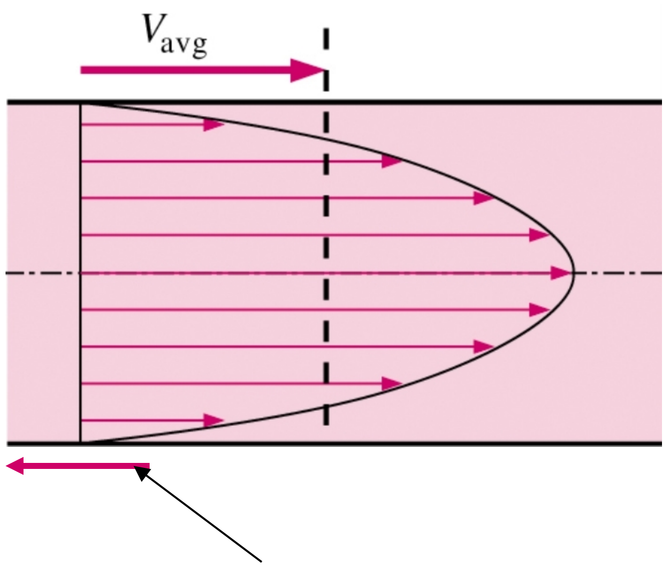
Friction Losses
Flow through Conduits

Incompressible Flow

Goals

- Calculate frictional losses for laminar and turbulent flow through circular and non-circular pipes
- Define the friction factor in terms of flow properties
- Calculate the friction factor for laminar and turbulent flow
- Define and calculate the Reynolds number for different flow situations
- Derive the Hagen-Poiseuille equation

Introduction

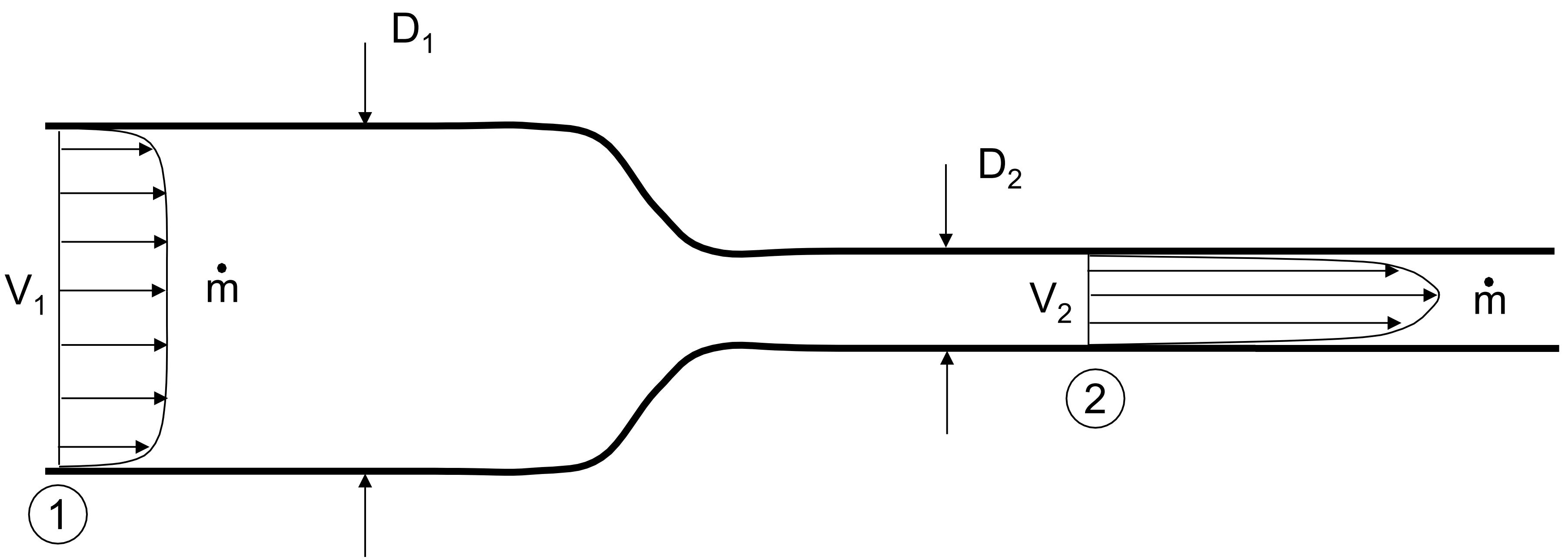


Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls

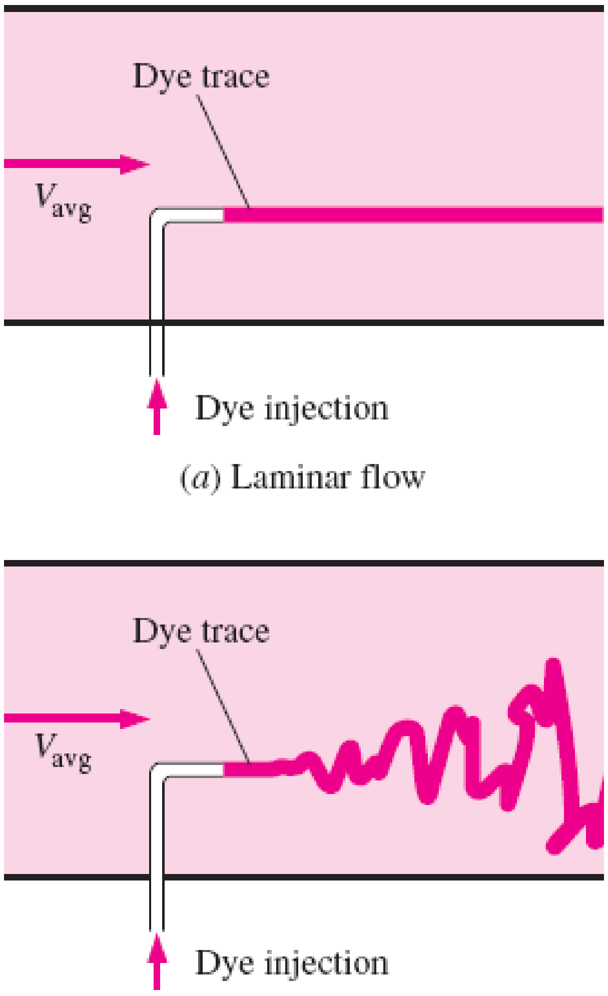
Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



LAMINAR AND TURBULENT FLOWS

- **Laminar flow:** characterized by *smooth streamlines* and *highly ordered motion.*
- **Turbulent flow:** characterized by *velocity fluctuations* and *highly disordered motion.*
- The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.



(*a*) Laminar flow

(*b*) Turbulent flow

Reynolds Number

- The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature,* and *type of fluid,* among other things.
- British engineer Osborne Reynolds (1842–1912) discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid.
- The ratio is called the **Reynolds number** and is expressed for internal flow in a circular pipe as

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}}D}{\nu} = \frac{\rho V_{\text{avg}}D}{\mu}$$

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Reynolds Number

- At large Reynolds numbers, the inertial forces are large relative to the viscous forces $\Rightarrow$ Turbulent Flow
- At *small* or *moderate* Reynolds numbers, the viscous forces are large enough to suppress these fluctuations $\Rightarrow$ Laminar Flow
- The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number,** Re_{cr}.
- The value of the critical Reynolds number is different for different geometries and flow conditions. For example, $Re_{cr} = 2300$ for internal flow in a circular pipe.

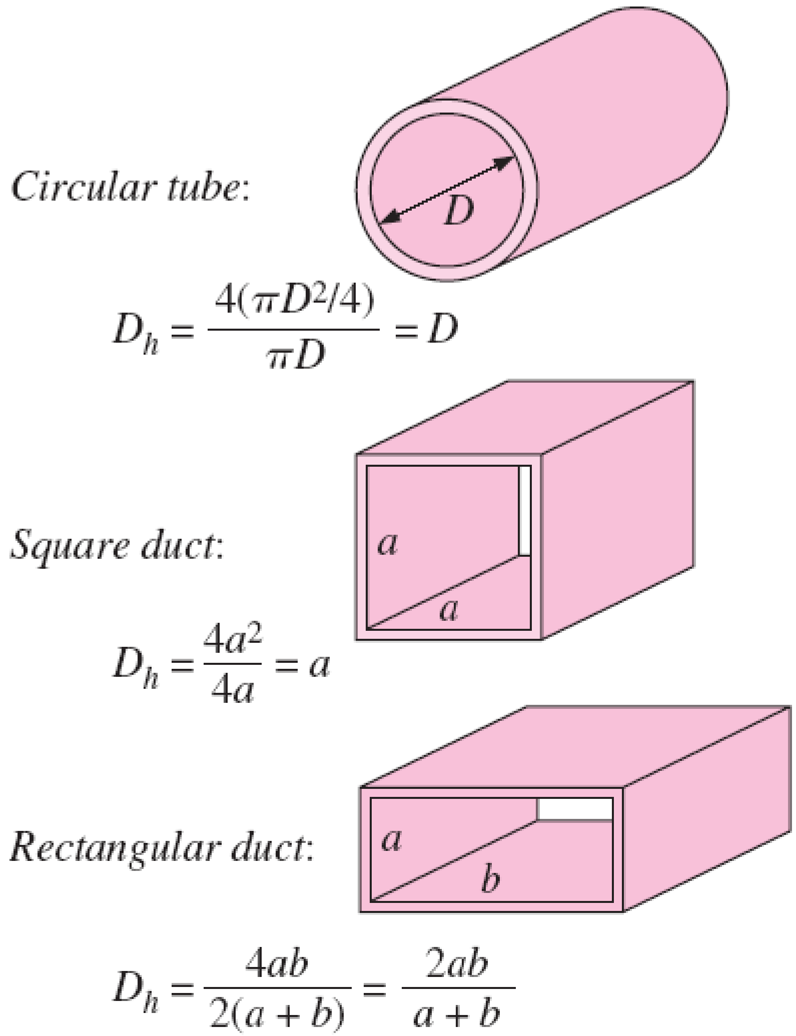
Reynolds Number

- For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter** D_h defined as

$$D_h = \frac{4A_c}{p}$$

A_c = cross-section area
P = wetted perimeter

- The transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and *fluctuations in the flow.*

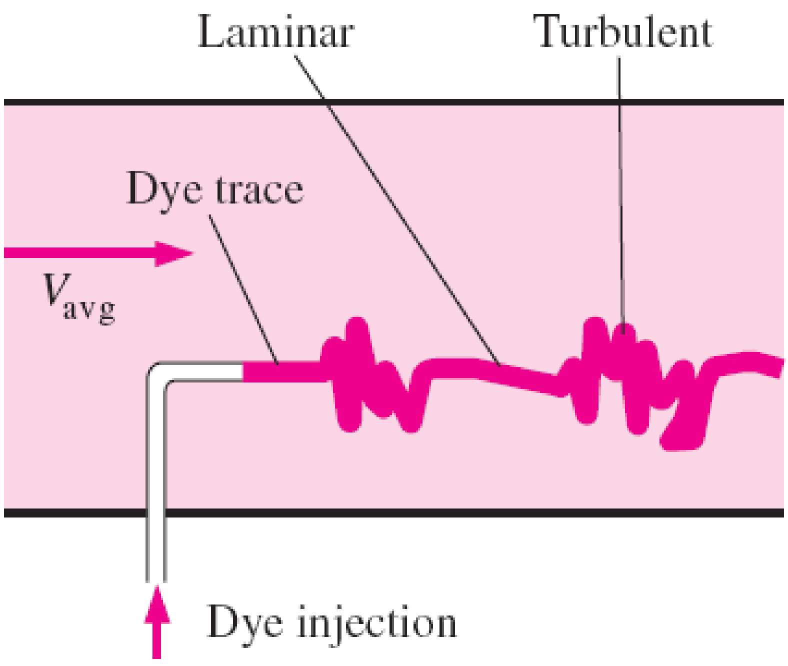


Reynolds Number

- Under most practical conditions, the flow in a circular pipe is

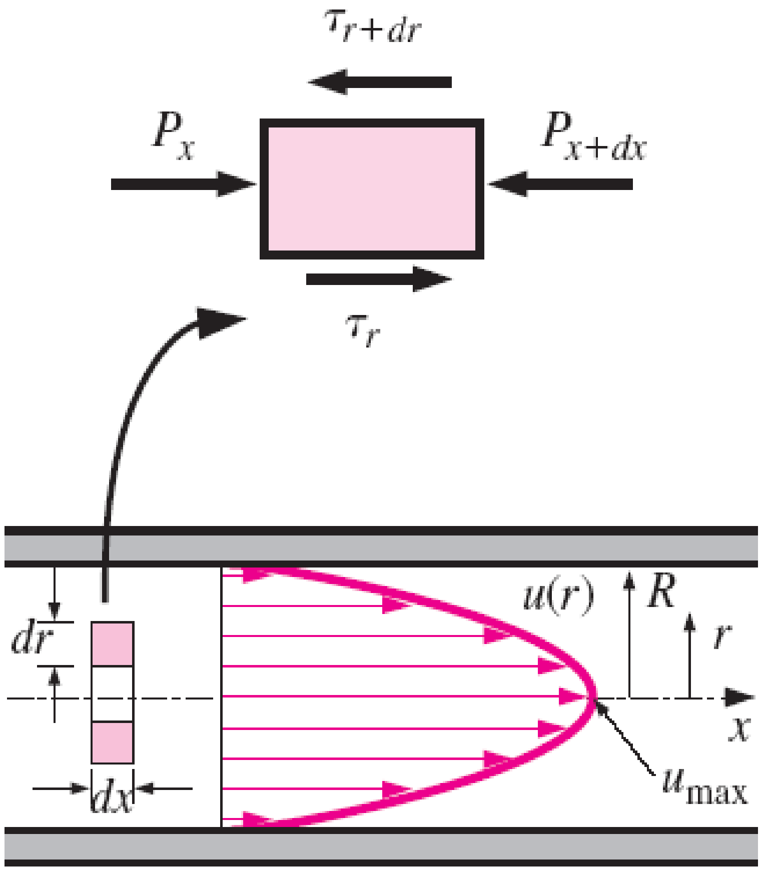
$$Re \lesssim 2300 \qquad \text{laminar flow}$$
$$2300 \lesssim Re \lesssim 4000 \qquad \text{transitional flow}$$
$$Re \gtrsim 4000 \qquad \text{turbulent flow}$$

- In transitional flow, the flow switches between laminar and turbulent randomly.



