LECTURE HANDOUTS

L1

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Sources & effects of electromagnetic fields and Vector fields

Introduction :

- Electric and magnetic fields, commonly called electromagnetic fields (EMFs), are invisible areas of energy that result from an electric charge. EMFs are linked to the use of electrical power and various forms of natural and artificial lighting. The energy area or waves given off by EMFs are called radiation. EMFs with low-level radiation are generally considered harmless to humans.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electromagnetic fields (EMF)

- Electric fields are created by differences in voltage: the higher the voltage, the stronger will be the resultant field.
- Magnetic fields are created when electric current flows: the greater the current, the stronger the magnetic field. An electric field will exist even when there is no current flowing.
- If current does flow, the strength of the magnetic field will vary with power consumption but the electric field strength will be constant.
- Electromagnetic fields are a combination of invisible electric and magnetic fields of force. They are generated by natural phenomena like the Earth’s magnetic field but also by human activities, mainly through the use of electricity.
- Mobile phones, power lines and computer screens are examples of equipment that generates electromagnetic fields.
- Electric fields are produced by the local build-up of electric charges in the atmosphere associated with thunderstorms.
- The earth's magnetic field causes a compass needle to orient in a North-South direction and is used by birds and fish for navigation.

Human-made sources of electromagnetic fields

- ✓ Besides natural sources the electromagnetic spectrum also includes fields generated by human-made sources: X-rays are employed to diagnose a broken limb after a sport accident.
- ✓ The electricity that comes out of every power socket has associated low frequency electromagnetic fields.
- ✓ And various kinds of higher frequency radiowaves are used to transmit information – whether via TV antennas, radio stations or mobile phone base stations.

Effects of electromagnetic fields

- Electrical currents exist in the human body due to the chemical reactions that occur as part of the normal bodily functions, even in the absence of external electric fields.
- For example, nerves relay signals by transmitting electric impulses. Most biochemical reactions from digestion to brain activities go along with the rearrangement of charged particles. Even the heart is electrically active - an activity that your doctor can trace with the help of an electrocardiogram.
- Low-frequency electric fields influence the human body just as they influence any other material made up of charged particles.
- When electric fields act on conductive materials, they influence the distribution of electric charges at their surface. They cause current to flow through the body to the ground.
- Low-frequency magnetic fields induce circulating currents within the human body. The strength of these currents depends on the intensity of the outside magnetic field. If sufficiently large, these currents could cause stimulation of nerves and muscles or affect other biological processes.
- Both electric and magnetic fields induce voltages and currents in the body but even directly beneath a high voltage transmission line, the induced currents are very small compared to thresholds for producing shock and other electrical effects.

Effects on general health

- Some members of the public have attributed a diffuse collection of symptoms to low levels of exposure to electromagnetic fields at home. Reported symptoms include headaches, anxiety, suicide and depression, nausea, fatigue and loss of libido.
- To date, scientific evidence does not support a link between these symptoms and exposure to electromagnetic fields.
- At least some of these health problems may be caused by noise or other factors in the environment, or by anxiety related to the presence of new technologies.

Cataracts

- General eye irritation and cataracts have sometimes been reported in workers exposed to high levels of radiofrequency and microwave radiation, but animal studies do not support the idea that such forms of eye damage can be produced at levels that are not thermally hazardous. There is no evidence that these effects occur at levels experienced by the general public.

Electromagnetic fields and cancer

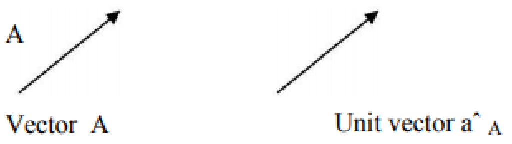
- Despite many studies, the evidence for any effect remains highly controversial. However, it is clear that if electromagnetic fields do have an effect on cancer, then any increase in risk will be extremely small.
- The results to date contain many inconsistencies, but no large increases in risk have been found for any cancer in children or adults.
- A number of epidemiological studies suggest small increases in risk of childhood leukemia with exposure to low frequency magnetic fields in the home.
- However, scientists have not generally concluded that these results indicate a cause-effect relation between exposure to the fields and disease (as opposed to artifacts in the study or effects unrelated to field exposure).
- In part, this conclusion has been reached because animal and laboratory studies fail to demonstrate any reproducible effects that are consistent with the hypothesis that fields cause or promote cancer.
- Large-scale studies are currently underway in several countries and may help resolve these issues.

Vector Analysis

- ✓ The Vector Analysis comprises of Vector Algebra and Vector Calculus.
- ✓ Any physical quantity may be Scalar quantity or Vector quantity.
- ✓ A Scalar quantity is specified by magnitude only while for a Vector quantity requires both magnitude and direction to be specified.
- ✓ A vector is represented graphically by a directed line segment.

Scalar - A quantity that is characterized only by magnitude is called a scalar.
Vector - A quantity that is characterized both by magnitude and direction is called a vector.
Unit vector - A Unit vector' is a vector of unit magnitude and directed along that vector'. aˆ A is a Unit vector along the direction of A .

- ✓ Thus, the graphical representation of A and aˆ A are



Also $\hat{a}_A = A / |A|$ or $A = \hat{a}_A |A|$

Video Content / Details of website for further learning :
https://corrosion-doctors.org/Voltage/electromagnetic-effects.htm

Important Books/Journals for further learning including the page nos.: 01-13
Grzegorz Redlarski, Arkadiusz Żak, Natalia Ziółkowska, Barbara Przybylska-Gornowicz and Marek Krawczuk, "Influence of Electric, Magnetic, and Electromagnetic Fields on the Circadian System: Current Stage of Knowledge", BioMed Research International, Vol.6, PP 1-13, 2014.

Course Faculty

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L2

LECTURE HANDOUTS

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Different Co-ordinate Systems

Introduction :

- A coordinate system is a way of uniquely specifying the location of any position in space with respect to a reference origin. Any point is defined by the intersection of three mutually perpendicular surfaces.

Prerequisite knowledge for Complete understanding and learning of Topic:

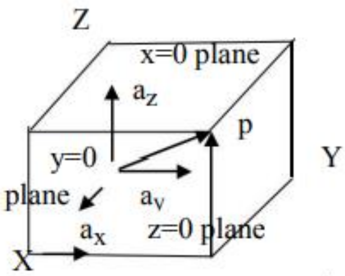
- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Different Co-ordinate Systems

- The coordinate axes are then defined by the normals to these surfaces at the point. Of course the solution to any Problem is always independent of the choice of coordinate system used, but by taking advantage of symmetry, computation can often be simplified by proper choice of coordinate description.
- The coordinate systems are
  1. Rectangular (Cartesian) coordinate system,
  2. Circular (cylindrical) coordinate system, and
  3. Spherical coordinate system.

