MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS

L-01

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Rigid Bodies and Deformable bodies Simple Stresses and strains

Introduction : (Maximum 5 sentences)

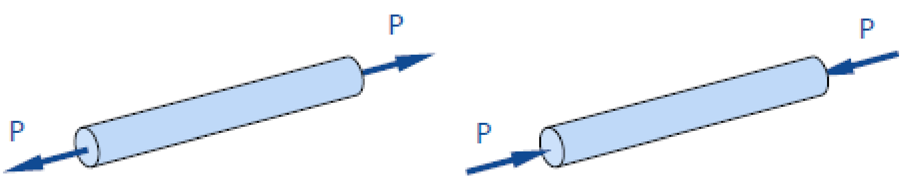
- A rigid body is a solid body in which deformation is zero or so small it can be neglected.
- The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it.
- A deformable body that changes its shape and/or volume while being acted upon by any kind of external force.

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour

Detailed content of the Lecture:

1. Stress is defined as the strength of a material per unit area or unit strength. It is the force on a member divided by area, which carries the force, formerly express in psi, now in N/mm^2 or MPa. Mathematically it is given by $\sigma = P/A$. where P is the applied normal load in Newton and A is the area in mm^2. The maximum stress in tension or compression occurs over a section normal to the load.
2. Normal stress is either tensile stress or compressive stress. Member's subject to pure tension (or tensile force) is under tensile stress, while compression members (members subject to compressive force) are under compressive stress.

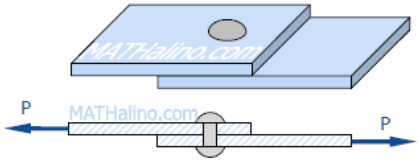


Bar in Tension

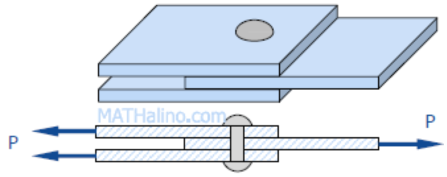
Bar in Compression

Compressive force will tend to shorten the member. Tension force on the other hand will tend to lengthen the member.

- Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.



Single Shear



Double Shear

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. A text book of Strength of Materials – Dr. R.K.Bansal – 2-5

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LECTURE HANDOUTS

L-02

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Elastic Constants, Elasticity, Hooke's law.

Introduction : (Maximum 5 sentences)

- Elastic modulus- Young's modulus, or the Young modulus, is a mechanical property that measures the stiffness of a solid material
- Bulk modulus- the relative change in the volume of a body produced by a unit compressive or tensile stress acting uniformly over its surface.
- Modulus of rigidity- Shear modulus also known as Modulus of rigidity is the measure of the rigidity of the body, given by the ratio of shear stress to shear strain

Prerequisite knowledge for Complete understanding and learning of Topic:
(Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour

Detailed content of the Lecture:

1. In engineering, **elasticity** is the ability of a body **to resist a distorting influence** and to return to its original size and shape when that influence or force is removed. Solid objects will deform when adequate forces are applied to them. If the material is elastic, the object will return to its initial shape and size when these forces are removed.

2. Elastic limit, **maximum stress or force per unit area within a solid material that can arise** before the onset of permanent deformation.

3. When an elastic body is subjected to stress, a proportionate amount of strain is produced. The ratio of the applied stresses to the strains generated will always be constant and is known as elastic constant. Elastic constant represents the elastic behaviour of objects.

4. Hooke's law states that **within the elastic limit the stress is directly proportional to the strain.**

5. The ratio of applied stress to the strain is constant and is known as Young's modulus or modulus of elasticity. Mathematically it is given by **E = Stress /Strain**. Young's modulus is denoted by letter "E". The unit of modulus of elasticity is the same as the unit of stress which is megapascal (MPa). **1 MPa is equal to 1 N/mm^2.**

6. When a body is subjected to mutually perpendicular direct stresses which are alike and equal, within its elastic limits, the ratio of direct stress to the corresponding volumetric strain is found to be constant. This ratio is called bulk modulus and is represented by letter "K". Unit of Bulk modulus is MPa.

7. When a body is subjected to shear stress the shape of the body gets changed, the ratio of shear stress to the corresponding shear strain is called rigidity modulus or modulus of rigidity. It is denoted by the letters “G” or “C” or “N”. Unit of rigidity modulus is MPa.

8. When a body is subjected to simple tensile stress within its elastic limits then there is a change in the dimensions of the body in the direction of the load as well as in the opposite direction. When these changed dimensions are divided with their original dimensions, longitudinal strain and lateral strain are obtained.

9. Relationship between elastic constants:

- The relationship between Young’s modulus (E), rigidity modulus (G) and Poisson’s ratio (μ) is expressed as : $E = 2G (1+\mu)$
- The relationship between Young’s modulus (E), bulk modulus (K) and Poisson’s ratio (μ) is expressed as : $E = 3K (1-2\mu)$
- Young’s modulus can be expressed in terms of bulk modulus (K) and rigidity modulus (G) as :

$$E = \frac{9KG}{(3K + G)}$$

- Poisson’s ratio can be expressed in terms of bulk modulus (K) and rigidity modulus (G) as :

$$\mu = \frac{(3K - 2G)}{(6K + 2G)}$$

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. A text book of Strength of Materials – Dr. R.K.Bansal – 2-6

LECTURE HANDOUTS

L-03

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Analysis of bars, thermal stresses, volumetric strain

Introduction : (Maximum 5 sentences)

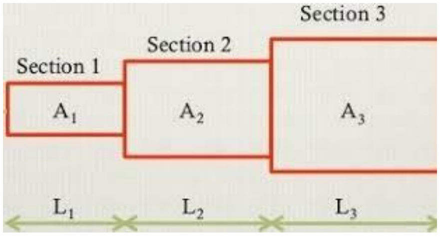
- A bar of different lengths and of different diameters and hence of different cross-sectional areas)
- The principle of superposition says that when a number of loads are acting on a body, the resulting strain, according to the principle of superposition, will be the algebraic sum of strains caused by individual loads.
- Thermal stress is stress created by any change in temperature to a material.
- The volumetric strain is the unit change in volume, i.e. the change in volume divided by the original volume.

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

Detailed content of the Lecture:

1. A bar of having different length and different cross-sectional area and bar is subjected with an axial load P.

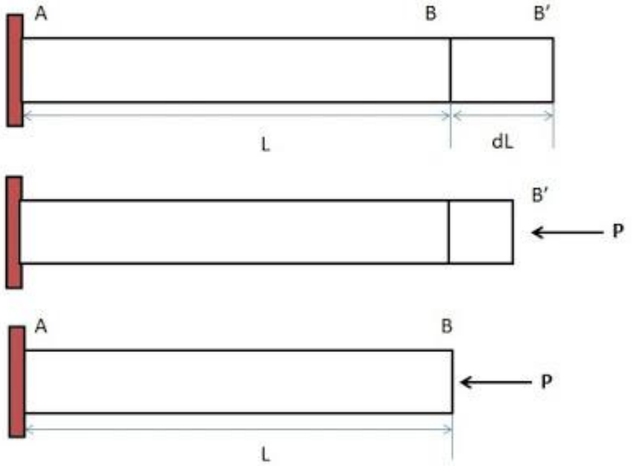


2. Length and cross-sectional areas of each section of bar is different and therefore stress induced, strain and change in length too will be different for each section of bar.
3. Young's modulus of elasticity of each section might be same or different depending on the material of the each section of bar.
4. Axial load for each section will be same i.e. P. To determine the total change in length of the bar of varying sections, have to add change in length of each section of bar.
5. P = Bar is subjected here with an axial Load ;A_1, A_2 and A_3 = Area of cross section of section 1, section 2 and section 3 respectively; L_1, L_2 and L_3 = Length of section 1, section 2 and

section 3 respectively; σ_1, σ_2 and σ_3 = Stress induced for the section 1, section 2 and section 3 respectively; ε_1, ε_2 and ε_3 = Strain developed for the section 1, section 2 and section 3 respectively; E= Young's Modulus of the bar.

6. Let us see here stress and strain produced for the section 1 Stress, $\sigma_1 = P / A_1$; Strain, $\varepsilon_1 = \sigma_1/E$; Strain, $\varepsilon_1 = P / A_1E$. Similarly for Section 2 & 3.
7. Change in length for each section with the help of definition of strain: $\Delta L_1 = \varepsilon_1 L_1$; $\Delta L_2 = \varepsilon_2 L_2$; $\Delta L_3 = \varepsilon_3 L_3$.
8. Total change in length of the bar of varying sections $\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3$

Thermal Stress: Stress which is induced in a body due to change in the temperature is known as thermal stress and the corresponding strain is called thermal strain. Thermal stress induces in a body when the temperature of the body is raised or lowered and the body is not allowed to expand or contract freely. **No stress will be induces in a body, when it is allowed to expand or contract freely.**



Let

L = Original length of the rod,

T = Rise in temperature,

E = Young's modulus,

α = Coefficient of linear expansion and

dL = Extension produced in the rod; Due to the increase in the temperature, there is an extension produced in the rod.

When the rod is allowed to expand freely, the; extension produced in the rod is given by

$$Extension\ produced = \alpha TL$$

$$dL = \alpha TL$$

Volumetric Strain: The ratio of change in volume to the original volume of a body is called volumetric strain. Mathematically, volumetric strain is given by dV/V

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. A text book of Strength of Materials – Dr. R.K.Bansal – 14-15 ; & 42

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LECTURE HANDOUTS

L-04

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Analysis of bars of composite sections, thermal stresses in composite bars

Introduction : (Maximum 5 sentences)

- A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other
- The extension or compression in each bar is equal
- The total external load on the composite bar is equal to the sum of the loads carried by each different material.
- In case of thermal stresses in composite bar, for the equilibrium of the system, compression in one material should be equal to tension in other material.

Prerequisite knowledge for Complete understanding and learning of Topic:
(Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

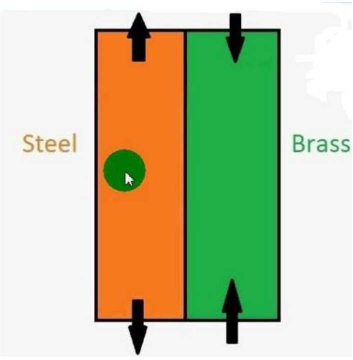
Analysis of bars of composite sections:

1. Composite bar behaves as single unit for compression and extension against compressive and tensile load.
2. Strains will be same for each bar of composite bar and hence stress produced in each bar of composite bar will be dependent over the young modulus of elasticity of respective material.
3. Total external load which will be acting over the composite bar will be shared by the each bar of composite bar and hence we can say that total external load on composite bar will be equal to the addition of the load shared by each bar of composite bar.
4. A_1 and A_2 = Area of cross section of bar 1 and bar 2 respectively; E_1 and E_2 = Young's modulus of elasticity for material of bar 1 and material of bar 2 respectively; P_1 and P_2 = Load shared by bar 1 and bar 2 respectively; σ_1 and σ_2 = Stress induced in bar 1 and bar 2 respectively;
5. As we have already discussed that total external load which will be acting over the composite bar will be shared by the each bar of composite bar and therefore we will have following equation. $P = P_1 + P_2$
6. Stress induced in bar 1, $\sigma 1= P_1 / A_1$; Stress induced in bar 1, $\sigma 2= P_2 / A_2$; $P = \sigma_1 A_1 + \sigma_2 A_2$
7. Strain in bar 1, $\varepsilon_1= \sigma_1/ E_1$; Strain in bar 2, $\varepsilon_2 = \sigma_2/ E_2$ .From above statement that strains will be same for each bar of composite bar, we will have following equation. $\sigma_1/ E_1 = \sigma_2/ E_2$;

Thermal stresses in composite bars:

1. Let us consider that we have one composite bar consisting two bars of different materials i.e. one bar of brass and other bar of steel Let us assume that we are now going to heat the

composite bars up to some temperature.



2. There will be free expansion or free contraction in the material according to the rising or lowering of temperature of the material.
3. If free expansion or free contraction of the material due to change in temperature is restricted partially or completely, there will be stress induced in the material and this stress will be termed as thermal stress.
4. If brass and steel both are not in composite in nature, the change in dimensions separately for steel and brass due to change in temperature, may be able to say easily that change in length of the brass bar will be more than the change in length of the steel bar for similar rise in temperature of the bar and this is basically due to **different co-efficient of linear expansion i.e. α for brass and steel.**
5. When brass and steel are in composite state and therefore both members of composite bars will not be able to expand freely. Hence the expansion of the composite bar, as a whole, will be less than that for brass but more than that for steel.
6. It has been observed that brass will be subjected with compressive load or compressive stress because steel will restrict the brass to expand up to the limit up to which brass could be expanded.
7. And similarly, steel will be subjected with tensile load or tensile stress because brass will force the steel to expand beyond the limit up to which steel could be expanded.
8. In simple words, it can be concluded that both members of composite bars will be under stress but one will be in tensile stress and other will be in compressive stress i.e. steel will be under tensile stress and brass will be under compressive stress.
9. $\alpha_s T + \sigma_s/E_s = \alpha_b T - \sigma_b/E_b$

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. A text book of Strength of Materials – Dr. R.K.Bansal – 30 & 44

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Estd. 2000

LECTURE HANDOUTS

L-05

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Stresses inclined plane, principal stress, principal planes, Mohr's Circle

Introduction : (Maximum 5 sentences)

- Tensile, Compressive, shear stresses were acting in a plan, which was at right angles to the line of action of the force.
- The planes, which have no shear stresses, are known as principal planes. Hence principal planes are the planes of **zero shear stresses.**
- The normal stresses, acting on a principal plane, are known as principal stresses.
- Mohr's circle is a graphical method of finding normal, tangential and resultant stresses on an oblique plane.

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

1. The most general state of stress at a point may be represented by 6 components: Normal stresses (σ_x, σ_y, σ_z) and tangential stresses (τ_{xy}, τ_{yz}, τ_{zx})
2. Plane Stress-state of stress in which two faces of the cubic element are free of stress.

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

3. The maximum and minimum principal stresses can be evaluated using the above formula
4. For determining stresses on oblique section the following two cases will be considered.
   - A member subjected to a direct stress in one plane

- The member is subjected to like direct stresses in two mutually perpendicular directions.

5. When a member is subjected to a direct stress in one plane, then the stresses on an oblique plane are given by $\sigma_n = \sigma \cos^2\theta$; $\sigma_t = \sigma/2 * \sin 2\theta$
6. Maximum normal stress = σ
7. Maximum shear stress = σ/2

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. A text book of Strength of Materials – Dr. R.K.Bansal – 85-139

LECTURE HANDOUTS **L-06**

RA **II/IV**

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Solutions to the Problems based on stress, Strain, Young's Modulus.

Introduction : (Maximum 5 sentences)

- Problems based on elastic constants.
- The concept of Hooke's law has been applied

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

Detailed content of the Lecture:

1. A steel rod 1 m long and 20mm x 20mm in cross section is subjected to a tensile force of 40 KN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 Gpa.

Given data:

Length (l) = 1m = $1x10^3$mm

Cross sectional area (A) = 20x20=400mm^2

Tensile force (P) = 40KN= $40x10^3$N

Young's modulus (E) = 200 Gpa = $200x10^3$ N/mm^2

To find:

Elongation (δl) =?

Solution:

Strain (e)=δl/l (Also)

δl=Pl/AE

= $40x10^3x1x10^3 / 400x200x10^3$ = **0.5mm**

Result:

δl = 0.5mm.

2. A circular rod of diameter 16 mm and 500 mm long is subjected to a tensile force 40kN. The modulus of elasticity for steel may be taken as 200 kN/mm^2. Find stress, strain and elongation of the bar due to applied load.

***Solution*:** Load $P = 40$ kN $= 40 \times 1000$ N

$E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2

$L = 500$ mm

Diameter of the rod $d = 16$ mm

Therefore, sectional area $A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 16^2$

$= 201.06$ mm^2

Stress $p = \frac{P}{A} = \frac{40 \times 1000}{201.06} =$ **198.94 N/mm²**

Strain $e = \frac{p}{E} = \frac{198.94}{200 \times 10^3} =$ **0.0009947**

Elongation $\Delta = \frac{PL}{AE} = \frac{4.0 \times 1000 \times 500}{201.06 \times 200 \times 10^3} =$ **0.497 mm**

3. A specimen of steel 20 mm diameter with a gauge length of 200 mm is tested to destruction. It has an extension of 0.25 mm under a load of 80 kN and the load at elastic limit is 102 kN. The maximum load is 130 kN. The total extension at fracture is 56 mm and diameter at neck is 15 mm. Find (i) The stress at elastic limit. (ii) Young's modulus. (iii) Percentage elongation. (iv) Percentage reduction in area. (v) Ultimate tensile stress.

***Solution*:** Diameter $d = 20$ mm

Area $A = \frac{\pi d^2}{4} = 314.16$ mm^2

(*i*) **Stress at elastic limit** $= \frac{\text{Load at elastic limit}}{\text{Area}}$

$= \frac{102 \times 10^3}{314.16} =$ **324.675 N/mm²**

(*ii*) **Young's modulus** $E = \frac{\text{Stress}}{\text{Strain}}$ within elastic limit

$= \frac{P/A}{\Delta/L} = \frac{80 \times 10^3/314.16}{0.25/200}$

$=$ **203718 N/mm²**

(*iii*) **Percentage elongation** $= \frac{\text{Final extension}}{\text{Original length}}$

$= \frac{56}{200} \times 100 =$ **28**

(*iv*) **Percentage reduction in area**

$= \frac{\text{Initial area} - \text{Final area}}{\text{Initial area}} \times 100$

$= \frac{\frac{\pi}{4} \times 20^2 - \frac{\pi}{4} \times 15^2}{\frac{\pi}{4} \times 20^2} \times 100 =$ **43.75**

$$(v)\ \textbf{Ultimate Tensile Stress} = \frac{\text{Ultimate Load}}{\text{Area}}$$

$$= \frac{130 \times 10^3}{314.16} = \mathbf{413.80\ N/mm^2.}$$

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. Strength of Materials by R.K.Bansal, Page no: 5-7, 9-11.
2. Mechanics of Solids by S.S.Bhavikatti Page No: 245

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LECTURE HANDOUTS

L-07

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Problems- Analysis of bars- Principle of Superposition

Introduction : (Maximum 5 sentences)

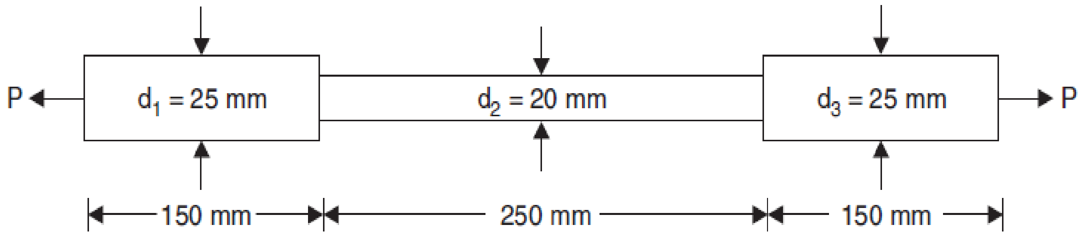
- To Calculate the stresses in the bars of Varying cross section.
- To apply the concept of principle of superposition

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

Detailed content of the Lecture:

1. The bar shown in Fig. is tested in universal testing machine. It is observed that at a load of 40 kN the total extension of the bar is 0.280 mm. Determine the Young's modulus of the material.



Solution: Extension of portion 1,

$$\frac{PL_1}{A_1E} = \frac{40 \times 10^3 \times 150}{\frac{\pi}{4} \times 25^2 E}$$

Extension of portion 2,

$$\frac{PL_2}{A_2E} = \frac{40 \times 10^3 \times 250}{\frac{\pi}{4} \times 20^2 E}$$

Extension of portion 3, $\frac{PL_3}{A_3E} = \frac{40 \times 10^3 \times 150}{\frac{\pi}{4} \times 25^2 E}$

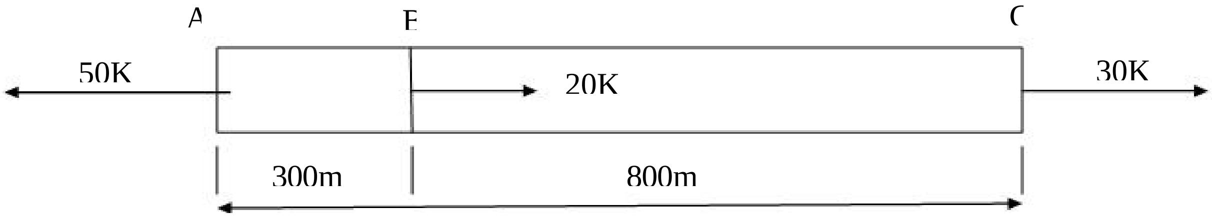
$$\text{Total extension} = \frac{40 \times 10^3}{E} \times \frac{4}{\pi}\left\{\frac{150}{625} + \frac{250}{400} + \frac{150}{625}\right\}$$

$$0.280 = \frac{40 \times 10^3}{E} \times \frac{4}{\pi} \times \frac{1.112}{E}$$

$$\boldsymbol{E = 200990 \text{ N/mm}^2}$$

2. **A steel bar of cross sectional area 200mm^2 is loaded as shown in fig. find the change in length of the bar. Take E as 200Gpa.**

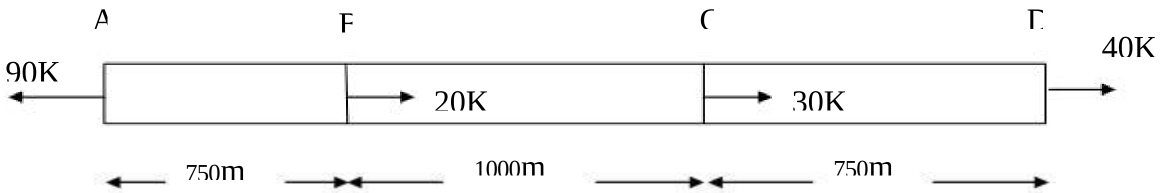
   Given data:



To find: δl = ?
Solution:

We Know That:
$\delta l = 1/AE\ [P_1L_1+P_2L_2+P_3L_3]$
= 1/(200X200X103)[(20X103X300)+(30X103X800)]
= 0.75 mm
Result:
Change in length δl = 0.75 mm

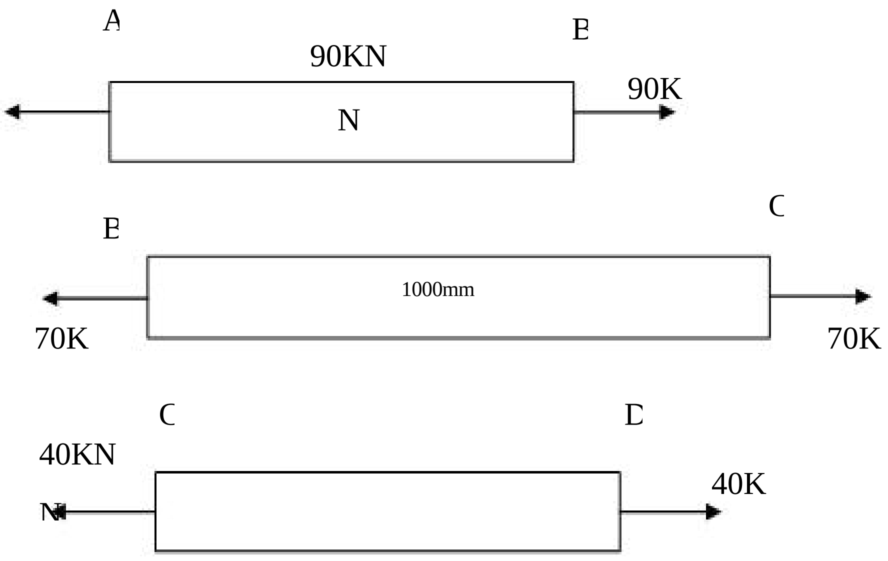
3. **A steel bar of 600mm2 cross sectional area is carrying loads as shown in fig. Determine the elongation of the bar. Take E for the steel as 200 GPa.**



   Cross sectional area (A) = 600mm^2
   Young's modulus (E) = 200 Gpa = 200x10^3 N/mm^2

To find: δl =?

Solution:



We Know That,

$\delta l = 1/AE\,[P_1L_1+P_2L_2+P_3L_3]$

$= 1/\,(600X200X10^{3)}[(90X10^3X750)+(70X10^3X1000)\ +(40X10^3X750)]$

$= 1.395$ mm

Result:

Change in length δl = 1.395 mm.

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. Strength of Materials by R.K.Bansal, Page no: 14-16.
2. Mechanics of Solids by S.S.Bhavikatti Page No: 246-248.

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LECTURE HANDOUTS

L-08

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Problems- Composite bars

Introduction : (Maximum 5 sentences)

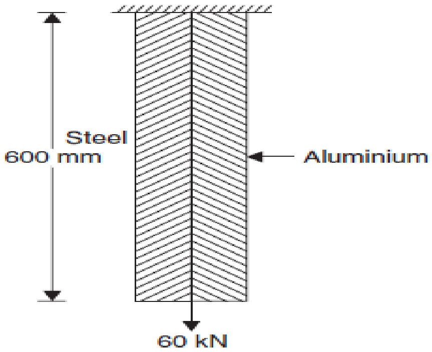
- To Introduce the concept of Compound Bars and to Calculate the Stresses developed in the materials.

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

Detailed content of the Lecture:

1. **A compound bar of length 600 mm consists of a strip of aluminium 40 mm wide and 20 mm thick and a strip of steel 60 mm wide × 15 mm thick rigidly joined at the ends. If elastic modulus of aluminium and steel are 1×105 N/mm2 and 2×105 N/mm2, determine the stresses developed in each material and the extension of the compound bar when axial tensile force of 60 kN acts**



Data available is

$$L = 600 \text{ mm}$$
$$P = 60 \text{ kN} = 60 \times 1000 \text{ N}$$
$$A_a = 40 \times 20 = 800 \text{ mm}^2$$
$$A_s = 60 \times 15 = 900 \text{ mm}^2$$
$$E_a = 1 \times 10^5 \text{ N/mm}^2,\ E_s = 2 \times 10^5 \text{ N/mm}^2.$$

Let the load shared by aluminium strip be P_a and that shared by steel be P_s. Then from equilibrium condition

$$P_a + P_s = 60 \times 1000 \quad ...(1)$$

From compatibility condition, we have

$$\Delta_a = \Delta_s$$

$$\frac{P_a L}{A_a E_a} = \frac{P_s L}{A_s E_s}$$

i.e.

$$\frac{P_a \times 600}{800 \times 1 \times 10^5} = \frac{P_s \times 600}{900 \times 2 \times 10^5}$$

$$P_s = 2.25 \ P_a \quad ...(2)$$

Substituting it in eqn. (1), we get

$$P_a + 2.25 \ P_a = 60 \times 1000$$

i.e. $P_a = 18462$ N.

$\therefore$ $P_s = 2.25 \times 18462 = 41538$ N.

$\therefore$ **Stress in aluminium strip** $= \frac{P_a}{A_a} = \frac{18462}{800}$

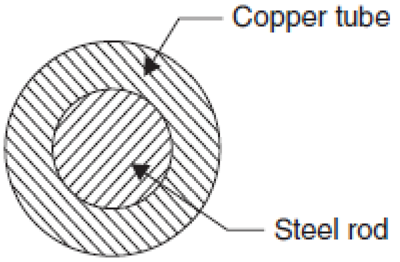
$$= \mathbf{23.08 \ N/mm^2}$$

Stress in steel strip $= \frac{P_s}{A_s} = \frac{41538}{900} = \mathbf{46.15 \ N/mm^2}$

Extension of the compound bar $= \frac{P_a L}{A_a E_a} = \frac{18462 \times 600}{800 \times 1 \times 10^5}$

$$\mathbf{\Delta l = 0.138 \ mm.}$$

2. **A compound bar consists of a circular rod of steel of 25 mm diameter rigidly fixed into a copper tube of internal diameter 25 mm and external diameter 40 mm as shown in Fig. If the compound bar is subjected to a load of 120 kN, find the stresses developed in the two materials. Take $E_s = 2 \times 10^5$ N/mm^2 and $E_c = 1.2 \times 10^5$ N/mm^2.**



Solution: Area of steel rod $A_s = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$

Area of copper tube $A_c = \frac{\pi}{4} (40^2 - 25^2) = 765.76 \text{ mm}^2$

From equation of equilibrium,

$$P_s + P_c = 120 \times 1000 \quad ...(1)$$

where P_s is the load shared by steel rod and P_c is the load shared by the copper tube.

From compatibility condition, we have

$$\Delta_s = \Delta_c$$

$$\frac{P_s L}{A_s E_s} = \frac{P_c L}{A_c E_c}$$

$$\frac{P_s}{490.87 \times 2 \times 10^5} = \frac{P_c}{765.76 \times 1.2 \times 10^5}$$

$$\therefore \quad P_s = 1.068\ P_c \quad \text{...(2)}$$

From eqns. (1) and (2), we get

$$1.068\ P_c + P_c = 120 \times 1000$$

$$\therefore \quad P_c = \frac{120 \times 1000}{2.068} = 58027 \text{ N}$$

$$\therefore \quad P_s = 1.068\ P_c = 61973 \text{ N}$$

$\therefore$ **Stress in copper** $= \frac{58027}{9765.76} =$ **75.78 N/mm²**

Stress in steel $= \frac{61973}{490.87} =$ **126.25 N/mm²**

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. Mechanics of Solids by S.S.Bhavikatti Page No: 264-266-248.