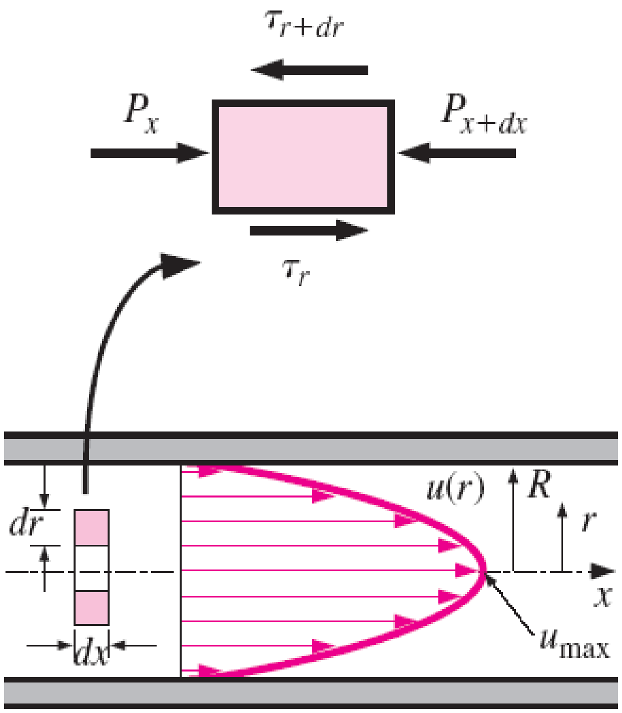
LAMINAR FLOW IN PIPES

- In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe.
- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and no motion in the radial direction such that no acceleration (since flow is steady and fully-developed).



LAMINAR FLOW IN PIPES

- Now consider a ring-shaped differential volume element of radius *r,* thickness *dr,* and length *dx* oriented coaxially with the pipe. A force balance on the volume element in the flow direction gives



$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

- Dividing by $2\pi drdx$ and rearranging,

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

LAMINAR FLOW IN PIPES

- Taking the limit as *dr, dx* → 0 gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

- Substituting $\tau = -\mu(du/dr)$ gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

- The left side of the equation is a function of *r*, and the right side is a function of *x*. The equality must hold for any value of *r* and *x*; therefore, $f(r) = g(x)$ = constant.

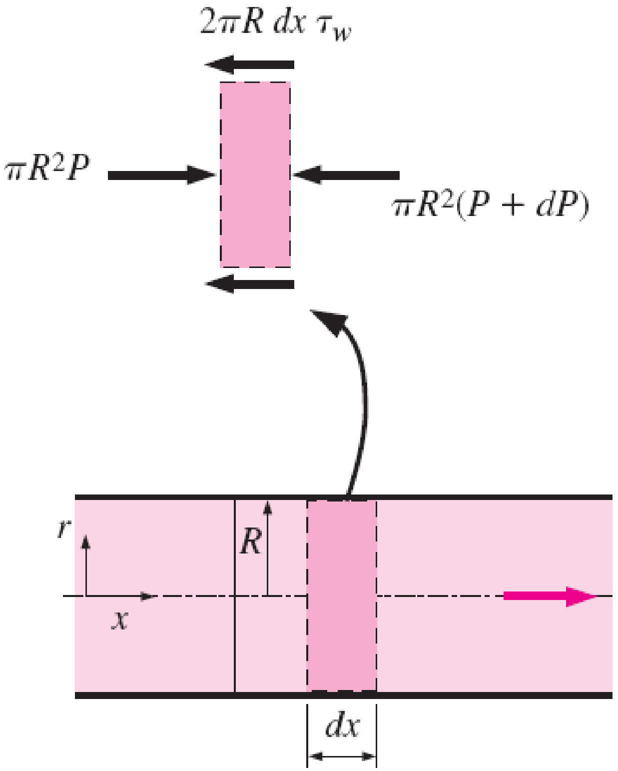
LAMINAR FLOW IN PIPES

- Thus we conclude that dP/dx = constant and we can verify that

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

- Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Then we solve the u(r) eq. by rearranging and integrating it twice to give

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

LAMINAR FLOW IN PIPES

- *Since* $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$, *then we can get* u(r)

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

- Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic.* Since u is positive for any r, and thus the dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).
- The average velocity is determined from

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

LAMINAR FLOW IN PIPES

- The velocity profile is rewritten as

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

- Thus we can get

$$u_{max} = 2V_{avg}$$

- Therefore, *the average velocity in fully developed laminar pipe flow is one half of the maximum velocity*.

Pressure Drop and Head Loss

- The *pressure drop* ΔP of pipe flow is related to the power requirements of the fan or pump to maintain flow. Since dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 give

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

- The pressure drop for laminar flow can be expressed as

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

- ΔP due to viscous effects represents an irreversible … lled **pressure loss** ΔP_L to

Pressure Drop and Head Loss

- In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L.

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

(Frictional losses due to viscosity)

Friction Losses

The resulting pressure (energy and head) losses are usually computed through the use of modified Fanning's friction factors:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}}$$

where F_k is the characteristic force, S is the friction surface area. This equation is general and it can be used for all flow processes.

Used for a pipe:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)\frac{D^2\pi}{4}}{(D\pi L)\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)D}{2L\rho v^2} = \frac{\Delta p}{L}\frac{D}{2\rho v^2}$$

where Fk is the press force, S is the area of curved surface. Rearranged, we get a form of pressure loss:

$$\Delta p_L = 4f\frac{L}{D}\frac{v^2\rho}{2} = \lambda\frac{L}{D}\frac{v^2\rho}{2} = \zeta\frac{v^2\rho}{2}$$

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Determination of Friction Factor with Dimensional Analysis

The Funning's friction factor is a function of Reynolds number, f = f(Re):

$$Re = \frac{vD}{\nu} = \frac{vD\rho}{\mu}$$

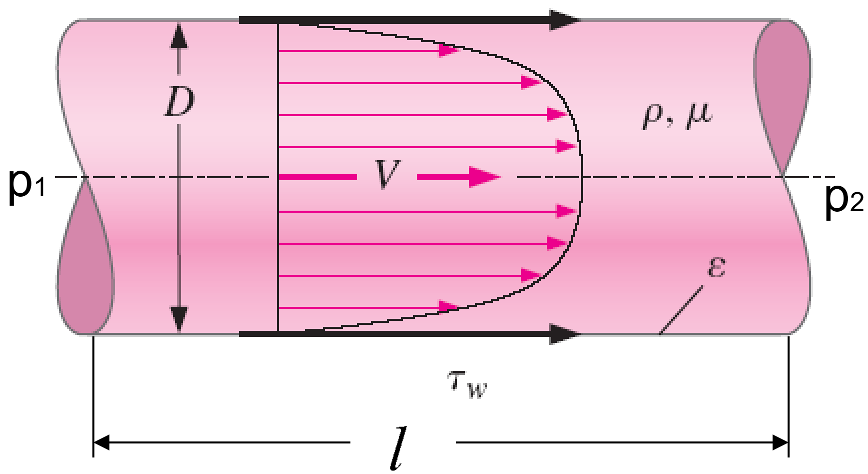
Many important chemical engineering problems cannot be solved completely by theoretical methods. For example, the pressure loss from friction losses in a long, round, straight, smooth pipe depends on all these variables: the length and diameter of pipe, the flow rate of the liquid, and the density and viscosity of the liquid.

If any one of these variables is changed, the pressure drop also changes. The empirical method of obtaining an equation relating these factors to pressure drop requires that the effect of each separate variable be determine in turn by systematically varying that variable while keeping all others constant.

It is possible to group many factors into a smaller number of dimensionless groups of variables. The groups themselves rather than separate factors appear in the final equation. These method is called dimensional analysis, which is an algebric treatment of the symbols for units considered

Determination of Pressure Difference by Dimensional

Many important chemical engineering problems cannot be solved completely by theoretical methods. For example, the pressure loss from friction losses (or the pressure difference $\Delta p = p_1 - p_2$ between two ends of a pipe) in a long, round, straight, smooth pipe a fluid is flowing depends on all these variables: pipe diameter d, pipe length l, fluid velocity v, fluid density ρ, and fluid viscosity μ.



Fluid Flow in Pipes

Goals: determination of friction losses of fluids in pipes or ducts, and of pumping power requirement.

The resulting pressure (energy and head) loss

$$\Delta p_L = (z_1 - z_2)\rho g + (p_1 - p_2) + \frac{(v_1^2 - v_2^2)\rho}{2}$$

is usually computed through the use of the modified Fanning friction factor:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}}$$

Used for a pipe:

$$f = \frac{F_k}{S\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)\frac{D^2\pi}{4}}{(D\pi L)\rho \frac{v^2}{2}} = \frac{(p_1 - p_2)D}{2L\rho v^2} = \frac{\Delta p}{L}\frac{D}{2\rho v^2}$$

where F_k is the press force, S is the area of curved surface. Rearranged, we get a form of pressure loss:

$$\Delta p_L = 4f\frac{L}{D}\frac{v^2\rho}{2} = \lambda\frac{L}{D}\frac{v^2\rho}{2} = \zeta\frac{v^2\rho}{2}$$

The Funning's friction factor is a function of Reynolds number, f = f(Re):

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The Funning's friction factor is a function of Reynolds number, f = f(Re):

$$Re = \frac{vD}{\nu} = \frac{vD\rho}{\mu}$$

Calculation of Pumping Power Requirement

The friction factors were determined with dimensional analysis for a smooth pipe :

laminar	$f = \frac{16}{Re}$	$Re < 2100$
turbulent	$f = 0.0791 Re^{-1/4}$	$4000 < Re < 10^5$
turbulent	$\frac{1}{\sqrt{f}} = 1.7372 \ln\left(Re\sqrt{f}\right) - 0.3946$	$4000 < Re \geq 10^7$

The pressure loss is directly calculated from Hagen-Poiseuille's equation for laminar flow:

$$\Delta p_L = \frac{32 \mu L v}{D^2} = \frac{32 \mu L v}{D^2}\left(\frac{2 \rho v}{2 \rho v}\right) = 4 \frac{16}{Re} \frac{L}{D} \frac{v^2 \rho}{2}$$

When the fluid flows in a duct which is not circle in cross-section then we have to use the hydraulic diameter, D_h:

$$D_h = 4 \frac{A_c}{P} = 4 \frac{(\text{cross} - \text{section area})}{(\text{wetted perimeter})}$$

The pumping power requirement (pump power equation):

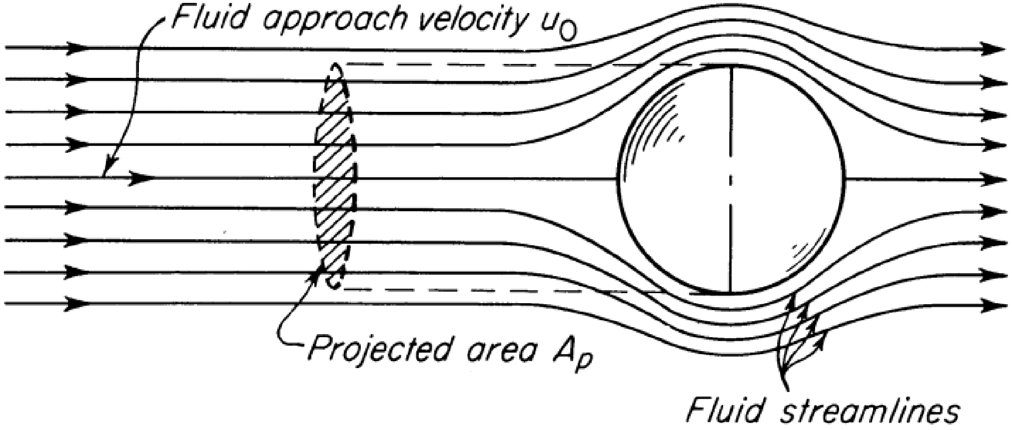
$$P = \frac{1}{\eta} \dot{V} \Delta p_{pump} = \frac{1}{\eta} \dot{V}\left(\Delta p_L + \Delta p_h + \Delta p_{pres}\right) = \frac{1}{\eta} \dot{V}\left[\left(1 + 4f \frac{L + \Sigma L_{eq.}}{D}\right)\frac{v^2 \rho}{2} + \left(z_2 - z_1\right)\rho g + p_2 - p_1\right]$$

Where P is the power (Watt), V is the quantity of flow (m3/s), L_{eq} is the equivalent pipe length of fittings, η is the efficiency of the pump.

6.2. Motion of Particles in Fluids.
Flow Around Objects

There are many processes that involve the motion of particles in fluids, or flow around objects:

- Sedimentation
- Liquid Mixing
- Food Industry
- Oil Reservoirs



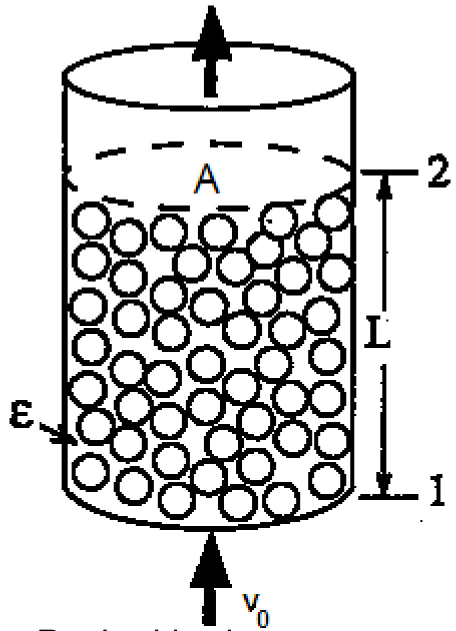
Flow around objects

7.1. Flow through Porous Media or Packed Bed

In many engineering systems, beds or packed columns, fluidization, filtration, are used in various processes.