1. Rectangular (Cartesian) coordinate system

- The most common and often preferred coordinate system is defined by the intersection of three mutually perpendicular planes



$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

$a_x \times a_y = a_z$

$a_y \times a_z = a_x$

$a_z \times a_x = a_y$

- If the point P is at a distance of r from O, then
- If the components of r along X, Y, Z are x, y, z then $r = x a_x + y a_y + z a_z = |r| a_r$

Differential length, surface and volume elements

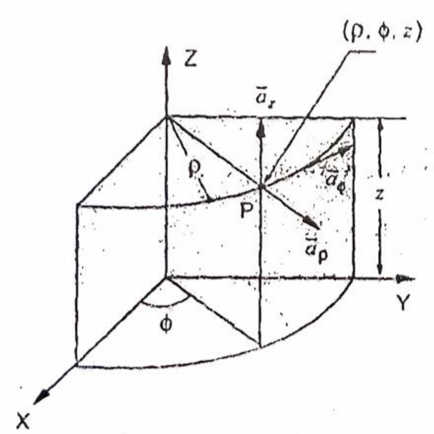
Differential length $| dr | = [dx^2 + dy^2 + dz^2]^{1/2}$

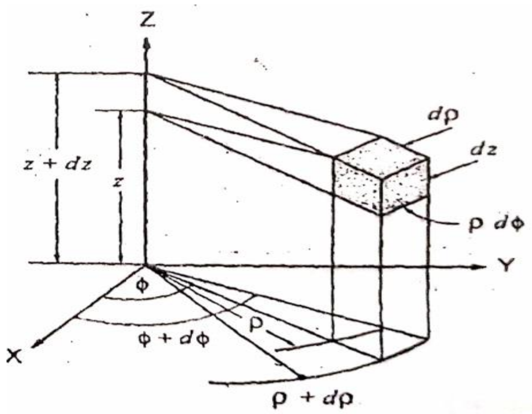
Differential surface $ds=dx\,dy +dy\,dz +dz\,dx$

Differential volume Dv= $dx\,dy\,dz$

2. Circular (cylindrical) coordinate system

- The circular cylindrical co-ordinate system is the three –dimensional version of polar coordinate of analytic geometry.





The differential length, $dl = \sqrt{(d\rho)^2 + (\rho\, d\phi)^2 + (dz)^2}$

The differential area, $ds = \rho\, d\rho\, d\phi$

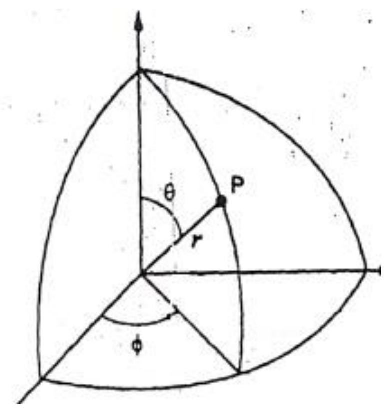
$= d\rho\, dz$

$= \rho\, d\phi\, dz$

The differential volume, $dv = \rho\, d\rho\, d\phi\, dz$

3. Spherical coordinate system

- The spherical coordinate system is most appropriate when dealing with problems having a degree of spherical symmetry.



The differential length, $dl = \sqrt{dr^2 + (r\, d\theta)^2 + (r \sin\theta\, d\phi)^2}$

The differential area, $ds = dr \cdot r\, d\theta = r\, dr\, d\theta$

$= dr \cdot r \sin\theta\, d\phi$

$= r \sin\theta\, d\phi\, dr$

$= r\, d\theta \cdot r \sin\theta\, d\phi$

$= r^2 \sin\theta\, d\theta\, d\phi$

The differential volume $dv = dr \cdot r\, d\theta \cdot r \sin\theta\, d\phi$

$dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$

Video Content / Details of website for further learning :

- http://www.crectirupati.com/sites/default/files/lecture_notes/EMF%20-%20II-I.pdf

Important Books/Journals for further learning including the page nos.: 5-6 and 12-18

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, pp., 2011.

Course Faculty

LECTURE HANDOUTS **L3**

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Tutorial Hour - Gradient, Divergence and Curl operation

Introduction :

- The Vector approach provides better insight into the various as sects of Electromagnetic phenomenon. Vector analysis is an essential tool. The Vector Analysis comprises of Vector Algebra and Vector Calculus

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

1. **gradient : ∇F=∂F∂xi+∂F∂yj+∂F∂zk.**
2. **divergence : ∇·f=∂f1∂x+∂f2∂y+∂f3∂z.**
3. **curl : ∇×f=(∂f3∂y−∂f2∂z)i+(∂f1∂z−∂f3∂x)j+(∂f2∂x−∂f1∂y)k.**

Problem 1

Given the two vector combinations

$$\mathbf{A}+\mathbf{B}=2i+3j-3k$$

and $$\mathbf{A}-\mathbf{B}=4i+j+k,$$

(*a*) *find A and B in vector form ;*

(*b*) *and also the dot products of* $\mathbf{A}+\mathbf{B}$ *and* $\mathbf{A}-\mathbf{B}$. *Also evaluate* $\mathbf{A}\cdot\mathbf{B}$.

Solution. $$\mathbf{A}+\mathbf{B}=2i+3j-3k \quad \ldots(i)$$
$$\mathbf{A}-\mathbf{B}=4i+j+k \quad \ldots(ii)$$

Adding (*i*) **and** (*ii*),

$$2\mathbf{A}=6i+4j-2k\ ;$$

(*a*) **whence** $$\mathbf{A}=3i+2j-k.$$

Similarly, subtracting (*ii*) **from** (*i*),

$$\mathbf{B}=-i+j-2k$$

(*b*) $$(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})=2i\cdot 4i+3j\cdot j+(-3k\cdot k)$$
$$=8+3-3$$
$$=8.$$
$$\mathbf{A}\cdot\mathbf{B}=-3+2+2=1.$$

Problem 2

Prove that the vectors

$$\mathbf{A} = 3i + j - 2k, \quad \mathbf{B} = 2i - 4j + k$$

and $$\mathbf{C} = 5i - 3j - k$$

form the sides of a right-triangle.

Solution. The three vectors can form the sides of a triangle if two of them added give the third one. Further, if any two of them are perpendicular to each other, the triangle is a right-angled one.

Examine the three vector relations :

$$\mathbf{A} = 3i + j - 2k \quad \text{...}(i)$$

$$\mathbf{B} = 2i - 4j + k \quad \text{...}(ii)$$

$$\mathbf{C} = 5i - 3j - k \quad \text{...}(iii)$$

Adding (*i*) and (*ii*) gives (*iii*)

Furthermore, $\mathbf{A} \cdot \mathbf{B} = 3 \times 2 - 1 \times 4 - 2 \times 1 = 0$, proving that the vectors A and B are perpendicular to each other. Hence the result.