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LECTURE HANDOUTS

L-09

RA

II/IV

Course Name with Code : **Strength of Materials (19RAC09)**

Course Faculty : **Dr.D.Velmurugan**

Unit : **I - Stress, Strain and Deformation of Solids**

Topic of Lecture: Problems- Thermal stresses

Introduction : (Maximum 5 sentences)

- To calculate the stress developed in the material due to change in temperature
- To apply the concept of elastic constants

Prerequisite knowledge for Complete understanding and learning of Topic: (Max. Four important topics)

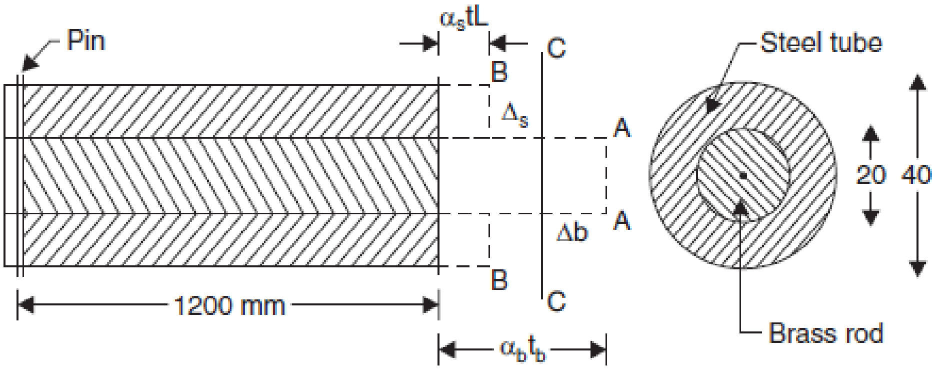
- Basics of Engineering Mechanics
- Basics of material behaviour
- Basics of simple mathematics

Detailed content of the Lecture:

1. **A bar of brass 20 mm is enclosed in a steel tube of 40 mm external diameter and 20 mm internal diameter. The bar and the tubes are initially 1.2 m long and are rigidly fastened at both ends using 20 mm diameter pins. If the temperature is raised by 60°C, find the stresses induced in the bar, tube and pins.**

Given:

$$E_s = 2 \times 10^5 \ N/mm^2$$
$$E_b = 1 \times 10^5 \ N/mm^2$$
$$\alpha_s = 11.6 \times 10^{-6}/°C$$
$$\alpha_b = 18.7 \times 10^{-6}/°C.$$



$$t = 60° \quad E_s = 2 \times 10^5 \text{ N/mm}^2 \quad E_b = 1 \times 10^5 \text{ N/mm}^2$$
$$\alpha_s = 11.6 \times 10^{-6}/°C \quad \alpha_b = 18.7 \times 10^{-6}/°C$$
$$A_s = \frac{\pi}{4}(40^2 - 20^2) \quad A_b = \frac{\pi}{4} \times 20^2$$
$$= 942.48 \text{ mm}^2 \quad = 314.16 \text{ mm}^2$$

$$t = 60° \quad E_s = 2 \times 10^5 \text{ N/mm}^2 \quad E_b = 1 \times 10^5 \text{ N/mm}^2$$
$$\alpha_s = 11.6 \times 10^{-6}/°C \quad \alpha_b = 18.7 \times 10^{-6}/°C$$
$$A_s = \frac{\pi}{4}(40^2 - 20^2) \quad A_b = \frac{\pi}{4} \times 20^2$$
$$= 942.48 \text{ mm}^2 \quad = 314.16 \text{ mm}^2$$

Since free expansion of brass ($\alpha_b\, tL$) is more than free expansion of steel ($\alpha_s\, tL$), compressive force *Pb* develops in brass and tensile force *Ps* develops in steel to keep the final position at CC

Horizontal equilibrium condition gives $P_b = P_s$, say P. From the figure, it is clear that

$$\Delta_s + \Delta_b = \alpha_b\, tL - \alpha_s tL = (\alpha_b - \alpha_s)tL.$$

where Δ_s and Δ_b are the changes in length of steels and brass bars.

$$\therefore \quad \frac{PL}{A_s E_s} + \frac{PL}{A_b E_b} = (18.7 - 11.6) \times 10^{-6} \times 60 \times 1200.$$

$$P \times 1200 \left[\frac{1}{942.48 \times 2 \times 10^5} + \frac{1}{314.16 \times 1 \times 10^5}\right] = 7.1 \times 10^{-6} \times 60 \times 1200$$

$$\therefore \quad P = 11471.3 \text{ N}$$

$$\therefore \quad \textbf{Stress in steel} = \frac{P}{A_s} = \frac{11471.3}{942.48} = \mathbf{12.17 \text{ N/mm}^2}$$

and
$$\textbf{Stress in brass} = \frac{P}{A_b} = \frac{11471.3}{314.16} = \mathbf{36.51 \text{ N/mm}^2}$$

The pin resist the force P at the two cross-sections at junction of two bars.

$$\therefore \quad \textbf{Shear stress in pin} = \frac{P}{2 \times \text{Area of pin}}$$
$$= \frac{11471.3}{2 \times \pi/4 \times 20^2} = \mathbf{18.26 \text{ N/mm}^2}$$

Problem *Determine the changes in length, breadth and thickness of a steel bar which is 4 m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Take E = 2 × 10⁵ N/mm² and Poisson's ratio = 0.3.*

Sol. Given :

Length of the bar,	$L = 4 \text{ m} = 4000 \text{ mm}$
Breadth of the bar,	$b = 30 \text{ mm}$
Thickness of the bar,	$t = 20 \text{ mm}$
∴ Area of cross-section,	$A = b \times t = 30 \times 20 = 600 \text{ mm}^2$
Axial pull,	$P = 30 \text{ kN} = 30000 \text{ N}$
Young's modulus,	$E = 2 \times 10^5 \text{ N/mm}^2$
Poisson's ratio,	$\mu = 0.3.$

Now strain in the direction of load (or longitudinal strain),

$$= \frac{\text{Stress}}{E} = \frac{\text{Load}}{\text{Area} \times E} \qquad \left(\because \text{ Stress} = \frac{\text{Load}}{\text{Area}}\right)$$

$$= \frac{P}{A.E.} = \frac{30000}{600 \times 2 \times 10^5} = 0.00025.$$

But longitudinal strain $= \frac{\delta L}{L}$.

$$\therefore \qquad \frac{\delta L}{L} = 0.00025.$$

$\therefore$ δL (or change in length) $= 0.00025 \times L$

$= 0.00025 \times 4000 =$ **1.0 mm.** Ans.

Using equation (2.3),

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$0.3 = \frac{\text{Lateral strain}}{0.00025}$$

$\therefore$ Lateral strain $= 0.3 \times 0.00025 = 0.000075.$

We know that

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}\left(\text{or } \frac{\delta t}{t}\right)$$

$\therefore$ $\delta b = b \times$ Lateral strain

$= 30 \times 0.000075 =$ **0.00225 mm.** Ans.

Similarly, $\delta t = t \times$ Lateral strain

$= 20 \times 0.000075 =$ **0.0015 mm.** Ans.

Video Content / Details of website for further learning (if any): Nil

Important Books/Journals for further learning including the page nos:

1. Mechanics of Solids by S.S.Bhavikatti Page No: 274-277.
2. Strength of Materials by R.K.Bansal, Page No:59-64

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L - 10

LECTURE HANDOUTS

RA | **II / IV**

Course Name with Code : **19RAC09- STRENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II -Transverse Loading on Beams and Stresses in Beam**

Topic of Lecture: Beams types- Transverse loading

Introduction :

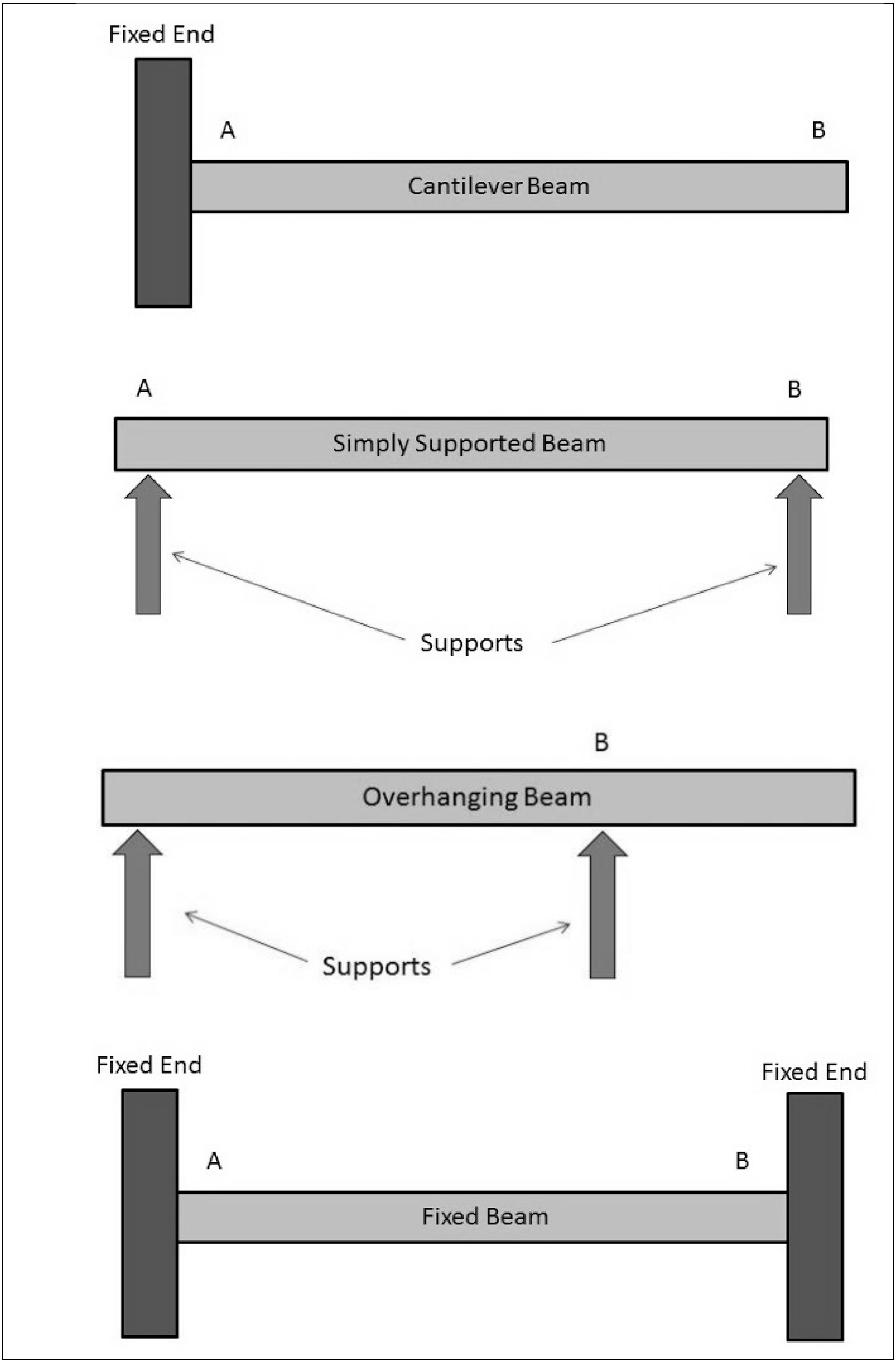
1. Horizontal structural member used to carry vertical load, shear load and sometimes horizontal load
2. Structure member which cross section is much smaller compare to its length and undergoes lateral load
3. A horizontal bar witch undergoes lateral load (or)
4. A couple which tends to bend it or a horizontal bar undergoes bending stress.

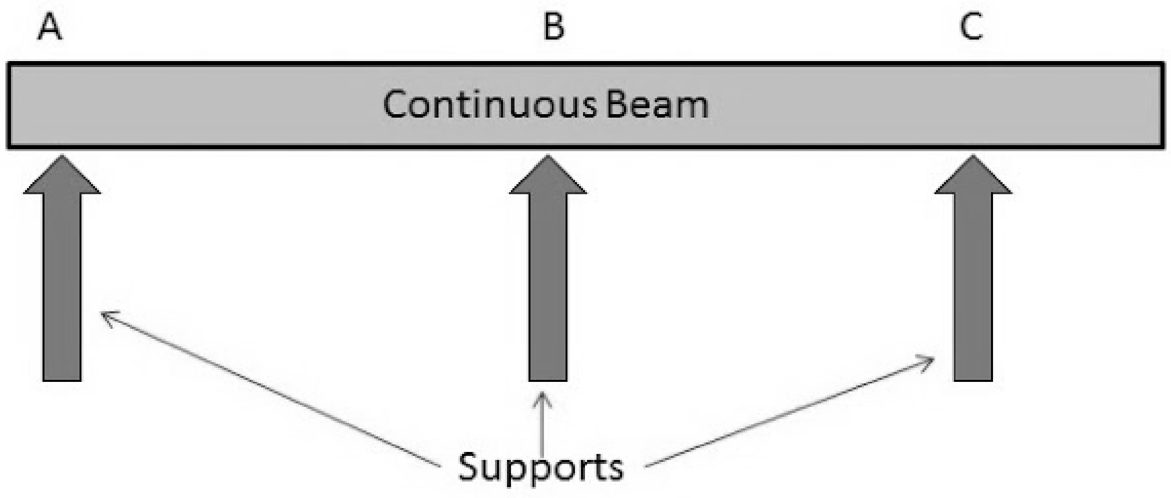
Prerequisite knowledge for Complete understanding and learning of Topic:

1. Load
2. Structure
3. Force
4. Stress
5. Free body diagram

Types of beams

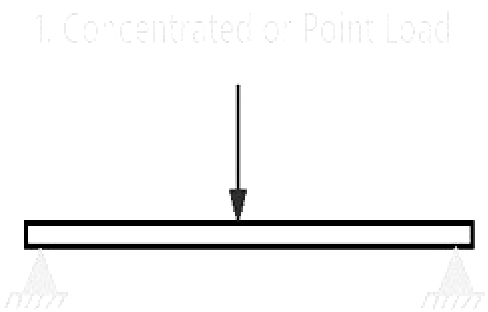
1. Cantilever beams
2. Simply supported beams
3. Overhanging beams
4. Fixed beams, and
5. Continuous beams


Fixed End
A
B
Cantilever Beam
A
B
Simply Supported Beam
Supports
B
Overhanging Beam
Supports
Fixed End
Fixed End
A
B
Fixed Beam

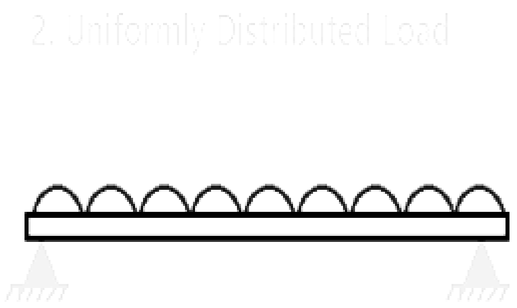


TYPES OF TRANSVERSE OF LOADING

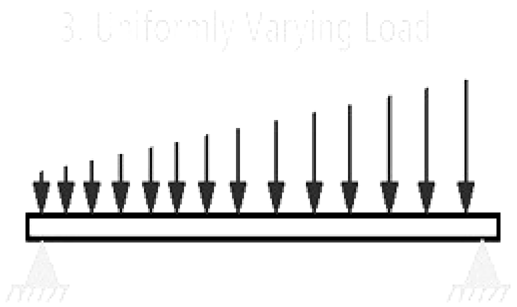
- Concentrated or Point Load: Act at a point.



- Uniformly Distributed Load: Load spread along the length of the Beam.



- Uniformly Varying Load: Load spread along the length of the Beam, Rate of varying loading point to point.



Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=sXt6YE09LPQ

Important Books/Journals for further learning including the page nos.:

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.256-258.

L - 11

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09- STRENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II-Transverse Loading on Beams and Stresses in Beam**

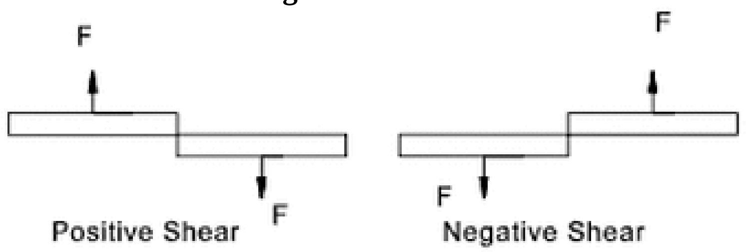
Topic of Lecture: Shear force and Bending moment diagrams – Cantilever beams

Introduction :
A shear force diagram is one which shows the variation of the shear force along the length of the beam. A bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

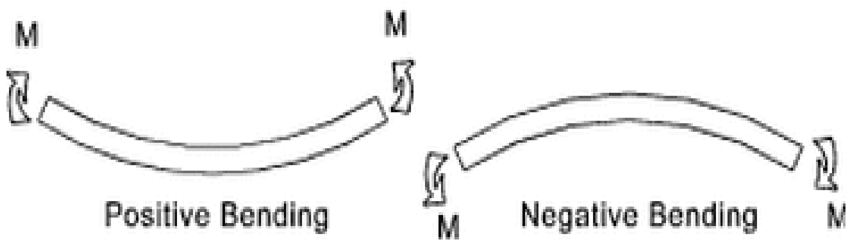
Prerequisite knowledge for Complete understanding and learning of Topic:

1. Force
2. Moment
3. Point load
4. Uniformly distributed load
5. Uniformly varying load

SHEAR FORCE
Algebraic sum of the vertical forces to the right or left of the section



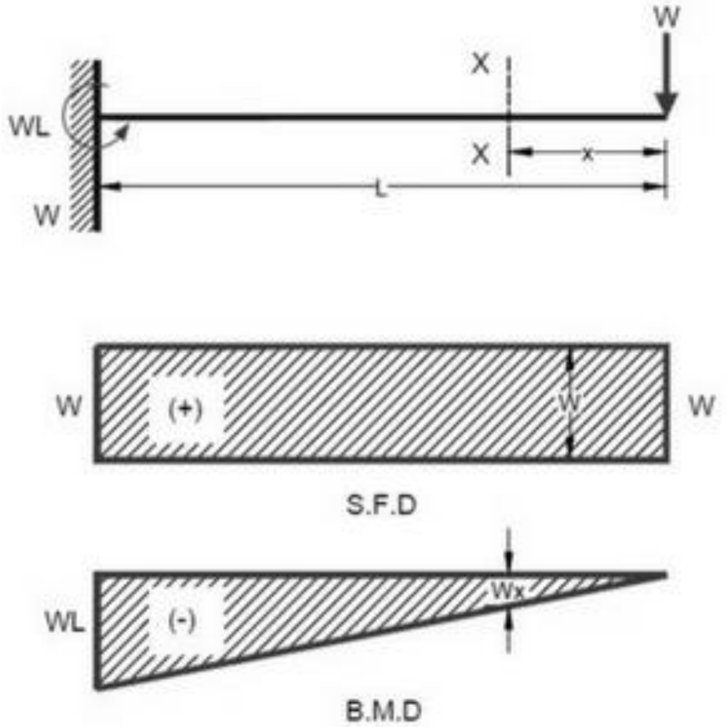
BENDING MOMENT
Algebraic sum of the moments of all the forces to the right or left of the section



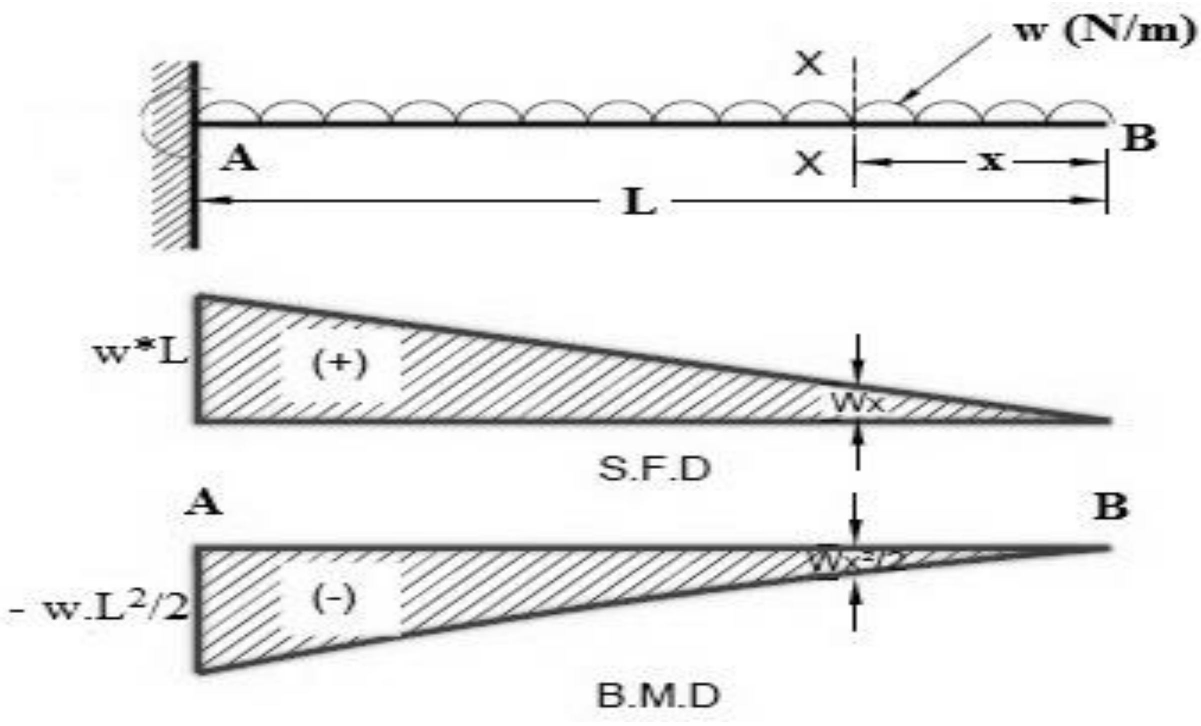
Shear force diagram – shows the variation of the shear force along the length of the beam

Bending moment diagram – shows the variation of the bending moment laong the length of the beam.

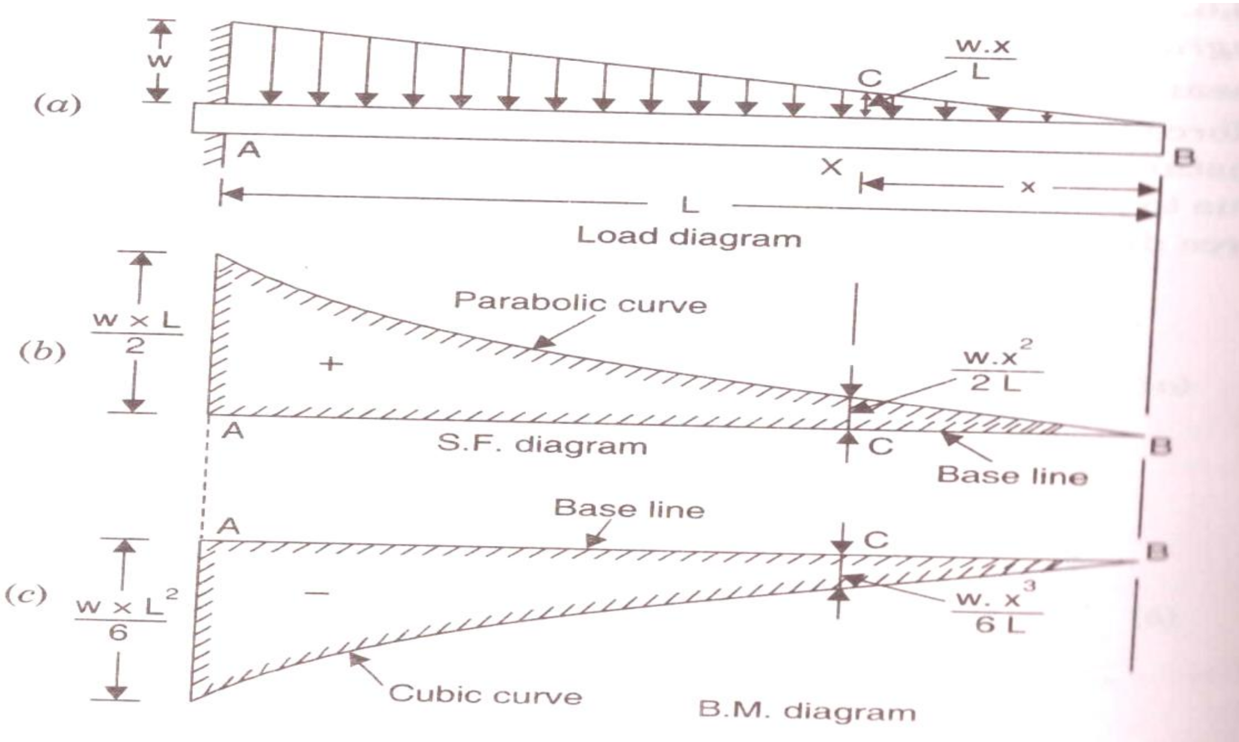
SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR CANTILEVER BEAM CARRYING POINT LOAD



SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR CANTILEVER BEAM CARRYING UNIFORMLY DISTRIBUTED LOAD



SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR CANTILEVER BEAM CARRYING UNIFORMLY VARYING LOAD



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=R0rW7ziiuZM

Important Books/Journals for further learning including the page nos.:

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.258-276.

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L - 12

LECTURE HANDOUTS

RA | **IV / II**

Course Name with Code : **19RAC09- STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II- Transverse Loading on Beams and Stresses in Beam**

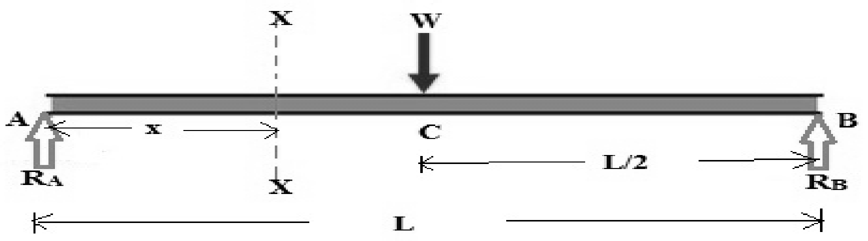
Topic of Lecture: Shear force and Bending moment diagrams – Simply supported beams

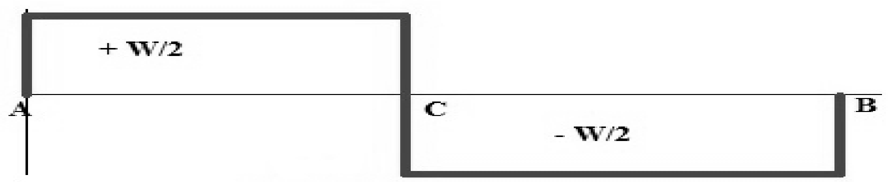
Introduction :
A shear force diagram is one which shows the variation of the shear force along the length of the beam. A bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

Prerequisite knowledge for Complete understanding and learning of Topic:

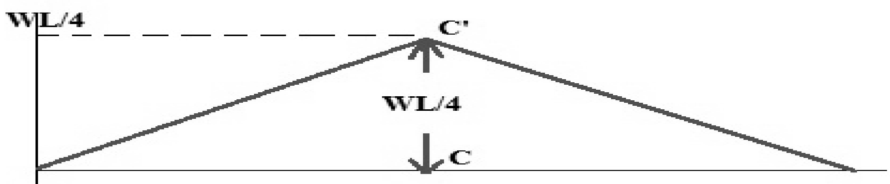
1. Shear force
2. Bending moment
3. Point load
4. Uniformly distributed load
5. Uniformly varying load

SHEAR FORCE & BENDING MOMENT DIAGRAM OF A SIMPLY SUPPORTED BEAM CARRYING POINT LOAD AT MID POINT:



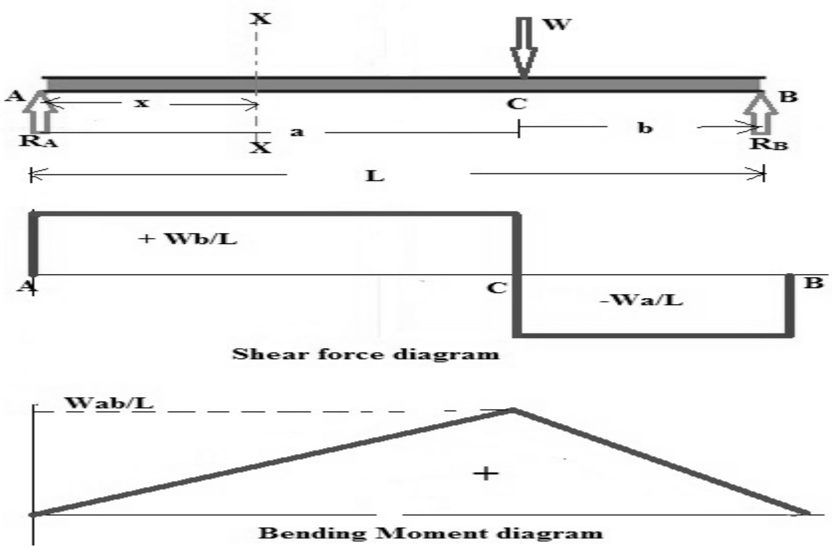


Shear force diagram

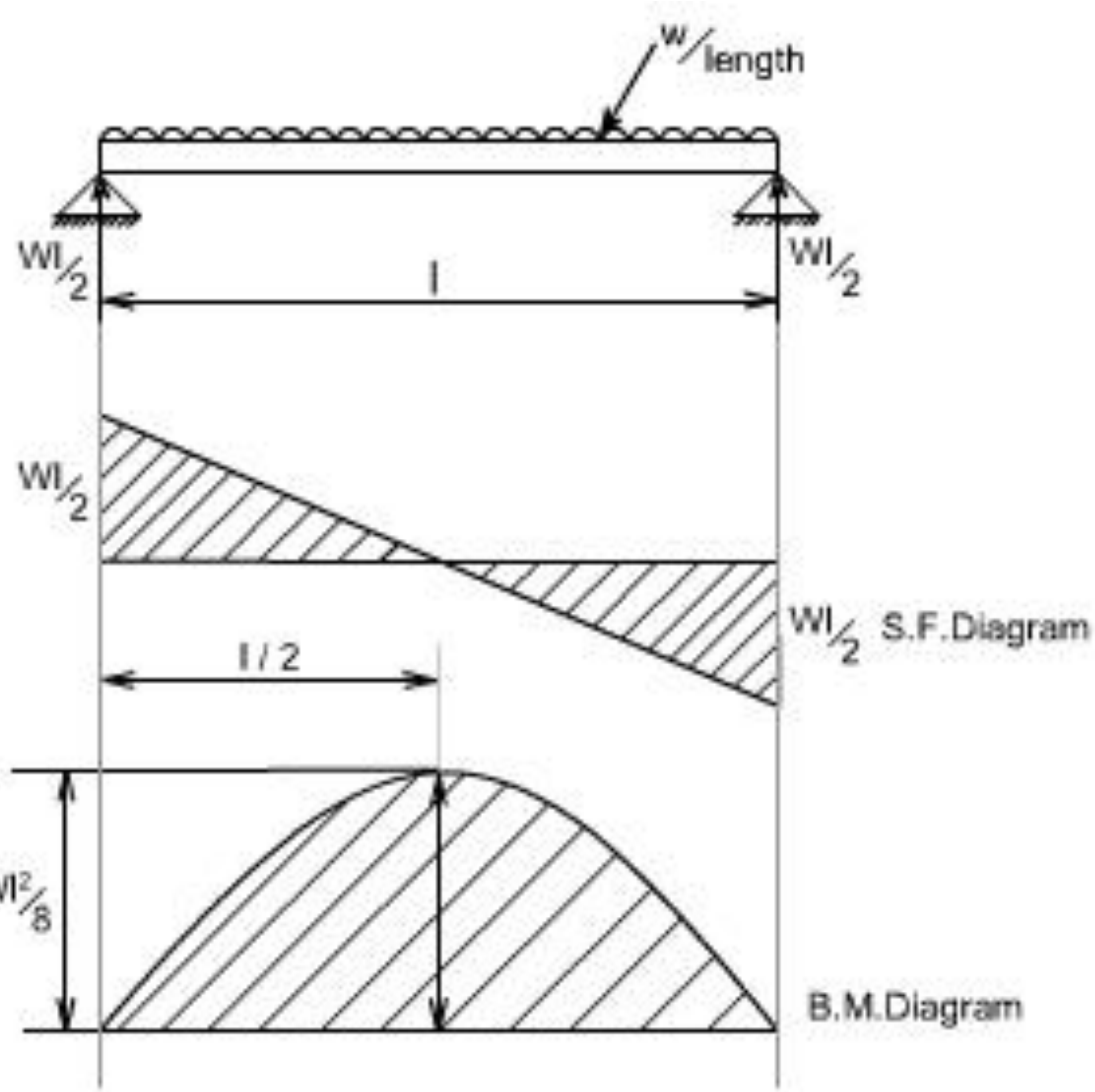


Bending Moment diagram

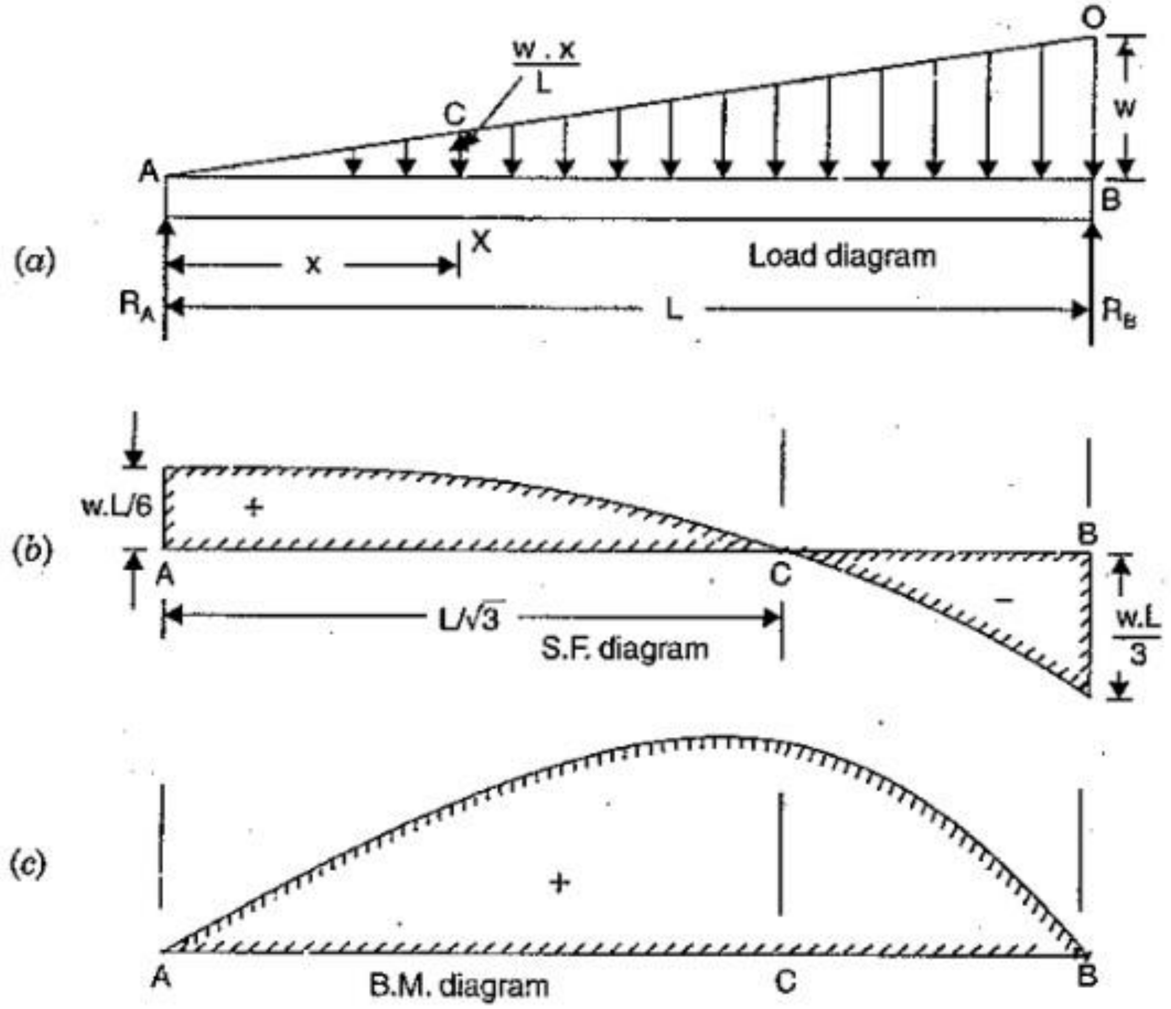
SHEAR FORCE & BENDING MOMENT DIAGRAM OF A SIMPLY SUPPORTED BEAM CARRYING ECCENTRIC POINT LOAD :



SHEAR FORCE & BENDING MOMENT DIAGRAM OF A SIMPLY SUPPORTED BEAM CARRYING UNIFORMLY DISTRIBUTED LOAD



SHEAR FORCE & BENDING MOMENT DIAGRAM OF A SIMPLY SUPPORTED BEAM CARRYING UNIFORMLY VARYING LOAD



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=IpAf9D9s3OU

Important Books/Journals for further learning including the page nos.:

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.276-298.