A typical packed bed is a cylindrical column that is filled with suitable spheres or other non-spherical packing material. Fluid flows between the particles in small diameter tortuous, winding channels.

	Fluid	Solids
fraction	ε	$(1-\varepsilon)$
volume	$\varepsilon(AL)$	$(1-\varepsilon)(AL)$
mass	$\varepsilon(AL)\rho_f$	$(1-\varepsilon)(AL)\rho_p$



Packed bed

Friction Coefficient for Packed Bed

Definition of Reynolds number for packed bed:

$$\mathrm{Re_p} = \frac{v_\varepsilon D_h \rho_f}{\mu} = \frac{v_0}{\varepsilon}\frac{D_h \rho_f}{\mu} = \frac{v_0}{\varepsilon}\frac{2}{3}\frac{\varepsilon D}{(1-\varepsilon)}\frac{\rho_f}{\mu} = \frac{2}{3}\frac{1}{(1-\varepsilon)}\frac{v_0 D \rho_f}{\mu}$$

$f_p = f_p(\mathrm{Re}_p)$, the results have been correlated in equations of form:

$$\text{laminar} \qquad f_p = \frac{150}{\mathrm{Re_p}} \qquad \mathrm{Re_p} < 10 \qquad \text{(Blake – Kozeny's eq.)}$$

$$\text{transitional} \qquad f_p = \frac{150}{\mathrm{Re_p}} + \frac{7}{4} \qquad 10 < \mathrm{Re_p} < 1000 \qquad \text{(Ergun's eq.)}$$

$$\text{turbulent} \qquad f_p = \frac{7}{4} \qquad \mathrm{Re_p} > 1000 \qquad \text{(Burke – Plummer's eq.)}$$

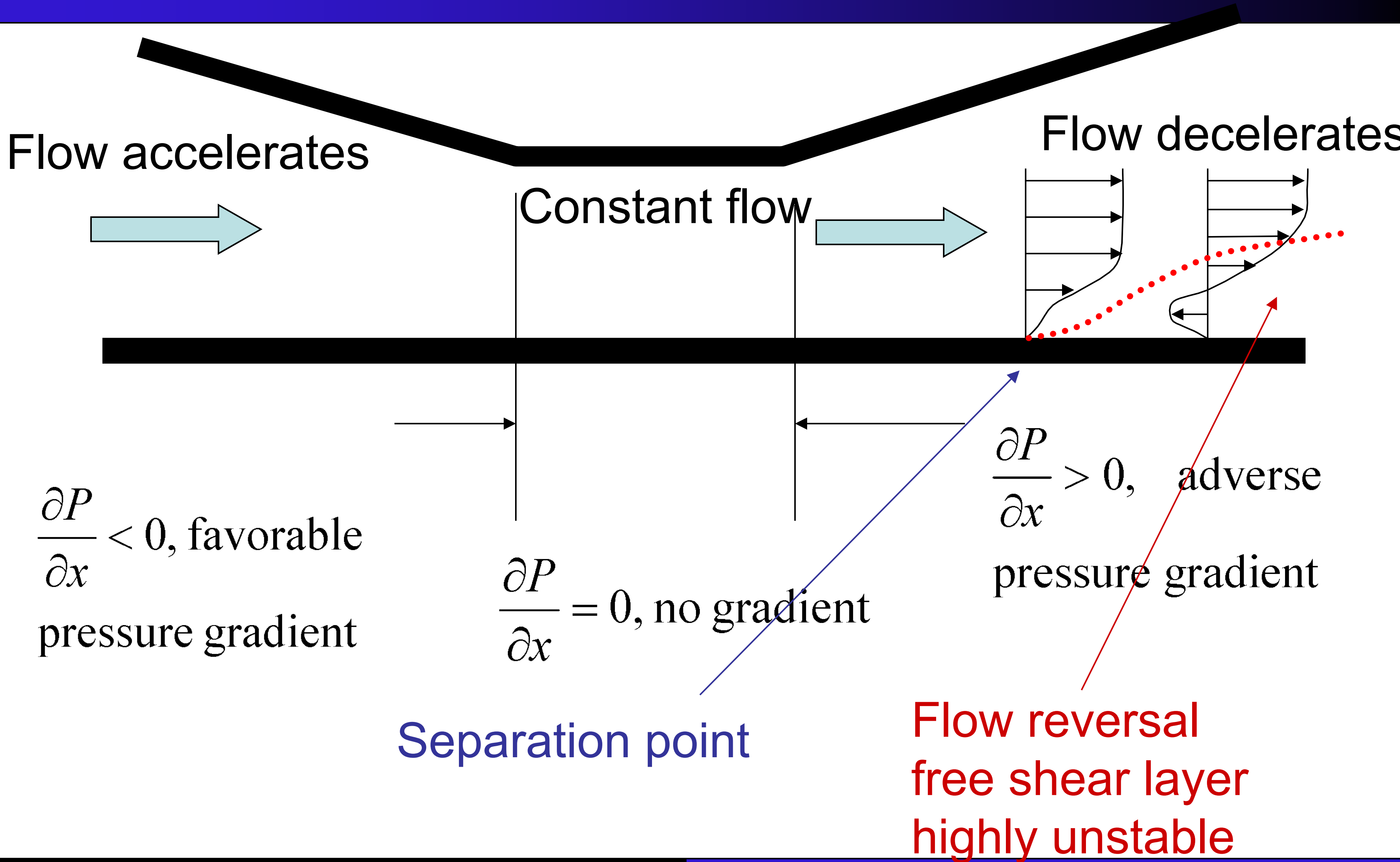
The Ergun's equation predicts the pressure drop (or flow) through porous media or packed columns quite well.

Pressure drop:

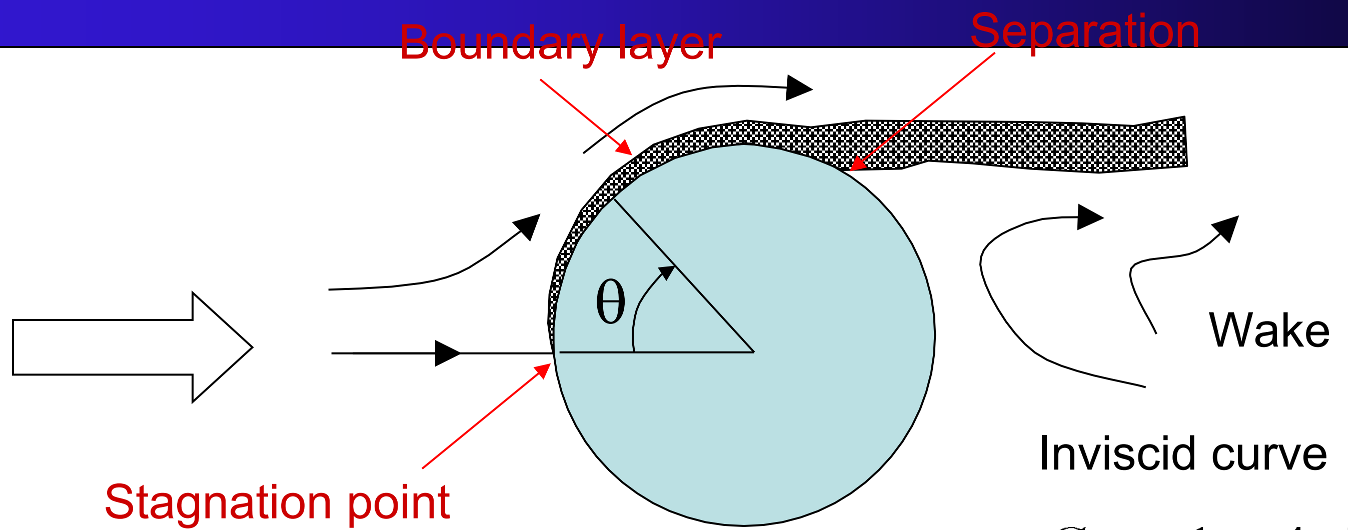
$$\Delta p = f_p \frac{L \rho_f}{D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right) v_0^2$$