Problem 3

Using the definition of "cross product", find the angle between the vectors

$$2\,\mathbf{u}_x - 2\mathbf{u}_y + \mathbf{u}_z \qquad \textit{and}$$

$$2\mathbf{u}_x - \mathbf{u}_y - 2\mathbf{u}_z.$$

Solution. The angle between two vectors **A** and **B** may be determined from the relation

$$\sin(\mathbf{A}, \mathbf{B}) = \frac{|\mathbf{A} \times \mathbf{B}|}{|\mathbf{A}|\,|\mathbf{B}|}$$

Let the vectors given be **A** and **B** respectively.

In matrix form,

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 2 & -2 & 1 \\ 2 & -1 & -2 \end{vmatrix}$$

$$= (4 + 1)\,\mathbf{u}_x + (2 + 4)\,\mathbf{u}_y + (-2 + 4)\,\mathbf{u}_z$$

$$= 5\mathbf{u}_x + 6\mathbf{u}_y + 2\mathbf{u}_z$$

$$|\mathbf{A} \times \mathbf{B}| = \sqrt{5^2 + 6^2 + 2^2}$$

$$= \sqrt{65}$$

$$|\mathbf{A}| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$|\mathbf{B}| = \sqrt{2^2 + 1^2 + 2^2} = 3$$

$$\sin(\mathbf{A}, \mathbf{B}) = \frac{\sqrt{65}}{(3)\,(3)} = 0.896$$

Angle between the vectors A and $B = \sin^{-1}(0.896) = \mathbf{63.6^\circ}$.

Video Content / Details of website for further learning :

- https://www.cis.rit.edu/class/simg303/Notes/Ch6-VectorCalculus.pdf

Important Books/Journals for further learning including the page nos.: 09-12

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS | **L4**

EEE | **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Divergence theorem & Stoke's theorem

Introduction :

- Divergence is defined as the net outward flow of the flux per unit volume over a closed incremental surface

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Divergence theorem

- The volume integral of the divergence of a vector field over a closed surface S enclosing volume V is equal to the volume integral of the divergence of integral taken through out the volume V.

$$\iint_s A.ds = \iiint_v \nabla.A dv$$

- It is the integral over the region enclosing a volume V, Surface integral is the surface of the surface of the region, It is denoted by S. · The divergence of any vector A is given

$$\nabla.A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

dv = dx dy dz

$$\iiint_v \nabla.A dv = \iiint_v \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right)$$

- Taking volume integral on both sides
- Consider element in x direction

$$= \iint_v \left[\frac{\partial A_x}{\partial x} dx \right] dydz \qquad \int \frac{\partial A_x}{\partial x} dx = Ax_1 - Ax_2 = Ax$$

- But x_1 and x_2 be limits for x direction

Then
$$\iiint \frac{\partial Ax}{\partial x} dxdydz = \iint_s Axdydz \qquad = \iint_s Axds_x$$

Similarly the following terms

$$\iiint_v \frac{\partial Ay}{\partial y} dxdydz = \iint_s Aydsy \quad , \quad \iiint_v \frac{\partial Az}{\partial z} dxdydz = \iint_s Azdsz$$

$$\iiint_v \nabla.Adv = \iiint_v \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dxdydz$$

$$= \iint_s (Axdsx + Aydsy + Azdsz)$$

$$\iiint_v \nabla.Adv = \iint_s A.ds$$

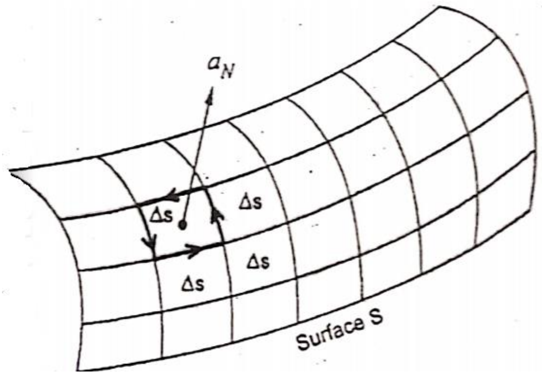
Stoke's theorem

- The surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector of along the contour bounding the surface

$$(\nabla \text{ x H}).\text{ds} = \text{H.dl}$$

Proof:

- Consider an arbitrary surface this is broken up into incremental surfaces of area Ds as shown in the figure. If H is any field vector, the by definition of the curl to one of these incremental surfaces.



$$\frac{\oint H.dl\Delta s}{\Delta s} = (\Delta x H)N$$

- Where, N indicates normal to the surface and dl D_s indicates that the closed path of an incremental area D_s. The curl of H normal to the surface can be written as

$$\frac{\oint H.dl\Delta s}{\Delta s} = (\Delta x H)a_N$$

or

$$\oint H.dl\Delta s = (\nabla x H)a_N \Delta s \quad = (\nabla x H).\Delta s$$

- Where a_N is a unit vector normal to D_s. The closed integral for whole surface s is given by the surface integral of the normal component of curl H

$$\oint H.dl = \iint_s \nabla x H.ds$$

Video Content / Details of website for further learning :

- http://files.teceee-in.webnode.in/200000080-99f329aec5/EMT-Lecture%20Notes-TEC.pdf

Important Books/Journals for further learning including the page nos.: 56-58

Purcell, Edward M.; David J. Morin, "Electricity and Magnetism" Cambridge Univ. Press., 2013

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LECTURE HANDOUTS

L5

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Coulomb's Law and applications & Electric field intensity

Introduction :

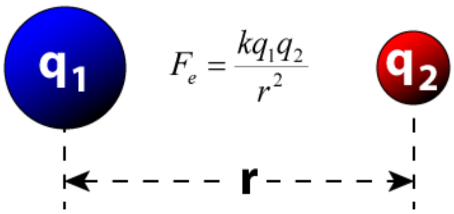
- One of the most important of these is Coulomb's law, which describes the electric force between charged objects. Both gravitational and electric forces decrease with the square of the distance between the objects, and both forces act along a line between them.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Coulomb's Law

- Coulomb's Law gives an idea about the force between two point charges. By the word point charge, we mean that in physics, the size of linear charged bodies is very small as against the distance between them.
- Therefore, we consider them as point charges as it becomes easy for us to calculate the force of attraction/ repulsion between them.