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L -13

LECTURE HANDOUTS

RA

IV / II

Course Name with Code : **19RAC09- STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II- Transverse Loading on Beams and Stresses in Beam**

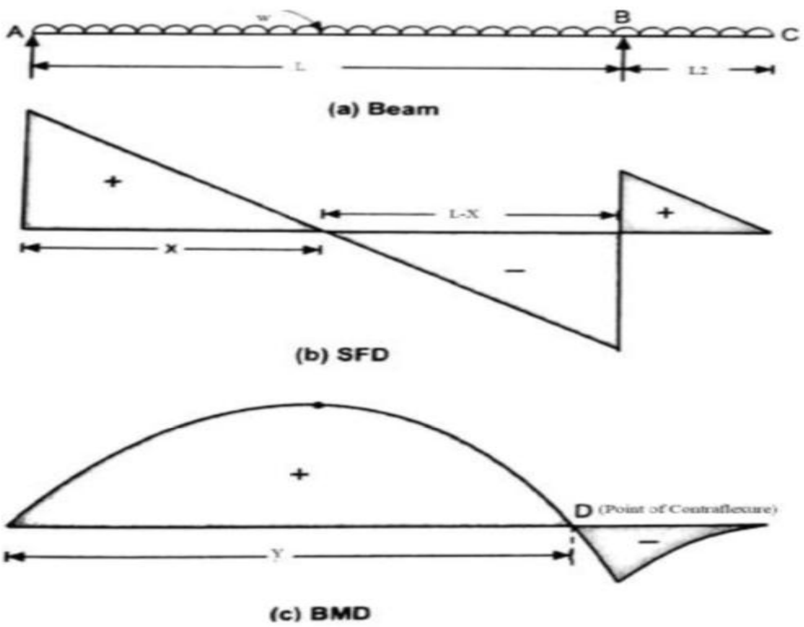
Topic of Lecture: Shear force and Bending moment diagrams – Overhanging beams

Introduction :

1. If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam.
2. In case of overhanging beams, the B.M. is positive between the two supports, whereas the B.M. is negative for the overhanging portion.
3. Hence at some point the B.M. is zero after changing its sign from positive to negative or vice versa.
4. That point is known as the point of contra flexure or point of inflexion.

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Types of beams
2. Transverse loading
3. Shear force
4. Bending moment



1. Draw the SFD and BMD for the beam which is loaded as shown in the fig. Determine the point of contraflexure within the span AB.

Solution:

Taking moments about A, we have

$$R_B \times 8 + 800 \times 3 = 2000 \times 5 + 1000(8 + 2)$$

or $$8R_B + 2400 = 10000 + 10000$$

$\therefore$ $$R_B = \frac{20000 - 2400}{8} = \frac{17600}{8} = 2200 \text{ N}$$

and $$R_A = \text{Total load} - R_B = 3800 - 2200 = 1600$$

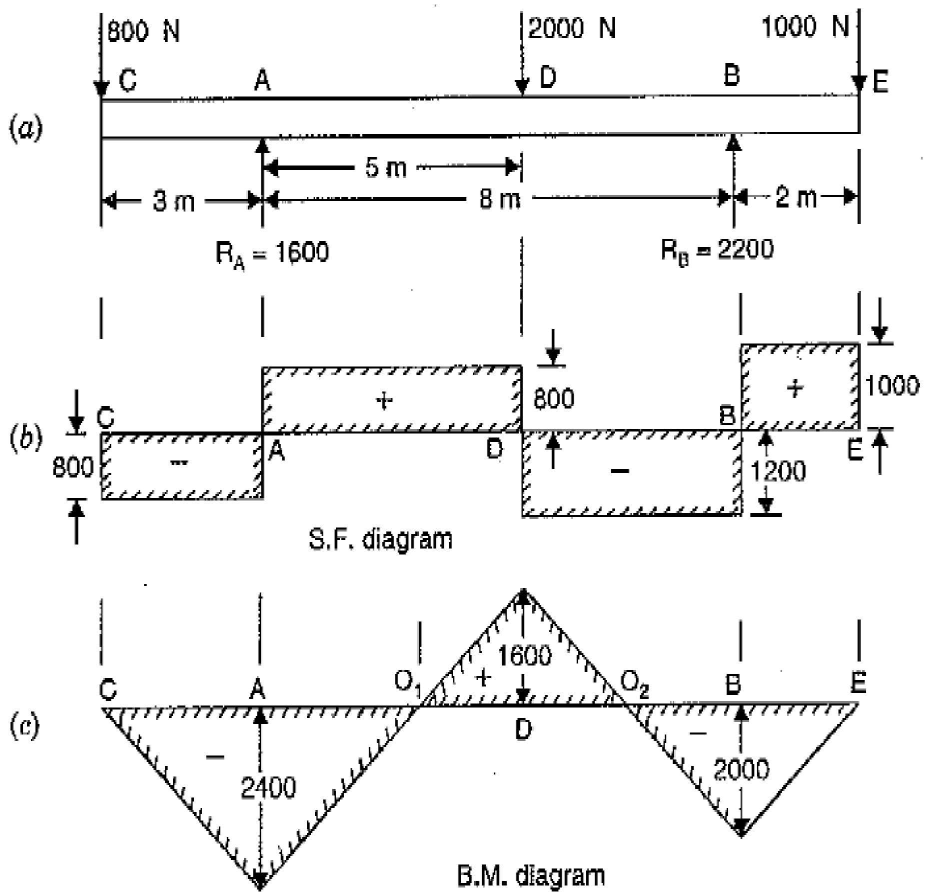


Fig. 6.38

S.F. Diagram

S.F at C $= -800$ N

S.F. between C and A remains -800 N

S.F. at A $= -800 + R_A = -800 + 1600 = +800$ N

S.F. between A and D remains $+800$ N

S.F. at D $= +800 - 2000 = -1200$ N

S.F. between D and B remains -1200 N

S.F. at B $= -1200 + R_B = -1200 + 2200 = +1000$ N

S.F. between B and E remains $+1000$ N

B.M. Diagram

B.M. at C $= 0$

B.M. at A $= -800 \times 3 = -2400$ Nm

B.M. at D $= -800 \times (3+5) + R_A \times 5$

$= -800 \times 8 + 1600 \times 5$

$= -6400 + 8000 = +1600$ Nm

B.M. at B $= -1000 \times 2 = -2000$ Nm

B.M. at E $= 0$

The B.M. diagram is drawn as shown in Fig. 6.38 (*c*).

Points of Contraflexure

There will be two points of contraflexure O_1 and O_2, where B.M. becomes zero after changing its sign. Point O_1 lies between A and D, whereas the point O_2 lies between D and B.

(*i*) Let the point O_1 is x metre from A.

Then B.M. at O_1 $= -800(3+x) + R_A \times x = -800(3+x) + 1600x$

$= -2400 - 800x + 1600x = -2400 + 800x$

But B.M. at O_1 is zero

$$\therefore \quad O = -2400 + 800x \quad \text{or} \quad x = \frac{2400}{800} = 3 \text{ m.} \quad \textbf{Ans.}$$

(*ii*) Let the point O be x metre from B.

Then B.M. at O_2 $= 1000(x+2) - R_B \times x = 1000x + 2000 - 2200 \times x = 2000 - 1200x$

But B.M. at O_2 $= 0$

$$\therefore \quad O = 2000 - 1200x$$

$$\therefore \quad x = \frac{2000}{1200} = \frac{5}{3} = 1.67 \text{ m from B.} \quad \textbf{Ans.}$$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=m6P18bc0xS8

Important Books/Journals for further learning including the page nos.:
A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.298-311.

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L - 14

LECTURE HANDOUTS

RA

IV / II

Course Name with Code : **19RAC09- STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II – Transverse Loading on Beams & Stresses in Beam**

Topic of Lecture: Theory of simple bending

Introduction :

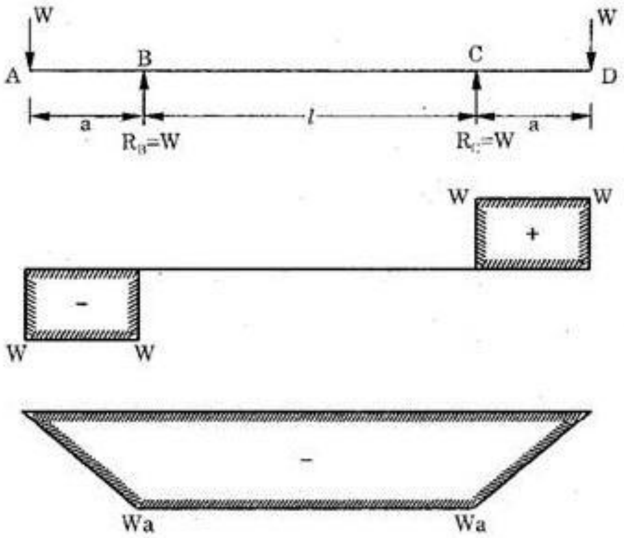
If a length of a beam is subjected to a constant bending moment and no shear force , then the stresses will be setup in that length of beam due to bending moment only and that length of the beam is said to be in pure bending or simple bending**.**

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Types of beams
2. Transverse loads
3. Shear force
4. Bending moment

PURE BENDING OR SIMPLE BENDING:

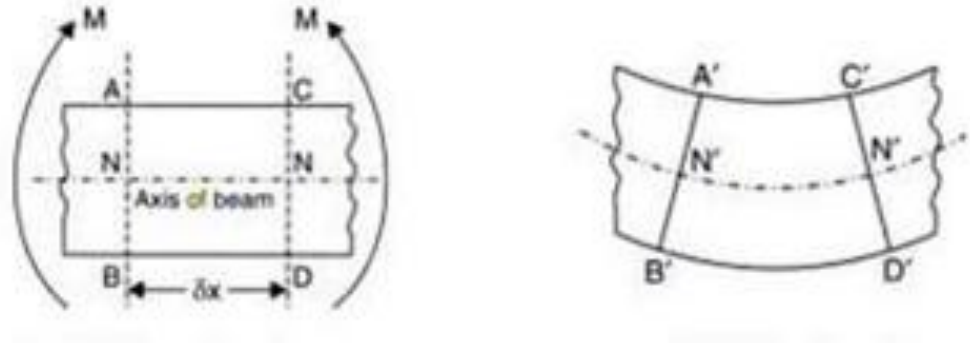
1. Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the overhanging portion
2. In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
3. Hence the portion AB is said to be subjected to pure bending or simple bending



THEORY OF SIMPLE BENDING ASSUMPTIONS MADE

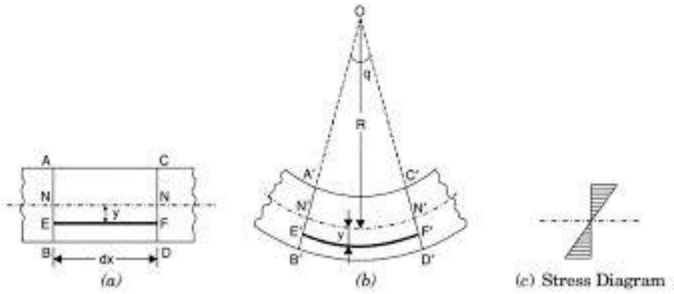
1. Beam is initially straight, and has a constant cross-section.
2. Beam is made of homogeneous material and the beam has a longitudinal plane of symmetry.
3. Resultant of the applied loads lies in the plane of symmetry.
4. The geometry of the overall member is such that bending not buckling is the primary cause of failure.
5. Elastic limit is nowhere exceeded and 'E' is same in tension and compression.
6. Plane cross – sections remains plane before and after bending.

THEORY OF SIMPLE BENDING



1. Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam and parallel to each other
2. Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
3. The layers of the beam are not of the same length before bending and after bending.
4. The layer AC is shortened to A'C'. Hence it is subjected to compressive stress
5. The layer BD is elongated to B'D'. Hence it is subjected to tensile stresses.
6. Hence the amount of shortening decrease from the top layer towards bottom and the amount of elongation decreases from the bottom layer towards top
7. Therefore there is a layer in between which neither elongates nor shortens. This layer is called neutral layer.
8. The filaments/ fibers of the material are subjected to neither compression nor to tension
9. The line of intersection of the neutral layer with transverse section is called neutral axis (N-N).
10. Hence the theory of pure bending states that the amount by which a layer in a beam subjected to pure bending, increases or decreases in length, depends upon the position of the layer w.r.t neutral axis N-N.

EXPRESSION FOR BENDING STRESS



f/ y = E / R

NEUTRAL AXIS(N.A.)

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section.

MOMENT OF RESISTANCE

Due to the tensile and compressive stresses, forces are exerted on the layers of a beam subjected to simple bending

These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section

Bending equation

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

M is expressed in N mm ; I in mm^4

F is expressed in N/mm^2 ; y in mm

E is expressed N/mm^2; R in mm

Condition of simple bending

In actual practice, a member is subjected to such loading that the B.M. varies from section to section and also the shear force is not zero. but shear force is zero at a section where bending moment is maximum.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=H96aaks551k

Important Books/Journals for further learning including the page nos.:

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.326-332.

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LECTURE HANDOUTS

RA | **IV/III**

Course Name with Code : **19RAC09- STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II- Transverse Loading on Beams and Stresses in Beam**

Topic of Lecture: Bending stress distribution

Introduction :

1. Bending stress across the depth of any cross section, whether solid or hollow, depends on the strain distribution.
2. Strain distribution across the depth of cross section is linear (at least according to the basic assumption of the theory of simple bending).
3. This means that stress distribution across the depth of a hollow cross section is identical to that in a solid section but because there is no material in a hollow section at certain places, its moment of resistance is much lesser.
4. However, this is based on the assumption that the position of the hollow does not alter the location of the neutral axis.
5. If neutral axis is shifted due to the hollow portion, then strain distribution and consequently stress distribution would be different but could be calculated using the same basic principles

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Simple bending / pure bending
2. Neutral axis
3. Neutral layer
4. Moment of resistance
5. Tensile & compressive stresses

Bending stress distribution

Let M be the moment of resistance of a beam section and I be the moment of inertia of the beam section about the neutral axis.

The bending stress, σ at any layer in the beam section, distance y from the neutral axis is given by the expression $\sigma = \frac{M}{I} \times y$. The maximum bending stress occurs at the outermost layer.

Let y_{max} be the distance of the outermost layer from the neutral axis and the maximum bending stress at that layer be σ_{max}.

Thus,

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$\therefore \; M = \frac{I}{y_{max}} \times \sigma_{max}$$

$$= Z \times \sigma_{max}$$

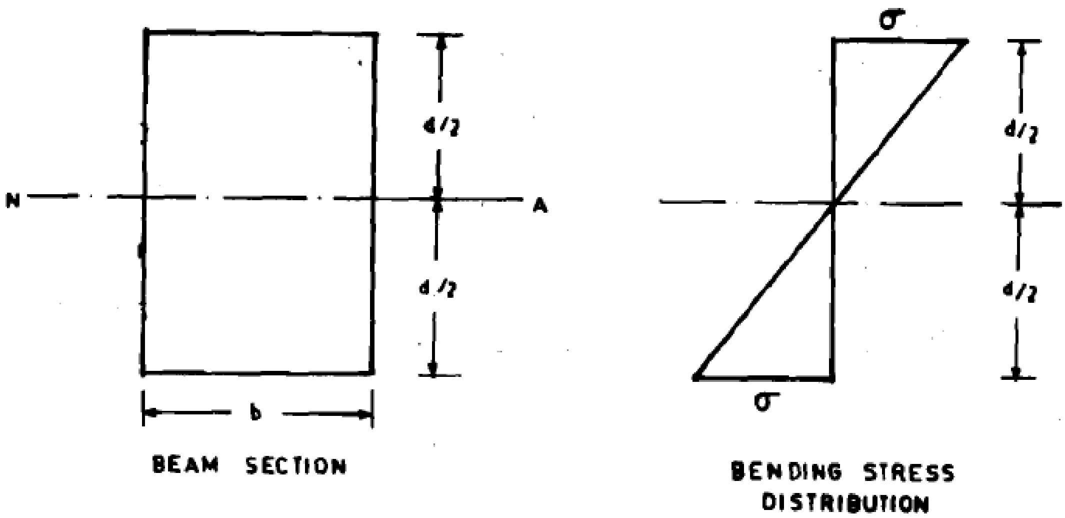
where Z is known as the section modulus. So the section modulus is the ratio of the moment of inertia of the beam section about the neutral axis to the distance of the outermost layer of the beam from the neutral axis.

If σ_{max} is equal to the permissible bending stress, then M represents the greatest moment of resistance of the beam section.

Solid rectangular section

Let b be the breadth and d be the depth of a rectangular beam section.

The neutral axis coincides with the centroidal axis of the beam.



Moment of inertia, $I = \frac{1}{12} bd^3$

Distance of outermost layer from the neutral axis, $y_{max} = \frac{d}{2}$

$$\therefore \text{Section modulus, } Z = \frac{I}{y_{max}}$$

$$= \frac{1}{12} bd^3 \times \frac{2}{d}$$

$$= \frac{1}{6} bd^2$$

Let σ be the maximum bending stress at the outermost layer.

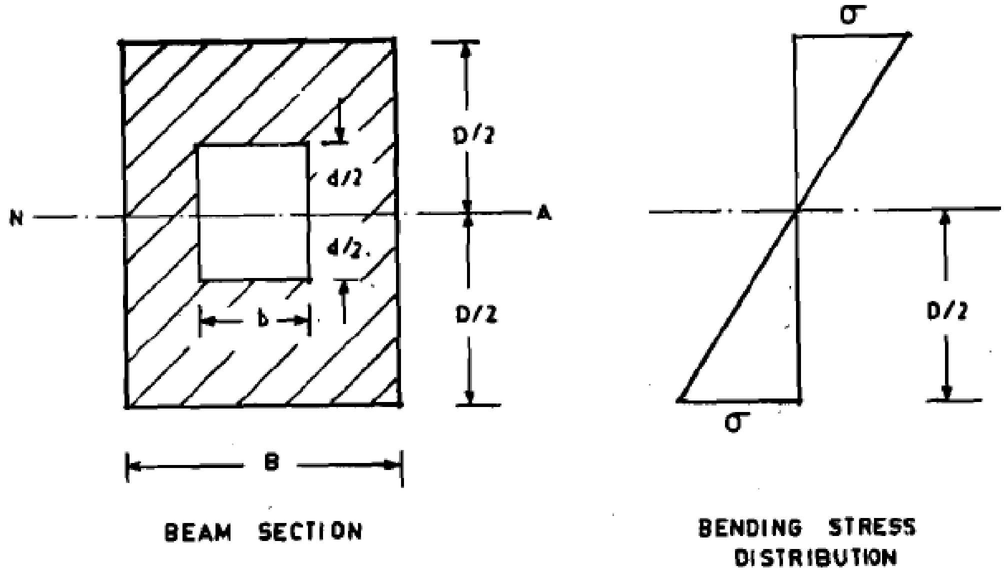
$\therefore$ Moment of resistance, $M = \sigma \times Z = \frac{1}{6} \sigma \, bd^2$

Hollow rectangular section

Let the breadth and depth of the central rectangular hole be b and d, respectively.

Moment of inertia, $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

Distance of outermost layer from the neutral axis, $y_{max} = \frac{D}{2}$



$$\therefore \text{Section modulus, } Z = \frac{I}{y_{max}}$$

$$= \frac{1}{12}\left[BD^3 - bd^3\right] \times \frac{2}{D}$$

$$= \left[\frac{BD^3 - bd^3}{6D}\right]$$

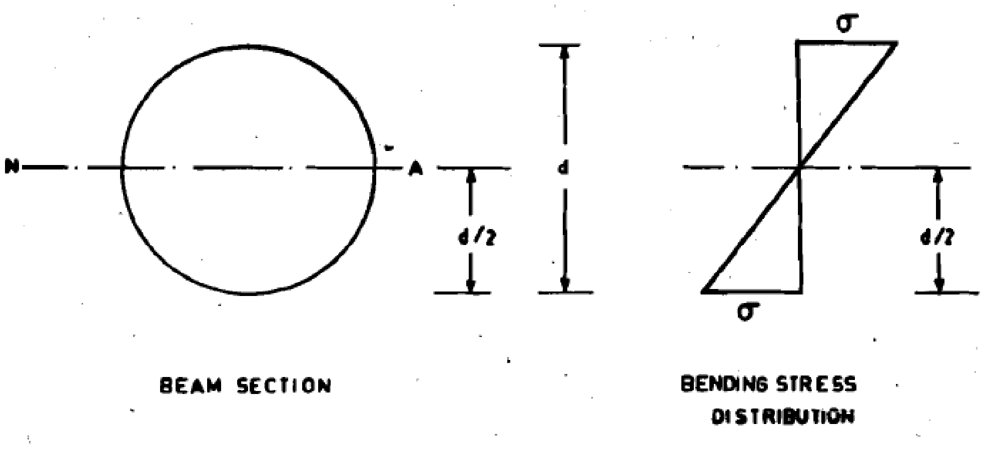
Let σ be the maximum bending stress at the outermost layer.

$$\therefore \text{Moment of resistance,} \quad M = \sigma \times Z = \frac{\sigma}{6D}\left[BD^3 - bd^3\right]$$

Solid circular section

Moment of inertia about the neutral axis, $I = \frac{\pi d^4}{64}$

Distance of outermost layer from the neutral axis, $y_{max} = \frac{d}{2}$



$$\text{Section modulus, } Z = \frac{I}{y_{max}}$$

$$= \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$= \frac{\pi d^3}{32}$$

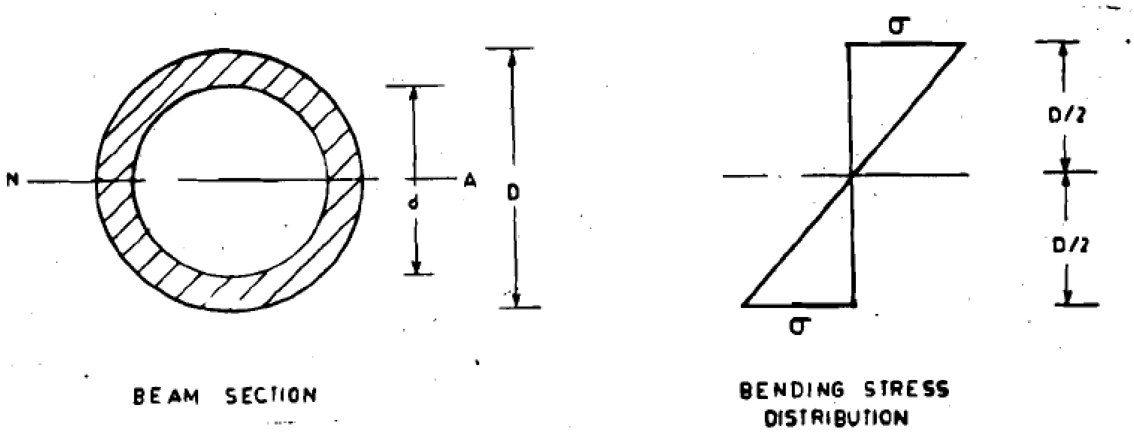
Let σ be the maximum bending stress at the outermost layer.

$$\therefore \text{ Moment of resistance, } M = \sigma \times Z$$

$$= \sigma \frac{\pi d^3}{32}$$

Hollow circular section

Let the external and internal diameter of the hollow circular section be D and d respectively.



$$\text{Moment of inertia about the neutral axis, } I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$\text{Distance of outermost layer from the neutral axis, } y_{max} = \frac{D}{2}$$

$$\therefore \text{ Section modulus, } Z = \frac{I}{y_{max}}$$

$$= \frac{\pi}{64}\left[D^4 - d^4\right] \times \frac{2}{D}$$

$$= \frac{\pi}{32D}\left[D^4 - d^4\right]$$

Let σ be the maximum bending stress at the outermost layer.

$\therefore$ Moment of resistance, $M = \sigma \times Z$

$$= \frac{\sigma\pi}{32D}\left[D^4 - d^4\right]$$

Video Content / Details of website for further learning (if any):

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.333.

Important Books/Journals for further learning including the page nos.:

https://www.youtube.com/watch?v=-ecNfGxSv-k

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LECTURE HANDOUTS

RA | **IV / II**

Course Name with Code : **16 MED07 STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II-Transverse Loading on Beams and Stresses in Beam**

Topic of Lecture: Load carrying capacity of beams

Introduction :

The load carrying capacity of a beam can be found by

1. Calculate Max. Moment due to it and apply M/ Z
2. Where M is max. Moment and Z is modulus of section, which is bearing capacity of section.
3. Section modulus (Z) is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.
4. Maximum bending moment, M is the moment of resistance offered by the section.
5. M is maximum when Z is maximum. Hence section modulus represents the strength of a section

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Moment of inertia
2. Bending moment
3. Shear force
4. Bending stresses in beams
5. Theory of simple bending

Section modulus for various shapes of beam sections

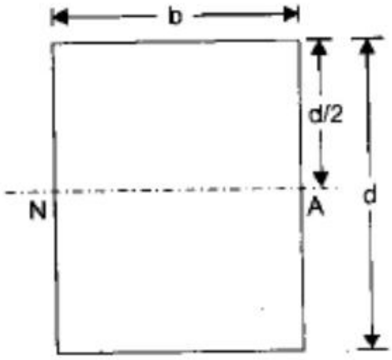
1. Rectangular section
2. Hollow rectangular section
3. Circular section
4. Hollow circular section

1. Rectangular Section

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{bd^2}{6}$$

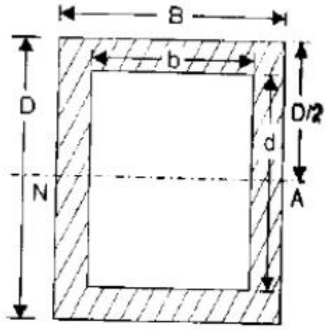


2. Rectangular Hollow Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \left(\frac{D}{2}\right)$$

$$Z = \frac{1}{6D}\left[BD^3 - bd^3\right]$$

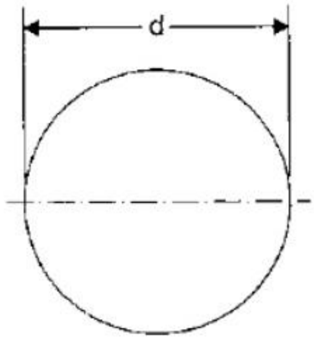


3. Circular Section

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\pi}{32} d^3$$

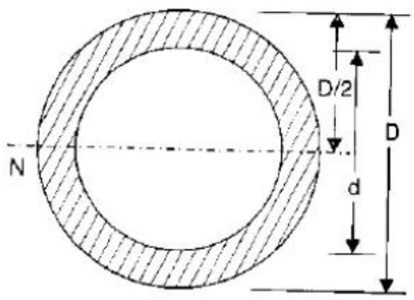


4. Circular Hollow Section

$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{32D}\left[D^4 - d^4\right]$$



1. A cantilever of length 2 meter fails when a load of 2 kN is applied at the free end. If the section of the beam is 40mm×60 mm, find the stress at the failure.

Given:

Length , L = 2m = 2×10^3mm

Load , W = 2kN = 2000N.

Section of beam is 40 mm × 60 mm.

∴ Width of beam, $b = 40$ mm

Depth of beam, $d = 60$ mm

Fig. 7.10

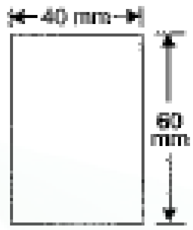


Fig. 7.10 (a)

Fig. 7.10 (a) shows the section of the beam.

Section modulus of a rectangular section is given by equation (7.7).

$$\therefore \quad Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

Maximum bending moment for a cantilever shown in Fig. 7.10 is at the fixed end.

$$\therefore \quad M = W \times L = 2000 \times 2 \times 10^3 = 4 \times 10^6 \text{ Nmm}$$

Let σ_{max} = Stress at the failure

Using equation (7.6), we get

$$M = \sigma_{max} \cdot Z$$

$$\therefore \quad \sigma_{max} = \frac{M}{Z} = \frac{4 \times 10^6}{24000} = \mathbf{166.67 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=QpjM319bejw

Important Books/Journals for further learning including the page nos.:

A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.333-351

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LECTURE HANDOUTS

RA

IV / II

Course Name with Code : **19RAC09- STENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Unit : **II- Transverse Loading on Beams and Stresses in Beam**

Topic of Lecture: Proportioning of sections

Introduction :

1. In symmetrical sections, the neutral axis passes through the geometrical centre of the section.
2. But in case of unsymmetrical sections, the N.A. does not pass through the geometrical centre of the section.
3. Hence the value of y for the topmost or bottom most layer of the section from neutral axis will not be the same.
4. N.A. **passes** through the centre of gravity of the section
5. So bending stresses is different.

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Centre of gravity
2. Centroid
3. Centroid /centre of gravity of simple plane figures
4. Method of moments
5. Moment of inertia

Detailed content of the Lecture:

1. A cast iron bracket subject to bending has the cross0section of I-form with unequal flanges.The dimensions of the section are shown in the figure. Find the posision of the Neutral axis and moment of inertia of the section about the N.A.. If the maximum bending moment on the section is 40 MNmm, determne the maximum bending stress. What is the nature of the stress?