Unit-IV : BOUNDARY LAYER

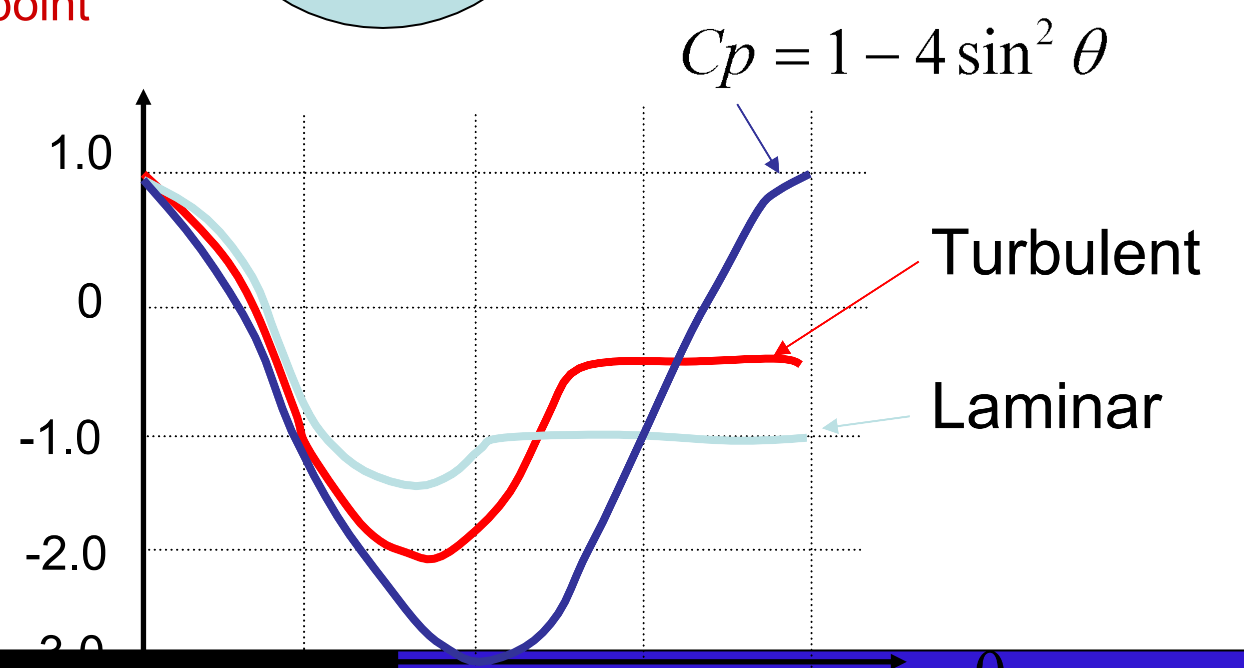
UNIT-IV
Boundary Layer and separation



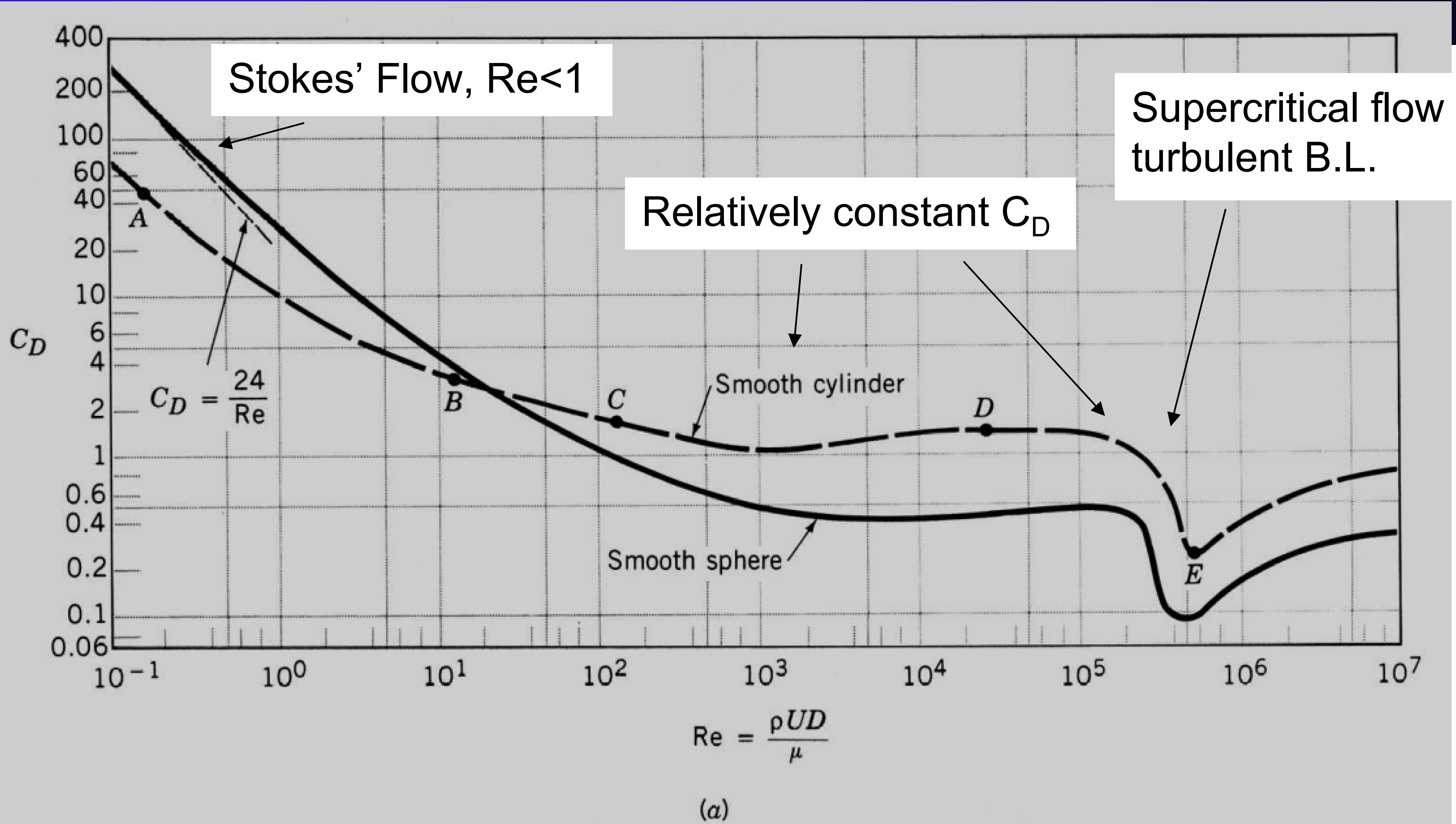
Flow Separation



$$Cp = \frac{P - P_\infty}{1/2\,\rho U_\infty^2}$$

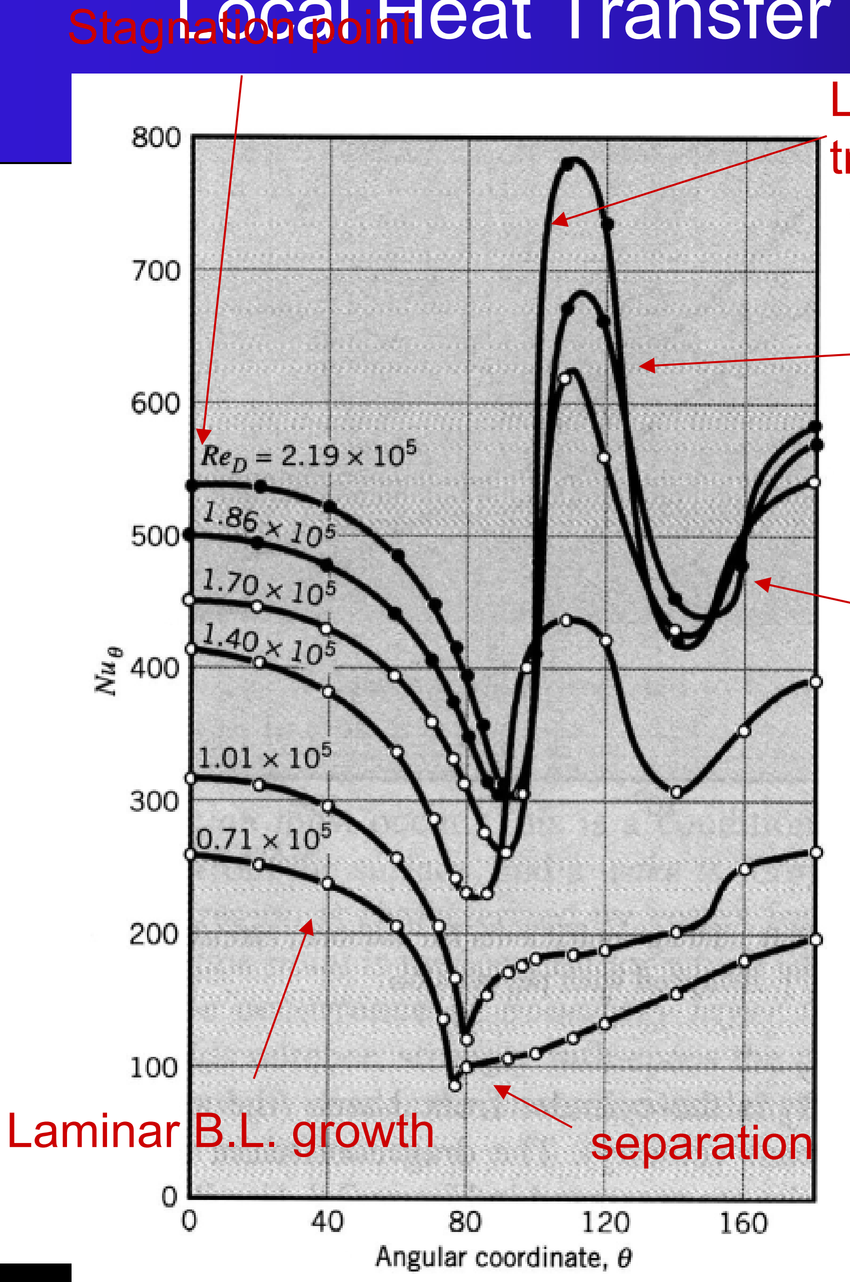


Drag Coefficient: C_D

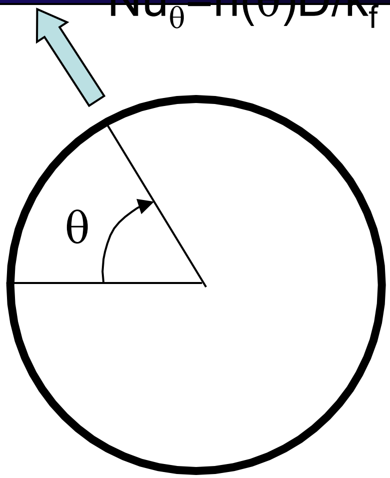


■ **FIGURE 9.23** (*a*) **Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.**

Local Heat Transfer Distribution



$q_\theta'' = h(\theta)(T_s - T_\infty)$,
$Nu_\theta = h(\theta)D/k_f$



Local Nusselt number for airflow normal to a circular cylinder.
(figure 10-22 from the ITHT text)

Averaged Nusselt Number Correlations of Cylinders in Cross Flows

Note 1: averaged Nusselt number correlations for the circular cylinder flows can be found in chapter 10-5. Correlations for other noncircular cylinders in cross flow can also be found in this chapter (see Table 10-3).

Note 2: Heat transfer between a tube bank (tube bundle) and cross flow is given in many HT textbooks (for example: see chapter 7 of "Introduction to Heat Transfer" by Incropera & DeWitt. The configuration is important for many practical applications, for example, the multiple pass heat exchanger in a condenser unit. The use of tube bank can not only save the operating space but also can enhance heat transfer. The wake flows behind each row of tubes are highly turbulent and can greatly enhance the convective heat transfer. In general, one can find an averaged convection coefficient using empirical correlation.

Note 3: Because of its compactness, pressure drop across a tube bank can be also significant and warrants careful design consideration.

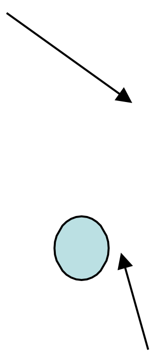
Example

A hot-wire anemometer is a flow device used to measure flow velocity based on the principle of convective heat transfer. Electric current is passing through a thin cylindrical wire to heat it up to a high temperature, that is why it is called "hot-wire". Heat is dissipated to the fluid flowing the wire by convection heat transfer such that the wire can be maintained at a constant temperature. Determine the velocity of the airstream (it is known to be higher than 40 m/s and has a temperature of 25°C), if a wire of 0.02 mm diameter achieved a constant temperature of 150°C while dissipating 50 W per meter of electric energy.

25°C, U>40 m/s

Hot-wire, 0.02 mm dia.

Air (87.5°C), Pr=0.707, $\nu=15\times10^{-6}$ m^2/s, k=0.026 W/m.K



Constant temperature 150°C

Example (cont.)

$$q = \bar{h}A(T_S - T_\infty)$$

$$50 = \bar{h}(\pi DL)(150 - 25)$$

$$\bar{h} = \frac{50}{\pi(0.02\times10^{-3})(1)(125)} = 6369(W/m^2.K)$$

$$Nu = \frac{\bar{h}D}{k} = \frac{6369(0.00002)}{0.026} = 4.90$$

$$\mathrm{Re} > \frac{\mathrm{VD}}{\nu} = \frac{(40)(0.00002)}{15\times10^{-6}} = 53,$$

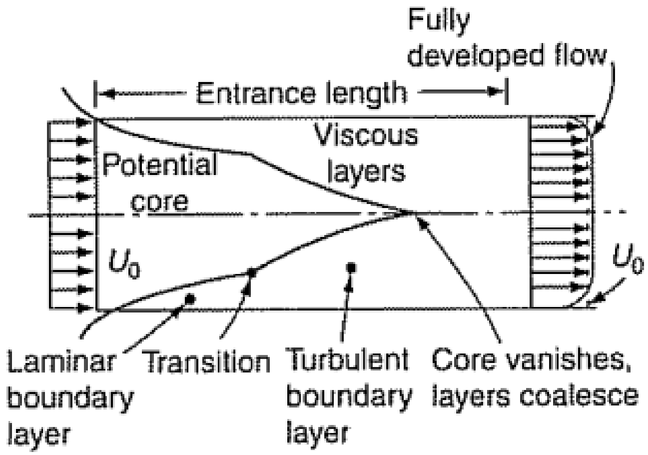
assume 4000 > Re > 40, use equation (10-37)

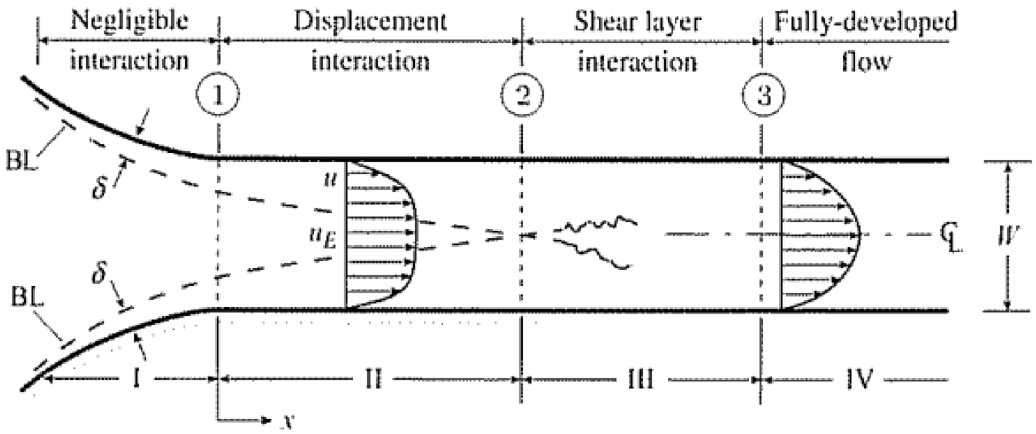
$$Nu = (0.683)\,\mathrm{Re}^{0.466}\ \mathrm{Pr}^{1/3}$$

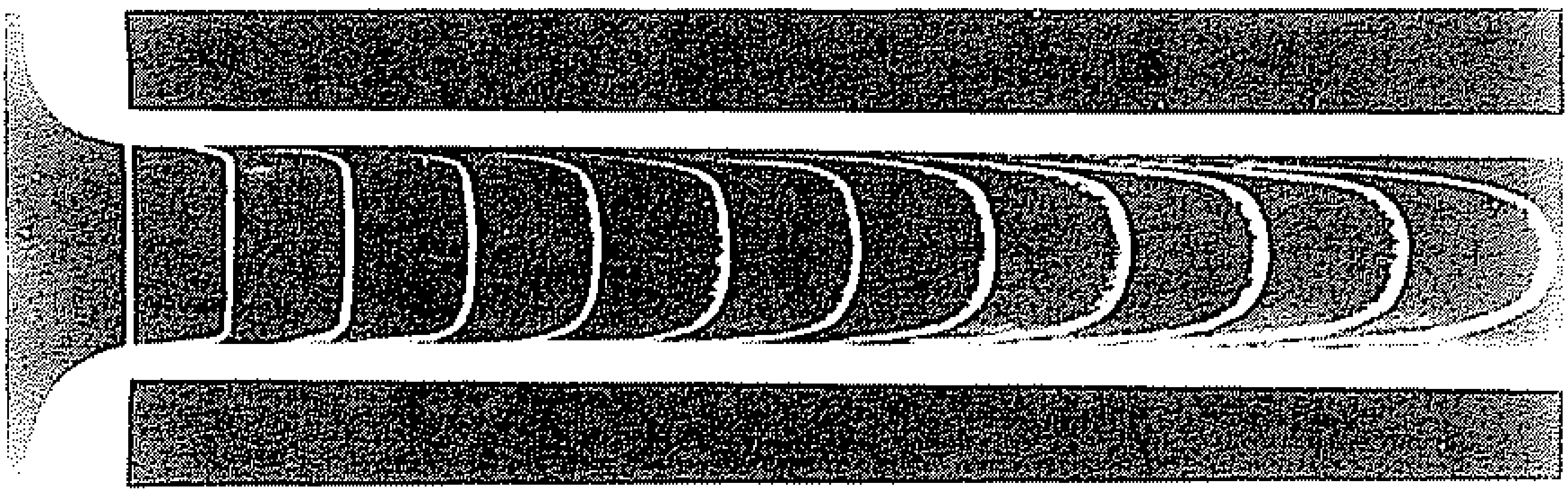
$$4.90 = (0.683)\left(VD/\nu\right)^{0.466}(0.707)^{1/3}$$