- Charles-Augustin de Coulomb, a French physicist in 1784, measured the force between two point charges and he came up with the theory that the force is inversely proportional to the square of the distance between the charges.
- He also found that this force is directly proportional to the product of charges (magnitudes only).
- We can show it with the following explanation. Let's say that there are two charges q_1 and q_2. The distance between the charges is 'r', and the force of attraction/repulsion between them is 'F'. Then

$$F \propto q_1q_2$$

$$F \propto 1/r^2$$

$F = k\, q_1 q_2 / r^2$

where

k is proportionality constant and equals to $1/4\,\pi\,\varepsilon_0$.

ε_0 is the epsilon naught and it signifies permittivity of a vacuum.

The value of k comes $9 \times 10^9\ Nm^2/\ C^2$ when we take the S.I unit of value of ε_0 is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

- According to this theory, like charges repel each other and unlike charges attract each other. This means charges of same sign will push each other with repulsive forces while charges with opposite signs will pull each other with attractive force.

Vector Form of Coulomb's Law

- The physical quantities are of two types namely scalars (with the only magnitude) and vectors (those quantities with magnitude and direction).
- Force is a vector quantity as it has both magnitude and direction. The Coulomb's law can be re-written in the form of vectors.
- Remember we denote the vector "F" as F, vector r as r and so on.
- Let there be two charges q1 and q2, with position vectors r1 and r2 respectively.
- Now, since both the charges are of the same sign, there will be a repulsive force between them. Let the force on the q_1 charge due to q_2 be F_{12} and force on q_2 charge due to q_1 charge be F_{21}. The corresponding vector from q_1 to q_2 is r_{21} vector.

$$r_{21} = r_2 - r_1$$

- Now, the force on charge q_2 due to q_1, in vector form is:

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1\, q_2}{r_{21}^2}\,\hat{r}_{21}$$

- The above equation is the vector form of Coulomb's Law.

Coulomb's Law applications

- Coulomb's Law has a great many applications to modern life, from
  - Xerox machines to laser printers,
  - to powder coating

Electric field intensity

- The space around an electric charge in which its influence can be felt is known as the **electric field**. The **electric field Intensity** at a point is the force experienced by a unit positive charge placed at that point.
- Electric Field Intensity is a vector quantity. It is denoted by 'E'. Formula: Electric Field is

  $E = F/q$.   Unit of E is NC^{-1} or Vm^{-1}.

- The difference between electric field and electric field intensity is the electric field is a region around a charge in which it exerts electrostatic force on another charges. While the strength of electric field at any point in space is called electric field intensity.

- The value of this force depends upon the distance between the two charges. The electric field intensity due to a positive charge is always directed away from the charge and the intensity due to a negative charge is always directed towards the charge.

- Due to a point charge q, the intensity of the electric field at a point d units away from it is given by the expression:

  $E = q/[4\pi\varepsilon d^2]$ NC^{-1}

- The intensity of the electric field at any point due to a number of charges is equal to the vector sum of the intensities produced by the separate charges.

- Force Experienced by a Charge in Electric Field

  The force experienced by a charge in an electric field is given by, \vec{F}=Q\vec{E}*F*=*QE*. where E is the electric field intensity.

Special Cases:

- If **Q is a positive charge**,
  Acceleration
  a = F/m = QE/m.

- If **Q is a negative charge**,
  Acceleration
  a = F/m = QE/m

- A charge in an electric field experiences a force whether it is at rest or moving. The electric force is independent of the mass and velocity of the charged particle, it depends upon the charge.

Video Content / Details of website for further learning:
https://www.toppr.com/guides/physics/electric-charges-and-fields/coulombs-law/

Important Books/Journals for further learning including the page nos.: 35-38

- Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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LECTURE HANDOUTS

L6

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction

Date of Lecture:

Topic of Lecture: Tutorial Hour

Introduction :

- One of the most important of these is Coulomb's law, which describes the electric force between charged objects. Both gravitational and electric forces decrease with the square of the distance between the objects, and both forces act along a line between them.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Coulomb's Law

Problem 1

Find the force of interaction between two charges spaced 10 cm apart in a vacuum. The charges are 4×10^{-8} and 6×10^{-5} coulomb respectively. If the same charges are separated by the same distance in kerosene ($\varepsilon_r = 2$), what is the corresponding force of interaction ?

Solution. Refer to Fig. 2.1,

$$q_1 = 4 \times 10^{-8} \text{ Coulomb.}$$
$$q_2 = 6 \times 10^{-5} \text{ Coulomb.}$$
$$r_{12} = 10 \text{ cm} = 0.1 \text{ m.}$$
$$\varepsilon_o = 1/(36\pi\, 10^9)$$
$$k = \frac{1}{4\pi\, \varepsilon_0} = \frac{1}{4\pi} \cdot 36\pi\, 10^9 = 9 \times 10^9.$$

Substituting the values in equation (2.2),

$$\mathbf{F}_{12} = 9 \times 10^9 \times \frac{4 \times 10^{-8} \times 6 \times 10^{-5}}{(0.1)^2} = \mathbf{u}_{12}$$
$$\mathbf{F}_{12} = 2.16 \text{ Nw.}$$

For kerosene, $\varepsilon_r = 2$

$$\mathbf{F}_{12} = \frac{2.16}{2} = 1.08 \text{ Nw.}$$

Problem 2

Three equal positive charges of 4×10^{-9} Coulomb each are located at three corners of a square, side 20 cm. Determine the magnitude and direction of the electric field at the vacant corner point of the square.

Solution. See Fig. 2.5.

The three charges q_1, q_2, q_3 are located at the corners 1, 2, 3 respectively of the square, with the corner 4 vacant. We have to find the resultant field **E** at 4.

The principle of superposition is applied for evaluation of the resultant electric field intensity **E** at the vacant corner 4. Thus $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$.

The fields E_1 and E_3 are equal in magnitude but perpendicular to each other, directed along 1-4 and 3-4 respectively.

$$E_1 = \left(\frac{q_1}{4\pi\varepsilon_0 a^2}\right) = 9 \times 10^9 \left(\frac{q_1}{a^2}\right)$$

(where a = side of the square = 0.2 m)

$$= (9 \times 10^9) \times \frac{4 \times 10^{-9}}{(0.2)^2}$$

$$|\mathbf{E}_1| = 900 \text{ V/m.} = |\mathbf{E}_3|$$

$\mathbf{E}_2$ is along the diagonal 2.4.

$$|\mathbf{E}_2| = (9 \times 10^9) \times \left[\frac{q_2}{(\sqrt{2}a)^2}\right] = 9 \times 10^9 \times \left[\frac{4 \times 10^{-9}}{2 \times (0.2)^2}\right]$$

$$= 450 \text{ V/m} \quad \text{(along the diagonal)}$$

$\mathbf{E}_1 + \mathbf{E}_3$ = vector directed along the diagonal, in the same direction as $\mathbf{E}_2$.