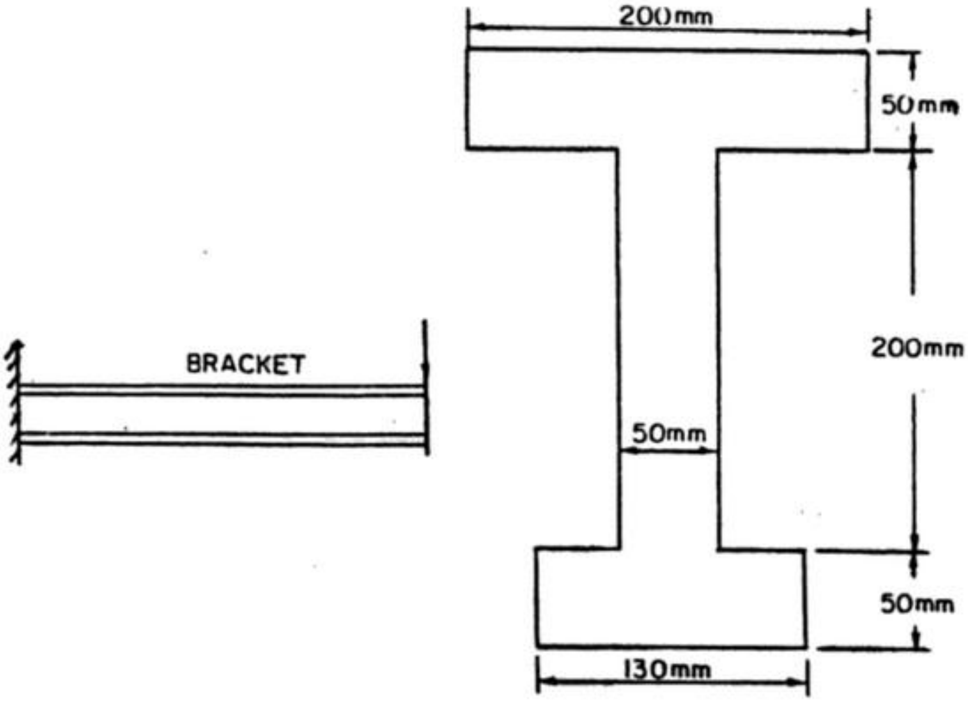
Solution : Given

Max. B.M., $M = 40$ MN mm $= 40 \times 10^6$ Nmm

Let us first calculate the C.G. of the section. Let $\bar{y}$ is the distance of the C.G. from the bottom face. The section is symmetrical about y-axis and hence $\bar{y}$ is only to be calculated. Then,

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{(A_1 + A_2 + A_3)}$$

where A_1 = Area of bottom flange
$= 130 \times 50 = 6500 \text{ mm}^2$



y_1 = Distance of C.G. of A_1 from bottom face

$= \frac{50}{2} = 25$ mm

A_2 = Area of web

$= 200 \times 50 = 10000 \text{ mm}^2$

y_2 = Distance of C.G. of A_2 from bottom face

$= 50 + \frac{200}{2} = 150$ mm

A_3 = Area of top flange

$= 200 \times 50 = 10000 \text{ mm}^2$

y_3 = Distance of C.G. of A_3 from bottom face

$= 50 + 200 + \frac{50}{2} = 275$ mm.

$$\therefore \qquad \bar{y} = \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000}$$

$$= \frac{162500 + 1500000 + 2750000}{26500}$$

$$= \frac{4412500}{26500} = 166.51 \text{ mm}$$

Hence neutral axis is at a distance of 166.51 mm from the bottom face. **Ans.**

Moment of inertia of the section about the N.A.

$$I = I_1 + I_2 + I_3$$

where I_1 = M.O.I. of bottom flange about N.A.

= M.O.I. of bottom flange about an axis passing through its C.G. $+ A_1 \times$ (Distance of its C.G. from N.A.)^2

$$= \frac{130 \times 50^3}{12} + 6500 \times (166.51 - 25)^2$$

$$= 1354166.67 + 130163020$$

$$= 131517186.6 \text{ mm}^4$$

Similarly I_2 = M.O.I. of web about N.A.

$$= \frac{50 \times 200^3}{12} + A_2 . (166.51 - y_2)^2$$

$$= \frac{50 \times 200^3}{12} + 10000\,(166.51 - 150)^2$$

$$= 33333333.33 + 272580.1$$

$$= 33605913.43 \text{ mm}^4$$

and I_3 = M.O.I. of top flange about N.A.

$$= \frac{200 \times 50^3}{12} + A_3 . (y_3 - 166.51)^2$$

$$= \frac{200 \times 50^3}{12} + 10000 \times (275 - 166.51)^2$$

$$= 2083333.33 + 117700801$$

$$= 119784134.3 \text{ mm}^4$$

$\therefore$ $I = I_1 + I_2 + I_3$

$$= 131517186.6 + 33605913.43 + 119784134.3$$

$$= 284907234.9 \text{ mm}^4. \quad \textbf{Ans.}$$

Now distance of C.G. from the upper top fibre

$$= 300 - \bar{y} = 300 - 166.51 = 133.49 \text{ mm}$$

and the distance of C.G. from the bottom fibre

$$= \bar{y} = 166.51 \text{ mm}$$

Hence we shall take the value of $y = 166.51$ mm for maximum bending stress.

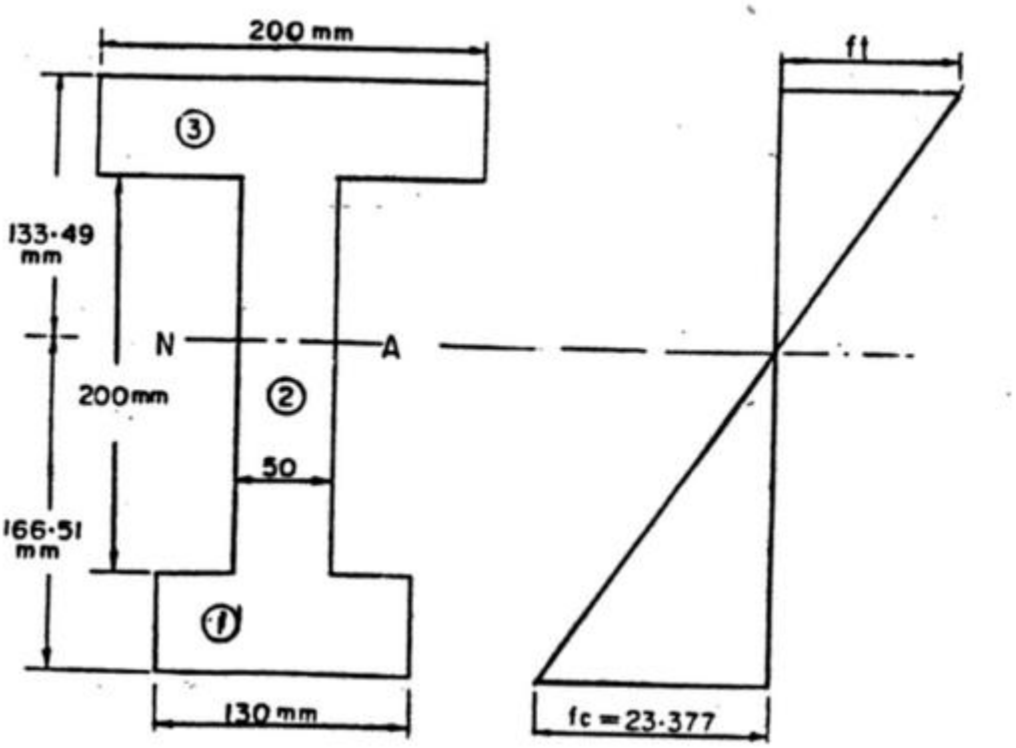
Now using the equation

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \quad f = \frac{M}{I} \times y = \frac{40 \times 10^6}{284907234.9} \times 166.51 = 23.377 \text{ N/mm}^2$$

$\therefore$ Maximum bending stress

$$= 23.377 \text{ N/mm}^2. \quad \textbf{Ans.}$$



This stress will be compressive. In case of cantileers , upper layer is subjected to tensile stress, whereas the lower layer is subjected to compressive stress.

Video Content / Details of website for further learning (if any):
https://nptel.ac.in/content/storage2/courses/114106043/Tutorial%205.pdf

Important Books/Journals for further learning including the page nos.:
A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.245-255.

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LECTURE HANDOUTS

RA

IV / II

Course Name with Code : 19RAC09- STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Unit : II- Transverse Loading on Beams and Stresses in Beam

Topic of Lecture: Shear stress distribution

Introduction :

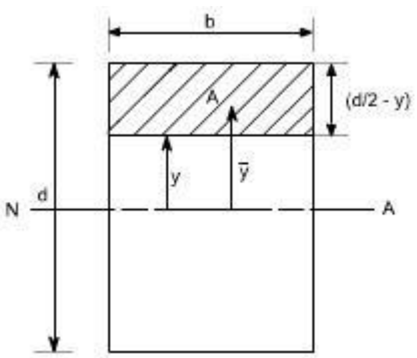
1. In actual practice, a beam is subjected to a bending moment which varies from section to section.
2. Also the shear force acting on the section is not zero.
3. It also varies from section to section
4. Hence beam will also be subjected to shear stresses
5. In this chapter the distribution of the shear stress across the various sections will be determined

Prerequisite knowledge for Complete understanding and learning of Topic:

1. Types of beams
2. Transverse loading of beams
3. Stresses on beams
4. Shear forces on beams
5. Bending moment on beams

Shearing stress distribution in typical cross-sections:

Rectangular section

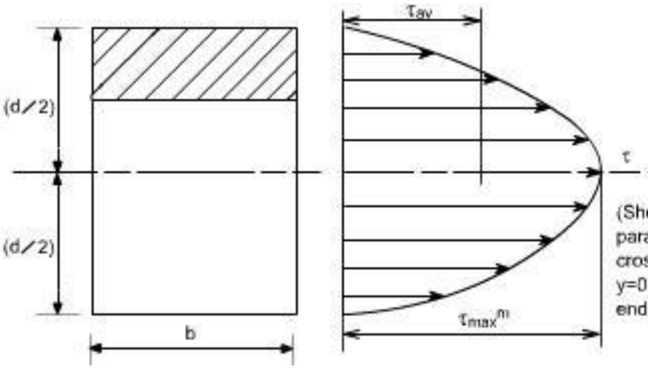


Shear stress , τ

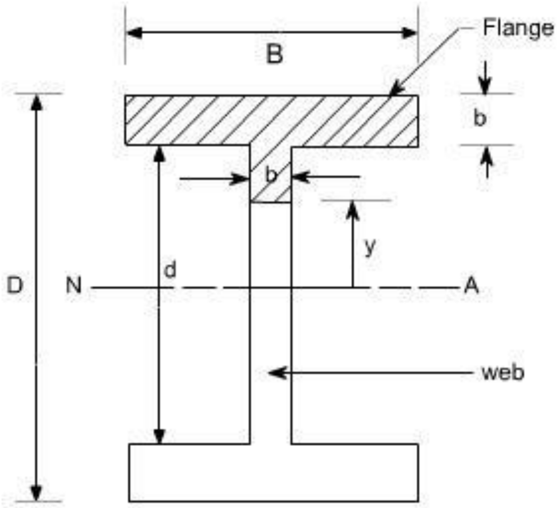
$$\tau = \frac{6.F.\left\{\left(\frac{d}{2}\right)^2 - y^2\right\}}{b\,d^3}$$

Maximum shear stress , τ_{max} = 3F / 2 b.d

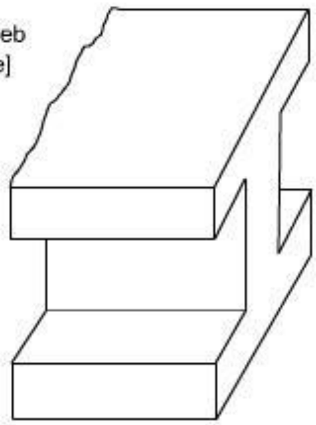
Shear stress distribution



“ I “ – section



[Here flange and web thickness are same]



Shear stress,

$$\tau = \frac{F}{b\,I}\left[\frac{B\left(D^2 - d^2\right)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

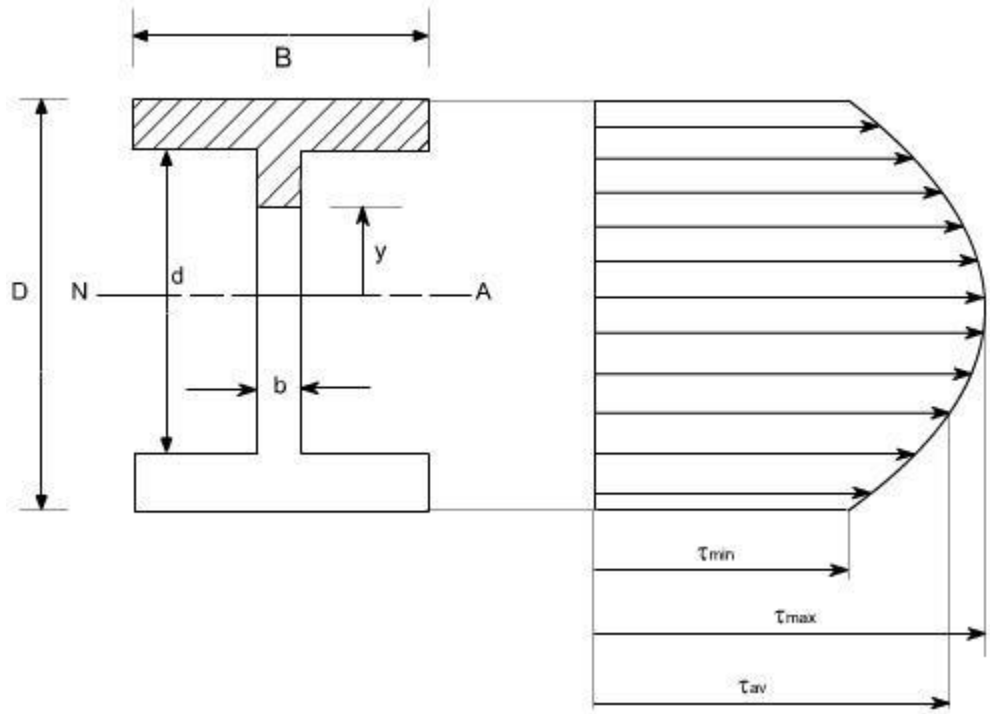
Maximum shear stress ,

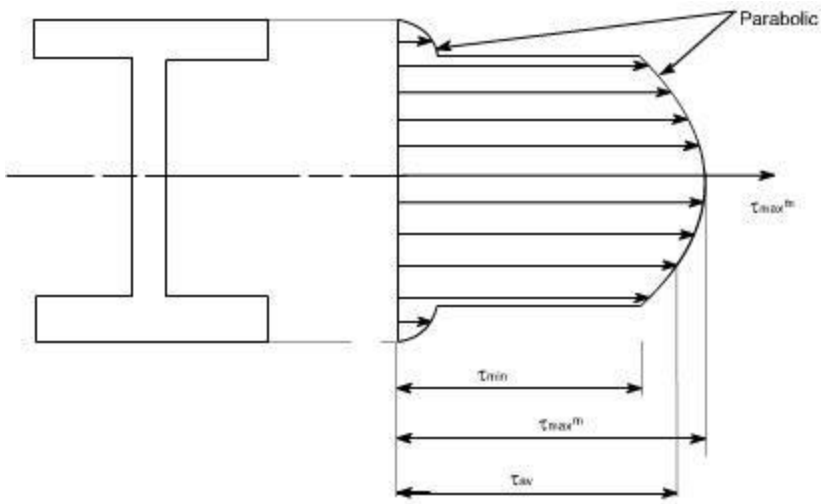
$$\tau_{max} \text{ at } y = 0 = \frac{F}{8\,b\,I}\left[B\left(D^2 - d^2\right) + bd^2\right]$$

Minimum shear stress,

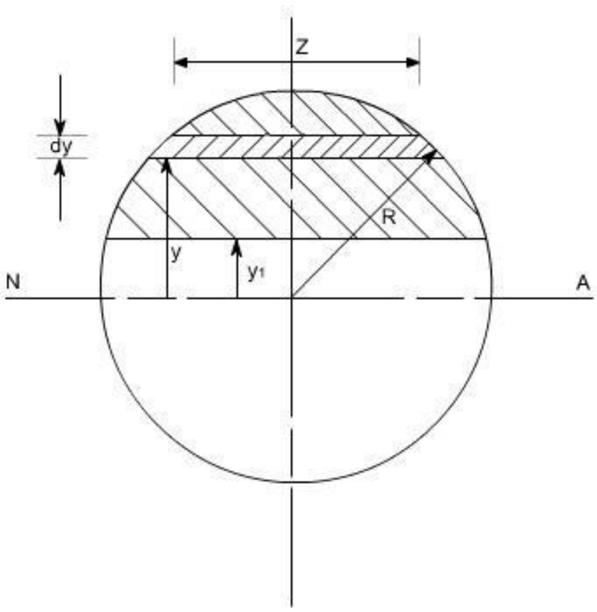
$$\tau_{min} \text{ at } y = d/2 = \frac{F}{8\,b\,I}\left[B\left(D^2 - d^2\right)\right]$$

Shear stress distribution



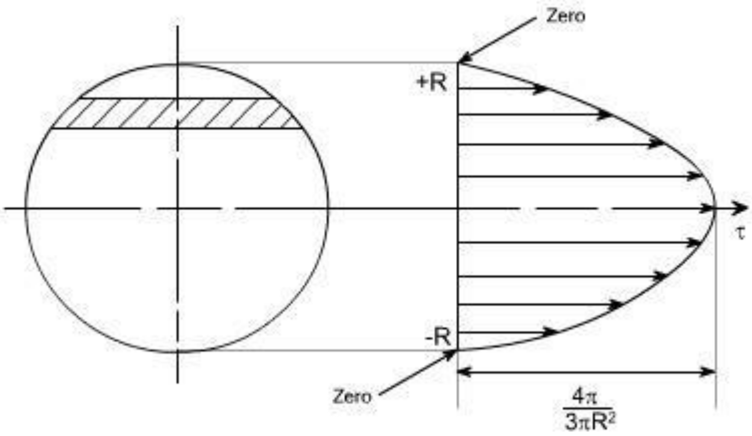


Circular section



Shear stress, $\tau = \frac{4F(R^2 - y_1^2)}{3\pi R^4}$

Shear stress distribution



Video Content / Details of website for further learning (if any):
http://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/homepage.htm

Important Books/Journals for further learning including the page nos.:
A Textbook of Strength of materials, Dr.R.K.Bansal, Lakshmi publications (p) Ltd., 2002. Pp.389-430.

LECTURE HANDOUTS

RA | **II / IV**

Course Name with Code : **19RAC09- STRENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Torsion in circular Shaft- formulation stresses

Introduction :

1. Torsion is the twisting of an object due to an applied torque.
2. A common example of torsion in engineering is when a transmission drive shaft (such as in an automobile) receives a turning force from its power source (the engine).
3. Torsional shear stress or Torsional stress is the shear stress produced in the shaft due to the twisting. This twisting in the shaft is caused by the couple acting on it.
4. The theory of Torsion is based on the following Assumptions: The material in the shaft is uniform throughout. The twist along the shaft is uniform.
5. Torsional stresses arise when a twisting force acts on a body due to the applied torque. Shafts are mostly subjected to torsional stresses. Shear stresses arise when forces act tangential to the surface of the body in opposite directions.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Torsion: When a pair of force of equal magnitude, but opposite direction acting on body it tends to twist the body.

Modulus of Elasticity in Shear: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear or Modulus of Rigidity.

Shaft: The shaft is the machine element which is used to transmit power in machines.

Twisting Moment: The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration.

Modulus of Elasticity in Shear: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear or Modulus of Rigidity.

Polar Modulus: It is the ratio between polar moment of inertia and radius or shaft.

Stiffness: The stiffness of spring is defined as the load required to produce unit deflection.

Definition of Torsion: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.

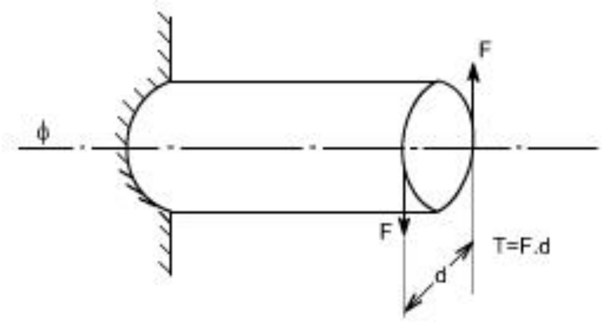


Fig. Shaft in Torsion

Assumption:

(i) The materiel is homogenous i.e. of uniform elastic properties exists throughout the material.
(ii) The material is in elastic, follows Hook's law, with shear stress proportional to shear strain.
(iii) The stress does not exceed the elastic limit.
(iv) The circular section remains constant
(v) Cross section remains plane.
(vi) Cross section rotates as if rigid i.e. every diameter rotates through the same angle of twist

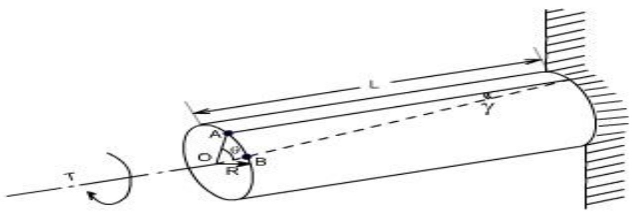


Fig. Torsional shaft

Effects of Torsion: The effects of a torsional load applied to a bar are

(i) To impart an angular displacement of one end cross section with respect to the other end.

(ii) To setup shear stresses on any cross section of the bar perpendicular to its axis

We want to find the maximum shear stress τ_{max} which occurs in a circular shaft of radius c due to the application of a torque T. Using the assumptions above, we have, at any point r inside the shaft, the shear stress is $\tau_r = r/c\ \tau_{max}$.

$\int \tau_r dA\ r = T$

$\int r^2/c\ \tau_{max}\ dA = T$

$\tau_{max}/c \int r^2\ dA = T$

Now, we know,

$J = \int r^2\ dA$

is the polar moment of inertia of the cross sectional area..

Thus, the maximum shear stress

$\tau_{max} = Tc/J$

The above equation is called the *torsion formula*.

Now, for a solid circular shaft, we have,

$J = \pi/32(d)^4$

Further, for any point at distance r from the center of the shaft, we have, the shear stress τ is given by

$\tau = Tr/J$

We only consider the torsional loading of simple circular shafts in this analysis, i.e. cylinders or non-eccentric tubes without splits. Circular shafts are most commonly used as torque carrying members in machinery with rotating parts (like drive shafts of motors). This is fortuitous, as the analysis of non circular members under torsion is not simple to perform analytically.

Video Content / Details of website for further learning (if any):

1.https://www.youtube.com/watch?v=pRK9hsarRN8

Important Books/Journals for further learning including the page nos.:

1. A Text Book of Theory of Machines By J. S. Brar, R. K. Bansal Page No: 692.

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L - 20

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09- STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Deformation in circular, hollow shafts

Introduction :

1. A shaft excavated as a cylinder. The circular shaft is equally strong at all points; convenient for concrete lining and tubbing, both of which can be made relatively watertight; and offers the least resistance to airflow.
2. Generally for power transmission, circular shafts are used because there is uniform stress distribution along any radius of the shaft.
3. Plane sections of shaft remain plane after the application of twisting moment, as a result there is no distortion in the sections of shafts and change in volume of the shaft is zero.
4.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Analysis of torsion of circular bars:

In workshops and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied either to the rim of a pulley, keyed to the shaft or at any other suitable point at some distance from the axis of the shaft. The product of this turning force and the distance between the point of application of the force and the axis of the shaft is known as torque, turning moment or twisting moment. And the shaft is said to be subjected to torsion. Due to this torque, every cross section of the shaft is subjected to some shear stress.

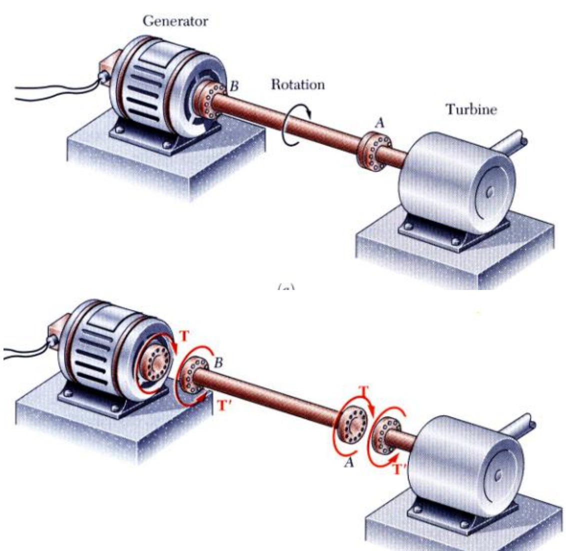

Fig. Torsion in shaft with transmission

Circular Bars or Shafts:

The shafts are usually cylindrical in section, solid or hollow. They are made of mild steel, alloy steel and copper alloys. Shaft may be subjected to the following loads:

1. Torsional load
2. Bending load
3. Axial load
4. Combination of above three loads.

The shafts are designed on the basis of strength and rigidity.

Duty of Shafts:

Shafts are also used to transmit power from a motor to a pump or compressor, from an engine or turbine to a generator, and from an engine to axle in automobiles. During power transmission, the shaft is subjected to torque which causes twist of the shaft.

Differentiate between torque and torsion in shafts:

When a structural or machine member is subjected to a moment about its longitudinal axis, the member twists and shear stress is induced in every cross section of the member. Such a mode of loading is called **Torsion.** And the twisting moment is referred to as **Torque**.

Pure Torsion:

A circular shaft is said to be in a state of pure torsion when it is subjected to torque only, without being acted upon by any bending moment or axial force. Otherwise if the shaft is subjected to two opposite turning moment it is said to be in pure torsion. and it will exhibit the tendency of shearing off at every cross-section which is perpendicular to longitudinal axis.

Generally two types of stresses are induced in a shaft.

1. Torsional (Shear) stresses due to transmission of torque.
2. Bending stresses due to weight of pulley, gear etc mounted on shaft.

Polar Moment of Inertia (J):

The M.I. of a plane about an axis perpendicular to the plane of the area is called polar moment of inertia of the area with respect to the point at which the axis intersects the plane.

Polar moment of inertia of solid body (J) = Ixx + Iyy = $\frac{\pi}{32}d^4$ mm^4

Polar moment of inertia of hollow body (J) = Ixx + Iyy = $\frac{\pi}{32}\left(D^4 - d^4\right)$ mm^4

Section Modulus (Z):

Section modulus is the ratio of MI about the neutral axis divided by the most distant point from the neutral axis. Z = $\frac{I}{y_{max}}$ mm^3

Section modulus for circular solid shaft (Z) = $\left(\frac{\left(\pi/64\right)D^4}{D/2}\right) = \frac{\pi}{32}D^3$

Section modulus for circular hollow shaft (Z) = $\left[\frac{\left(\pi/64\right)\left(D^4 - d^4\right)}{D/2}\right] = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$

Section modulus for rectangular section (Z) = $\left[\frac{bd^3/12}{d/2}\right] = \frac{bd^2}{6}$

Polar Modulus (Zp):

It is the ratio of polar moment of inertia to outer radius. Zp = $\frac{J}{R}$ mm^3

Polar Modulus of solid body (Zp) = $\left(\frac{\left(\pi/32\right)D^4}{D/2}\right) = \frac{\pi}{16}D^3$

Polar modulus of hollow body (Zp) = $\left[\frac{\left(\pi/32\right)\left(D^4 - d^4\right)}{D/2}\right] = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)$

Video Content / Details of website for further learning (if any):
1. https://www.youtube.com/watch?v=pRK9hsarRN8 2. https://www.youtube.com/watch?v=ICDZ5uLGrI4
Important Books/Journals for further learning including the page nos. :
1. A Text Book of Theory of Machines by J. S. Brar, R. K. Bansal Page No: 692. 2. A Text Book of Machine Design by R.S.KHURMI and J.K.GUPTA Page No: 120.

Course Teacher

Verified by HoD

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L - 21

LECTURE HANDOUTS

RA | **II / IV**

Course Name with Code : 19RAC09 STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Deformation hollow shafts / Stepped , simple problems

Introduction :

1. In the hollow shaft, the material at the center is removed and spread at large radius. Therefore, hollow shafts are stronger than solid shaft having the same weight.
2. The stiffness of the hollow shaft is more than the solid shaft with the same weight.
3. Hollow shafts are much lighter than solid shafts and can transmit same torque like solid shafts of the same dimensions. Moreover less energy is necessary to acceleration and deceleration of hollow shafts. Therefore hollow shafts have great potential for use in power transmission in automotive industry.
4. Stepped Shaft refers to tubular steel shafts which taper in a series of steps rather than one continuous narrowing.
5. Shafts are manufactured in stepped as well as plain form. The main advantage of stepped shaft over plain shaft is its high Torsional Rigidity.

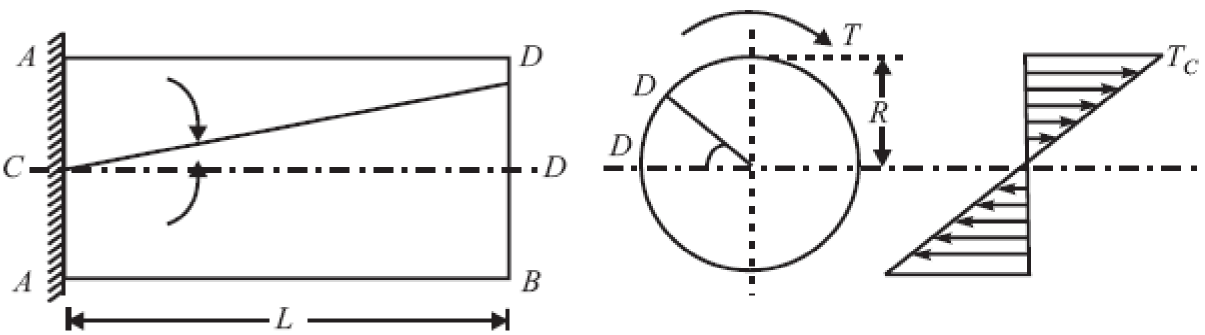
Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Assumptions for Shear Stress in a Circular Shaft Subjected to Torsion:

1. The material of the shaft is uniform throughout.
2. The shaft circular in section remains circular after loading.
3. A plane section of shaft normal to its axis before loading remains plane after the torques has been applied.
4. The twist along the length of shaft is uniform throughout.
5. The distance between any two normal cross-sections remains the same after the application of torque.
6. Maximum shear stress induced in the shaft due to application of torque does not exceed its elastic limit value. Torsion equation is derived as follows



Let,
T =

Maximum twisting torque or twisting moment

D = Diameter of the shaft

R = Radius of the shaft

J = Polar moment of Inertia

τ = Max. Permissible Shear stress (Fixed for a given material)

G = Modulus of rigidity

θ = Angle of twist (Radians) = angle *D'OD*

L = Length of the shaft.

$\emptyset$ = Angle *D'CD* = Angle of Shear strain

Than **Torsion equation is:** $\frac{T}{J} = \frac{\tau}{R} = \frac{G.\theta}{L}$

Let the shaft is subjected to a torque or twisting moment '*T*'. And hence every *C.S.* of this shaft will be subjected to shear stress.

Now distortion at the outer surface = *DD'*

$$\text{Shear strain at outer surface} = \frac{Distortion}{Unit\ length}$$

$$\tan \emptyset = \frac{DD'}{CD}$$

$$\text{i. e. shear stress at the outer surface}, \tan \emptyset = \frac{DD'}{CD}$$

Now *DD'* = R.θ or $\emptyset$ = R . θ/L

Now *G* = Shar stress induced/shear strain produced

$$G = \frac{\tau}{\frac{R.\theta}{l}} \quad or \quad \frac{\tau}{R} = \frac{G.\theta}{L}$$

This equation is called Stiffness equation:

Hear *G*, θ, *L* are constant for a given torque '*T*'.

i.e., τ is proportional to *R*

If τ be the intensity of shear stress at any layer at a distance '*r*' from canter of the shaft, then;

$$\frac{\tau}{R} = \frac{G.\theta}{L}$$

Now Torque in terms of Polar Moment of Inertia,

From the fig

Area of the ring $(dA) = 2\,\pi r \cdot dr$

Since, $\tau r = (\tau/R) \cdot r$

Turning force on Elementary Ring $= (\tau/R) \cdot r \cdot 2\pi r dr.$

$= (\tau/R).2\,\pi r^2.dr$ ...(*i*)

Turning moment $dT = (\tau/R).2\,\pi r^2.dr.r$

$dT = (\tau/R).\,r^2 \cdot 2\,\pi.r \cdot dr = (\tau/R).\,r^2 \cdot dA$

$$T = (\tau/R)\int_0^R r^2 dA$$

$\int_0^R r^2 dA = M.I.$ of elementary ring about an axis perpendicular to the plane passing through center of circle.