$V = 65.9(m/s)$, Re $= 87.9$ satisfy the range of validity

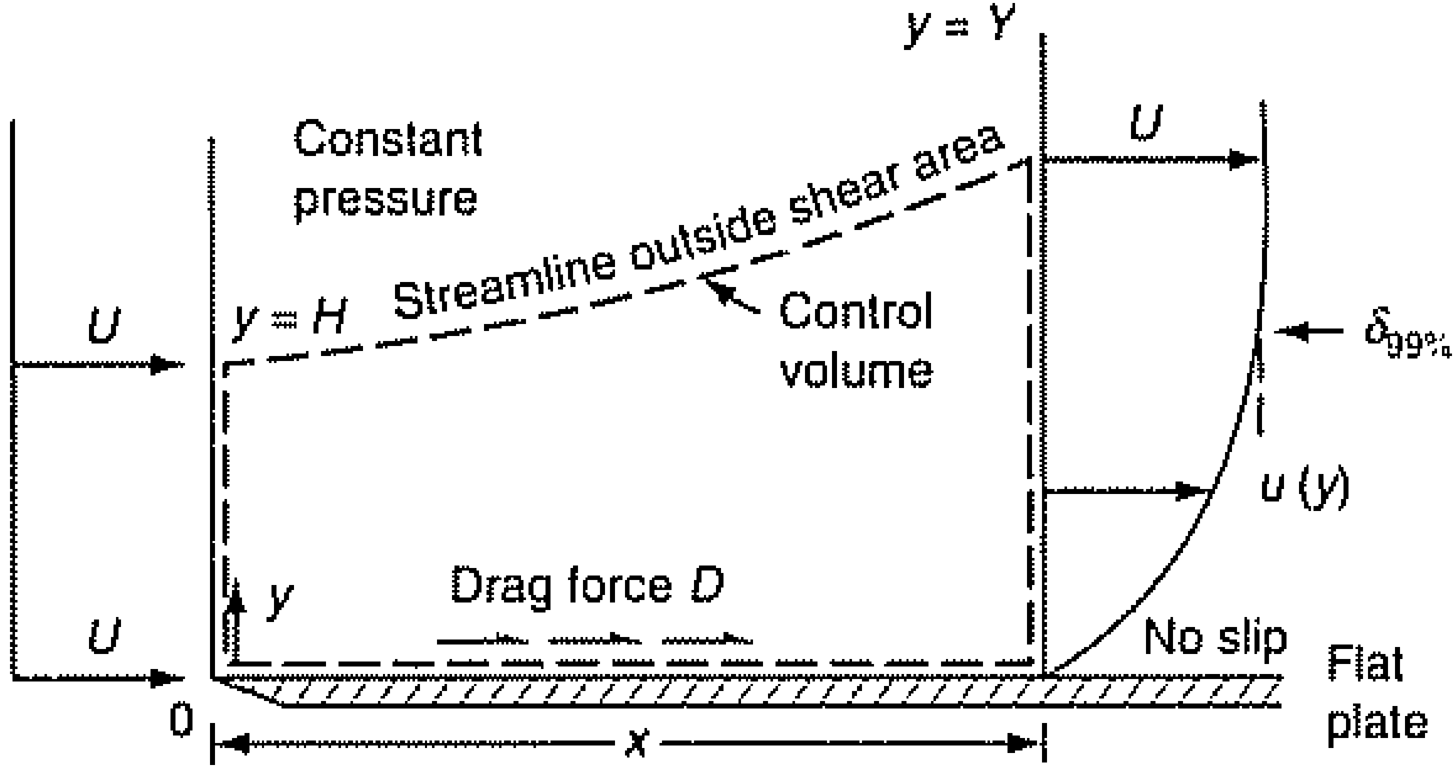
EFFECTS OF VISCOUS FORCES ON FLOW REGIMES IN A CHANNEL





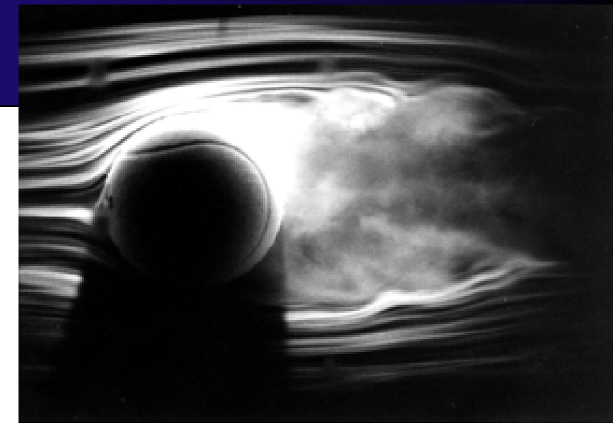


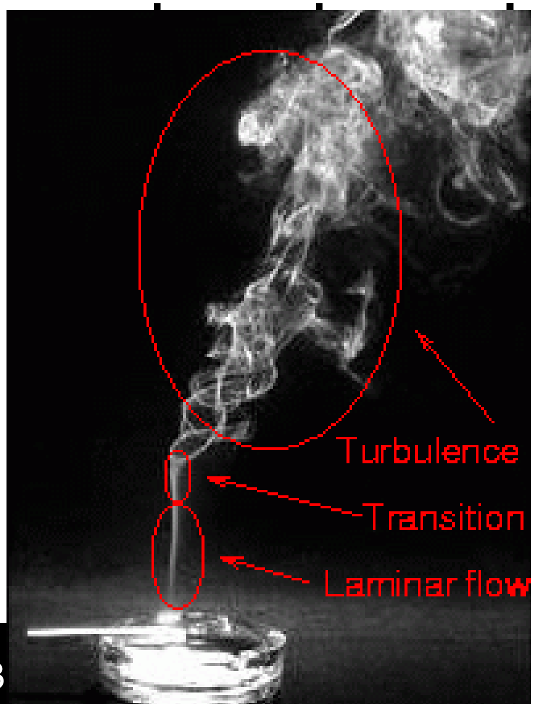
FLAT PLATE ANALYSIS



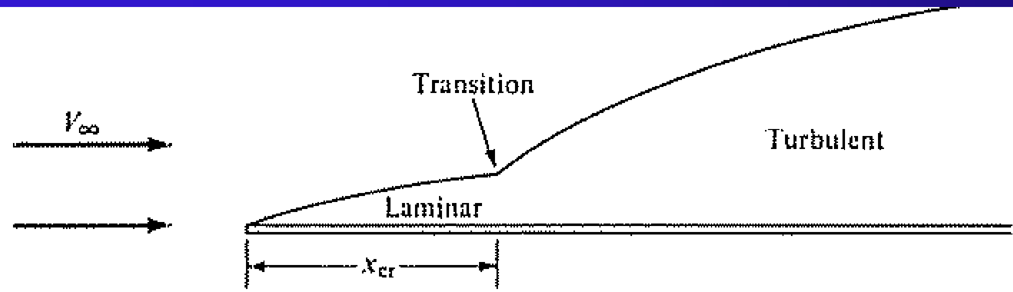
LAMINAR VERSUS TURBULENT FLOW

- Two types of viscous flows
  - Laminar: streamlines are smooth and reglar and a fluid element moves smoothly along a streamline
  - Turbulent: streamlines break up and fluid elements move in a ...ar, and chaotic





LAMINAR VERSUS TURBULENT FLOW



All B.L.'s transition from laminar to turbulent

Turbulent velocity profiles are 'fuller'

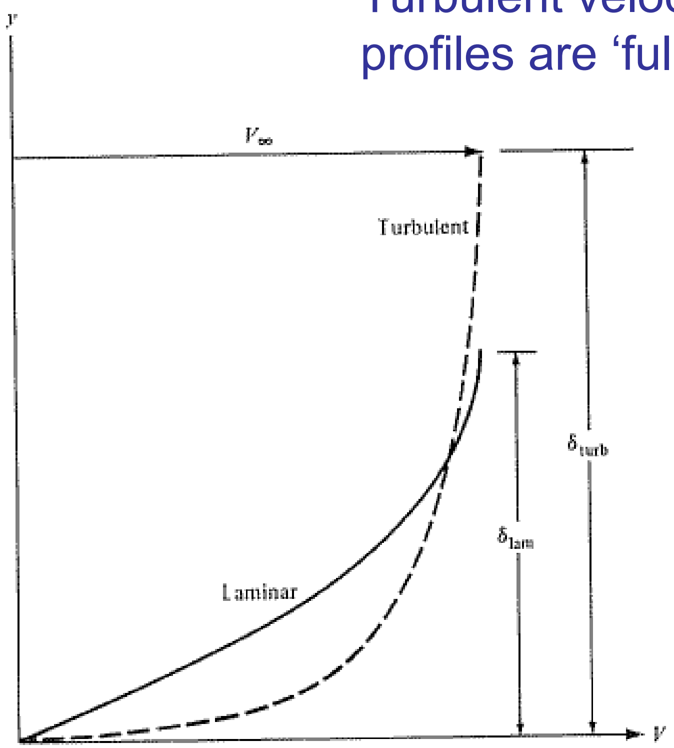
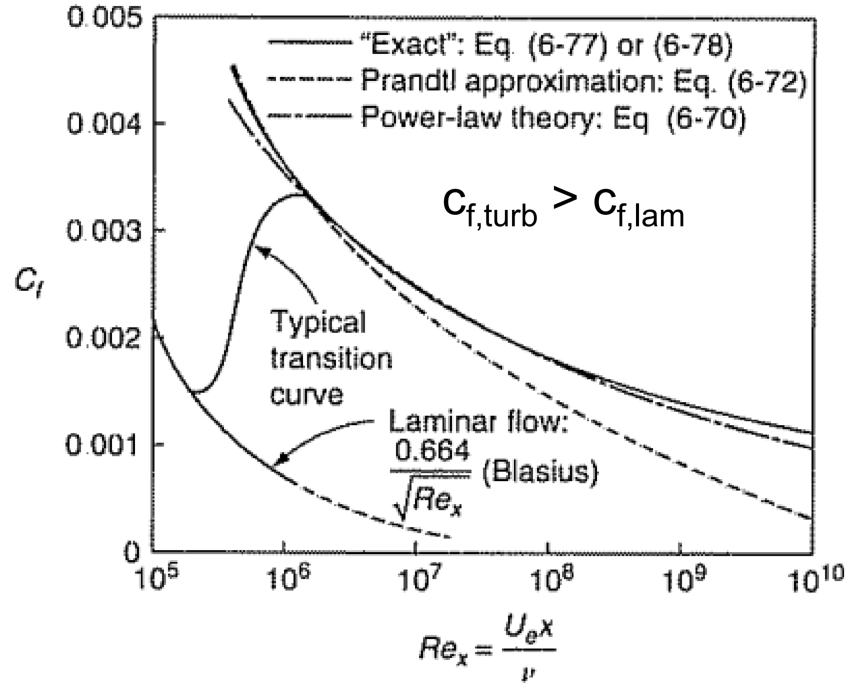
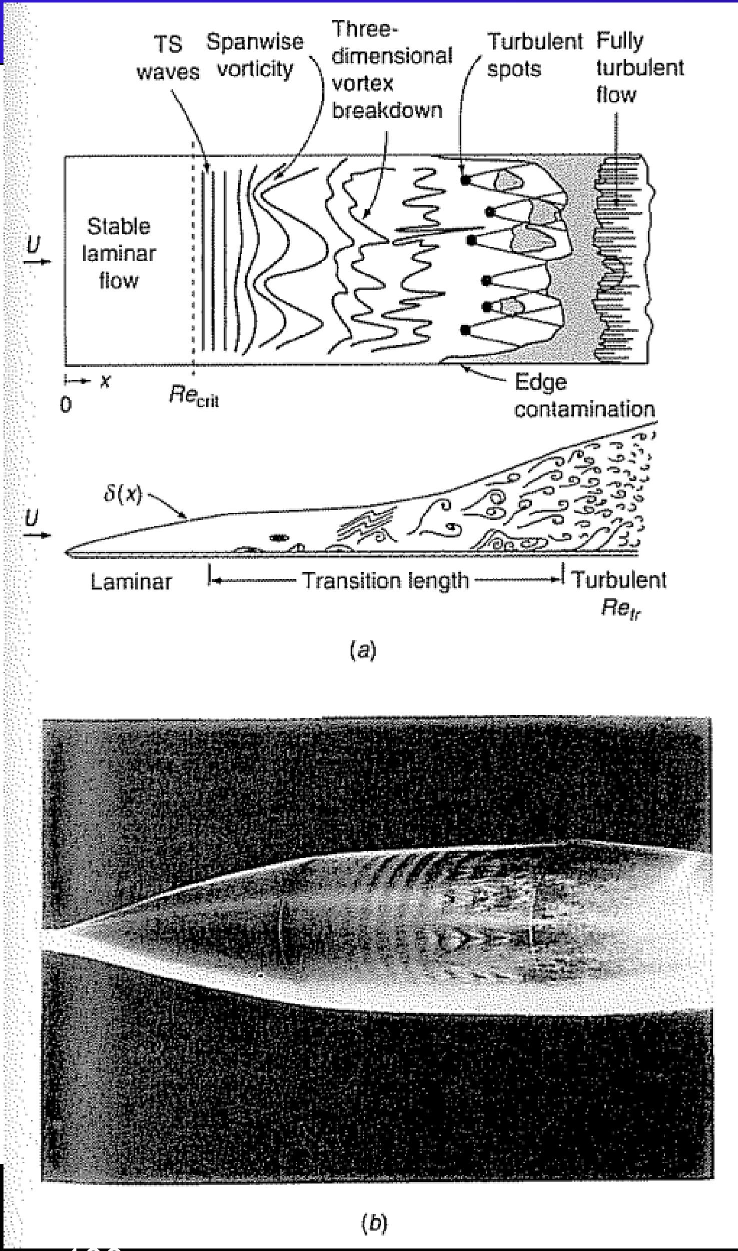


Figure 4.37 Velocity profiles for laminar and turbulent boundary layers. Note that the turbulent boundary layer thickness is larger than the laminar boundary layer thickness