$$|\mathbf{E}_1 + \mathbf{E}_3| = \sqrt{2}E_1 = 900\sqrt{2}.$$

Hence the resultant field at 4 is directed along the diagonal and is, therefore, inclined at 45° to the horizontal axis (or to E_1).

$$\mathbf{E} = (900\sqrt{2} + 450) \underline{/+45^\circ} = \mathbf{1725 \angle 45^\circ \text{ V/m.}}$$

Video Content / Details of website for further learning :

- https://www.toppr.com/guides/physics/electric-charges-and-fields/coulombs-law/

Important Books/Journals for further learning including the page nos.: 37-38

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L7

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Field due to point & continuous charges

Introduction :

- The concept of electric field due to a continuous charge distribution. ... Coulomb's law gives the expression of on elctrostatic force on a charge due to other charge in it's vicinity.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Field due to point & continuous charges

- The charge distributions we have seen so far have been discrete: made up of individual point particles. This is in contrast with a **continuous charge distribution**, which has at least one nonzero dimension.

- If a charge distribution is continuous rather than discrete, we can generalize the definition of the electric field. We simply divide the charge into infinitesimal pieces and treat each piece as a point charge.

- For a line charge, a surface charge, and a volume charge, the summation in the definition of an Electric field. These are generalizations of the expression for the field of a point charge. They implicitly include and assume the principle of superposition.

- The "trick" to using them is almost always in coming up with correct expressions for dl, dA, or dV, as the case may be, expressed in terms of *r*, and also expressing the charge density function appropriately.

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\left(\frac{q_i}{r^2}\right)\hat{r}}_{\text{Point charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0}\int_{line}\left(\frac{\lambda\, dl}{r^2}\right)\hat{r}}_{\text{Line charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{surface} \left(\frac{\sigma\, dA}{r^2}\right) \hat{r}}_{\text{Surface charge}}$$

$$\vec{E}(P) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{volume} \left(\frac{\rho\, dV}{r^2}\right) \hat{r}}_{\text{Volume charge}}$$

- It may be constant; it might be dependent on location. Note carefully the meaning of r in these equations: It is the distance from the charge element (qi, λdl, σdA, ρdV) to the location of interest, P(x,y,z) (the point in space where you want to determine the field).

- However, don't confuse this with the meaning of r^; we are using it and the vector notation E⃗ to write three integrals at once. That is, Equation

$$E_x(P) = \frac{1}{4\pi\epsilon_0} \int_{line} \left(\frac{\lambda\, dl}{r^2}\right)_x,$$

$$E_y(P) = \frac{1}{4\pi\epsilon_0} \int_{line} \left(\frac{\lambda\, dl}{r^2}\right)_y,$$

$$E_z(P) = \frac{1}{4\pi\epsilon_0} \int_{line} \left(\frac{\lambda\, dl}{r^2}\right)_z$$

Video Content / Details of website for further learning :

- https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics /Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_II_ Thermodynamics%2C_Electricity%2C_and_Magnetism_(OpenStax)/05%3A Calculating_Electric_Fields_of_Charge_Distributions

Important Books/Journals for further learning including the page nos.: 40-42

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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L8

LECTURE HANDOUTS

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Electric flux density, Gauss's law and applications

Introduction :

- Electric flux density is electric flux passing through a unit area perpendicular to the direction of the flux. Electric flux density is a measure of the strength of an electric field generated by a free electric charge, corresponding to the number of electric lines of force passing through a given area.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric flux

- **Electric flux** is the rate of flow of the **electric** field through a given area. **Electric flux** is proportional to the number of **electric** field lines going through a virtual surface. When it comes to the study of science and the functioning of electricity, there is boundless knowledge and information that one stands to gain.
- The concept of Electric flux is one such field of study of science. It is pertinent to the understanding of electric force and its behavior. Electric flux is a property of an electric field. It may be thought of as the number of forces that intersect a given area.
- Electric field lines are usually considered to start on positive electric charges and to end on negative charges. Field lines directed into a closed surface are considered negative; those directed out of a closed surface are positive.
- If there is no given net charge within a given closed surface then every field line directed into the given surface continues through the interior and is usually directed outward elsewhere on the surface.
- The negative flux just equals in magnitude the positive flux, so that the net or total, electric flux is zero.
- If a net charge is contained inside a closed surface, the total flux through the surface is proportional to the enclosed charge, positive if it is positive, negative if it is negative.

Electric flux density

- **Electric flux density** is defined as the amount of flux passes through unit surface area in the space imagined at right angle to the direction of electric field.
- The expression of electric field at a point is given by

$$E = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}$$

- Where, Q is the charge of the body by which the field is created. R is the distance of the point from the center of the charged body.

$$Q = \Psi$$

- The above equation can be rewritten as,

$$E = \frac{\psi}{4\pi\epsilon_0\epsilon_r r^2} \Rightarrow \epsilon_0\epsilon_r E = \frac{\psi}{4\pi r^2}$$

- This is the expression of flux per unit area since, $4\pi r^2$ is the surface area of the imaginary spare of radius r.
- This is the flux passing through per unit area at a distance r from the center of the charge. This is called **electric flux density** at the said point.
- We generally denote it with English letter D. Therefore,

$$D = \epsilon_0\epsilon_r E$$

- From, the above expression of D, it is clear that electric field intensity and **electric field density** are in same phasor.

- As the number of electric lines of force emanated from a charged body is equal to the quantity of charge of the body measured in coulombs.

Gauss's Law

- According to the Gauss law, **the total flux linked with a closed surface is $1/\varepsilon_0$ times the charge enclosed by the closed surface.**
- For example, A point charge q is placed inside a cube of edge 'a'. Now as per the **Gauss law**, the flux through each face of the cube is $q/6\varepsilon_0$.
- The electric field is the basic concept to know about electricity. Generally, the electric field of the surface is calculated by applying Coulomb's law, but to calculate the electric field distribution in a closed surface, we need to understand the concept of Gauss law.
- It explains about the electric charge enclosed in a closed or the electric charge present in the enclosed closed surface.

Gauss Law Formula

- As per the Gauss theorem, the total charge enclosed in a closed surface is proportional to the total flux enclosed by the surface. Therefore, If ϕ is total flux and ϵ_0 is electric constant, the total electric charge Q enclosed by the surface is;

$$Q = \phi\,\epsilon_0$$

- The **Gauss law** formula is expressed by;

$$\phi = Q/\epsilon_0$$

Where,

Q = total charge within the given surface,

ε_0 = the electric constant.