$\int_0^R r^2 dA = J$ Polar Moment of Inertia

Now from equation (*ii*) $T = (\tau/R) \cdot J$

$$\frac{T}{J} = \frac{\tau}{R}$$

Combined equation *A* and *B*; we get

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G.\theta}{L}$$

This equation is called Torsion equation.

Strength of Solid Circular Shaft (T_{max}):

Strength of a shaft may be defined as the maximum torque which can be applied to the Shaft without exceeding allowable shear stress and angle of twist.

FOR SOLID SHAFT

From the torsion equation: $T/J = \tau/R = G.\theta/L$

Since $T = \tau.J/R$

$= \tau. [(\pi D^4/32)/(D/2)]$

$= \tau. (\pi/16)D^3$

$$\mathbf{T_{max} = (\pi/16)\ \tau_{max}.D^3}$$

Again since $T/J = G.\theta/L$

$T = J.G.\theta/L = (ÀD^4/32)\ .G.\theta/L$

$$\mathbf{T_{max} = (\pi D^4/32)\ .G.\theta_{max}/L}$$

Where,

T_{max} = Maximum torque

θ_{max} = Maximum angle of twist

τ_{max} = Maximum shear stress

Note: The strength of the shaft is the minimum value of T_{max} from equation (*i*) and (*ii*). And for finding out diameter of shaft we take maximum value of dia obtained from equation (*i*) and (*ii*).

Strength of Hollow Circular Shaft (T_{max}):

From the torsion equation: $T/J = \tau/R = G.\theta/L$

Since $T = \tau.J/R$

$= \tau. [(\pi(D^4 - d^4)/32)/(D/2)]$

$= \tau. (\pi/16)\ ((D^4 - d^4)/D)$

$$\mathbf{T_{max} = (\pi/16)\ \tau_{max}.\ ((D^4 - d^4)/D)}$$

As the same for hollow shaft, $\mathbf{Tmax = (\pi/16)\ \tau_{max}.(D^4 - d^4)/D}$

$= (\pi/32)\ (D^4 - d^4)\ .G.\theta_{max}/L$

1. A hollow shaft of external diameter 120 mm transmits 30 kW power at 200 rpm. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2.

Solution:

Since we have

Power transmitted = 2π NT/60

300,000 = 2π × 200T/60

T = 14323.9 Nm

Also we have

$T = \pi/16.\ \tau_{max}.(d_0^4 - d_i^4 / d_0)$

$14323.9 \times 10^3 = \pi/16 * 60\ [(120)^4 - d_i^4 / 120]$

$d_i^4 = 61458000$

$d_i = 88.5$ mm

2. A round steel bar, 4 cm in diameter is subjected to a twisting torque of a kN.cm over a length of 1.5 meter find the maximum shearing stress and twist in degrees.

Solution:

Since we have

$T/J = \tau /r = G\Theta/L$

$\tau = T/J.r = 8000/t/32 * (4)^4\ (4/2) = 637$ N/cm^2

$\Theta = T.L/G.J = 8000*150 / 80 \times 10^5 * \Theta/32 * (4)^4$

$= 0.006$ rad. $= 0.006 * 180/\pi$

$= 0.34^o$

3. A solid shaft of mild steel 200 mm in diameter is to be replaced by hollow shaft of allowable shear stress is 22% greater. If the power to be transmitted is to be increased by 20% and the speed of rotation increased by 6%, determine the maximum internal diameter of the hollow shaft. The external diameter of the hollow shaft is to be 200 mm.

Solution:

Given that:

Diameter of solid shaft $d = 200$ mm

For hollow shaft diameter, $d_0 = 200$ mm

Shear stress; $\tau_H = 1.22\ \tau_s$

Power transmitted; $P_H = 1.20\ P_s$

Speed $N_H = 1.06\ N_s$

As the power transmitted by hollow shaft

$P_H = 1.20\ P_s$

$(2\pi.N_H.T_H)/60=(2\pi.N_s.T_s)/60\times1.20$

$N_H.T_H = 1.20\ N_s.T_s$

$\rightarrow$ $1.06\ N_s.T_H = 1.20\ N_sT_s$

$\rightarrow$ $1.06/1.20\ T_H = T_s$

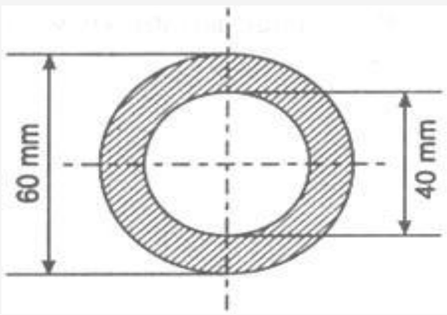
$\rightarrow$ $1.06/1.20 \times \pi/16\ t_H\ [(d_0)^4 - (d_i)^4/d_0] = \pi/16\ t_s.[d]^3$

$\rightarrow$ $1.06/1.20 \times 1.22\ t_s\ [(200)^4 - (d_i)^4/200] = t_s \times [200]^3$

$\rightarrow$ $d_i = 104$ mm

4. For one propeller drive shaft, compute the torsional shear stress when it is transmitting a torque of 1.76 kNm. The shaft is a hollow tube having an outside diameter of 60 mm and an inside diameter of 40 mm. Find the stress at both the outer and inner surfaces.

Solution: Given that



Torque transmitted 'T' = 1.76 kNm

Outside diameter of tube $D_0 = 60$ mm

Inside diameter of tube Di = 40 mm

Now, using the torsion equation

T/j =τ/r

where J is polar moment of inertia

For hollow shaft

$$J = \frac{\pi}{2}(r_0^4 - r_i^4) \; or \; \frac{\pi}{32}(D_0^4 - D_i^4)$$

$$\tau = \frac{T}{J}.r$$

Torsional stress on outer surface is;

τ_0 = T/J (D_0/2) = (1.76 × 1000 × 1000) * (60/2) / p/32 ((60)^4 – (40)^4)

→ τ_i = 34.475 N/mm^2

Thus, the stresses on outer and inner surfaces are 51.713 N/mm^2 and 34.475 N/mm^2 respectively.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=hiLnnTg4C8g
2. https://www.youtube.com/watch?v=pd_1Kwe_vKY
3 https://www.youtube.com/watch?v=yJXKoBTN0
4 https://www.youtube.com/watch?v=iaNnY2bR4wI

Important Books/Journals for further learning including the page nos.:

1. A Text Book of Theory of Machines by J. S. Brar, R. K. Bansal Page No: 689- 690; 693-694.

2. A Text Book of Machine Design by R.S.KHURMI and J.K.GUPTA Page No: 121-123.

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L - 22

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : 19RAC09 STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Stepped Shafts & Deflection of the shaft at fixed end

Introduction :

1. In the hollow shaft, the material at the center is removed and spread at large radius. Therefore, hollow shafts are stronger than solid shaft having the same weight.
2. The stiffness of the hollow shaft is more than the solid shaft with the same weight.
3. Hollow shafts are much lighter than solid shafts and can transmit same torque like solid shafts of the same dimensions. Moreover less energy is necessary to acceleration and deceleration of hollow shafts. Therefore hollow shafts have great potential for use in power transmission in automotive industry.
4. Stepped Shaft refers to tubular steel shafts which taper in a series of steps rather than one continuous narrowing.
5. Shafts are manufactured in stepped as well as plain form. The main advantage of stepped shaft over plain shaft is its high Torsional Rigidity.

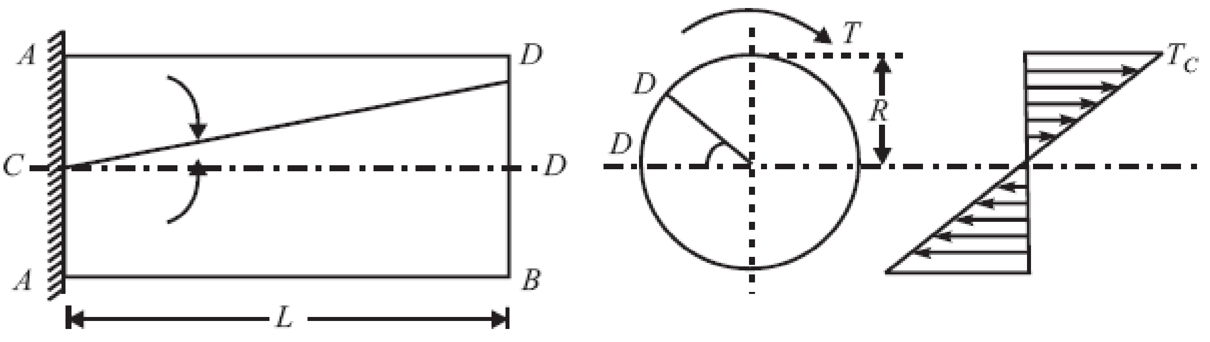
Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Assumptions for Shear Stress in a Circular Shaft Subjected to Torsion:

1. The material of the shaft is uniform throughout.
2. The shaft circular in section remains circular after loading.
3. A plane section of shaft normal to its axis before loading remains plane after the torques has been applied.
4. The twist along the length of shaft is uniform throughout.
5. The distance between any two normal cross-sections remains the same after the application of torque.
6. Maximum shear stress induced in the shaft due to application of torque does not exceed its elastic limit value. Torsion equation is derived as follows



Let,

T =

Maximum twisting torque or twisting moment
D = Diameter of the shaft
R = Radius of the shaft
J = Polar moment of Inertia
τ = Max. Permissible Shear stress (Fixed for a given material)
G = Modulus of rigidity
θ = Angle of twist (Radians) = angle $D'OD$
L = Length of the shaft.
$\emptyset$ = Angle $D'CD$ = Angle of Shear strain

Than **Torsion equation is:** $\frac{T}{J} = \frac{\tau}{R} = \frac{G.\theta}{L}$

Let the shaft is subjected to a torque or twisting moment 'T'. And hence every $C.S.$ of this shaft will be subjected to shear stress.

Now distortion at the outer surface = DD'

$$\text{Shear strain at outer surface} = \frac{Distortion}{Unit\ length}$$

$$\tan \emptyset = \frac{DD'}{CD}$$

$$\text{i.e. shear stress at the outer surface}, \tan \emptyset = \frac{DD'}{CD}$$

Now DD' = R.θ or $\emptyset$ = R . θ/L

Now G = Shar stress induced/shear strain produced

$$G = \frac{\tau}{\frac{R.\theta}{l}} \quad or \quad \frac{\tau}{R} = \frac{G.\theta}{L}$$

This equation is called Stiffness equation:

Hear G, θ, L are constant for a given torque 'T'.

i.e., τ is proportional to R

If τ be the intensity of shear stress at any layer at a distance 'r' from canter of the shaft, then;

$$\frac{\tau}{R} = \frac{G.\theta}{L}$$

Now Torque in terms of Polar Moment of Inertia,

From the fig

Area of the ring $(dA) = 2\ \pi r \cdot dr$

Since, $\tau r = (\tau/R) \cdot r$

Turning force on Elementary Ring $= (\tau/R) \cdot r \cdot 2\pi r dr$.
$= (\tau/R).2\ \pi r^2.dr$ ...(i)

Turning moment $dT = (\tau/R).2\ \pi r^2.dr.r$
$dT = (\tau/R).\ r^2 \cdot 2\ \pi.r \cdot dr = (\tau/R).\ r^2 \cdot dA$

$$T = (\tau/R) \int_0^R r^2 dA$$

$\int_0^R r^2 dA = M.I.$ of elementary ring about an axis perpendicular to the plane passing through center of circle.

$\int_0^R r^2 dA = J$ Polar Moment of Inertia

Now from equation (ii) $T = (\tau/R) \cdot J$

$$\frac{T}{J} = \frac{\tau}{R}$$

Combined equation A and B; we get

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G.\theta}{L}$$

This equation is called Torsion equation.

Strength of Solid Circular Shaft (T_{max}):

Strength of a shaft may be defined as the maximum torque which can be applied to the Shaft without exceeding allowable shear stress and angle of twist.

FOR SOLID SHAFT

From the torsion equation: $T/J = \tau/R = G.\theta/L$

Since $T = \tau.J/R$

$= \tau. [(\pi D^4/32)/(D/2)]$

$= \tau. (\pi/16)D^3$

$$\boldsymbol{T_{max} = (\pi/16)\, \tau_{max}.D^3}$$

Again since $T/J = G.\theta/L$

$T = J.G.\theta/L = (ÀD^4/32)\, .G.\theta/L$

$$\boldsymbol{T_{max} = (\pi D^4/32)\, .G.\theta_{max}/L}$$

Where,

T_{max} = Maximum torque

θ_{max} = Maximum angle of twist

τ_{max} = Maximum shear stress

Note: The strength of the shaft is the minimum value of T_{max} from equation (*i*) and (*ii*). And for finding out diameter of shaft we take maximum value of dia obtained from equation (*i*) and (*ii*).

Strength of Hollow Circular Shaft (T_{max}):

From the torsion equation: $T/J = \tau/R = G.\theta/L$

Since $T = \tau.J/R$

$= \tau. [(\pi(D^4 - d^4)/32)/(D/2)]$

$= \tau. (\pi/16)\, ((D^4 - d^4)/D)$

$$\boldsymbol{T_{max} = (\pi/16)\, \tau_{max}.\, ((D^4 - d^4)/D)}$$

As the same for hollow shaft, $\boldsymbol{Tmax = (\pi/16)\, \tau_{max}.(D^4 - d^4)/D}$

$= (\pi/32)\, (D^4 - d^4)\, .G.\theta_{max}/L$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=hiLnnTg4C8g
2. https://www.youtube.com/watch?v=pd_1Kwe_vKY

Important Books/Journals for further learning including the page nos.:

1. A Text Book of Theory of Machines by J. S. Brar, R. K. Bansal Page No: 693-694.

2. A Text Book of Machine Design by R.S.KHURMI and J.K.GUPTA Page No: 121-123.

MUTHAYAMMAL ENGINEERING COLLEGE

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L - 23

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09- STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Stresses –Helical Spring

Introduction :

1. The Helical springs are made up of wire coiled in the form of helix and is primarily intended for compressive or tensile loads. The cross section of wire from which the spring is made it may be either circular or squared.
2. The two forms of helical springs are compression and tension helical spring. These springs are said to be “Closely Coiled” when the spring is coiled so close that the plane containing each turn is nearly at right angle to the axis of helix and wire is subjected to tension.
3. In closely coiled helical spring the helix angle is very small it is usually less than 10 degrees. The major stress produced in helical springs is shear stresses due to twisting.
4. The load applied is either parallel or along spring. In “Open Coiled” helical springs the spring wire is coiled in such a way that there is gap between 2 consecutive turns, as a result of which the helix angle is large.

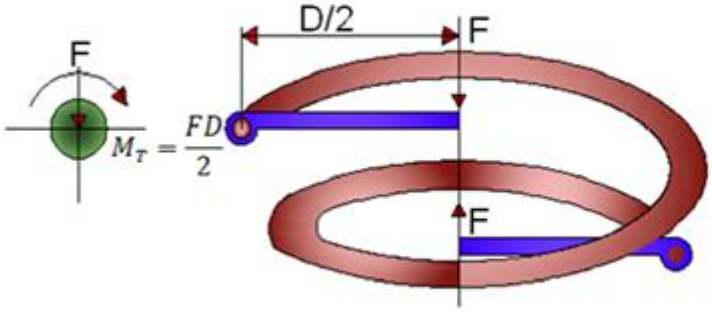
Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.
-

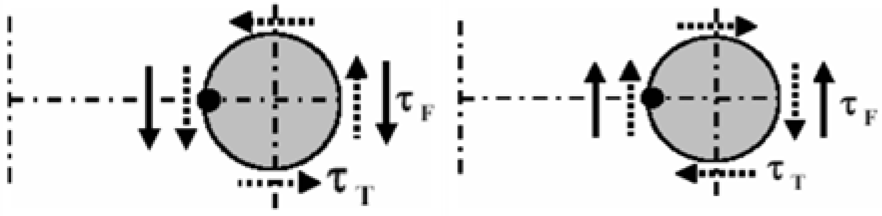
Detailed content of the Lecture:

Stresses in the helical spring wire:

From the free body diagram, we have found out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil.



The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the figure.



The broken arrows show the shear stresses arising due to the torsion T and solid arrows show the shear stresses due to the force F.

It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crake, is always initiated from the inner radius of the spring.

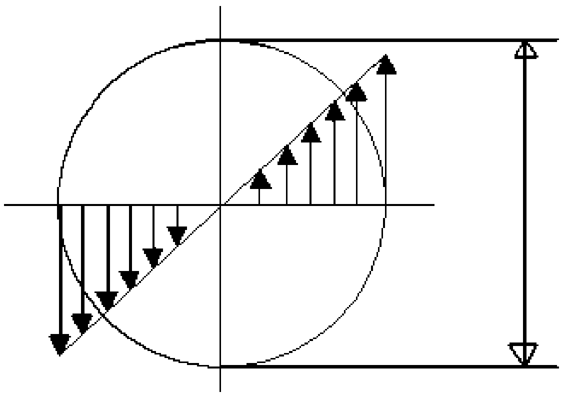
The radius of the spring is given by **D/2**. Note that D is the mean diameter of the spring. The torque T acting on the spring is

$$T = F x \frac{D}{2}$$

If d is the diameter of the coil wire and polar moment of inertia,

$$I_p = \frac{\pi d^4}{32}$$

The shear stress in the spring wire due to torsion is



$$\tau_T = \frac{Tr}{I_p} = \frac{F x \frac{D}{2} x \frac{d}{2}}{\frac{\pi d^4}{32}}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=hiLnnTg4C8g
2. https://www.youtube.com/watch?v=pd_1Kwe_vKY

Important Books/Journals for further learning including the page nos.:

1. A Text Book of Theory of Machines by J. S. Brar, R. K. Bansal Page No: 721-723.

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09- STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Helical Spring Problems

Introduction :

1. The Helical springs are made up of wire coiled in the form of helix and is primarily intended for compressive or tensile loads. The cross section of wire from which the spring is made it may be either circular or squared.
2. The two forms of helical springs are compression and tension helical spring. These springs are said to be “Closely Coiled” when the spring is coiled so close that the plane containing each turn is nearly at right angle to the axis of helix and wire is subjected to tension.
3. In closely coiled helical spring the helix angle is very small it is usually less than 10 degrees. The major stress produced in helical springs is shear stresses due to twisting.
4. The load applied is either parallel or along spring. In “Open Coiled” helical springs the spring wire is coiled in such a way that there is gap between 2 consecutive turns, as a result of which the helix angle is large.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.
-

Detailed content of the Lecture:

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm2 .if the number of active turns or active coils is 8.Estimate the following:

(i) wire diameter

(ii) mean coil diameter

(iii) Weight of the spring.

Assume G = 83,000 N/mm2 ; = 7700 kg/m3

Solution:

(i) for wire diameter if W is the axial load, then

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{\tau_{max^m}}{d/2}$$

$$D = \frac{400}{d/2} \cdot \frac{\pi d^4}{32} \cdot \frac{2}{W}$$

$$D = \frac{400.\pi d^3.2}{5000.16}$$

$$D = 0.0314\, d^3$$

Further, deflection is given as

$$x = \frac{8wD^3.n}{G.d^4}$$

on substituting the relevant parameters we get

$$50 = \frac{8.5000.(0.0314d^3)^3.8}{83{,}000.d^4}$$

$$d = 13.32 mm$$

Therefore,

D = .0314 x (13.317)3mm

=74.15mm

D = 74.15 mm

Weight:

mass or weight = volume . density

= area . length of the spring . density of spring material

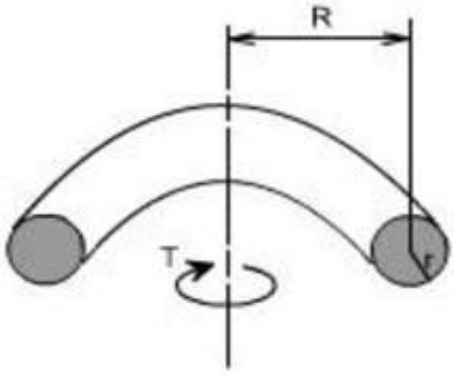
$$= \frac{\pi d^2}{4} . \pi D n . \rho$$

On substituting the relevant parameters we get

Weight = 1.996 kg

≈ 2.0kg

Close coiled helical spring subjected to axial torque T or axial couple



In this case the material of the spring is subjected to pure bending which tends to reduce Radius R of the coils. In this case the bending moment is constant throughout the spring and is equal to the applied axial Torque T. The stresses i.e. maximum bending stress may thus be determined from the bending theory.

$$\sigma_{max} = \frac{M.y}{I}$$

$$= \frac{T.d/2}{\frac{\pi d^4}{64}}$$

$$\sigma_{max} = \frac{32T}{\pi d^3}$$

Close-coiled helical springs. Close-coiled helical springs are the springs in which helix angle is very small or in other words the pitch between two adjacent turns is small. A close-coiled helical spring carrying an axial load is shown in Fig. 16.13. As the helix angle in case of close-coiled helical springs are small, hence the bending effect on the spring is ignored and we assume that the coils of a close-coiled helical springs are to stand purely torsional stresses.

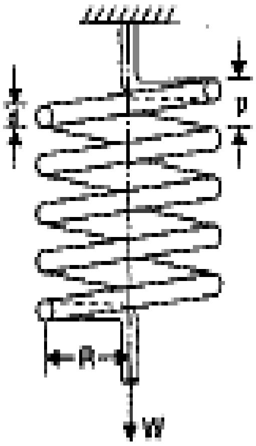


Fig. 16.13

Expression for max. shear stress induced in wire. Fig. 16.13 shows a close-coiled helical spring subjected to an axial load.

Let d = Diameter of spring wire

p = Pitch of the helical spring

n = Number of coils

R = Mean radius of spring coil

W = Axial load on spring

C = Modulus of rigidity

τ = Max. shear stress induced in the wire

θ = Angle of twist in spring wire, and

δ = Deflection of spring due to axial load

l = Length of wire.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=46quOD7V-cQ
2. https://www.youtube.com/watch?v=nRwa8msCbP0

Important Books/Journals for further learning including the page nos.:

Strength of Materials by R.K.Bansal, Page no: 724-725.

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09- STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Problems on Helical Spring

Introduction :

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Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm2 .if the number of active turns or active coils is 8.Estimate the following:

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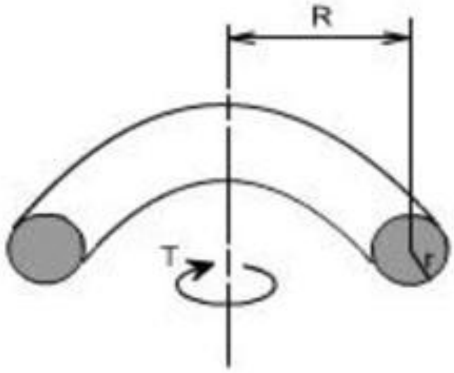
Close coiled helical spring subjected to axial torque T or axial couple



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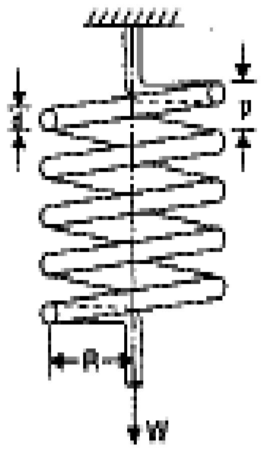

Fig. 16.13

Expression for max. shear stress induced in wire. Fig. 16.13 shows a close-coiled helical spring subjected to an axial load.

Let d = Diameter of spring wire

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n = Number of coils

R = Mean radius of spring coil

W = Axial load on spring

C = Modulus of rigidity

τ = Max. shear stress induced in the wire

θ = Angle of twist in spring wire, and

δ = Deflection of spring due to axial load

l = Length of wire.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=46quOD7V-cQ
2. https://www.youtube.com/watch?v=nRwa8msCbP0

Important Books/Journals for further learning including the page nos.:

Strength of Materials by R.K.Bansal, Page no: 724-725.

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L - 26

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09 STRENGTH OF MATERIALS

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Helical Spring, Carriage Spring

Introduction :

1. A leaf spring is a simple form of spring commonly used for the suspension in wheeled vehicles. Originally called a laminated or carriage spring, and sometimes referred to as a semi-elliptical spring, elliptical spring, or cart spring, it is one of the oldest forms of springing.
2. A leaf spring takes the form of a slender arc-shaped length of spring steel of rectangular cross-section.
3. The leaf spring has seen a modern development in cars. The new Volvo XC90 (from 2016 year model and forward) has a transverse leaf spring in high tech composite materials, a solution that is similar to the latest Chevrolet Corvette.
4. This means a straight leaf spring, that is tightly secured to the chassis, and the ends of the spring bolted to the wheel suspension, to allow the spring to work independently on each wheel. This means the suspension is smaller, flatter and lighter than a traditional setup.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed content of the Lecture:

Applications of springs:

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring-loaded valves.
3. To control motion by maintaining contact between two element as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

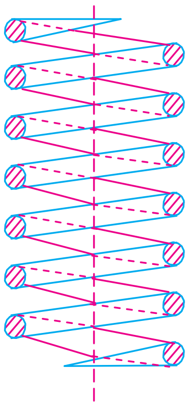
Definition:

A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released.
Important types of springs are:

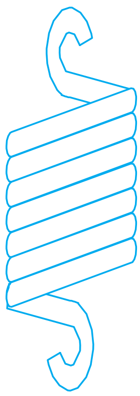
There are various types of springs such as

Helical Spring:
They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting. They are both used in tension and compression.

Fig. Helical Spring



(*a*) Compression helical spring.



(*b*) Tension helical spring.

Spiral springs:

They are made of flat strip of metal wound in the form of spiral and loaded in torsion. In this the major stresses are tensile and compression due to bending.

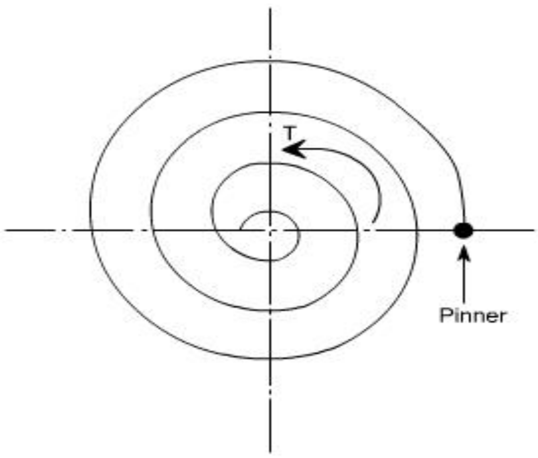


Fig. Spiral spring

Leaf springs:
They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency. Leaf springs may be full elliptic, semi elliptic or cantilever types, In these type of springs the major stresses which come into picture are tensile & compressive.

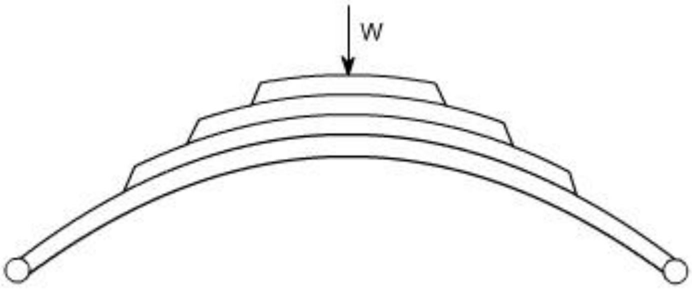


Fig. Leaf spring

These type of springs are used in the automobile suspension system.

Uses of springs :
(a) To apply forces and to control motions as in brakes and clutches.
(b) To measure forces as in spring balance.
(c) To store energy as in clock springs.
(d) To reduce the effect of shock or impact loading as in carriage springs.
(e) To change the vibrating characteristics of a member as inflexible mounting of motors.

Video Content / Details of website for further learning (if any):

1. www.youtube.com/watch?v=tTBnW5gAieM
2. https://www.youtube.com/watch?v=ftW3UgjHWBg

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 721-724.

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L - 27

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 STRENGTH OF MATERIALS**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Problems on Helical and Carriage Springs

Introduction :

1. A leaf spring is a simple form of spring commonly used for the suspension in wheeled vehicles. Originally called a laminated or carriage spring, and sometimes referred to as a semi-elliptical spring, elliptical spring, or cart spring, it is one of the oldest forms of springing.
2. A leaf spring takes the form of a slender arc-shaped length of spring steel of rectangular cross-section.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics, Mathematics and Engineering Mechanics.
- Basic concept on shafts and springs.

Detailed Content of the Lecture

Problem *A close coiled helical spring of 10 cm mean diameter is made up of 1 cm diameter rod and has 20 turns. The spring carries an axial load of 200 N. Determine the shearing stress. Taking the value of modulus of rigidity = 8.4 × 10⁴ N/mm², determine the deflection when carrying this load. Also calculate the stiffness of the spring and the frequency of free vibration for a mass hanging from it.* (AMIE, Winter 1982)

Sol. Given :

Mean diameter of coil, $D = 10$ cm $= 100$ mm

∴ Mean radius of coil, $R = 5$ cm $= 50$ mm

Diameter of rod, $d = 1$ cm $= 10$ mm

Number of turns, $n = 20$

Axial load, $W = 200$ N

Modulus of rigidity, $C = 8.4 \times 10^4$ N/mm²

Let τ = Shear stress in the material of the spring

δ = Deflection of the spring due to axial load

s = Stiffness of spring

τ = Frequency of free vibration.

Using equation (16.24),

$$\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 200 \times 50}{\pi \times 10^3} = \mathbf{50.93\ N/mm^2}. \quad \textbf{Ans.}$$

Deflection of the spring

Using equation (16.26),

$$\delta = \frac{64\,WR^3 \times n}{Cd^4} = \frac{64 \times 200 \times 50^3 \times 20}{8.4 \times 10^4 \times 10^4} = \mathbf{38.095\ mm.} \quad \textbf{Ans.}$$

Stiffness of the spring

$$\text{Stiffness} = \frac{\text{Load on spring}}{\text{Deflection of spring}} = \frac{200}{38.095} = \mathbf{5.25\ N/mm.} \quad \textbf{Ans.}$$

Frequency of free vibration

$$\delta = 38.095 \text{ mm} = 3.8095 \text{ cm}$$

Using the relation, $$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{981}{3.8095}} = \mathbf{2.55\ cycles/sec.} \quad \textbf{Ans.}$$

Problem 16.39. *A closely coiled helical spring of mean diameter 20 cm is made of 3 cm diameter rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take $C = 8 \times 10^4$ N/mm².*

Sol. Given :

Mean dia. of coil, $D = 20$ cm $= 200$ mm

∴ Mean radius of coil, $R = \frac{200}{2} = 100$ mm

Dia. of spring rod, $d = 3$ cm $= 30$ mm

Number of turns, $n = 16$

Weight dropped, $W = 3$ kN $= 3000$ N

Compression of the spring, $\delta = 18$ cm $= 180$ mm

Modulus of rigidity, $C = 8 \times 10^4$ N/mm²

Let h = Height through which the weight W is dropped

W = Gradually applied load which produces the compression of spring equal to 180 mm.

Now using equation (16.26),

$$\delta = \frac{64W.R^3.n}{Cd^4}$$

or $$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

or $$W = \frac{180 \times 8 \times 10^4 \times 30^4}{64 \times 100^3 \times 16} = 11390 \text{ N}$$

Work done by the falling weight on spring

$$= \text{Weight falling } (h + \delta) = 3000\,(h + 180) \text{ N-mm}$$

Energy stored in the spring $= \frac{1}{2}\, W \times \delta$

$$= \frac{1}{2} \times 11390 \times 180 = 1025100 \text{ N-mm.}$$

Equating the work done by the falling weight on the spring to the energy stored in the spring, we get

$$3000(h + 180) = 1025100$$

or $$h + 180 = \frac{1025100}{3000} = 341.7 \text{ mm}$$

∴ $$h = 341.7 - 180 = \mathbf{161.7\ mm.} \quad \textbf{Ans.}$$

Video Content / Details of website for further learning (if any):

1. www.youtube.com/watch?v=tTBnW5gAieM
2. https://www.youtube.com/watch?v=ftW3UgjHWBg

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 725-727.