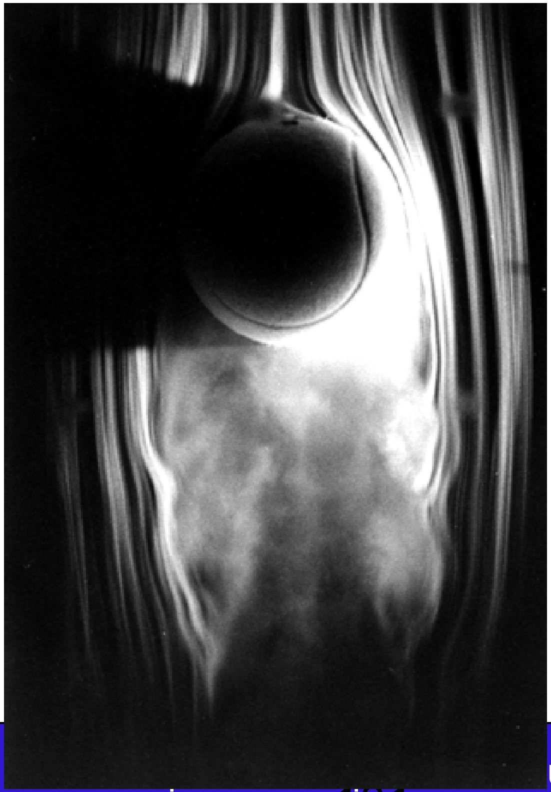
LAMINAR TO TURBULENT TRANSITION



1. Stable laminar flow near leading edge
2. Unstable 2D Tollmien-Schlichting waves
3. Development of 3D unstable waves and 'hairpin' eddies
4. Vortex breakdown at regions of high localized shear
5. Cascading vortex breakdown into fully 3D fluctuations
6. Formation of turbulent spots at locally intense fluctuations
7. Coalescence of spots into fully turbulent flow

EXAMPLE: FLOW SEPARATION

- Key to understanding: Friction causes flow separation within boundary layer
- Separation then creates another form of drag called pressure drag due to separation



RELEVANCE OF FRICTION ON AN AIRFOIL

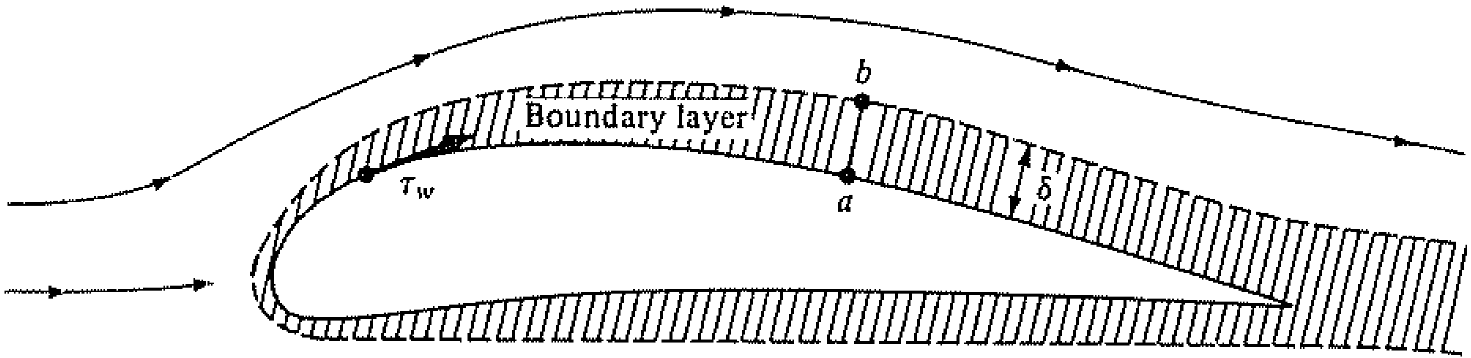
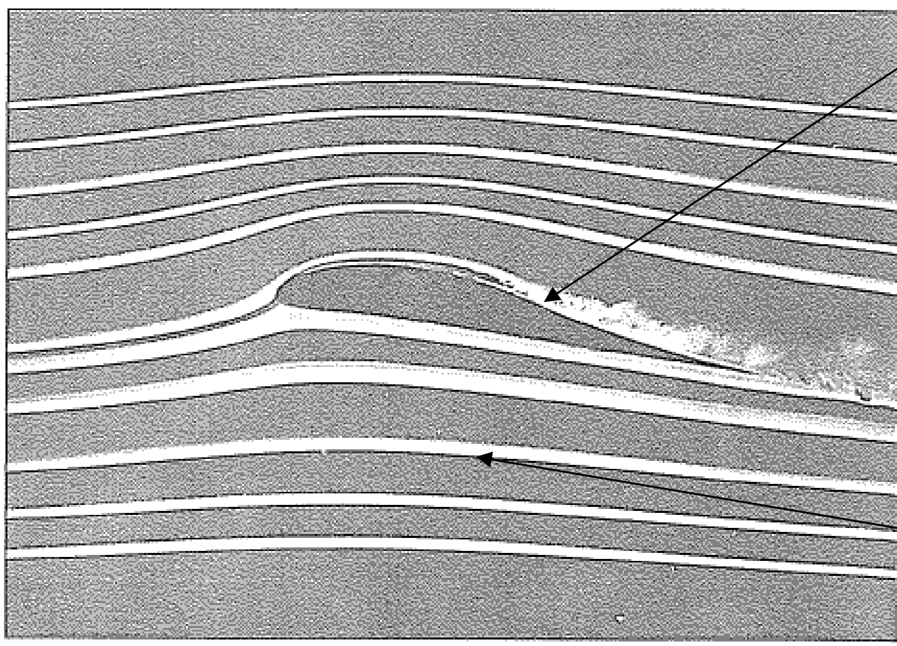


Figure 4.32 Flow in real life, with friction. The thickness of the boundary layer is greatly overemphasized for clarity



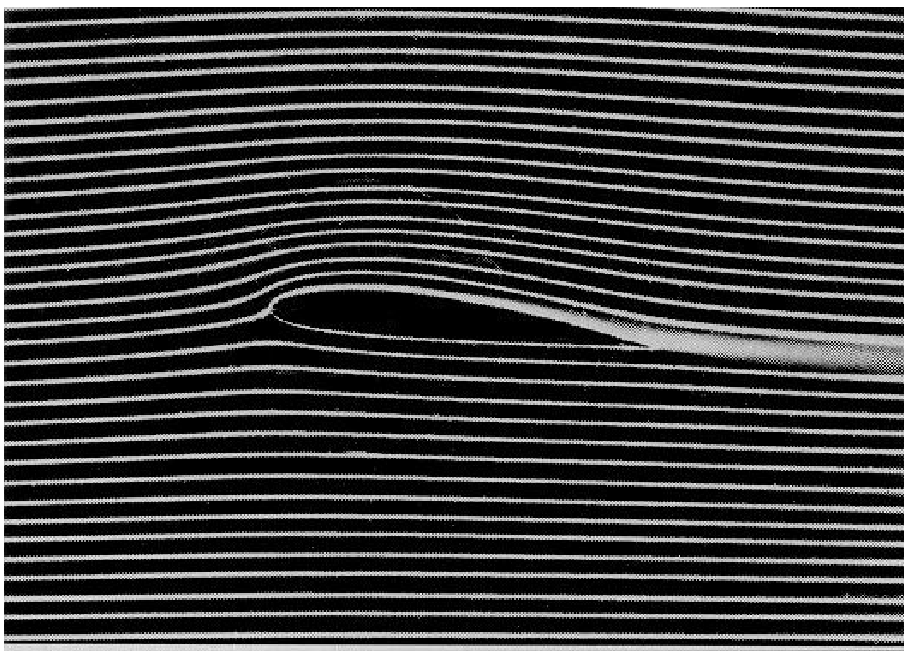
Flow very close to surface of airfoil is Influenced by friction and is viscous (boundary layer flow)

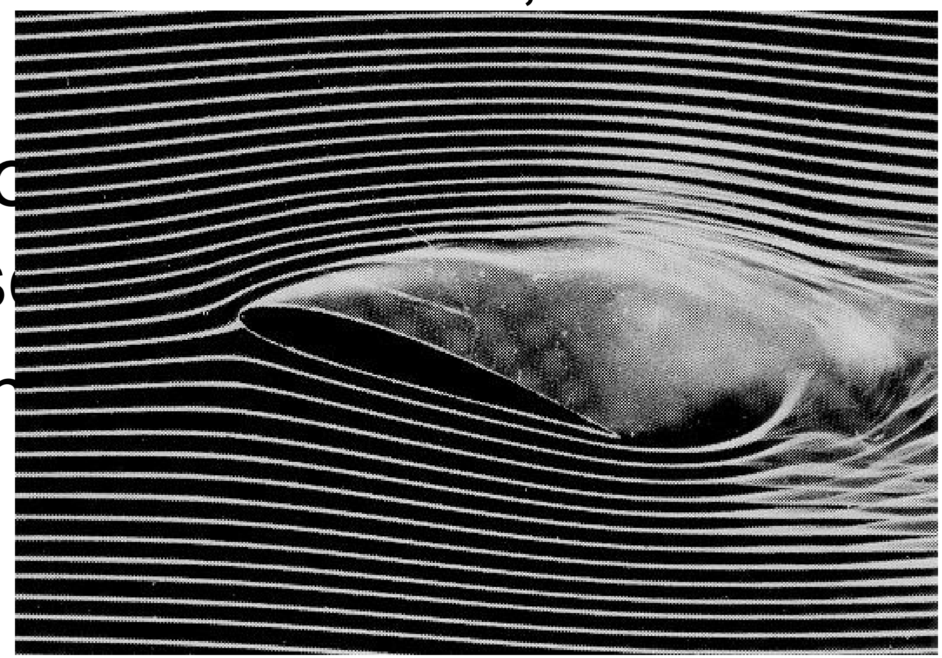
Stall (separation) is a viscous phenomena

Flow away from airfoil is not influenced by friction and is wholly inviscid

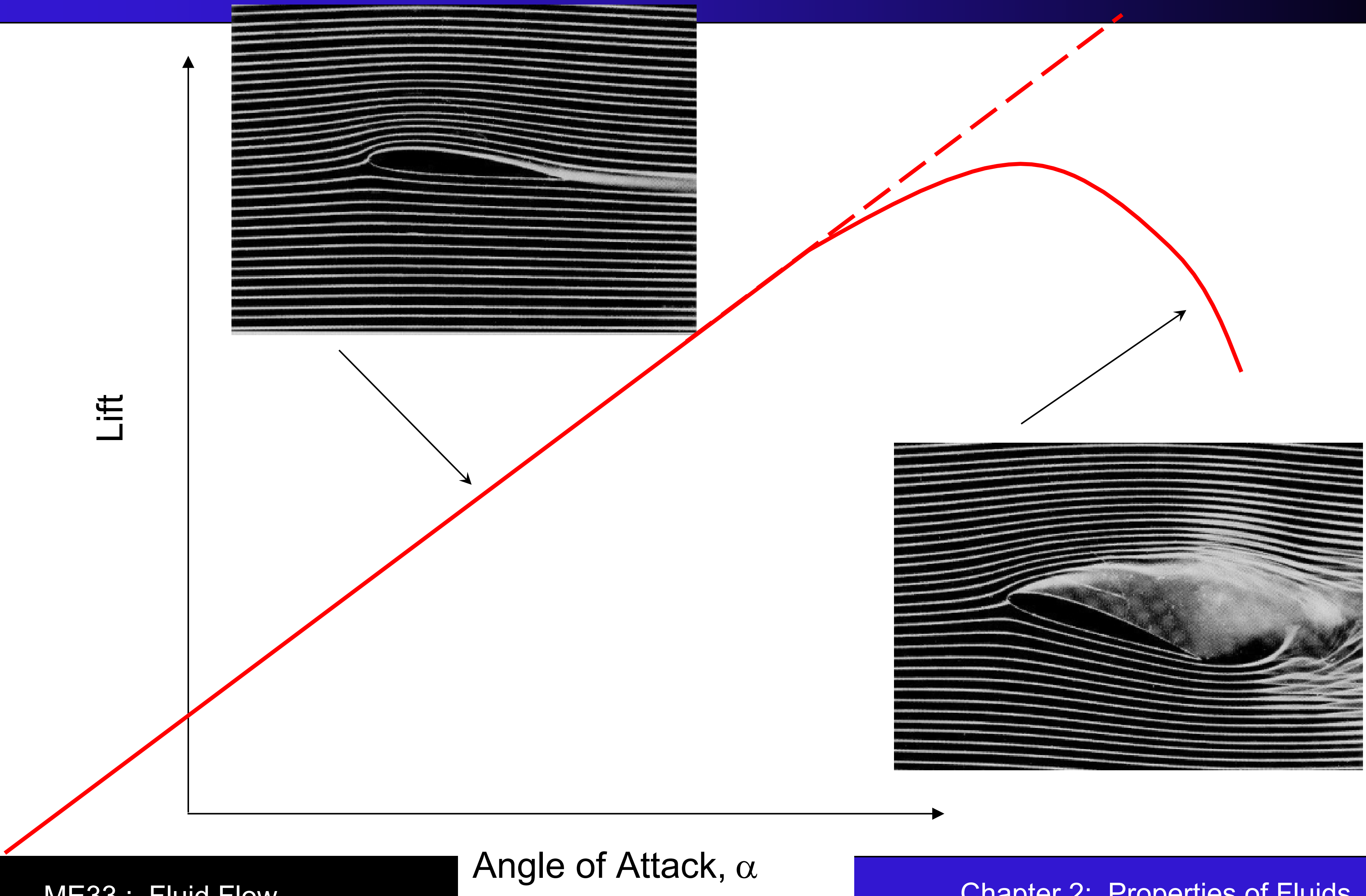
EXAMPLE: AIRFOIL STALL

- Key to understanding: Friction causes flow separation within boundary layer
  1. B.L. either laminar or turbulent
  2. All laminar B.L. → turbulent B.L.
  3. Turbulent B.L. 'fuller' than laminar B.L., more