Gauss Theorem

- The net flux through a closed surface is directly proportional to the net charge in the volume enclosed by the closed surface

$$A = q_{net}/\varepsilon_0$$

- In simple words, the **Gauss theorem** relates the 'flow' of electric field lines (flux) to the charges within the enclosed surface. If there are no charges enclosed by a surface, then the net electric flux remains zero. This means that the number of electric field lines entering the surface is equal to the field lines leaving the surface.

Applications of Gauss Law

- In the case of a charged ring of radius R on its axis at a distance x from the centre of the ring. In case of an infinite line of charge, at a distance 'r'.

$$E = (1/4 \times \pi r\varepsilon_0)(2\pi/r) = \lambda/2\pi r\varepsilon_0.$$

Where

λ is the linear charge density.

- The intensity of the electric field near a plane sheet of charge is $E = \sigma/2\varepsilon 0K$
  where σ = surface charge density.
- The intensity of the electric field near a plane charged conductor $E = \sigma/K\varepsilon 0$ in a medium of dielectric constant K. If the dielectric medium is air, then $Eair = \sigma/\varepsilon 0$.
  The field between two parallel plates of a condenser is $E = \sigma/\varepsilon_0$, where σ is the surface charge density.

Video Content / Details of website for further learning :

- https://www.electrical4u.com/electric-flux/

Important Books/Journals for further learning including the page nos.: 69-71

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

L9

LECTURE HANDOUTS

EEE

II/III

Course Name with Code : 16EED03/ Electromagnetic Theory

Course Faculty : Ms V.Deepika

Unit : I - Introduction **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The electric field intensity is the measure of the strength of an electric field at any point. It is equal to the electric force per unit charge experienced by a test charge which is placed at that point. The electric field intensity between two points is the vector sum of all the electric fields acting at that point

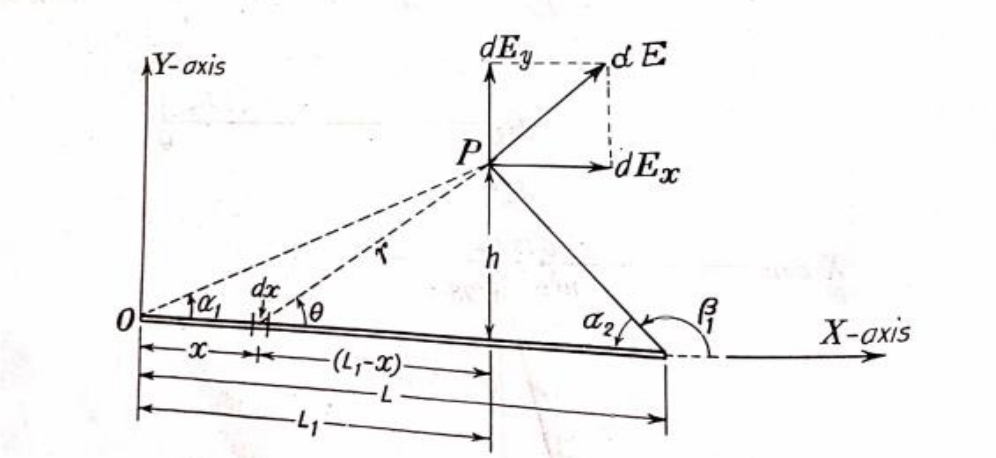
Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric field intensity

Electric field due to a straight, uniformly charged wire at a linear density of $+\lambda$ Coulomb per metre length. To find the field intensity at any point P as shown in Fig. 2.9, at a distance h.

Solution. Consider a uniformly charged wire (L), the charge being assumed to be uniformly distributed at the rate



$$\left.\begin{aligned} dE_x &= dE \cos\theta \\ dE_y &= dE \sin\theta \end{aligned}\right\}$$

$$dE = \frac{\lambda\, dx}{4\pi\, \varepsilon_0\, r^2}$$

Thus $$dE_x = \frac{\lambda \cos\theta\, dx}{4\pi\, \varepsilon_0\, r^2}$$

Referring to Fig. 2.9,

$$L_1 - x = h \cot\theta$$

Differentiating,

$$-dx = -h \operatorname{cosec}^2\theta\, d\theta$$

Also, $$r = h \operatorname{cosec}\theta$$

Substituting in 2.15 (a),

$$dE_x = \frac{\lambda}{4\pi\, \varepsilon_0\, h} \cos\theta\, d\theta$$

$$E_x = \int_{\theta=\alpha_1}^{\beta_1} dE_x = \frac{\lambda}{4\pi\, \varepsilon_0\, h} \int_{\alpha_1}^{\pi-\alpha_2} \cos\theta\, d\theta$$

$$E_x = \frac{\lambda}{4\pi\, \varepsilon_0\, h} (\sin\alpha_2 - \sin\alpha_1)$$

Similarly, as $$dE_y = \frac{\lambda\, dx}{4\pi\, \varepsilon_0\, r^2} \sin\theta$$

we obtain the Y-component of the field at P as

$$E_y = \frac{\lambda}{4\pi\, \varepsilon_0\, h} (\cos\alpha_1 + \cos\alpha_2)$$

Case (i)

$\alpha_1 = \alpha_2 = \alpha$, say, we have

$$E = E_y = \frac{\lambda}{2\pi\, \varepsilon_0\, h} \cos\alpha$$

$$\boxed{\mathbf{E} = \hat{n} \frac{\lambda}{2\pi\, \varepsilon_0\, h} \cos\alpha}$$

Case (ii)

Infinity of conductor

$$\mathbf{E} = \hat{n} \frac{\lambda}{2\pi\, \varepsilon_0\, h} (\text{as } \alpha = 0)$$

Video Content / Details of website for further learning :

- http://www.yourdictionary.com › electric-flux-density

Important Books/Journals for further learning including the page nos.: 46-48

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L10

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field

Date of Lecture:

Topic of Lecture: Electric potential

Introduction :

- The work per unit of charge required to move a charge from a reference point to a specified point, measured in joules per coulomb or volts. The static electric field is the negative of the gradient of the electric potential.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric potential

- The potential difference between two points is defined as the work required to be done for bringing a unit positive charge from one point to other point. When a body is charged, it can attract an oppositely charged body and can repulse a similar charged body. That means, the charged body has ability of doing work. That ability of doing work of a charged body is defined as electrical potential of that body.

- If two electrically charged bodies are connected by a conductor, the electrons starts flowing from lower potential body to higher potential body, that means current starts flowing from higher potential body to lower potential body depending upon the potential difference of the bodies and resistance of the connecting conductor.

- So, electric potential of a body is its charged condition which determines whether it will take from or give up electric charge to other body. Electric potential is graded as electrical level, and difference of two such levels, causes current to flow between them. This level must be measured from a reference zero level. The earth potential is taken as zero level.