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L - 28

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Slop & Deflection of Beam

Introduction :

1. The Beam is a long piece of a body capable of holding the load by resisting the bending. The deflection of the beam towards a particular direction when force is applied on it is called Beam deflection.
2. There is a range of beam deflection equations that can be used to calculate a basic value for deflection in different types of beams.
3. Generally, deflection can be calculated by taking the double integral of the Bending Moment Equation, M(x) divided by EI (Young's Modulus x Moment of Inertia)
4. One of the most important applications of beam deflection is to obtain equations with which we can determine the accurate values of beam deflections in many practical cases.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

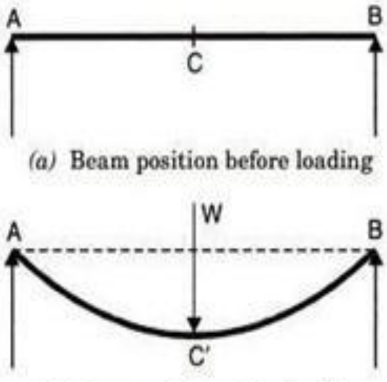
Detailed content of the Lecture :

INTRODUCTION:

Deflection and slope of a beam subjected to uniform bending moment

If a beam carries uniformly distributed load or a point load, the beam is deflected from its original position. In this chapter, we shall study the amount by which a beam is deflected from its position. Due to the loads acting on the beam, it will be subjected to bending moment. The radius of curvature of the deflected beam is given by the equation $\frac{M}{I} = \frac{E}{R}$. The radius of curvature will be constant if $R = \frac{I \times E}{M}$ = constant. The term $(I \times E)/M$ will be constant, if the beam is subjected to a constant bending moment M. This means that a beam for which, when loaded, the value of $(E \times I)/M$ is constant, will bend in a circular arc.

Fig. 4.1 (*a*) shows the beam position before any load is applied on the beam whereas Fig. 4.1 (*b*) shows the beam position after loading.

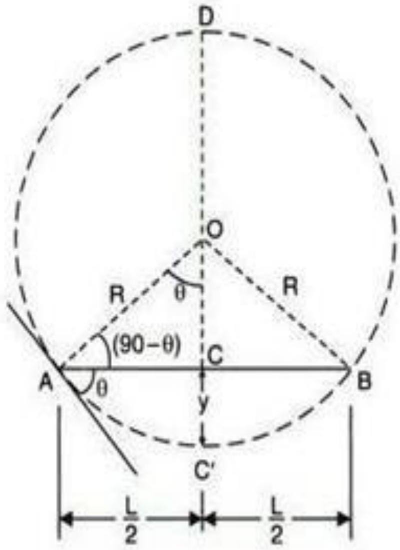


(*a*) Beam position before loading

(*b*) Beam position after loading

:

A beam AB of length L is subjected to a uniform bending moment M as shown in Fig. 4.1 (c). As the beam is subjected to a constant bending moment, hence it will bend into a circular arc. The initial position of the beam is shown by ACB, whereas the deflected position is shown by $AC'B$.



Let R = Radius of curvature of the deflected beam,

y = Deflection of the beam at the centre (*i.e.*, distance CC'),

I = Moment of inertia of the beam section,

E = Young's modulus for the beam material, and

θ = Slope of the beam at the end A (*i.e.*, the angle made by the tangent at A with the beam AB). For a practical beam the deflection y is a small quantity.

Hence $\tan\theta = \theta$ where θ is in radians. Hence θ becomes the slope as slope is

$$\frac{dy}{dx} = \tan\theta = \theta.$$

Now $$AC = BC = \frac{L}{2}$$

Also from the geometry of a circle, we know that

$$AC \times CB = DC \times CC'$$

$$\frac{L}{2} \times \frac{L}{2} = (2R - y) \times y \qquad (\because \; DC = DC' - CC' = 2R - y)$$

or $$\frac{L^2}{4} = 2Ry - y^2$$

For a practical beam, the deflection y is a small quantity. Hence the square of a small quantity will be negligible. Hence neglecting y^2 in the above equation, we get

$$\frac{L^2}{4} = 2Ry$$

$\therefore$ $$y = \frac{L^2}{8R} \qquad ...(i)$$

But from bending equation, we have

$$\frac{M}{I} = \frac{E}{R}$$

or $$R = \frac{E \times I}{M} \qquad ...(ii)$$

Substituting the value of R in equation (*i*), we get

$$y = \frac{L^2}{8 \times \frac{EI}{M}}$$

or $$y = \frac{ML^2}{8EI} \qquad ...(4.1)$$

Equation (4.1) gives the central deflection of a beam which bends in a circular arc.

Value of Slope (θ)

From triangle AOB, we know that

$$\sin\theta = \frac{AC}{AO} = \frac{\left(\frac{L}{2}\right)}{R} = \frac{L}{2R}$$

Since the angle θ is very small, hence sin θ = θ (in radians)

$$\therefore \quad \theta = \frac{L}{2R}$$

$$= \frac{L}{2 \times \frac{EI}{M}} \qquad \left(\because \; R = \frac{EI}{M} \text{ from equation } (ii)\right)$$

$$= \frac{M \times L}{2EI} \qquad \text{...(4.2)}$$

Equation (4.2) gives the slope of the deflected beam at *A* or at *B*.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=JZ1T-UkJo4U

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 511-512.

MUTHAYAMMAL ENGINEERING COLLEGE

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L - 29

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Slope & Deflection and its reflections

Introduction :

1. The Beam is a long piece of a body capable of holding the load by resisting the bending. The deflection of the beam towards a particular direction when force is applied on it is called Beam deflection.
2. There is a range of beam deflection equations that can be used to calculate a basic value for deflection in different types of beams.
3. Generally, deflection can be calculated by taking the double integral of the Bending Moment Equation, M(x) divided by EI (Young's Modulus x Moment of Inertia)
4. One of the most important applications of beam deflection is to obtain equations with which we can determine the accurate values of beam deflections in many practical cases.

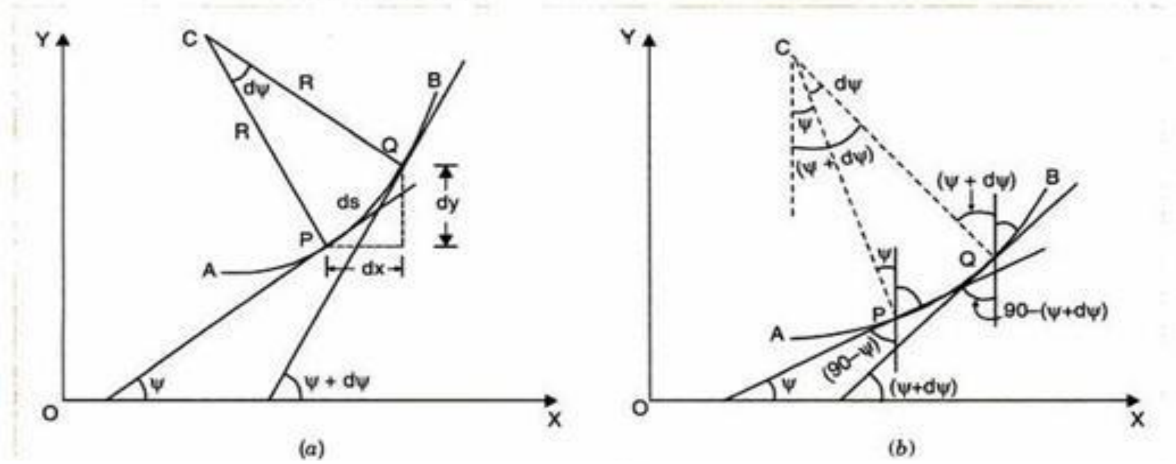
Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

Let the curve *AB* represents the deflection of a beam as shown in Fig. 4.2 (*a*). Consider a small portion *PQ* of this beam. Let the tangents at *P* and *Q* make angle ψ and $\psi + d\psi$ with *x*-axis. Normal at *P* and *Q* will meet at *C* such that

$$PC = QC = R$$



The point C is known as centre of curvature of the curve PQ.

Let the length of PQ is equal to ds.

From Fig. 4.2 (b), we see that

$$\text{Angle } PCQ = d\psi$$

$$\therefore \quad PQ = ds = R.d\psi$$

or
$$R = \frac{ds}{d\psi} \quad ...(i)$$

But if x and y be the co-ordinates of P, then

$$\tan\psi = \frac{dy}{dx} \quad ...(ii)$$

$$\sin\psi = \frac{dy}{ds}$$

and
$$\cos\psi = \frac{dx}{ds}$$

Now equation (i) can be written as

$$R = \frac{ds}{d\psi} = \frac{\left(\frac{ds}{dx}\right)}{\left(\frac{d\psi}{dx}\right)} = \frac{\left(\frac{1}{\cos\psi}\right)}{\left(\frac{d\psi}{dx}\right)}$$

or
$$R = \frac{\sec\psi}{\left(\frac{d\psi}{dx}\right)} \quad ...(iii)$$

Differentiating equation (ii) w.r.t. x, we get

$$\sec^2\psi \,.\, \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

or
$$\frac{d\psi}{dx} = \frac{\left(\frac{d^2y}{dx^2}\right)}{\sec^2\psi}$$

Substituting this value of $\frac{d\psi}{dx}$ in equation (iii), we get

$$R = \frac{\sec\psi}{\left(\frac{\frac{d^2y}{dx^2}}{\sec^2\psi}\right)} = \frac{\sec\psi \,.\, \sec^2\psi}{\frac{d^2y}{dx^2}} = \frac{\sec^3\psi}{\left(\frac{d^2y}{dx^2}\right)}$$

Taking the reciprocal to both sides, we get

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\sec^3\psi} = \frac{\frac{d^2y}{dx^2}}{(\sec^2\psi)^{3/2}}$$

$$= \frac{\frac{d^2y}{dx^2}}{(1+\tan^2\psi)^{3/2}}$$

For a practical beam, the slope $\tan\psi$ at any point is a small quantity. Hence $\tan^2\psi$ can be neglected.

$$\therefore \quad \frac{1}{R} = \frac{d^2y}{dx^2} \quad ...(iv)$$

From the bending equation, we have

$$\frac{M}{I} = \frac{E}{R}$$

or
$$\frac{1}{R} = \frac{M}{EI} \quad ...(v)$$

Equating equations (iv) and (v), we get

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$\therefore$ $$M = EI\frac{d^2y}{dx^2} \quad ...(4.3)$$

Differentiating the above equation w.r.t. x, we get

$$\frac{dM}{dx} = EI\frac{d^3y}{dx^3}$$

But $\frac{dM}{dx} = F$ shear force

$\therefore$ $$F = EI\frac{d^3y}{dx^3} \quad ...(4.4)$$

Differentiating equation (4.4) w.r.t. x, we get

$$\frac{dF}{dx} = EI\frac{d^4y}{dx^4}$$

But $\frac{dF}{dx} = w$ the rate of loading

$\therefore$ $$w = EI\frac{d^4y}{dx^4} \quad ...(4.5)$$

Hence, the relation between curvature, slope, deflection etc. at a section is given by :

Deflection $= y$

Slope $= \frac{dy}{dx}$

Bending moment $= EI\frac{d^2y}{dx^2}$

Shearing force $= EI\frac{d^3y}{dx^3}$

The rate of loading $= EI\frac{d^4y}{dx^4}$.

Units. In the above equations, E is taken in N/mm^2

I is taken in mm^4, y is taken in mm,

M is taken in Nm and x is taken in m.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 513-515.

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : 19RAC09- Strength of Materials

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Double Integration Method

Introduction :

1. The **double integration method** is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.
2. When a structure is placed under load it will bend, deflect or displace. The deflection will depend on the following factors: 1. Geometry of the structure, including shape and flexural rigidity of member. 2. Flexibility/rigidity of the material used. 3. Restraint of the supports. 4. Load pattern.
3. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

The followings are the important methods for finding the slope and deflection at a section in a loaded beam :

(*i*) Double integration method
(*ii*) Moment area method, and
(*iii*) Macaulay's method

Incase of double integration method, the equation used is

$$M = EI\frac{d^2y}{dx^2} \quad \text{or} \quad \frac{d^2y}{dx^2} = \frac{M}{EI}$$

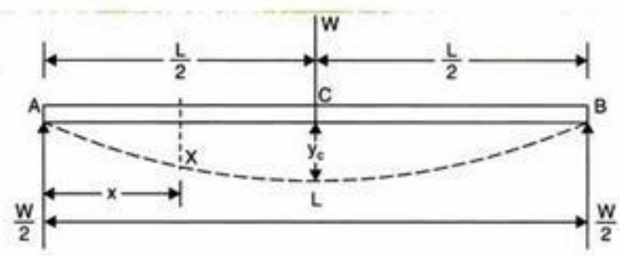
First integration of the above equation gives the value of $\frac{dy}{dx}$ or slope. The second integration gives the value of y or deflection.

The first two methods are used for a single load whereas the third method is used for several loads.

DEFLECTION OF A SIMPLY SUPPORTED BEAM CARRYING A POINT LOAD AT THE CENTRE

A simply supported beam AB of length L and carrying a point load W at the centre is shown in Fig.

As the load is symmetrically applied the reactions R_A and R_B will be equal. Also the maximum deflection will be at the centre.



Now $$R_A = R_B = \frac{W}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_x = R_A \times x$$

$$= \frac{W}{2} \times x$$

(Plus sign is as B.M. for left portion at X is clockwise)

But B.M. at any section is also given by equation (4.3) as

$$M = EI \frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \times x \qquad ...(i)$$

On integration, we get

$$EI \frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1 \qquad ...(ii)$$

where C_1 is the constant of integration. And its value is obtained from boundary conditions. The boundary condition is that at $x = \frac{L}{2}$, slope $\left(\frac{dy}{dx}\right) = 0$ (As the maximum deflection is at the centre, hence slope at the centre will be zero). Substituting this boundary condition in equation (*ii*), we get

$$0 = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_1$$

or $$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in equation (*ii*), we get

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \qquad ...(iii)$$

The above equation is known the *slope equation*. We can find the slope at any point on the beam by substituting the values of x. Slope is maximum at A. At A, $x = 0$ and hence slope at A will be obtained by substituting $x = 0$ in equation (*iii*).

$$\therefore \quad EI \left(\frac{dy}{dx}\right)_{\text{at } A} = \frac{W}{4} \times 0 - \frac{WL^2}{16}$$

$\left[\left(\frac{dy}{dx}\right)_{\text{at } A}\right.$ is the slope at A and is represented by $\left.\theta_A\right]$

or $$EI \times \theta_A = -\frac{WL^2}{16}$$

$$\therefore \quad \theta_A = -\frac{WL^2}{16EI}$$

The slope at point B will be equal to θ_A, since the load is symmetrically applied.

$$\therefore \quad \theta_B = \theta_A = -\frac{WL^2}{16EI} \qquad ...(4.6)$$

Equation (4.6) gives the slope in radians.

Deflection at any point

Deflection at any point is obtained by integrating the slope equation (*iii*). Hence integrating equation (*iii*), we get

$$EI \times y = \frac{W}{4} \cdot \frac{x^3}{3} - \frac{WL^2}{16} x + C_2 \quad \text{...(iv)}$$

where C_2 is another constant of integration. At A, $x = 0$ and the deflection (y) is zero.

Hence substituting these values in equation (*iv*), we get

$$EI \times 0 = 0 - 0 + C_2$$

or

$$C_2 = 0$$

Substituting the value of C_2 in equation (*iv*), we get

$$EI \times y = \frac{Wx^3}{12} - \frac{WL^2 \cdot x}{16} \quad \text{...(v)}$$

The above equation is known as *the deflection equation*. We can find the deflection at any point on the beam by substituting the values of x. The deflection is maximum at centre point C, where $x = \frac{L}{2}$. Let y_c represents the deflection at C. Then substituting $x = \frac{L}{2}$ and $y = y_c$ in equation (*v*), we get

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16} \times \left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{96} - \frac{WL^3}{32} = \frac{WL^3 - 3WL^3}{96}$$

$$= -\frac{2WL^3}{96} = -\frac{WL^3}{48}$$

$$\therefore \quad y_c = -\frac{WL^3}{48EI}$$

(Negative sign shows that deflection is downwards)

$$\therefore \quad \text{Downward deflection, } y_c = \frac{WL^3}{48EI} \quad \text{...(4.7)}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=W0u0AVa-xjg

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 515-517.

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L – 31

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Problems on Double Integration Method

Introduction :

1. The **double integration method** is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.
2. When a structure is placed under load it will bend, deflect or displace. The deflection will depend on the following factors: 1. Geometry of the structure, including shape and flexural rigidity of member. 2. Flexibility/rigidity of the material used. 3. Restraint of the supports. 4. Load pattern.
3. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

PROBLEM : 1. *A beam 6 m long, simply supported at its ends, is carrying a point load of 50 kN at its centre. The moment of inertia of the beam (i.e. I) is given as equal to 78×10^6 mm^4. If E for the material of the beam = 2.1×10^5 N/mm^2, calculate : (i) deflection at the centre of the beam and (ii) slope at the supports.*

Given :

Length, $L = 6 \text{ m} = 6 \times 1000 = 6000$ mm

Point load, $W = 50 \text{ kN} = 50{,}000$ N

M.O.I., $I = 78 \times 10^6 \text{ mm}^4$

Value of $E = 2.1 \times 10^5 \text{ N/mm}^2$

Let y_c = Deflection at the centre and

θ_A = Slope at the support.

(*i*) Using equation (12.7) for the deflection at the centre, we get

$$y_c = \frac{WL^3}{48EI}$$

$$= \frac{50000 \times 6000^3}{48 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= \mathbf{13.736 \text{ mm.}}$$

(*ii*) Using equation (12.6) for the slope at the supports, we get

$$\theta_B = \theta_A = -\frac{WL^2}{16EI}$$

$$= \frac{WL^2}{16EI} \quad \text{(Numerically)}$$

$$= \frac{50000 \times 6000^2}{16 \times 2.1 \times 10^5 \times 78 \times 10^6} \text{ radians}$$

$$= 0.06868 \text{ radians}$$

$$= 0.06868 \times \frac{180}{\pi} \text{ degree} \qquad \left(\because 1 \text{ radian} = \frac{180}{\pi} \text{ degree}\right)$$

$$= \mathbf{3.935°.}$$

PROBLEM : 2 *A beam 4 metre long, simply supported at its ends, carries a point load W at its centre. If the slope at the ends of the beam is not to exceed 1°, find the deflection at the centre of the beam.*

Given :

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Point load at centre $= W$

Slope at the ends, $\theta_A = \theta_B = 1° = \frac{1 \times \pi}{180} = 0.01745$ radians

Let y_c = Deflection at the centre

Using equation (12.6), for the slope at the supports, we get

$$\theta_A = \frac{WL^2}{16EI} \quad \text{(Numerically)}$$

or

$$0.01745 = \frac{WL^2}{16EI} \quad \text{...(i)}$$

Now using equation (12.7), we get

$$y_c = \frac{WL^3}{48EI}$$

$$= \frac{WL^2}{16EI} \times \frac{L}{3} \quad \left(\because \frac{WL^3}{48EI} = \frac{WL^2}{16EI} \times \frac{L}{3}\right)$$

$$= 0.01745 \times \frac{4000}{3} \quad \left[\because \frac{WL^2}{16EI} = 0.01745 \text{ from equation } (i)\right]$$

$$= \textbf{23.26 mm.}$$

PROBLEM :3 *A beam 3 m long, simply supported at its ends, is carrying a point load W at the centre. If the slope at the ends of the beam should not exceed 1°, find the deflection at the centre of the beam.*

Given :

Length, $L = 3 \text{ m} = 3 \times 1000 = 3000 \text{ mm}$

Point load at centre $= W$

Slope at the ends, $\theta_A = \theta_B = 1°$

$$= \frac{1 \times \pi}{180} = 0.01745 \text{ radians}$$

Let y_c = Deflection at the centre

Using equation (12.6), we get

$$\theta_A = \frac{WL^2}{16EI} \quad \text{or} \quad 0.01745 = \frac{WL^2}{16EI} \quad \text{...(i)}$$

Now using equation (12.7), we get

$$y_c = \frac{WL^3}{48EI} = \frac{WL^2}{16EI} \times \frac{L}{3}$$

$$= 0.01745 \times \frac{L}{3} \quad \left(\because \frac{WL^2}{16EI} = 0.01745\right)$$

$$= 0.01745 \times \frac{3000}{3} \quad (\because L = 3000 \text{ mm})$$

$$= \textbf{17.45 mm.}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 518-519.

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L – 32

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : 19RAC09 Strength of Materials

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Macaulay's Method
Introduction : 1. Macaulay's method (the double integration method) is a technique used in structural analysis to determine the deflection of Euler-Bernoulli beams. Use of Macaulay's technique is very convenient for cases of discontinuous and/or discrete loading. 2. Typically partial uniformly distributed loads (u.d.l.) and uniformly varying loads (u.v.l.) over the span and a number of concentrated loads are conveniently handled using this technique. 3. Macaulay enables one continuous expression for bending moment to be obtained, and provided that certain rules are followed the constants of integration will be the same for all sections of the beam. 4. It is advisable to deal with each different type of load separately.
Prerequisite knowledge for Complete understanding and learning of Topic: • Basic Knowledge in Physics and Mathematics. • Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

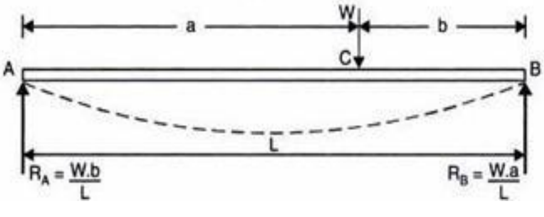
The procedure of finding slope and deflection for a simply supported beam with an eccentric point load as mentioned in Art. 4.5, is a very laborious. There is a convenient method for determining the deflections of the beam subjected to point loads.

This method was devised by Mr. M.H. Macaulay and is known as Macaulay's method. This method mainly consists in the special manner in which the bending moment at any section is expressed and in the manner in which the integrations are carried out.

Deflection of a simply Supported Beam with an Eccentric Point Load:

A simply supported beam *AB* of length *L* and carrying a point load *W* at a distance '*a*' from left support and at a distance '*b*' from right support is shown in Fig. 4.7. The reactions at *A* and *B* are given by,

$$R_A = \frac{W.b}{L} \quad \text{and} \quad R_B = \frac{W.a}{L}$$



The bending moment at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x = \frac{W.b}{L} \times x$$

The above equation of B.M. holds good for the values of x between 0 and 'a'. The B.M. at any section between C and B at a distance x from A is given by,

$$M_x = R_A.x - W \times (x - a)$$
$$= \frac{W.b}{L} . x - W(x - a)$$

The above equation of B.M. holds good for all values of x between $x = a$ and $x = b$.

The B.M. for all sections of the beam can be expressed in a single equation written as

$$M_x = \frac{W.b}{L} x \quad \vdots \quad - W(x - a) \qquad ...(i)$$

Stop at the dotted line for any point in section AC. But for any point in section CB, add the expression beyond the dotted line also.

The B.M. at any section is also given by equation (4.3) as

$$M = EI \frac{d^2y}{dx^2} \qquad ...(ii)$$

Hence equating (i) and (ii), we get

$$EI \frac{d^2y}{dx^2} = \frac{W.b}{L}.x \quad \vdots \quad - W(x - a) \qquad ...(iii)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \frac{W.b}{L} \frac{x^2}{2} + C_1 \quad \vdots \quad - \frac{W(x-a)^2}{2} \qquad ...(iv)$$

Integrating equation (iv) once again, we get

$$EIy = \frac{W.b}{2L} . \frac{x^3}{3} + C_1x + C_2 \quad \vdots \quad - \frac{W}{2} \frac{(x-a)^3}{3} \qquad ...(v)$$

where C_2 is another constant of integration. This constant is written after C_1x. The integration of $(x - a)^2$ will be $\left(\frac{x-a}{3}\right)^3$. This type of integration is justified as the constant of integrations C_1 and C_2 are valid for all values of x.

The values of C_1 and C_2 are obtained from boundary conditions. The two boundary conditions are :

(i) At $x = 0, y = 0$ and $\qquad$ (ii) At $x = L, y = 0$

(i) At A, $x = 0$ and $y = 0$. Substituting these values in equation (v) upto dotted line only, we get

$$0 = 0 + 0 + C_2$$

$$\therefore \quad C_2 = 0$$

(ii) At B, $x = L$ and $y = 0$. Substituting these values in equation (v), we get

$$0 = \frac{W.b}{2L} . \frac{L^3}{3} + C_1 \times L + 0 - \frac{W}{2} \frac{(L-a)^3}{3}$$

($\because$ $C_2 = 0$. Here complete Eq. (v) is to be taken)

$$= \frac{W.b.L^2}{6} + C_1 \times L - \frac{W}{2} \frac{b^3}{3} \qquad (\because \; L - a = b)$$

$$\therefore \quad C_1 \times L = \frac{W}{6} . b^3 - \frac{W.b.L^2}{6} = - \frac{W.b}{6} (L^2 - b^2)$$

$$\therefore \quad C_1 = - \frac{W.b}{6L} (L^2 - b^2) \qquad ...(vi)$$

Substituting the value of C_1 in equation (iv), we get

$$EI \frac{dy}{dx} = \frac{W.b}{L} \frac{x^2}{2} + \left[- \frac{W.b}{6L} \left(L^2 - b^2\right) \right] \quad \vdots \quad - \frac{W(x-a)^2}{2}$$

$$= \frac{W.b.x^2}{2L} - \frac{W.b}{6L} \left(L^2 - b^2\right) \quad \vdots \quad - \frac{W(x-a)^2}{2} \qquad ...(vii)$$

Substituting the values of C_1 and C_2 in equation (v), we get

$$EIy = \frac{W.b}{6L}.x^3 + \left[-\frac{Wb}{6L}(L^2 - b^2)\right]x + 0 \quad \vdots \quad -\frac{W}{6}(x-a)^3 \qquad ...(viii)$$

Equation ($viii$) gives the deflection at any point in the beam. To find the deflection y_c under the load, substitute $x = a$ in equation ($viii$) and consider the equation upto dotted line as point C lies in AC). Hence, we get

$$EIy_c = \frac{W.b}{6L}.a^3 - \frac{W.b}{6L}(L^2 - b^2)a = \frac{W.b}{6L}.a(a^2 - L^2 + b^2)$$

$$= -\frac{W.a.b}{6L}(L^2 - a^2 - b^2)$$

$$= -\frac{W.a.b}{6L}[(a+b)^2 - a^2 - b^2] \qquad (\because \; L = a + b)$$

$$= -\frac{W.a.b}{6L}[a^2 + b^2 + 2ab - a^2 - b^2]$$

$$= -\frac{W.a.b}{6L}[2ab] = -\frac{Wa^2.b^2}{3L}$$

$$\therefore \qquad y_c = -\frac{Wa^2.b^2}{3EIL} \qquad ...\text{(same as before)}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=qN7g9K-8nNw

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 531-533.

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : 19RAC09 Strength of Materials

Course Teacher : Dr.D.Velmurugan

Topic of Lecture: Problems on Macaulay’s Method

Introduction :

1. Macaulay's method (the double integration method) is a technique used in structural analysis to determine the deflection of Euler-Bernoulli beams. Use of Macaulay's technique is very convenient for cases of discontinuous and/or discrete loading.
2. Typically partial uniformly distributed loads (u.d.l.) and uniformly varying loads (u.v.l.) over the span and a number of concentrated loads are conveniently handled using this technique.
3. Macaulay enables one continuous expression for bending moment to be obtained, and provided that certain rules are followed the constants of integration will be the same for all sections of the beam.
4. It is advisable to deal with each different type of load separately.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

PROBLEM : *A beam of length 6 m is simply supported at its ends and carries a point load of 40 kN at a distance of 4 m from the left support. Find the deflection under the load and maximum deflection. Also calculate the point at which maximum deflection takes place. Given M.O.I. of beam = 7.33 × 10⁷ mm⁴ and E = 2 × 10⁵ N/mm².*

Given :

Length, $L = 6 \text{ m} = 6000 \text{ mm}$

Point load, $W = 40 \text{ kN} = 40{,}000 \text{ N}$

Distance of point load from left support, $a = 4 \text{ m} = 4000 \text{ mm}$

∴ $b = L - a = 6 - 4 = 2 \text{ m} = 2000 \text{ mm}$

Let y_c = Deflection under the load

y_{max} = Maximum deflection

Using equation $y_c = -\dfrac{W \cdot a^2 \cdot b^2}{3EIL}$

∴ $$y_c = -\frac{40000 \times 4000^2 \times 2000^2}{3 \times 2 \times 10^5 \times 7.33 \times 10^7 \times 6000}$$

$= -$ **9.7 mm.**

PROBLEM : *A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find :*

(i) deflection under each load,

(ii) maximum deflection, and

(iii) the point at which maximum deflection occurs.

Given $E = 2 \times 10^5$ N/mm² and $I = 85 \times 10^6$ mm⁴.

Given :

$$I = 85 \times 10^5 \text{ mm}^4 \text{ ; } E = 2 \times 10^5 \text{ N/mm}^2$$

First calculate the reactions R_A and R_B.

Taking moments about A, we get

$$R_B \times 6 = 48 \times 1 + 40 \times 3 = 168$$

$$\therefore \quad R_B = \frac{168}{6} = 28 \text{ kN}$$

$$\therefore \quad R_A = \text{Total load} - R_B = (48 + 40) - 28 = 60 \text{ kN}$$

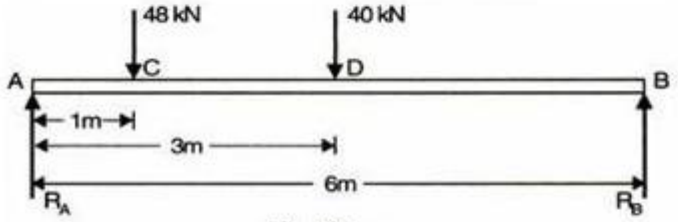

Fig. 4.8

Consider the section X in the last part of the beam (*i.e.*, in length DB) at a distance x from the left support A. The B.M. at this section is given by,

$$EI \frac{d^2y}{dx^2} = R_A.x \;\vdots\; -48(x-1) \;\vdots\; -40(x-3)$$

$$= 60x \;\vdots\; -48(x-1) \;\vdots\; -40(x-3)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \frac{60x^2}{2} + C_1 \;\vdots\; -48\frac{(x-1)^2}{2} \;\vdots\; -40\frac{(x-3)^2}{2}$$

$$= 30x^2 + C_1 \;\vdots\; -24(x-1)^2 \;\vdots\; -20(x-3)^2 \qquad ...(i)$$

Integrating the above equation again, we get

$$EIy = \frac{30x^3}{3} + C_1x + C_2 \;\vdots\; \frac{-24(x-1)^3}{3} \;\vdots\; \frac{-20(x-3)^3}{3}$$

$$= 10x^3 + C_1x + C_2 \;\vdots\; -8(x-1)^3 \;\vdots\; -\frac{20}{3}(x-3)^3 \qquad ...(ii)$$

To find the values of C_1 and C_2, use two boundary conditions. The boundary conditions are:

(*i*) at $x = 0, y = 0$, and (*ii*) at $x = 6$ m, $y = 0$.