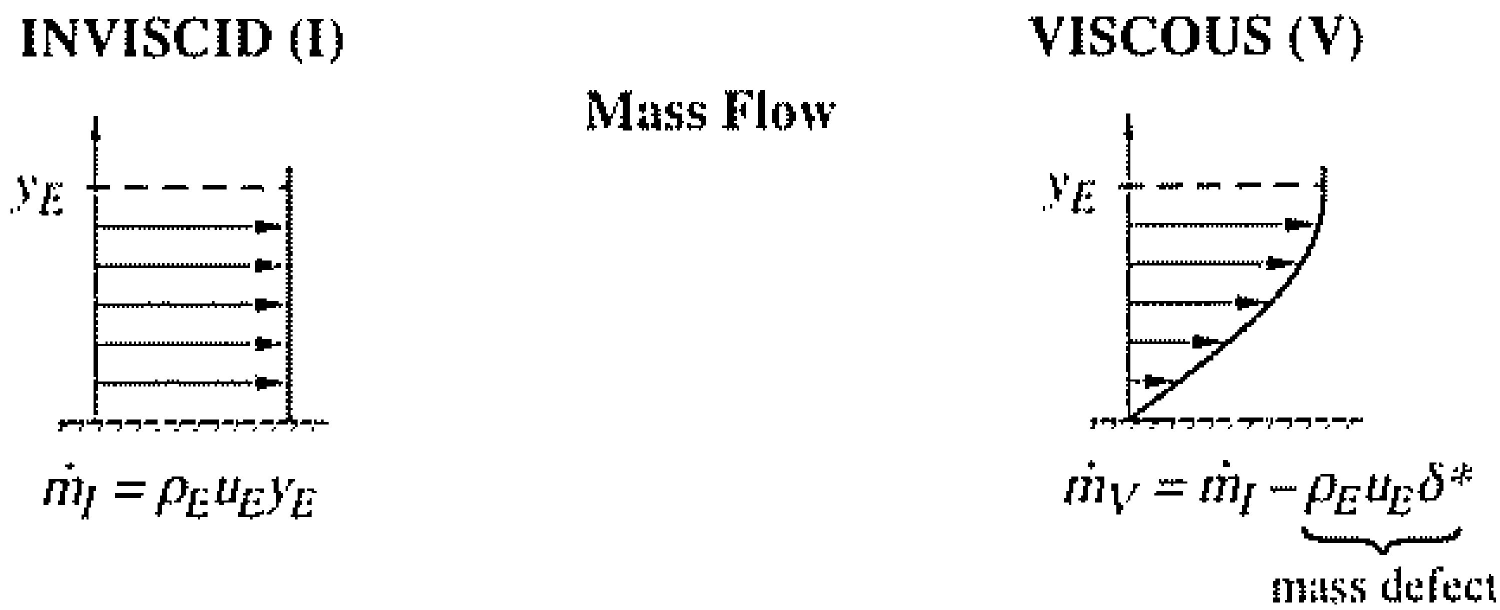




EXAMPLE: AIRFOIL STALL

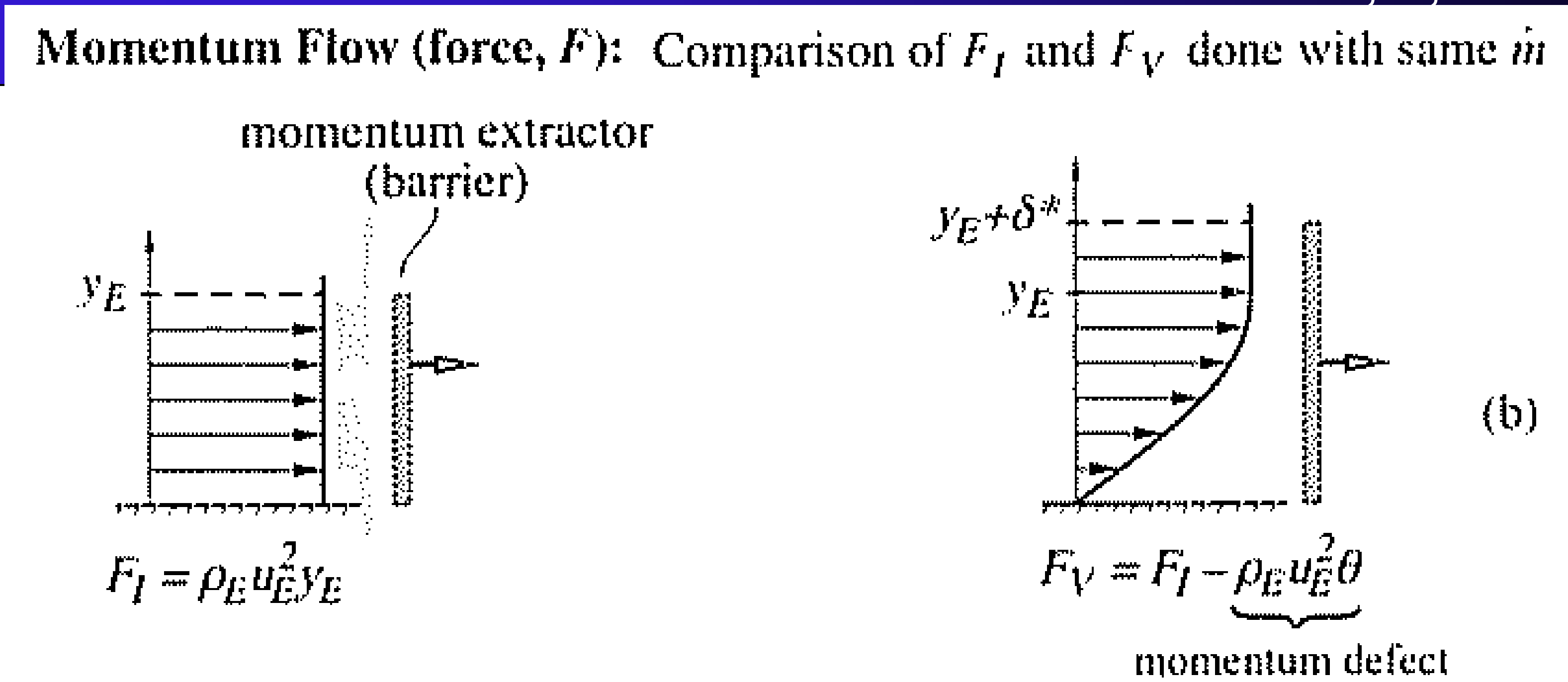


ALTERNATE PHYSICAL INTERPRETATIONS OF δ^*, θ, and θ^*



- Physical interpretation of displacement thickness, δ^* by considering mass flow rate that would occur in an inviscid flow which has velocity U_E and density ρ_E, and comparing this to actual, viscous, situation
- In figure $\rho_E U_E \delta^*$ is the defect in mass flow due to flow retardation in boundary layer
- Effect on flow outside boundary layer is equivalent to displacing the surface outwards, in the normal direction, a distance δ^*
- For a given $\rho_E U_E$, effective width of a 2D channel is reduced by sum of δ^*_{upper} and δ^*_{lower}

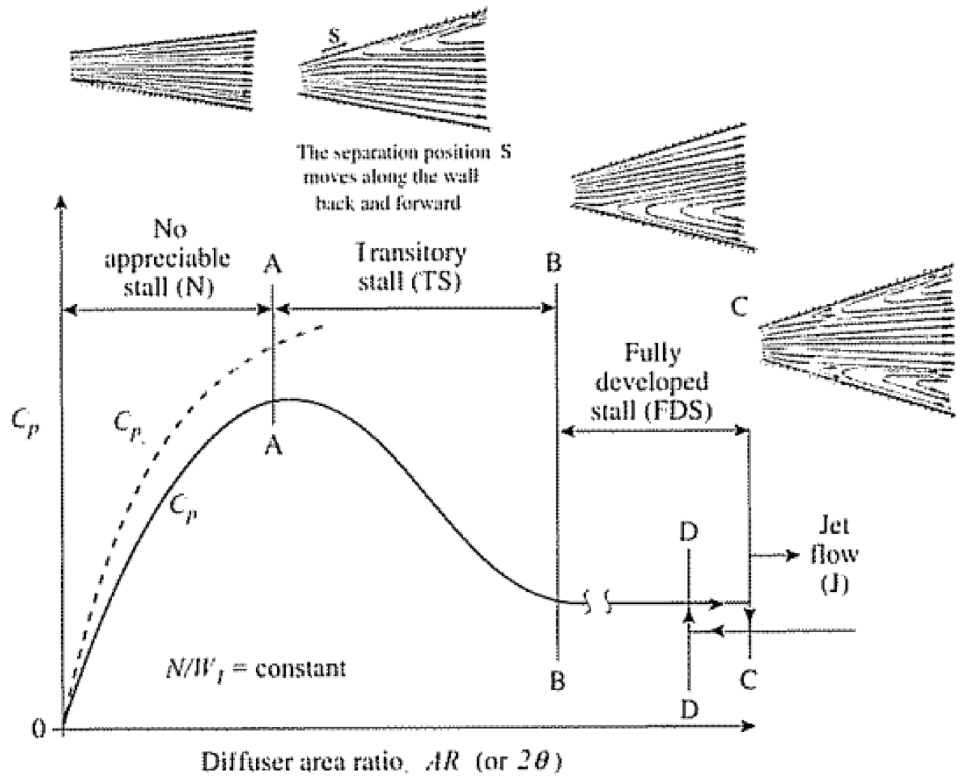
ALTERNATE PHYSICAL INTERPRETATIONS OF δ^*, θ, and θ^*

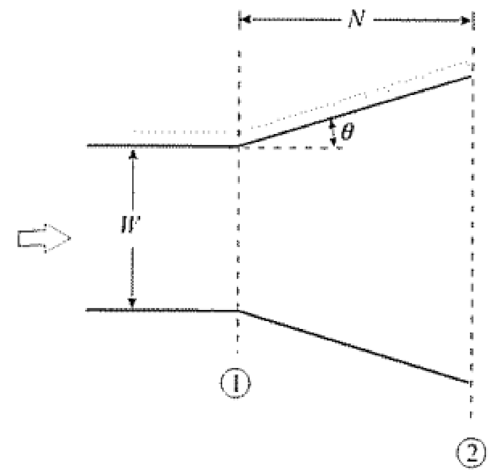


- Quantity $\rho_E U_E^2\theta$ represents defect in streamwise momentum flux between actual flow and a uniform flow having density ρ_E and velocity U_E outside boundary layer
- ...eing produced by

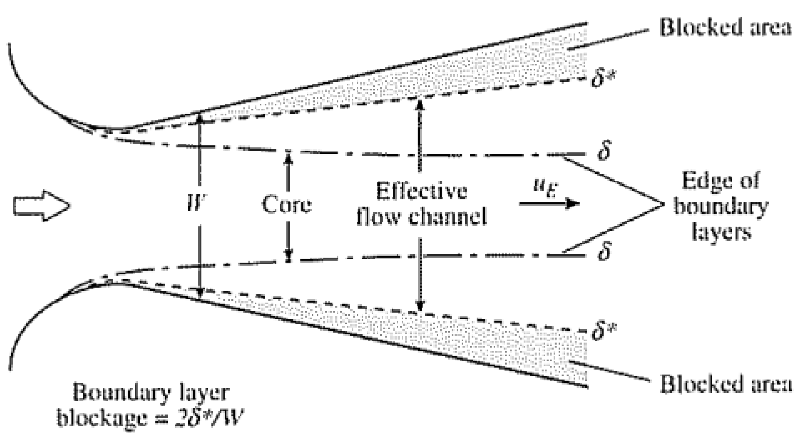
ALTERNATE PHYSICAL INTERPRETATIONS OF δ^*, θ, and θ^*

- Measures defect between flux of kinetic energy (mechanical power) in the actual flow and a uniform flow with U_E and ρ_E the same as outside the boundary layer
- Defect can be regarded as being produced by extraction of kinetic energy
- Power extracted is linked to device losses, and kinetic energy thickness is a key quantity in characterizing losses is internal flow devices

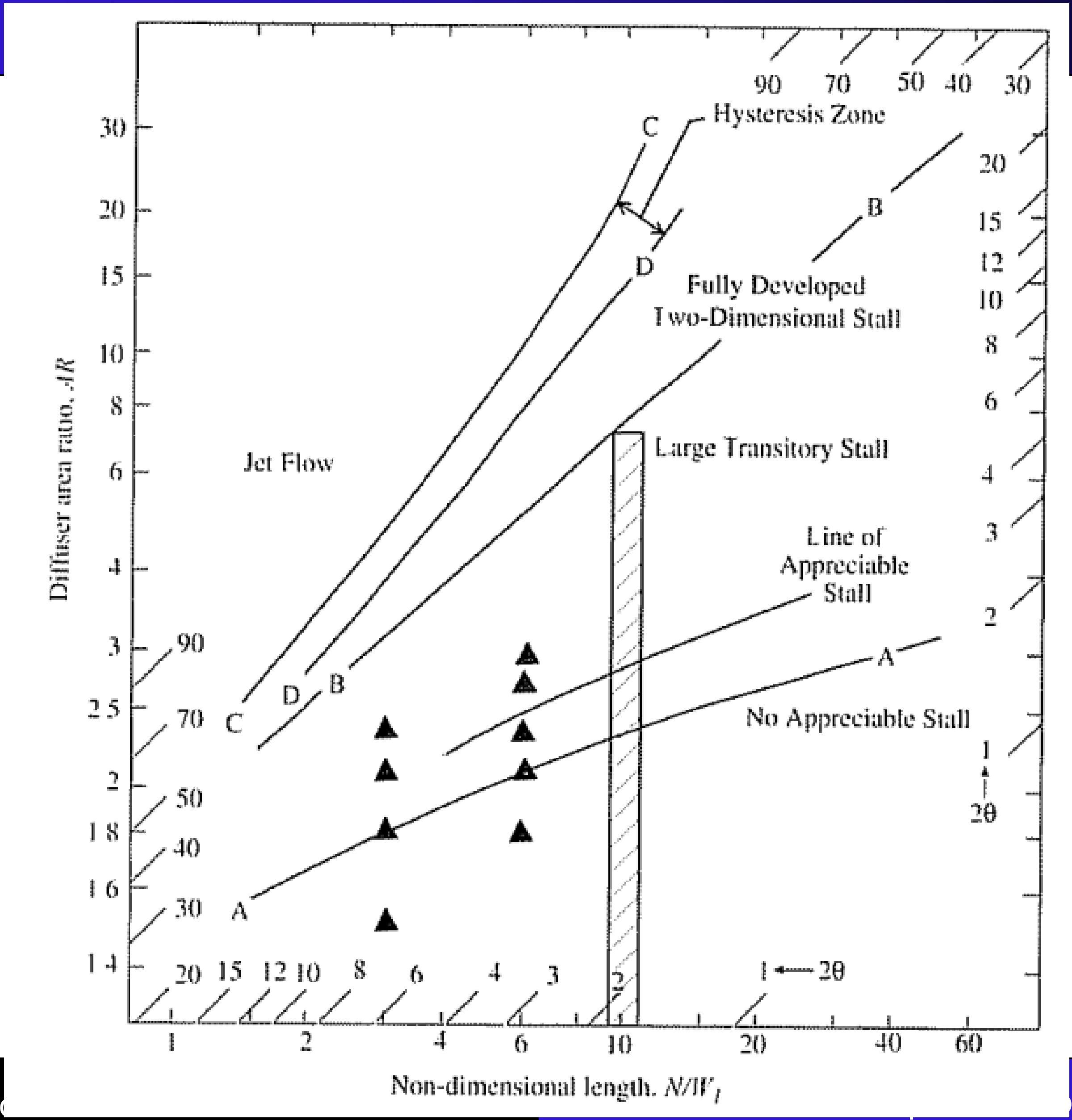




- Function of diffuser is to change a major fraction of flow KE into static pressure and to decrease velocity magnitude
- AR = W2/W1
- Non-dimensional length is N/W1
- Diffuser opening angle is tan(θ)=(AR-1)(2N/W1)
- For ideal flow, $C_{p,i}=1-1/AR^2$
- Compare prior to AA and after AA, significant deviation from predicted flow behavior



EXAMPLE: DIFFUSERS



Dimensional Analysis

It is a pure mathematical technique to establish a relationship between physical quantities involved in a fluid phenomenon by considering their dimensions.