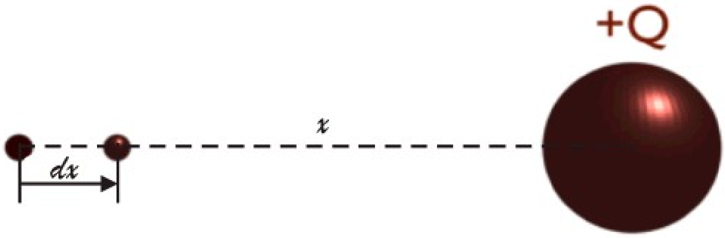
- Electric potential above the earth potential is taken as positive potential and the electric potential below the earth potential is negative. The unit of electric potential is volt. To bring a unit charge from one point to another, if one joule work is done, then the potential difference between the points is said to be one volt.

Potential at a Point due to Point Charge

- Let us take a positive charge + Q in the space. Let us imagine a point at a distance x from the said charge + Q. Now we place a unit positive charge at that point. As per Coulomb's law, the unit positive charge will experience a force,

$$F = \frac{Q}{4\pi\varepsilon_0\epsilon_r x^2}$$

Now, let us move this unit positive charge, by a small distance dx towards charge Q.



- During this movement the work done against the field is,

$$dw = -F.dx = -\frac{Q}{4\pi\epsilon_0\epsilon_r x^2}$$

- So, total work to be done for bringing the positive unit charge from infinity to distance x, is given by,

$$-\int_{\infty}^{x} dw = -\int_{\infty}^{x} \frac{Q}{4\pi\epsilon_0\epsilon_r x^2}.dx = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{x}\right]_{\infty}^{x} = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{x} - \frac{1}{\infty}\right] = \frac{Q}{4\pi\epsilon_0\epsilon_r x}$$

- As per definition, this is the electric potential of the point due to charge + Q. So, we can write,

$$V = \frac{Q}{4\pi\epsilon_0\epsilon_r x}$$

Video Content / Details of website for further learning :

- https://en.wikipedia.org/wiki/Electric_potential

Important Books/Journals for further learning including the page nos.: 80-84

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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Estd. 2000

LECTURE HANDOUTS

L11

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Electric field & equipotential plots and Relationship between E&V

Introduction :

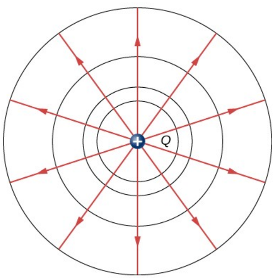
- The electric potential has magnitude but no direction; it is a scalar field. To represent this field graphically, lines of equal electric potential, called equipotential lines, are drawn around the source charge. As in the image below, the equipotential lines surrounding a positive point charge form concentric circles

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric field and equipotential plots

- We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. This is not surprising, since the two concepts are related. Consider Figure, which shows an isolated positive point charge and its electric field lines, which radiate out from a positive charge and terminate on negative charges.



- We use red arrows to represent the magnitude and direction of the electric field, and we use black lines to represent places where the electric potential is constant. These are called equipotential surfaces in three dimensions, or equipotential lines in two dimensions. The term equipotential is also used as a noun, referring to an equipotential line or surface.
- The potential for a point charge is the same anywhere on an imaginary sphere of radius *r* surrounding the charge. This is true because the potential for a point charge is given by

$$V=kq/r$$

- Thus has the same value at any point that is a given distance *r* from the charge. An equipotential sphere is a circle in the two-dimensional. Because the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.
- It is important to note that equipotential lines are always perpendicular to electric field lines. No work is required to move a charge along an equipotential, since ΔV=0. Thus, work is

$$W = -\Delta U = -q\Delta V = 0.$$

- Work is zero if the direction of the force is perpendicular to the displacement. Force is in the same direction as *E*, so motion along an equipotential must be perpendicular to *E*
- More precisely, work is related to the electric field by

$$W = \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{d}} = q\overrightarrow{\mathbf{E}} \cdot \overrightarrow{\mathbf{d}} = qEd\cos\theta = 0.$$

- Note that in this equation, *E* and *F* symbolize the magnitudes of the electric field and force, respectively. Neither *q* nor *E* is zero; *d* is also not zero.
- So cosθ must be 0, meaning θ must be 90º. In other words, motion along an equipotential is perpendicular to *E*.
- One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a conductor is an equipotential surface in static situations.
- There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at what we consider zero volts by connecting it to the earth with a good conductor a process called **grounding**. Grounding can be a useful safety tool.
- For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to Earth.
- Because a conductor is an equipotential, it can replace any equipotential surface. For example, in Figure, a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.
- Figure shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines.

Relationship between E&V

The relation is very simple. Electric field intensity is equal to the negative of rate of change of potential with respet to the distance or it can be defined as the negative of the rate of derivative of potential difference, V with respect to r, E = - dV/dr.21-Sep-2010

Video Content / Details of website for further learning :

- https://cnx.org/contents/GYAoAVIF@8/Equipotential-Surfaces-and-Conductors

Important Books/Journals for further learning including the page nos.: 100-102

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L12

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Electric field in free space, conductors and dielectrics

Introduction :

- The permittivity of free space (a vacuum) is a physical constant equal to approximately 8.85 x 10^{-12} farad per meter (F/m). It is symbolized. In general, permittivity is symbolized and is a constant of proportionality that exists between **electric** displacement and **electric** field intensity in a given medium.

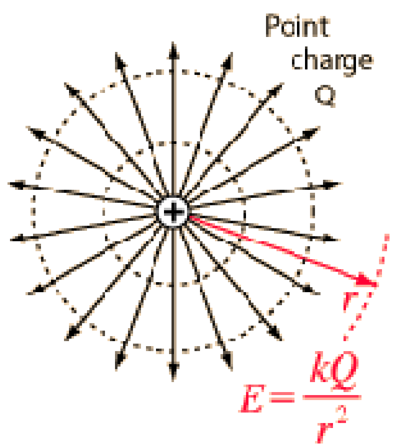
Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric field in free space

- The electric field of a point charge can be obtained from Coulomb's law

$$E = \frac{F}{q} = \frac{kQ_{source}q}{qr^2} = \frac{kQ_{source}}{r^2}$$



- The electric field is radially outward from the point charge in all directions. The circles represent spherical equipotential surfaces. The electric field is radially outward from the point charge in all directions. The circles represent spherical equipotential surfaces

- The electric field from any number of point charges can be obtained from a vector sum of the individual fields. A positive number is taken to be an outward field; the field of a negative charge is toward it

- In the equations describing electric fields and their propagation, three constants are normally used. One is the speed of light c, and the electric permittivity of free space ε0 This contains the force unit N for Newton and the unit A is the Ampere, the unit of electric current