(*i*) Substituting the first boundary condition *i.e.*, at $x = 0$, $y = 0$ in equation (*ii*) and considering the equation upto first dotted line (as $x = 0$ lies in the first part of the beam), we get

$$0 = 0 + 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*ii*) Substituting the second boundary condition *i.e.*, at $x = 6$ m, $y = 0$ in equation (*ii*) and considering the complete equation (as $x = 6$ lies in the last part of the beam), we get

$$0 = 10 \times 6^3 + C_1 \times 6 + 0 - 8(6-1)^3 - \frac{20}{3}(6-3)^3 \qquad (\because \; C_2 = 0)$$

or

$$0 = 2160 + 6C_1 - 8 \times 5^3 - \frac{20}{3} \times 3^3$$

$$= 2160 + 6C_1 - 1000 - 180 = 980 + 6C_1$$

$$\therefore \quad C_1 = \frac{-980}{6} = -163.33$$

Now substituting the values of C_1 and C_2 in equation (*ii*), we get

$$EIy = 10x^3 - 163.33x \;\vdots\; -8(x-1)^3 \;\vdots\; -\frac{20}{3}(x-3)^3 \qquad ...(iii)$$

(*i*) (*a*) *Deflection under first load i.e., at point C*. This is obtained by substituting $x = 1$ in equation (*iii*) upto the first dotted line (as the point C lies in the first part of the beam). Hence, we get

$$EI.y_c = 10 \times 1^3 - 163.33 \times 1$$
$$= 10 - 163.33 = -153.33 \text{ kNm}^3$$
$$= -153.33 \times 10^3 \text{ Nm}^3$$
$$= -153.33 \times 10^3 \times 10^9 \text{ Nmm}^3$$
$$= -153.33 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \quad y_c = \frac{-153.33 \times 10^{12}}{EI} = \frac{-153.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} \text{ mm}$$
$$= \mathbf{-9.019 \text{ mm.}}$$

(Negative sign shows that deflection is downwards).

(*b*) *Deflection under second load i.e. at point D*. This is obtained by substituting $x = 3$ m in equation (*iii*) upto the second dotted line (as the point D lies in the second part of the beam). Hence, we get

$$EI.y_D = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)^3$$
$$= 270 - 489.99 - 64 = -283.99 \text{ kNm}^3$$
$$= -283.99 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \quad y_D = \frac{-283.99 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = \mathbf{-16.7 \text{ mm.}}$$

(*ii*) *Maximum Deflection*. The deflection is likely to be maximum at a section between C and D. For maximum deflection, $\frac{dy}{dx}$ should be zero. Hence equate the equation (*i*) equal to zero upto the second dotted line.

$$\therefore \quad 30x^2 + C_1 - 24(x-1)^2 = 0$$

or $\quad 30x^2 - 163.33 - 24(x^2 + 1 - 2x) = 0 \qquad (\because \; C_1 = -163.33)$

or $\quad 6x^2 + 48x - 187.33 = 0$

The above equation is a quadratic equation. Hence its solution is

$$x = \frac{-48 \pm \sqrt{48^2 + 4 \times 6 \times 187.33}}{2 \times 6} = 2.87 \text{ m.}$$

(Neglecting – ve root)

Now substituting $x = 2.87$ m in equation (*iii*) upto the second dotted line, we get maximum deflection as

$$EIy_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87-1)^3$$
$$= 236.39 - 468.75 - 52.31$$
$$= 284.67 \text{ kNm}^3 = -284.67 \times 10^{12} \text{ Nmm}^3$$

$$\therefore \quad y_{max} = \frac{-284.67 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = \mathbf{-16.745 \text{ mm.}}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 534-536.

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L – 34

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09- Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Moment Area Method

Introduction :

1. Area– moment method is a semi graphical solution that relates slopes and deflections of the elastic curve to the area under the "M/EI" diagram, and the moment of the area of the "M/EI" diagram respectively.
2. This method is particularly useful when deflection at a specific point on the beam is required.
3. This method is advantageous when we solve problems involving beams, especially for those subjected to a series of concentrated loadings or having segments with different moments of inertia.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture : Moment Area Method

Fig. 4.17 shows a beam *AB* carrying some type of loading, and hence subjected to bending moment as shown in Fig. 4.17 (*a*). Let the beam bent into AQ_1P_1B as shown in Fig. 4.17 (*b*).

Due to the load acting on the beam. Let *A* be a point of zero slope and zero deflection.

Consider an element *PQ* of small length *dx* at a distance *x* from *B*. The corresponding points on the deflected beam are P_1Q_1 as shown in Fig. 4.17 (*b*).

Let R = Radius of curvature of deflected part P_1Q_1

$d\theta$ = Angle subtended by the arc P_1Q_1 at the centre O

M = Bending moment between P and Q

P_1C = Tangent at point P_1

Q_1D = Tangent at point Q_1.

The tangent at P_1 and Q_1 are cutting the vertical line through *B* at points *C* and *D*. The angle between the normals at P_1 and Q_1 will be equal to the angle

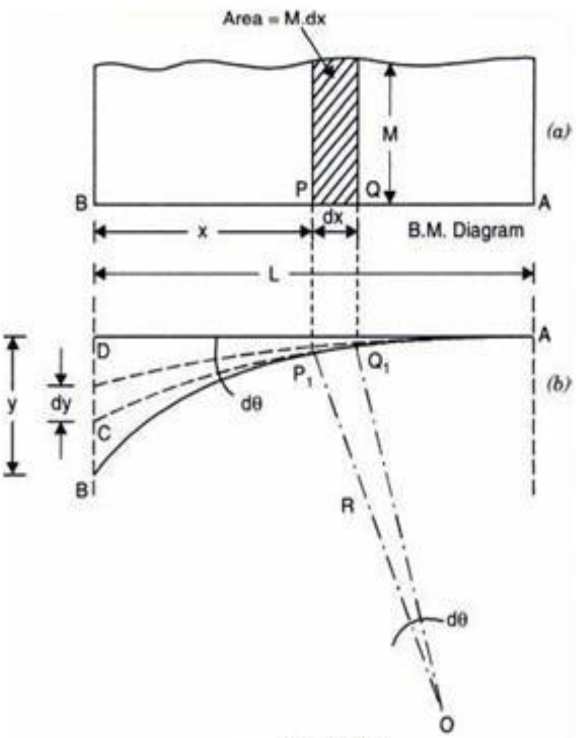


Fig. 4.17

Slope And Deflection Of Simply Supported Beam Carrying A Point Load At The Centre By Mohr's Theorem

Fig. 4.19 (*a*) shows a simply supported *AB* of length *L* and carrying a point load *W* at the centre of the beam *i.e.*, at point *C*. The B.M. diagram is shown in Fig. 4.19 (*b*). This is a case of symmetrical loading, hence slope is zero at the centre *i.e.*, at point *C*.

But the deflection is maximum at the centre.

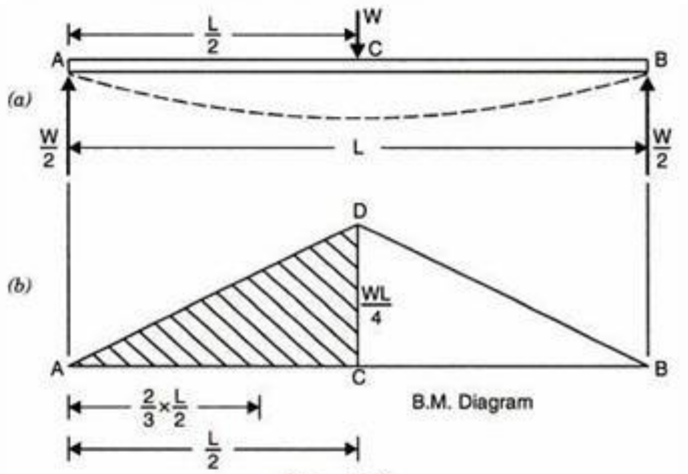


Fig. 4.19

Now using Mohr's theorem for slope, we get

Slope at $A = \dfrac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$

But area of B.M. diagram between *A* and *C*

$= \text{Area of triangle } ACD$

$= \dfrac{1}{2} \times \dfrac{L}{2} \times \dfrac{WL}{4} = \dfrac{WL^2}{16}$

$\therefore$ Slope at *A* or $\theta_A = \dfrac{WL^2}{EI}$

Now using Mohr's theorem for deflection, we get from equation (4.17) as

$$y = \frac{A\bar{x}}{EI}$$

where A = Area of B.M. Diagram between *A* and *C*

$= \dfrac{WL^2}{16}$

$\bar{x}$ = Distance of C.G. of area *A* from *A*

$= \dfrac{2}{3} \times \dfrac{L}{2} = \dfrac{L}{3}$

$$\therefore \qquad y = \frac{\dfrac{WL^2}{16} \times \dfrac{L}{3}}{EI} = \frac{WL^3}{48EI}.$$

Slope And Deflection Of Simply Supported Beam Carrying A Uniformly Distributed Load By Mohr's Theorem

Refer the following figure

Fig. 4.20 (*a*) shows a simply supported beam *AB* of length *L* and carrying a uniformly distributed load of *w*/unit length over the entire span. The B.M. diagram is shown in Fig. 4.20 (*b*). This is a case of symmetrical loading, hence slope is zero at the centre *i.e.*, at point *C*.

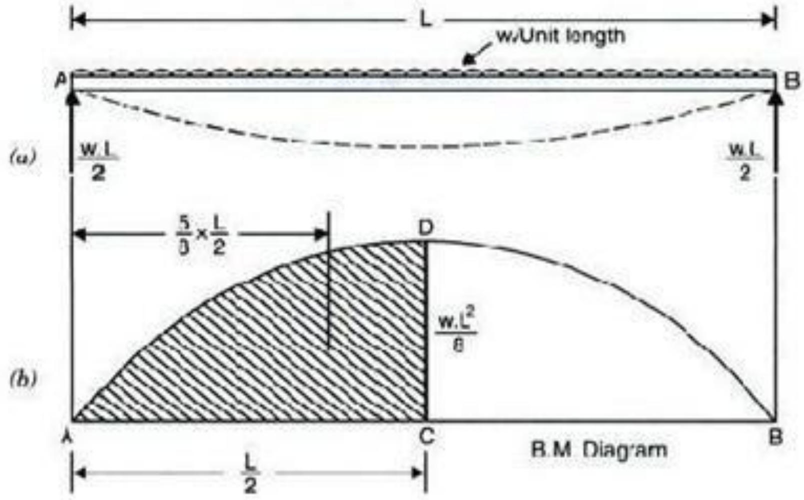


Fig. 4.20

(*i*) Now using Mohr's theorem for slope, we get

$$\text{Slope at } A = \frac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$$

But area of B.M. diagram between *A* and *C*

$$= \text{Area of parabola } ACD$$

$$= \frac{2}{3} \times AC \times CD$$

$$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8} = \frac{w \cdot L^3}{24}$$

$$\therefore \quad \text{Slope at } A = \frac{w \cdot L^3}{24EI}$$

(*ii*) Now using Mohr's theorem for deflection, we get from equation (4.17) as

$$y = \frac{A\bar{x}}{EI}$$

where A = Area of B.M. diagram between A and C

$$= \frac{w \cdot L^3}{24}$$

and $\bar{x}$ = Distance of C.G. of area *A* from *A*

$$= \frac{5}{8} \times AC = \frac{5}{8} \times \frac{L}{2} = \frac{5L}{16}$$

$$\therefore \quad y = \frac{\frac{w.L^3}{24} \times \frac{5L}{16}}{EI} = \frac{5}{384} \frac{w.L^4}{EI}.$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=x8AdLwAGplE

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 546-548.

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **IV- Deflection of Beams**

Topic of Lecture: Conjugate Beam Method

Introduction :

1. Conjugate beam is defined as the imaginary beam with the same dimensions (length) as that of the original beam but load at any point on the conjugate beam is equal to the bending moment at that point divided by EI.
2. The conjugate-beam method is an engineering method to derive the slope and displacement of a beam. Two theorems are important for Conjugate Beam Method
3. Theorem 1: The slope at a point in the real beam is numerically equal to the shear at the corresponding point in the conjugate beam.
4. Theorem 2: The displacement of a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam

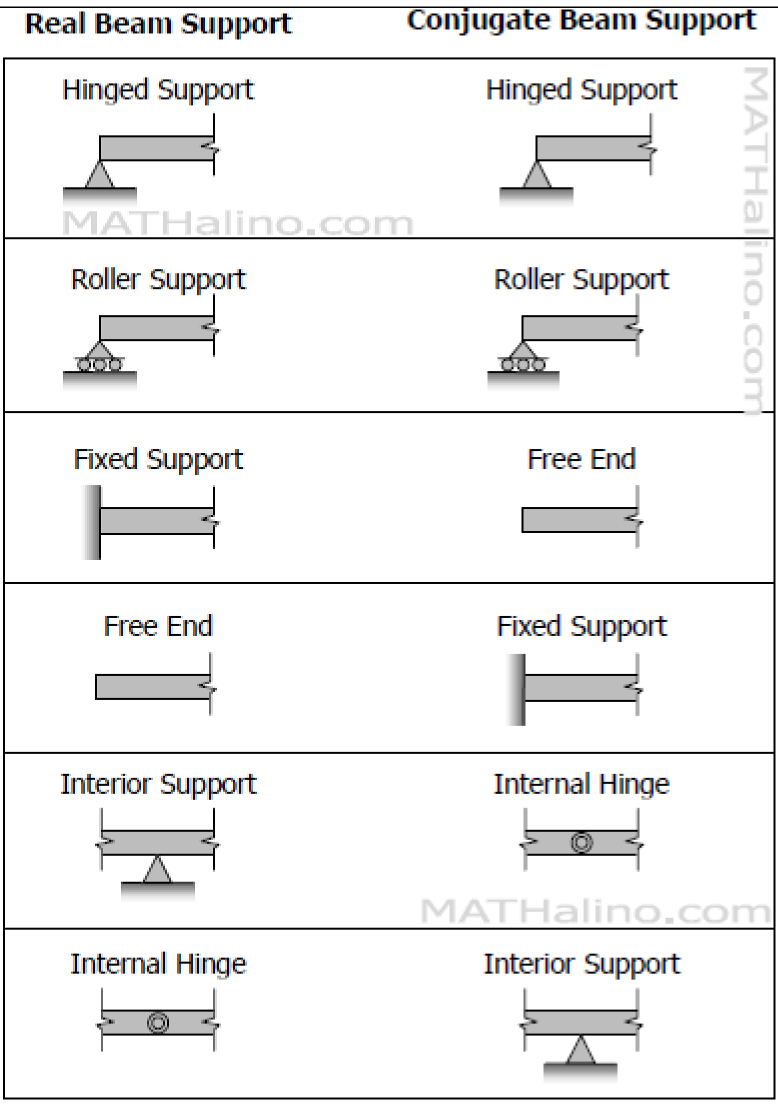
Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

Properties of Conjugate Beam

- The length of a conjugate beam is always equal to the length of the actual beam.
- The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
- A simple support for the real beam remains simple support for the conjugate beam.
- A fixed end for the real beam becomes free end for the conjugate beam.
- The point of zero shears for the conjugate beam corresponds to a point of zero slopes for the real beam.
- The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.
- Knowing that the slope on the real beam is equal to the shear on conjugate beam and the deflection on real beam is equal to the moment on conjugate beam, the shear and bending moment at any point on the conjugate beam must be consistent with the slope and deflection at that point of the real beam.
- Take for example a real beam with fixed support; at the point of fixed support there is neither slope nor deflection, thus, the shear and moment of the corresponding conjugate beam at that point must be zero. Therefore, the conjugate of fixed support is free end



Video Content / Details of website for further learning (if any):

1. https://www.mathalino.com/reviewer/strength-materials/conjugate-beam-method-beam-deflection

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 578-580.

Course Name with Code : **19RAC09 - Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Topic of Lecture: Conjugate Beam Method Problems

Introduction :

1. Conjugate beam is defined as the imaginary beam with the same dimensions (length) as that of the original beam but load at any point on the conjugate beam is equal to the bending moment at that point divided by EI.
2. The conjugate-beam method is an engineering method to derive the slope and displacement of a beam. Two theorems are important for Conjugate Beam Method
3. Theorem 1: The slope at a point in the real beam is numerically equal to the shear at the corresponding point in the conjugate beam.
4. Theorem 2: The displacement of a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

Compute the mid span value of EIδ for the beam shown in Fig. P-653. (Hint: Draw the M diagram by parts, starting from mid span toward the ends. Also take advantage of symmetry.

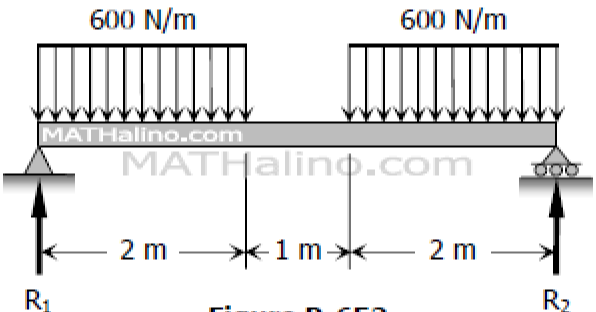
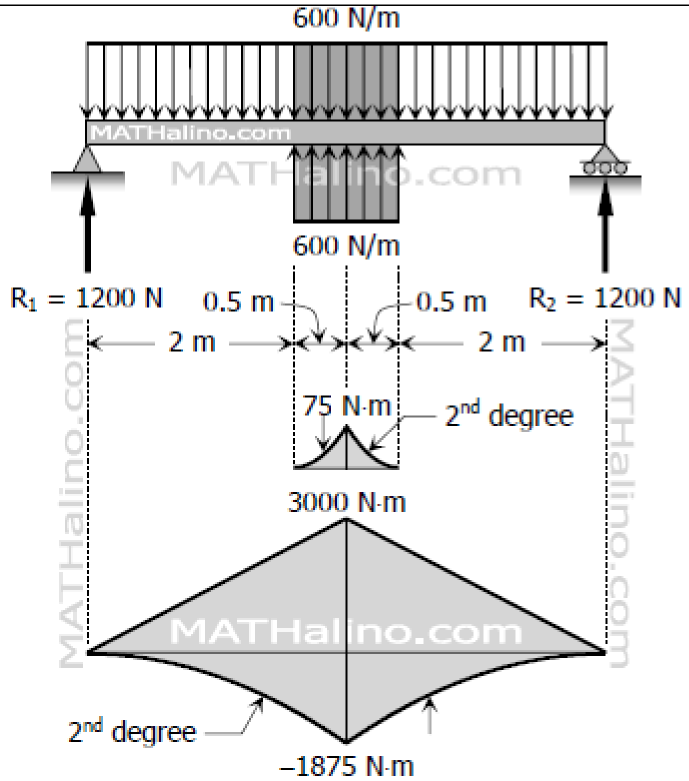
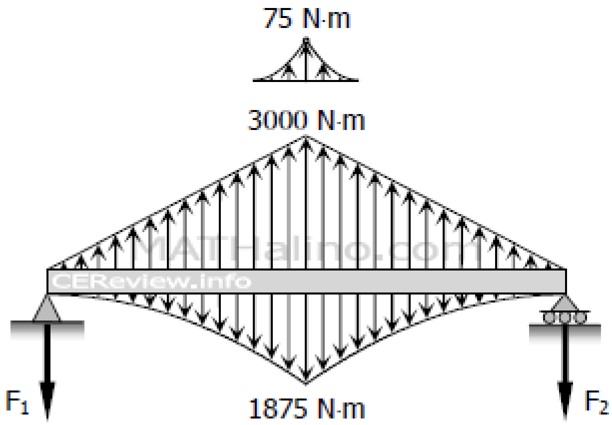


Figure P-653

By symmetry,
R1=R2=2(600)
R1=R2=1200 N



Moment Diagram by Parts



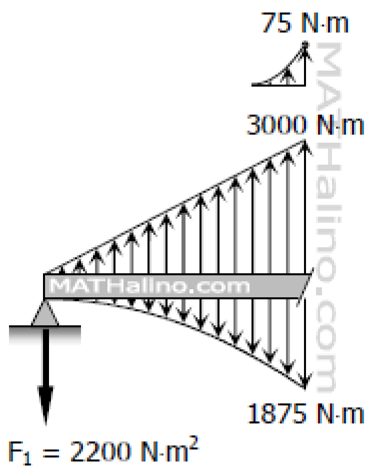
Conjugate Beam Loaded with M/EI Diagram

he loads of conjugate beam are symmetrical, thus,

F1=F2=12[12(5)(3000)+13(1)(75)−13(5)(1875)]

F1=F2=2200 N·m^2

For this beam, the maximum deflection will occur at the mid span.



Mmidspan=12(2.5)(3000)[13(2.5)]+13(0.5)(75)[14(0.5)]−13(2.5)(1875)[14(2.5)]−2200(2.5)
Mmidspan=−3350 $N\cdot m^3$

Therefore, the maximum deflection is
EI δmax=Mmidspan
EI δmax=−3350 $N\cdot m^3$
EI δmax=3350 $N\cdot m^3$ below the neutral axis Answer

Video Content / Details of website for further learning (if any):

1. https://www.mathalino.com/reviewer/strength-materials/problem-653-beam-deflection-conjugate-beam-method

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 584-586 for more problems

LECTURE HANDOUTS

RA **II / IV**

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Stresses in Cylindrical Shell

Introduction :

1. Pressure vessels are the vessels used to store or supply fluids under pressure. Stored fluid may undergo a change of state inside the pressure vessel e.g. in steam boiler or may undergo some chemical reaction. In nuclear / thermal power plants, chemical industries and various other industries, pressure vessels are used for storage and supply of different fluids, like water, gas, steam, air etc.
2. Pressure vessels are generally made of steel plates by bending them to desired shapes and joining the ends by welding. Pressure vessels have to be designed very carefully as their failure may cause dangerous accident due to explosion or leakage of fluid.
3. Pressure vessels with inner diameter to wall thickness ratio greater than 20 are called thin shell pressure vessels. These are used in boilers, tanks, pipes etc.
4. Pressure vessels with inner diameter to wall thickness ratio less than 20 are called thick shell pressure vessels. These are used in high pressure cylinders, tanks and gun barrels etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic Knowledge in Physics and Mathematics.
- Fundamental Concepts in Engineering Mechanics

Detailed content of the Lecture :

BIAXIAL STATE OF STRESSES:

This site is designed to help understand the stress and strain of materials under force by utilizing Free Body Diagrams, Statics, and various other methods.

Type of failure:

Such a component fails in since when subjected to an excessively high internal pressure. While it might fail by bursting along a path following the circumference of the cylinder. Under normal circumstance it fails by circumstances it fails by bursting along a path parallel to the axis. This suggests that the hoop stress is significantly higher than the axial stress.

In order to derive the expressions for various stresses we make following

Applications :

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

ANALYSIS : In order to analyse the thin walled cylinders, let us make the following assumptions :

• There are no shear stresses acting in the wall.

• The longitudinal and hoop stresses do not vary through the wall.

• Radial stresses sr which acts normal to the curved plane of the isolated element are neglibly small as compared to other two stresses especially when $\left[\frac{t}{R_i} < \frac{1}{20}\right]$

The state of tress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure, Actually a state of tri-axial stress exists on the inside of the vessel. However, for then walled pressure vessel the third stress is much smaller than the other two stresses and for this reason in can be neglected.

Thin Cylinders Subjected to Internal Pressure:

When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

• Circumferential or hoop stress

• The radial stress

• Longitudinal stress

now let us define these stresses and determine the expressions for them

Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.

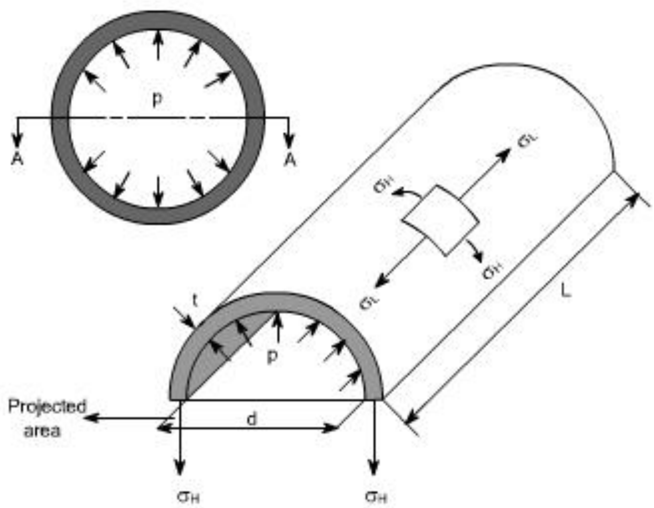


Fig .circumferential stress

In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure

d = inside diameter

L = Length of the cylinder

t = thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= p .d. L ------- (1)

The total resisting force owing to hoop stresses σ set up in the cylinder walls

= 2 .σ .L.t ---------(2)

Because σL.t. is the force in the one wall of the half cylinder.

the equations (1) & (2) we get

2 . σ L . t = p . d . L

σ = (p . d) / 2t

Circumferential or hoop Stress (σ) = (p .d)/ 2t

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=zqGWvGE2a8U
2. https://www.youtube.com/watch?v=iNG4bLMyeFA

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 740-742.

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres & Thick Cylinders**

Topic of Lecture: Longitudinal stress & Hoop Stress

Introduction:

1. Hoop stress is the stress induced in the hollow cylinder or hollow sphere which acts tangentially to the circumference of the cylinder and sphere in direction due to the action internal fluid pressure. It is tensile in nature is known as hoop stress.
2. The stresses induced in the cylinder due to the circumferential failure are called circumferential stress/ hoop stress. In thin cylinders, the pressure due to the fluid inside causes a bursting force on to the cylinder walls due to which the stress are induced in the cylinder.
3. Longitudinal stress can be defined as the stress acting on a pipe wall along the longitudinal direction. Longitudinal stress is usually produced by the fluid pressure in the pipe

Detailed content of the Lecture:

Longitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p. Then the walls of the cylinder will have a longitudinal stress as well as a circumferential stress.

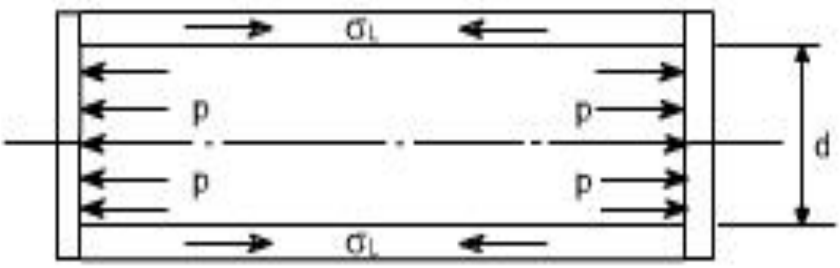

Fig. Longitudinal Stress

Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x π d2 /4

Area of metal resisting this force = pd.t. (approximately)

because pd is the circumference and this is multiplied by the wall thickness

Area of metal resisting this force = ltdt(approximate1y) ..

i.e. force d2/4 pd stress set up = - = p x - = - area ndt 4t pd longitudinal stress uL = - 4t

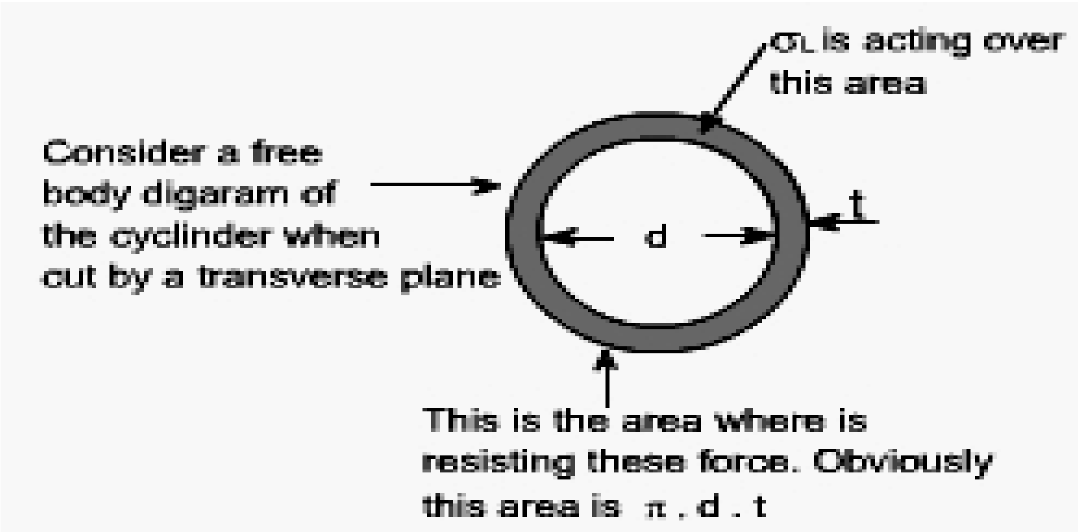

Fig. Longitudinal Stress

Volumetric Strain or Change in the Internal Volume:

When the thin cylinder is subjected to the internal pressure as we have already calculated that there is a change in the cylinder dimensions i.e, longitudinal strain and hoop strains come into picture. As a result of which there will be change in capacity of the cylinder or there is a change in the volume of the cylinder hence it becomes imperative to determine the change in volume or the volumetric strain.

The capacity of a cylinder is defined as

V = Area X Length

= πd2/4 x L

Let there be a change in dimensions occurs, when the thin cylinder is subjected to an internal pressure.

(i) The diameter d changes to δd + d

(ii) The length L changes to δL + L

Therefore, the change in volume = Final volume - Original volume

$$= \frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L$$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L}{\frac{\pi}{4}d^2.L}$$

$$\in_v = \frac{\{[d+\delta d]^2.(L+\delta L) - d^2.L\}}{d^2.L} = \frac{\{(d^2+\delta d^2+2d.\delta d).(L+\delta L) - d^2.L\}}{d^2.L}$$

simplifying and neglecting the products and squares of small quantities, i.e. δd & δL hence

$$= \frac{2d.\delta d.L + \delta L.d^2}{d^2 L} = \frac{\delta L}{L} + 2.\frac{\delta d}{d}$$

By definition $\frac{\delta L}{L}$ = Longitudnal strain

$\frac{\delta d}{d}$ = hoop strain, Thus

Volumetric strain = longitudnal strain + 2 x hoop strain

on substituting the value of longitudnal and hoop strains we get

$$\in_1 = \frac{pd}{4tE}[1-2\nu] \quad \& \quad \in_2 = \frac{pd}{4tE}[1-2\nu]$$

$$\text{or Volumetric} = \in_1 + 2\in_2 = \frac{pd}{4tE}[1-2\nu] + 2.\left(\frac{pd}{4tE}[1-2\nu]\right)$$

$$= \frac{pd}{4tE}\{1-2\nu+4-2\nu\} = \frac{pd}{4tE}[5-4\nu]$$

$$\text{Volumetric Strain} = \frac{pd}{4tE}[5-4\nu] \qquad \text{or} \qquad \boxed{\in_v = \frac{pd}{4tE}[5-4\nu]}$$

Therefore to find but the increase in capacity or volume, multiply the volumetric strain by original volume.

Hence

$$\boxed{\text{Increase in volume} = \frac{pd}{4tE}[5-4\nu]V}$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 741-742.

LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Deformation in thin & thick cylinders

Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

Detailed Content in the Lecture:

Effect of internal pressure on the dimension of a thin cylindrical shell :

When a fluid having internal pressure (p) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are :

(*i*) Hoop or circumferential stress (σ_1), acting on longitudinal section.

(*ii*) Longitudinal stress (σ_2) acting on the circumferential section.

These stresses are principal stresses, as they are acting on principal planes. The stress in the third principal plane is zero as the thickness (t) of the cylinder is very small. Actually the stress in the third principal plane is radial stress which is very small for thin cylinders and can be neglected.

Let p = Internal pressure of fluid

L = Length of cylindrical shell

d = Diameter of the cylindrical shell

t = Thickness of the cylindrical shell

E = Modulus of Elasticity for the material of the shell

σ_1 = Hoop stress in the material

σ_2 = Longitudinal stress in the material

μ = Poisson's ratio

δd = Change in diameter due to stresses set up in the material

δL = Change in length

δV = Change in volume.

The values of σ_1 and σ_2 are given by equations (17.1) and (17.2) as

$$\sigma_1 = \frac{pd}{2t}$$

$$\sigma_2 = \frac{p \times d}{4t}$$

Let e_1 = Circumferential strain,

e_2 = Longitudinal strain.