In dimensional analysis, from a general understanding of fluid phenomena, we first predict the physical parameters that will influence the flow, and then we group these parameters into dimensionless combinations which enable a better understanding of the flow phenomena. Dimensional analysis is particularly helpful in experimental work because it provides a guide to those things that significantly influence the phenomena; thus it indicates the direction in which experimental work should go.

Dimensional Analysis

Dimensional Analysis refers to the physical nature of the quantity (**Dimension**) and the type of unit used to specify it.

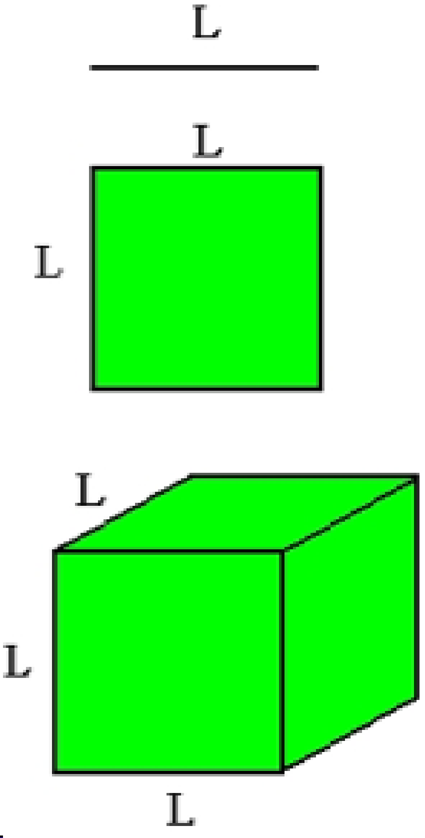
Distance has dimension L.

Area has dimension L^2.

Volume has dimension L^3.

Time has dimension T.

Speed has dimension L/T



Application of Dimensional Analysis

- Development of an equation for fluid phenomenon
- Conversion of one system of units to another
- Reducing the number of variables required in an experimental program
- Develop principles of hydraulic similitude for model study

Dimensional Reasoning & Homogeneity

- Principle of Dimensional Homogeneity

The fundamental dimensions and their respective powers should be identical on either side of the sign of equality.

- Dimensional reasoning is predicated on the proposition that, for an equation to be true, then both sides of the equation must be numerically and dimensionally identical.
- To take a simple example, the expression x + y = z when x =1, y =2 and z =3 is clearly numerically true but only if the dimensions of x, y and z are identical. Thus

1 elephant +2 aeroplanes =3 days is clearly nonsense but

is wholly accurate.

nsionally homogeneous if all

Fundamental Dimensions

- We may express physical quantities in either mass-length-time (MLT) system or force-length-time (FLT) system.

  This is because these two systems are interrelated through Newton's second law, which states that force equals mass times acceleration,

  $F = ma$ 2nd Law of motion

  $F = ML/T^2$

  $F = MLT^{-2}$

- Through this relation, we can covert from one system to the other. Other than convenience, it makes no difference which system we use, since the results are

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}	Θ^{-1}

Dimensions of Some Common Physical Quantities

[x], Length – L

[m], Mass – M

[t], Time – T

[v], Velocity – LT^{-1}

[a], Acceleration – LT^{-2}

[F], Force – MLT^{-2}

[Q], Discharge – L^3T^{-1}

[ρ], Mass Density – ML^{-3}

[P], Pressure – $ML^{-1}T^{-2}$

[*E*], Energy – ML^2T^{-2}

Basic Concepts

- All theoretical equations that relate physical quantities must be dimensionally homogeneous. That is, all the terms in an equation must have the same dimensions. For example

Q = A.V (homogeneous)

$L^3T^{-1} = L^3T^{-1}$

- We do, however sometimes use no homogeneous equation, the best known example in fluid mechanics being the Manning equation.

$$Q = VA = \left(\frac{1.49}{n}\right) AR^{\frac{2}{3}}\sqrt{S} \quad [U.S.]$$

$$Q = VA = \left(\frac{1.00}{n}\right) AR^{\frac{2}{3}}\sqrt{S} \quad [SI]$$

Mannings equation is an empirical equation. Generally the use of such equations is limited to specialized areas.

- To illustrate the basic principles of dimensional analysis, let us explore the equation for the speed V with which a pressure wave travels through a fluid. We must visualize the physical problem to consider physical factors probably influence the speed. Certainly the compressibility E_v must be factor; also the density and the kinematic viscosity of the fluid might be factors. The dimensions of these quantities, written in square brackets are

$$V=[LT^{-1}],\ E_v=[FL^{-2}]=[ML^{-1}T^{-2}],\ \rho=[ML^{-3}],\ v=[L^2T^{-1}]$$

Here we converted the dimensions of E_v into the MLT system using $F=[MLT^{-2}]$. Clearly, adding or subtracting such quantities will not produce dimensionally homogenous equations. We must therefore multiply them in such a way that their dimensions balance. So let us write

$$V=C\ E_v^{a}\ \rho^{b}\ v^{d}$$

Where C is a dimensionless constant, and let solve for the ... ting the dimensions, we get

$$(LT^{-1})=(ML^{-1}T^{-2})^a (ML^{-3})^b (L^2T^{-1})^d$$

To satisfy dimensional homogeneity, the exponents of each dimension must be identical on both sides of this equation. Thus

For M: $0 = a + b$

For L: $1 = -a -3b +2d$

For T: $-1 = -2a - d$

Solving these three equations, we get

$$a=1/2,\ b=-1/2,\ d=0$$

So that $V = C \sqrt{(E_v/\rho)}$

This identifies basic form of the relationship, and it also determines that the wave speed is not effected by the fluid's kinematic viscosity, v.

Dimensional analysis along such lines was developed by Lord Rayleigh

Methods for Dimensional Analysis

- Rayleigh's Method
- Buckingham's ∏-method

Rayleigh's Method

Functional relationship between variables is expressed in the form of an exponential relation which must be dimensionally homogeneous

if "*y*" is a function of independent variables $x_1, x_2, x_3, \ldots, x_n$, then

$$y = f(x_1, x_2, x_3, \ldots\ldots x_n)$$

In exponential form as

$$y = \phi[(x_1)^a, (x_2)^b, (x_3)^c, \ldots\ldots(x_n)^z]$$

Rayleigh's Method

Procedure

- Write fundamental relationship of the given data
- Write the same equation in exponential form
- Select suitable system of fundamental dimensions
- Substitute dimensions of the physical quantities
- Apply dimensional homogeneity
- Equate the powers and compute the values of the exponents
- Substitute the values of exponents
- Simplify the expression
- Ideal up to three independent variables, can be used for four.

Buckingham's ∏ method

- A more generalized method of dimension analysis developed by E. Buckingham and others and is most popular now. This arranges the variables into a lesser number of dimensionless groups of variables. Because Buckingham used ∏ (pi) to represent the product of variables in each groups, we call this method Buckingham pi theorem.
- "If 'n' is the total number of variables in a dimensionally homogenous equation containing 'm' fundamental dimensions, then they may be grouped into (n-m) ∏ terms.

$$f(X_1, X_2, \ldots\ldots X_n) = 0$$

then the functional relationship will be written as

$$\Phi\ (\Pi_1 , \Pi_2 , \ldots\ldots\ldots\ldots\Pi_{n-m}) = 0$$

The final equation obtained is in the form of:

$$\Pi_1 = f\ (\Pi_2, \Pi_3, \ldots\ldots\ldots\ldots\Pi_{n-m}) = 0$$

Buckingham's ∏ method Procedure

- List all physical variables and note 'n' and 'm'.

  n = Total no. of variables

  m = No. of fundamental dimensions (That is, [M], [L], [T])
- Compute number of ∏-terms by (n-m)
- Write the equation in functional form
- Write equation in general form
- Select repeating variables. Must have all of the 'm' fundamental dimensions and should not form a ∏ among themselves
- Solve each ∏-term for the unknown exponents by dimensional homogeneity.

Buckingham's Π method

Example:

- Let us apply Buckingham's Π method to an example problem that of the drag forces F_D exerted on a submerged sphere as it moves through a viscous fluid. We need to follow a series of following steps when applying Buckingham's Π theorem.
- **Step 1:** Visualize the physical problem, consider the factors that are of influence and list and count the n variables.

  We must first consider which physical factors influence the drag force. Certainly, the size of the sphere and the velocity of the sphere must be important. The fluid properties involved are the density ρ and the viscosity μ. Thus we can write

$$f\,(F_D, D, V, \rho, \mu) = 0$$

  Here we used D, the sphere diameter, to represent sphere size, and f stands for "some function". We see that n = 5. Note that the procedure cannot work if any relevant variables are … the procedure and experience … ables are relevant.

Buckingham's ∏ method

- **Step 2:** Choose a dimensional system (MLT or FLT) and list the dimensions of each variables. Find m, the number of fundamental dimensions involved in all the variables.

  Choosing the MLT system, the dimensions are respectively

  MLT^{-2} , L , LT^{-1} , ML^{-3} , $ML^{-1}T^{-1}$

  We see that M, L and T are involved in this example. So m = 3.

- **Step 3:** Determine n-m, the number of dimensionless ∏ groups needed. In our example this is 5 – 3 = 2, so we can write $\Phi(\Pi_1. \Pi_2) = 0$

- **Step 4:** Form the ∏ groups by multiplying the product of the primary (repeating) variables, with unknown exponents, by each of the remaining variables, one at a time. We choose ρ, D, and V as the primary variables. Then the ∏ terms are

$$\Pi_1 = D^a V^b \rho^c F_D$$

$$^a V^b \rho^c \mu^{-1}$$

Buckingham's ∏ method

- **Step 5:** To satisfy dimensional homogeneity, equate the exponents of each dimension on both sides on each pi equation and so solve for the exponents

$$\Pi_1 = D^a V^b \rho^c F_D = (L)^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0 L^0 T^0$$

Equate exponents:

L: $a + b - 3c + 1 = 0$

M: $c + 1 = 0$

T: $-b - 2 = 0$

We can solve explicitly for

$b = -2, \quad c = -1, \quad a = -2$

Therefore

$$\Pi_1 = D^{-2} V^{-2} \rho^{-1} F_D = F_D/(\rho V^2 D^2)$$

Buckingham's ∏ method

Finally, add viscosity to D, V, and ρ to find Π_2 . Select any power you like for viscosity. By hindsight and custom, we select the power -1 to place it in the denominator

$\Pi_1 = D^a V^b \rho^c \mu^{-1} = (L)^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^{-1} = M^0L^0T^0$

Equate exponents:

L: $a + b - 3c + 1 = 0$

M: $c - 1 = 0$

T: $-b + 1 = 0$

We can solve explicitly for

$b = 1, \quad c = 1, \quad a = 1$

Therefore,

$\Pi_2 = D^1 V^1 \rho^1 \mu^{-1} = (D V \rho)/(\mu) = R$ = Reynolds Number

R = Reynolds Number= Ratio of inertia forces to viscous forces

mensionless

Buckingham's $\prod$ method

Rearrange the pi groups as desired. The pi theorem states that the $\prod_s$ are related. In this example hence

$$F_D/(\rho V^2 D^2) = \Phi (R)$$

So that $F_D = \rho V^2 D^2 \Phi (R)$

We must emphasize that dimensional analysis does not provide a complete solution to fluid problems. It provides a partial solution only. The success of dimensional analysis depends entirely on the ability of the individual using it to define the parameters that are applicable. If we omit an important variable. The results are incomplete, and this may lead to incorrect conclusions. Thus, to use dimensional analysis successfully, one must be familiar with the fluid phenomena involved.

Similitude and Dimensional Analysis

CE30460 - Fluid Mechanics

Diogo Bolster

Goals of Chapter

- Apply Pi Theorem
- Develop dimensionless variables for a given flow situation
- Use dimensional variables in data analysis
- Apply concepts of modeling and similitude

Basic Principles

- Dimensional Homogeneity
- In a system with several variables one can construct a series of numbers that do not have dimensions. This inherently tells you something about the scale invariance or lack thereof of a problem…..

Units and Dimensions Important in Fluids

- Primary Dimensions
  - Length (L)
  - Time (T)
  - Mass (M)
  - Temperature (θ)
- For any relationship A=B
  - Units (A)=Units (B) Dimensional Homogeneity