Then **circumferential strain,**

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} \quad \text{...(17.5)}$$

$$= \frac{pd}{2tE} - \frac{\mu pd}{4tE} \quad \left(\because \ \sigma_1 = \frac{pd}{2t} \text{ and } \sigma_2 = \frac{pd}{4t}\right)$$

$$= \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right] \quad \text{...(17.6)}$$

and **longitudinal strain,**

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} \quad \text{...(17.7)}$$

$$= \frac{pd}{4tE} - \frac{\mu pd}{2tE} \quad \text{(substituting values of } \sigma_1 \text{ and } \sigma_2\text{)}$$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \quad \text{...(17.8)}$$

But circumferential strain is also given as,

$$e_1 = \frac{\text{Change in circumference due to pressure}}{\text{Original circumference}}$$

$$= \frac{\text{Final circumference} - \text{Original circumference}}{\text{Original circumference}}$$

$$= \frac{\pi(d + \delta d) - \pi d}{\pi d}$$

$$= \frac{\pi d + \pi\delta d - \pi d}{\pi d} = \frac{\pi\delta d}{\pi d}$$

$$= \frac{\delta d}{d}\left(\text{or} = \frac{\text{Change in diameter}}{\text{Original diameter}}\right) \quad \text{...(17.9)}$$

Equating the two values of e_1 given by equations (17.6) and (17.9), we get

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right] \quad \text{...(17.10)}$$

$\therefore$ Change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right) \quad \text{...(17.11)}$$

Similarly longitudinal strain is also given as,

$$e_2 = \frac{\text{Change in length due to pressure}}{\text{Original length}}$$

$$= \frac{\delta L}{L} \quad \text{...(17.12)}$$

Equating the two values of e_2 given by equations (17.8) and (17.12).

$$\therefore \quad \frac{\delta L}{L} = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \quad \text{...(17.13)}$$

$\therefore$ Change in length,

$$\delta L = \frac{p \times d \times L}{2tE}\left(\frac{1}{2} - \mu\right) \quad \text{...(17.14)}$$

∴ Change in volume (δV)

$$= \frac{\pi}{4}[d^2 L + 2\, dL\delta d + \delta L d^2] - \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4}[2d\, L\delta d + \delta L d^2]$$

∴ Volumetric strains $= \dfrac{\delta V}{V} = \dfrac{\frac{\pi}{4}[2d\, L\delta d + \delta L d^2]}{\frac{\pi}{4} d^2 \times L}$

$$= \frac{2\delta d}{d} + \frac{\delta L}{L} \quad \text{...(17.15)}$$

$$= 2e_1 + e_2 \qquad \left(\because \frac{\delta d}{d} = e_1, \frac{\delta L}{L} = e_2\right) \quad \text{...(17.16)}$$

$$= 2 \times \frac{pd}{2Et}\left[1 - \frac{\mu}{2}\right] + \frac{pd}{2Et}\left(\frac{1}{2} - \mu\right)$$

(Substituting the values of e_1 and e_2)

$$= \frac{pd}{2Et}\left(2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right)$$

$$= \frac{pd}{2Et}\left(2 + \frac{1}{2} - \mu - \mu\right)$$

$$= \frac{pd}{2Et}\left(\frac{5}{2} - 2\mu\right) \quad \text{...(17.17)}$$

Also change in volume $(\delta V) = V(2e_1 + e_2)$. ...(17.18)

Video Content / Details of website for further learning (if any):

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2. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 751-752.

LECTURE HANDOUTS

RA | **II / IV**

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Thin cylinder / Simple Problems

Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.

Detailed Content in the Lecture:

A thin cylinder 75 mm internal diameter, 250 mm long with walls 2.5 mm thick is subjected to an internal pressure of 7 MN/m². Determine the change in internal diameter and the change in length.

Solution

(a) From eqn. (9.5), change in diameter

$$= \frac{pd^2}{4tE}(2-\nu) = \frac{7\times10^6\times75^2\times10^{-6}}{4\times2.5\times10^{-3}\times200\times10^9}(2-0.3)$$

$$= 33.4\times10^{-6}\text{ m} = 33.4\ \mu\text{m}$$

(b) From eqn. (9.3), change in length

$$= \frac{pdL}{4tE}(1-2\nu) = \frac{7\times10^6\times75\times10^{-3}\times250\times10^{-3}}{4\times2.5\times10^{-3}\times200\times10^9}(1-0.6)$$

$$= 26.2\ \mu\text{m}$$

(c) Hoop stress

$$\sigma_H = \frac{pd}{2t} = \frac{7\times10^6\times75\times10^{-3}}{2\times2.5\times10^{-3}} = 105\text{ MN/m}^2$$

Longitudinal stress

$$\sigma_L = \frac{pd}{4t} = \frac{7\times10^6\times75\times10^{-3}}{2\times2.5\times10^{-3}}$$

$$= 52.5\text{ MN/m}^2$$

A cylinder has an internal diameter of 230 mm, has walls 5 mm thick and is 1 m long. It is found to change in internal volume by 12.0×10^{-6} m^3 when filled with a liquid at a pressure p. If $E = 200$ GN/m^2 and $\nu = 0.25$, and assuming rigid end plates, determine:

(a) the values of hoop and longitudinal stresses;
(b) the modifications to these values if joint efficiencies of 45% (hoop) and 85% (longitudinal) are assumed;
(c) the necessary change in pressure p to produce a further increase in internal volume of 15%. The liquid may be assumed incompressible.

Solution

(a) From eqn. (9.6)

$$\text{change in internal volume} = \frac{pd}{4tE}(5-4\nu)V$$

$$\text{original volume } V = \frac{\pi}{4} \times 230^2 \times 10^{-6} \times 1 = 41.6 \times 10^{-3}\ \text{m}^3$$

Then

$$\text{change in volume} = 12 \times 10^{-6} = \frac{p \times 230 \times 10^{-3} \times 41.6 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 200 \times 10^{9}}(5-1)$$

Thus

$$p = \frac{12 \times 10^{-6} \times 4 \times 5 \times 10^{-3} \times 200 \times 10^{9}}{230 \times 10^{-3} \times 41.6 \times 10^{-3} \times 4}$$

$$= \mathbf{1.25\ MN/m^2}$$

Hence,

$$\text{hoop stress} = \frac{pd}{2t} = \frac{1.25 \times 10^{6} \times 230 \times 10^{-3}}{2 \times 5 \times 10^{-3}}$$

$$= \mathbf{28.8\ MN/m^2}$$

$$\text{longitudinal stress} = \frac{pd}{4t} = \mathbf{14.4\ MN/m^2}$$

(b) Hoop stress, acting on the longitudinal joints (§9.6)

$$= \frac{pd}{2t\eta_L} = \frac{1.25 \times 10^{6} \times 230 \times 10^{-3}}{2 \times 5 \times 10^{-3} \times 0.85}$$

$$= \mathbf{33.9\ MN/m^2}$$

Longitudinal stress (acting on the circumferential joints)

$$= \frac{pd}{4t\eta_c} = \frac{1.25 \times 10^{6} \times 230 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 0.45}$$

$$= \mathbf{32\ MN/m^2}$$

(c) Since the change in volume is directly proportional to the pressure, the necessary 15% increase in volume is achieved by increasing the pressure also by 15%.

$$\text{Necessary increase in } p = 0.15 \times 1.25 \times 10^{6}$$

$$= \mathbf{1.86\ MN/m^2}$$

Problem : *Calculate : (i) the change in diameter, (ii) change in length and (iii) change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 3 N/mm². Take the value of E = 2 × 10⁵ N/mm² and Poisson's ratio, μ = 0.3.*

Sol. Given :

Diameter of shell, $d = 100$ **cm**

Thickness of shell, $t = 1$ **cm**

Length of shell, $L = 5 \text{ m} = 5 \times 100 = 500$ cm

Internal pressure, $p = 3 \text{ N/mm}^2$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.30$

(*i*) Change in diameter (δd) is given by equation (17.11) as

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$= \frac{3 \times 100^2}{2 \times 1 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.30\right]$$

$$= \frac{3}{40}[1 - 0.15] = \mathbf{0.06375\ cm.}\quad \textbf{Ans.}$$

(*ii*) Change in length (δL) is given by equation (17.14) as

$$\delta L = \frac{pdL}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{3 \times 100 \times 500}{2 \times 1 \times 2 \times 10^5}\left[\frac{1}{2} - 0.30\right]$$

$$= \frac{15}{40} \times 0.20 = \mathbf{0.075\ cm.}\quad \textbf{Ans.}$$

(*iii*) Change in volume (δV) is given by equation (17.18) as

$$\delta V = V\,[2e_1 + e_1]$$

$$= V\left[2\frac{\delta d}{d} + \frac{\delta L}{L}\right] \qquad \left(\because\ e_1 = \frac{\delta d}{d}, e_2 = \frac{\delta L}{L}\right)$$

Substituting the values of δd, δL, d and L, we get

$$\delta V = V\left[2 \times \frac{0.06375}{100} + \frac{0.075}{500}\right]$$

$$= V\,[0.001275 + 0.00015] = 0.001425\ V.$$

But $\quad V = \text{Original volume} = \frac{\pi}{4} d^2 L$

$$= \frac{\pi}{4} \times 100^2 \times 500 \text{ cm}^3 = 3926990.817 \text{ cm}^3$$

∴ $\quad \delta V = 0.001425 \times 3926990.817 = \mathbf{5595.96\ cm^3.}\quad$ **Ans.**

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=PKImDoXM-OI
2. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 751-753.

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LECTURE HANDOUTS

RA | **II / IV**

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Spherical Shells and Internal Pressure

Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

Detailed Content in the Lecture:

THIN SPHERICAL SHELLS

Fig. 1 shows a thin spherical shell of internal diameter 'd' and thickness 't' and subjected to an internal fluid pressure 'p'. The fluid inside the shell has a tendency to split the shell into two hemispheres along x-x axis.

The force (P) which has a tendency to split the shell

$$= p \times \frac{\pi}{4} d^2$$

The area resisting this force $= \pi.d.t$

$\therefore$ Hoop or circumferential stress (σ_1) induced in the material of the shell is given by,

$$\sigma_1 = \frac{\text{Force } P}{\text{Area resisting the force } P}$$

$$= \frac{p \times \frac{\pi}{4} d^2}{\pi.d.t} = \frac{p.d}{4t}$$

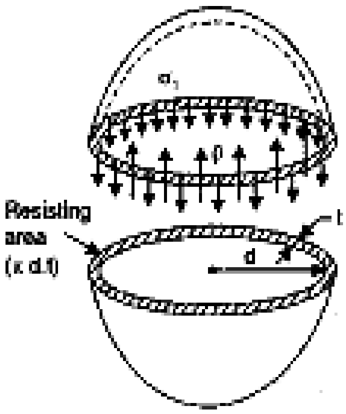

Fig. 1

The stress σ_1 is tensile in nature.

The fluid inside the shell is also having tendency to split the shell into two hemispheres along Y-Y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{p.d}{4t}$. Let this stress is σ_2.

$$\therefore \qquad \sigma_2 = \frac{p \times d}{4t}.$$

The stress σ_2 will be at right angles to σ_1.

Change in Dimensions of A Thin Spherical Shell Due to an Internal Pressure

In previous article, we have seen that the stresses σ_1 and σ_2 at any point are equal (each equal to $p.d/4t$) and like. There is no shear stress at any point in the shell. $\left(\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2} = \frac{p \times d}{4t} - \frac{p \times d}{4t} = 0\right)$. The stresses σ_1 and σ_2 are acting at right angles to each other.

$\therefore$ Strain in any one direction is given by,

$$e = \frac{\sigma_1}{E} - \frac{\mu \times \sigma_1}{E}$$

$$= \frac{\sigma_1}{E} - \frac{\mu \times \sigma_1}{E} \qquad \left(\because \ \sigma_1 = \sigma_2 = \frac{p \times d}{4t}\right)$$

$$= \frac{\sigma_1}{E}(1-\mu) = \frac{p \times d}{4tE}(1-\mu)$$

We know that strain in any direction is also

$$= \frac{\delta d}{d}$$

$$\therefore \quad \frac{\delta d}{d} = \frac{p \times d}{4tE}(1-\mu) \qquad \text{...(i)}$$

Volumetric strain $\left(\frac{\mathbf{dV}}{\mathbf{V}}\right)$

The ratio of change of volume to the original volume is known as volumetric strain. If V = original volume and dV = change in volume. Then $\frac{dV}{V}$ = volumetric strain.

Let V = Original volume

$$= \frac{\pi}{6} d^3 \qquad \left(\because \ \text{For a sphere, } V = \frac{4}{3}\pi r^3 = \frac{\pi}{6} d^3\right)$$

Taking the differential of the above equation, we get

$$dV = \frac{\pi}{6} \times 3d^2 \times d(d)$$

Hen

$$\frac{dV}{V} = \frac{\frac{\pi}{6} \times 3d^2 \times d(d)}{\frac{\pi}{6} \times d^3}$$

$$= 3\frac{d(d)}{d} \qquad \text{...(ii)}$$

But from equation (*i*), we have

$$\frac{\delta d}{d} \text{ or } \frac{d(d)}{d} = \frac{p \times d}{4tE}(1-\mu)$$

Substituting this value in equation (*ii*), we get

$$\frac{dV}{V} = \frac{3 \times p \times d}{4tE}(1-\mu).$$

Vessels subjected to fluid pressure

If a fluid is used as the pressurisation medium the fluid itself will change in volume as pressure is increased and this must be taken into account when calculating the amount of fluid which must be pumped into the cylinder in order to raise the pressure by a specified amount, the cylinder being initially full of fluid at atmospheric pressure.

Now the *bulk modulus* of a fluid is defined as follows:

$$\text{bulk modulus } K = \frac{\text{volumetric stress}}{\text{volumetric strain}}$$

where, in this case, volumetric stress = pressure p

and volumetric strain = $\frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$

$\therefore$

$$K = \frac{p}{\delta V / V} = \frac{pV}{\delta V}$$

i.e. **change in volume of fluid under pressure** $= \frac{pV}{K}$ (9.10)

The extra fluid required to raise the pressure must, therefore, take up this volume together with the increase in internal volume of the cylinder itself.

$\therefore$ extra fluid required to raise *cylinder* pressure by p

$$= \frac{pd}{4tE}[5 - 4\nu]V + \frac{pV}{K} \quad (9.11)$$

Similarly, for *spheres*, the extra fluid required is

$$= \frac{3pd}{4tE}[1 - \nu]V + \frac{pV}{K} \quad (9.12)$$

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=PKImDoXM-OI
2. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. **Strength of Materials by R.K.Bansal, Page no: 770-772.**

MUTHAYAMMAL ENGINEERING COLLEGE

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : 19RAC09 Strength of Materials

Course Teacher : Dr.D.Velmurugan

Unit : V- Thin Cylinders, Spheres and Thick Cylinders

Topic of Lecture: Deformation in Spherical Shell –Problems

Introduction:

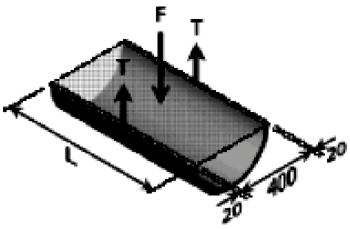
- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

Detailed Content in the Lecture:

Problem

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution



Longitudinal Section

(a) **Tangential stress** (longitudinal section):

$$F = 2T$$

$$pDL = 2(\sigma_t tL)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = 45 \text{ MPa}$$

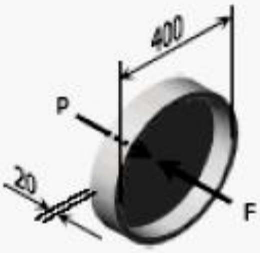
Longitudinal Stress (transverse section):

$$F = P$$

$$\frac{1}{4}\pi D^2 p = \sigma_l (\pi D t)$$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$$\sigma_l = 22.5 \text{ MPa}$$



Transverse Section

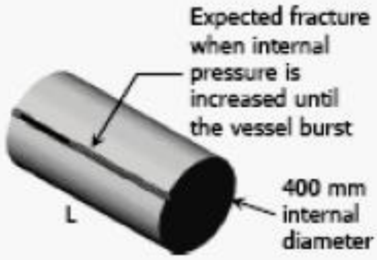
(b) From (a), $\sigma_t = \frac{pD}{2t}$ and $\sigma_l = \frac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$P = 12 \text{ MPa}$$

(c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.



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Course Faculty

Verified by HoD

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Deformation in Spherical Shell –Problems

Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

Detailed Content in the Lecture:

Problem

Calculate the minimum wall thickness for a cylindrical vessel that is to carry a gas at a pressure of 1400 psi. The diameter of the vessel is 2 ft, and the stress is limited to 12 ksi.

Solution

The critical stress is the tangential stress

$$\sigma_t = \frac{pD}{2t}$$

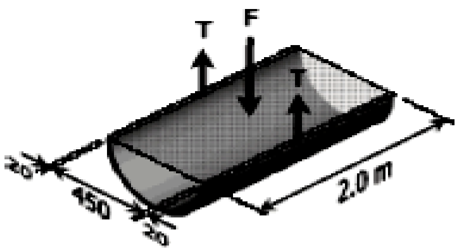
$$12\,000 = \frac{1400(2\times 12)}{2t}$$

$$t = 1.4 \text{ in}$$

Problem

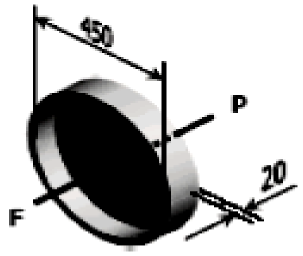
A cylindrical pressure vessel is fabricated from steel plating that has a thickness of 20 mm. The diameter of the pressure vessel is 450 mm and its length is 2.0 m. Determine the maximum internal pressure that can be applied if the longitudinal stress is limited to 140 MPa, and the circumferential stress is limited to 60 MPa.

Solution



Based on circumferential stress (tangential):

$$\Sigma F_V = 0$$
$$F = 2T$$
$$p(DL) = 2(\sigma_t Lt)$$
$$\sigma_t = \frac{pD}{2t}$$
$$60 = \frac{p(450)}{2(20)}$$
$$p = 5.33 \text{ MPa}$$



Based on longitudinal stress:

$$\Sigma F_H = 0$$
$$F = P$$
$$p\left(\tfrac{1}{4}\pi D^2\right) = \sigma_l(\pi D t)$$
$$\sigma_l = \frac{pD}{4t}$$

$$140 = \frac{p(450)}{4(20)}$$
$$p = 24.89 \text{ MPa}$$

Use $p = 5.33$ MPa

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=PKImDoXM-OI
2. https://www.youtube.com/watch?v=Mk8syrkI5Rs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 751-753.

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

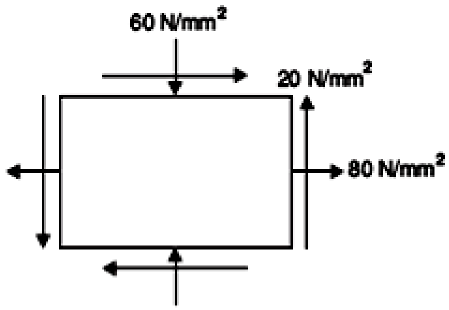
Topic of Lecture: Deformation in Spherical Shell –Problems

Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

Detailed Content in the Lecture:

Example . The state of stress in a material stressed to two-dimensional state of stress is as shown in Fig. 1 Determine principal stresses and maximum shear stress and the planes on which they act.



Solution:

$$p_{1,2} = \frac{p_x + p_y}{2} \pm \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

In this problem,

$$p_x = 80 \text{ N/mm}^2 \quad p_y = -60 \text{ N/mm}^2 \quad q = 20 \text{ N/mm}^2.$$

$$\therefore \quad p_{1,2} = \frac{80 + (-60)}{2} \pm \sqrt{\left(\frac{80 - (-60)}{2}\right)^2 + 20^2}$$

$$= 10 \pm \sqrt{70^2 + 20^2}$$

$$= 10 \pm 72.8$$

$$\therefore \quad p_1 = 82.8 \text{ N/mm}^2$$

and $\quad p_2 = -62.8 \text{ N/mm}^2$

$$q_{max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= 72.8 \text{ N/mm}^2$$

Let θ be the inclination of principal stress to the plane of p_x. Then,

$$\tan 2\theta = \frac{2q}{p_x - p_y} = \frac{2 \times 20}{80 - 60} = 2$$

$$\therefore \quad 2\theta = 63.44° \quad \text{or} \quad 63.44 + 180$$

$$\therefore \quad \theta = 31.72° \quad \text{or} \quad 121.72°$$

Planes of maximum shear make 45° to the above planes

$$\therefore \quad \theta' = 15.86° \quad \text{and} \quad 60.86°$$

Example . The state of stress in two-dimensionally stressed body at a point is as shown in Fig. 11.13(a). Determine the principal planes, principal stresses, maximum shear stress and their planes

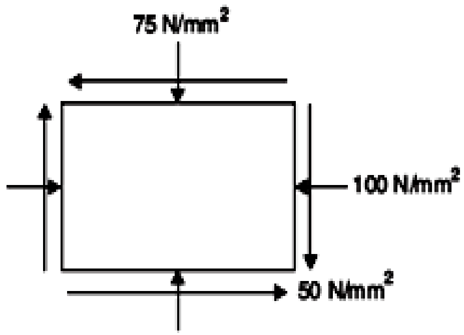


Fig. 11.13(a)

Solution: Let x and y directions be selected as shown in the figure. Then

$$p_x = -100 \text{ N/mm}^2, \quad p_y = -75 \text{ N/mm}^2, \quad q = -50 \text{ N/mm}^2$$

$$\therefore \quad p_1 = \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= \frac{-100-75}{2} + \sqrt{\left(\frac{-100+75}{2}\right)^2 + (-50)^2}$$

$$= -87.5 + 51.54$$

i.e., $\quad p_1 = -35.96 \text{ N/mm}^2$

$$p_2 = -\frac{p_x + p_y}{2} - \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= -87.5 - 51.54$$

i.e., $\quad p_2 = -139.04 \text{ N/mm}^2$

$$q_{max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

i.e., $\quad q_{max} = 51.51.$

Let principal plane of p_1 make angle θ with x-axis. Then

$$\tan 2\theta = \frac{2q}{p_x - p_y} = \frac{2(-50)}{-100+75} = 4$$

$$\therefore \quad 2\theta = 75.96 \quad \text{and} \quad 75.96 + 180$$

or $\quad \theta = 37.98° \quad$ and $\quad 127.98°$.

The planes of maximum shear stresses are at 45° to the principal planes. These planes are shown in Fig. 11.13(*b*).

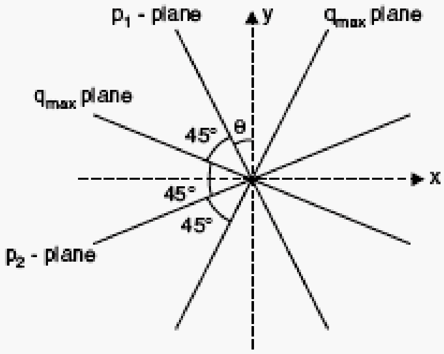


Fig. 11.13(b)

Video Content / Details of website for further learning (if any):

1. **https://www.youtube.com/watch?v=903QmNLVp18**

Important Books/Journals for further learning including the page nos.:

1. **Strength of Materials by R.K.Bansal, Page no: 110-112.**

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LECTURE HANDOUTS

RA

II / IV

Course Name with Code : **19RAC09 Strength of Materials**

Course Teacher : **Dr.D.Velmurugan**

Unit : **V- Thin Cylinders, Spheres and Thick Cylinders**

Topic of Lecture: Deformation in Spherical Shell –Problems, Lame's theorem.

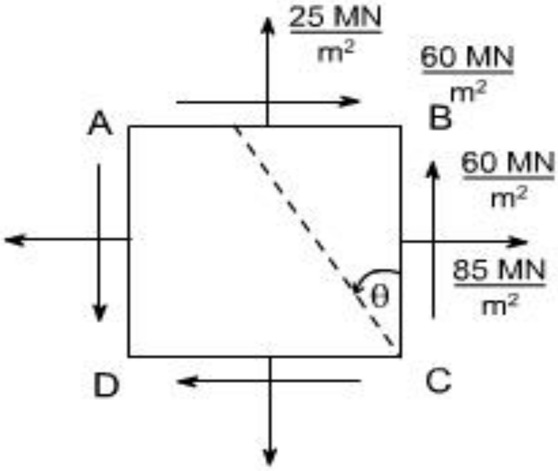
Introduction:

- Under the action of radial Pressures at the surfaces, the three Principal Stresses will be. These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.
-

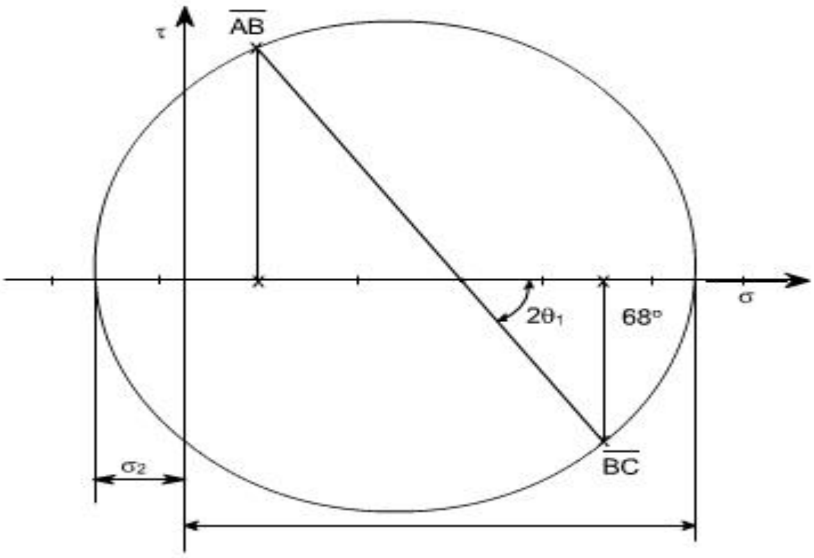
Detailed content of the Lecture:

GRAPHICAL SOLUTION:

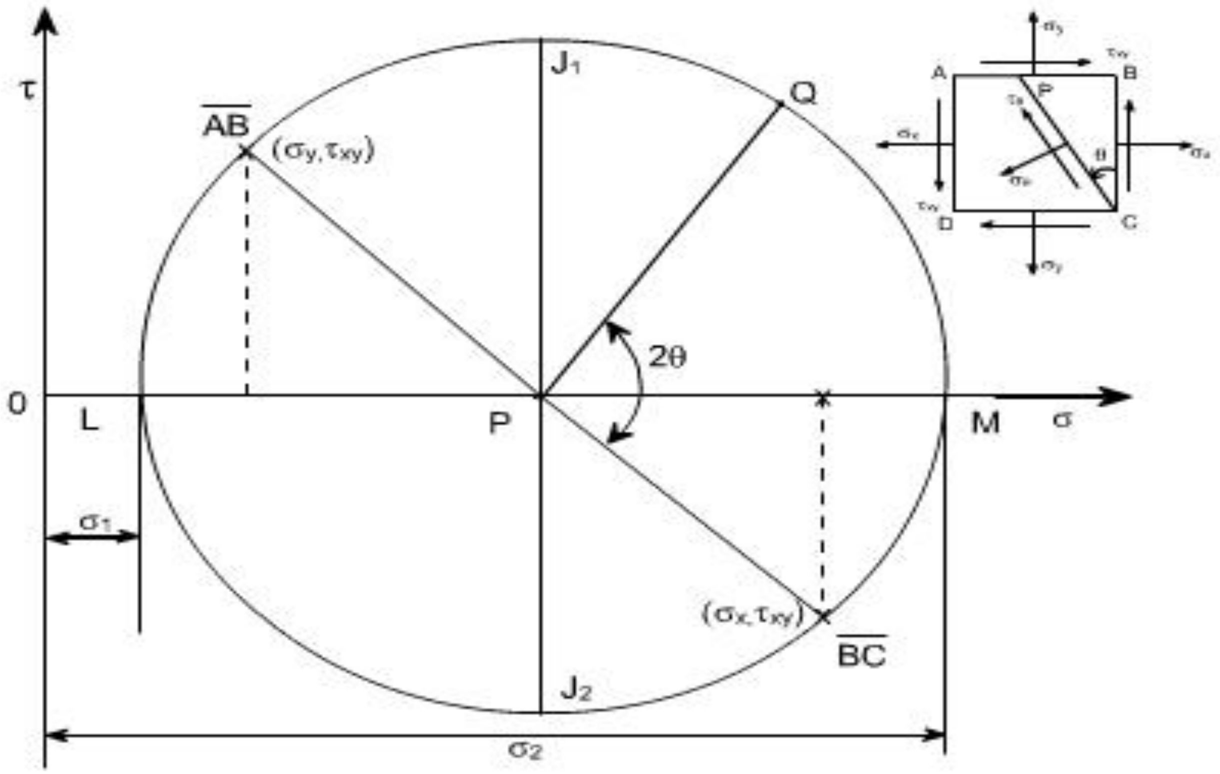
Mohr's Circle solution: The same solution can be obtained using the graphical solution i.e the Mohr's stress circle, for the first part, the block diagram becomes



Construct the graphical construction as per the steps given earlier.



Taking the measurements from the Mohr's stress circle, the various quantities computed are
$\sigma 1 = 120$ MN/m^2 tensile
$\sigma 2 = 10$ MN/m^2 compressive
$\sigma 1 = 340$ counter clockwise from BC
$\sigma 2 = 340 + 90 = 1240$ counter clockwise from BC
Part Second :
The required configuration i.e. the block diagram for this case is shown along with the stress circle.
By taking the measurements, the various quantities computed are given as
$\sigma 1 = 56.5$ MN/m^2 tensile
$\sigma 2 = 106$ MN/m^2 compressive
$\sigma 1 = 66015'$ counter clockwise from BC
$\sigma 2 = 156015'$ counter clockwise from BC



Salient points of Mohr's stress circle:
1. complementary shear stresses (on planes 90^0 apart on the circle) are equal in magnitude
2. The principal planes are orthogonal: points L and M are 180^0 apart on the circle (90^0 apart in material)

3. There are no shear stresses on principal planes: point L and M lie on normal stress axis.
4. The planes of maximum shear are 45^0 from the principal points D and E are 90^0 , measured round the circle from points L and M.
5. The maximum shear stresses are equal in magnitude and given by points D and E
6. The normal stresses on the planes of maximum shear stress are equal i.e. points D and E both have normal stress co-ordinate which is equal to the two principal stresses.

GRAPHICAL METHOD FOR ANALYSING IN PRINCIPAL STRESSES AND STRAINS

As we know that the circle represents all possible states of normal and shear stress on any plane through a stresses point in a material. Further we have seen that the co-ordinates of the point ‘Q' are seen to be the same as those derived from equilibrium of the element. i.e. the normal and shear stress components on any plane passing through the point can be found using Mohr's circle. Worthy of note:

1. The sides AB and BC of the element ABCD, which are 90^0 apart, are represented on the circle by $\overline{AB}\,P$ and $\overline{BC}\,P$ and they are 180^0 apart.
2. It has been shown that Mohr's circle represents all possible states at a point. Thus, it can be seen at a point. Thus, it can be seen that two planes LP and PM, 180^0 apart on the diagram and therefore 900 apart in the material, on which shear stress is zero. These planes are termed as principal planes and normal stresses acting on them are known as principal stresses.

Thus , $\sigma 1 = OL$

$\sigma 2 = OM$

3. The maximum shear stress in an element is given by the top and bottom points of the circle i.e. by points J1 and J2, Thus the maximum shear stress would be equal to the radius of the corresponding normal stress is obviously the distance OP, Further it can also be seen that the planes on which the shear stress is maximum are situated 90^0 from the principal planes (on circle), and 45^0 in the material.
4. The minimum normal stress is just as important as the maximum. The algebraic minimum stress could have a magnitude greater than that of the maximum principal stress if the state of stress were such that the centre of the circle is to the left of origin.

i.e. if $\sigma 1 = 20\ MN/m^2$ (say)

$\sigma 2 = 80\ MN/m^2$ (say)

Then $\sigma_{max} = 50\ MN/m^2$

If should be noted that the principal stresses are considered a maximum or minimum mathematically

5. Since the stresses on perpendicular faces of any element are given by the co-ordinates of two diametrically opposite points on the circle, thus, the sum of the two normal stresses for any and all orientations of the element is constant, i.e. Thus sum is an invariant for any particular state of stress.

Sum of the two normal stress components acting on mutually perpendicular planes at a point in a state of plane stress is not affected by the orientation of these planes.

Video Content / Details of website for further learning (if any):

1. https://www.youtube.com/watch?v=X96qgWsVsJs

Important Books/Journals for further learning including the page nos.:

1. Strength of Materials by R.K.Bansal, Page no: 123-125.