- This gives a value of free space permittivity

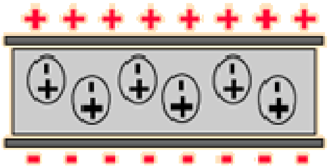
$$\varepsilon_0 = 8.854187817 \times 10^{-12} F/m \approx 8.85 \times 10^{-12} F/m$$

- which in practice is often used in the form

$$k = \frac{1}{4\pi\varepsilon_0} = 8.987552 \times 10^9 Nm^2/C^2 \text{ = Coulomb's constant}$$

- C for Coulomb, the unit of electric charge. Expressions for the electric fields in free space contain the electric permittivity ε_0 of free space

- The electric permittivity is connected to the energy stored in an electric field. It is involved in the expression for capacitance because it affects the amount of charge which must be placed on a capacitor to achieve a certain net electric field. In the presence of a polarizable medium, it takes more charge to achieve a given net electric field and the effect of the medium is often stated in terms of a relative permittivity



$$C = \frac{k\varepsilon_0 A}{d}$$

Electric field in conductors and dielectrics

- The **electric field** is zero inside a **conductor**. Just outside a **conductor**, the **electric field** lines are perpendicular to its surface, ending or beginning on charges on the surface. Any excess charge resides entirely on the surface or surfaces of a **conductor**.
- Conductor is one in which the outer electrons of an atom is easily detachable and migrate with application of weak Electric field. A dielectric is one in which the electrons are rigidly bounded to their nucleus, so the ordinary electric field will not be able to detach them away. The dielectric placed in electrostatic field will be subjected to electro static induction.
- The electric field will twisted and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely. If the electric field strength is

too high the dielectric will break down cease to beam insulator.

- The electric field in a conductor zero. Because in electrostatics free charges in a good conductor reside only on the surface. So the free charge inside the conductor is zero. So the field in it is caused by charges on the surface. Since charges are of the same nature and distribution is uniform, the electric fields cancel each other.

Types of Dielectrics:

1. Polar dielectrics
2. Non-polar dielectrics

1. Polar dielectrics:

- In polar dielectrics the molecules from dipoles even in absence of electric field. Even in absence of electric field, the dipoles are disposed at random the resultant electric field is zero.
- On the application of electric field the dipoles rearranged themselves so that their axes are aligned with the applied field. The electric field will twist and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely.
- These shifting results an instantaneous current called displacement current which causes in very small fraction of seconds. Eg: water, ether, ammonia

2. Non-Polar dielectrics:

- In these dielectrics the positive and negative elements in the uncharged conditions are closed to each other that their action is neutral.
- In the application of electric field will strict the positive and negative charges lightly with in the molecules to give rise to dipole. Eg: H, O etc

Video Content / Details of website for further learning :

- http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefie.html

Important Books/Journals for further learning including the page nos.: 93-94

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, pp., 2011.

Course Faculty

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LECTURE HANDOUTS

L13

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The work per unit of charge required to move a charge from a reference point to a specified point, measured in joules per coulomb or volts. The static electric field is the negative of the gradient of the electric potential

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Electric potential

Problem 1

A uniform electric field is directed along the positive X-axis and its magnitude is 30 Nw per Coulomb (or Volt per metre). What is the potential difference between the two points P_1 and P_2 on X-axis at distances $x_1 = 5$ cm and $x_2 = 20$ cm from the origin ?

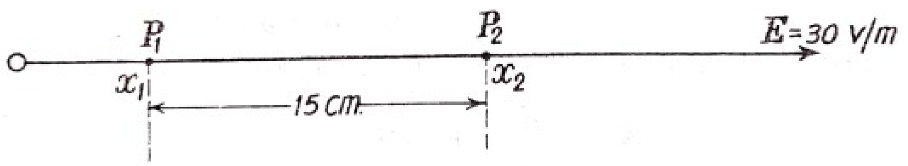


Fig. 3.13

If a positive test charge is moved from P_2 to P_1, work is done against the field (which is directed from P_1 to P_2) and so potential rises from P_2 to P_1 and is equal to $E(x_2 - x_1)$ $= 30 \times 0.15 = 4.5$ Volts. P_1 is at a higher potential than P_2 by **4.5 Volts.**

$$= 20 \times 1.5 = +30 \text{ V}.$$

The point b is at a potential of **30 V** above that of a.

Problem 2

Determine the potential difference between the points a and b which are at a distance of 0.5 m and 0.1 m respectively from a negative charge of 2.0×10^{-10} Coulomb.

Solution. Use equation (3.41) reproduced below, by taking

$$x_2 = a = 0.5 \text{ m}$$

and

$$x_1 = b = 0.1 \text{ m}$$

$$Q = -2 \times 10^{-10} \text{ Coulomb.}$$

$$V_{ba} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= 1.8 \times 8 = \mathbf{14.4 volts.}$$

The point a is at a higher potential than b by **14.4 volts.**

Problem 3

Explain and define the potential at a point in an electric field. Derive the potential at any point in a field due to a point charge.

Calculate the potential at a point P (x = 0, y = 0 metre) due to point charges Q_1 and Q_2. $Q_1 = +10^{-12}$ Coulomb at (x = 0.5, y = 0) and $Q_2 = -10^{-11}$ Coulomb at (x = −0.5, y = 0).

(Madras University, Nov. 1968)

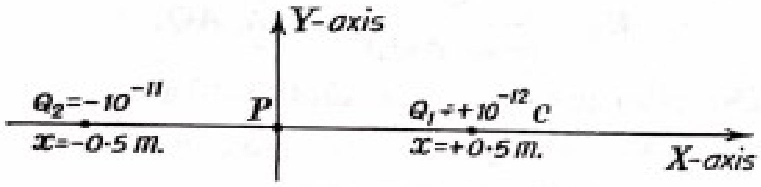


Fig. 3.15.

Solution. Potential at

P = Potential due to Q_1 + Potential due to Q_2

$$= \frac{Q_1}{4\pi\varepsilon_0 (0.5)} + \frac{Q_2}{4\pi\varepsilon_0 (0.5)}$$

Substituting for Q_1 and Q_2,

$$\mathbf{V = 0.162 Volt.}$$

Video Content / Details of website for further learning :

- https://en.wikipedia.org/wiki/Electric_potential

Important Books/Journals for further learning including the page nos.: 85-87

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

L14

LECTURE HANDOUTS

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Dielectric polarization, Electric field in multiple dielectrics, Boundary conditions

Introduction :

- Dielectric polarization is the term given to describe the behavior of a material when an external electric field is applied on it. A simple picture can be made using a capacitor

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Dielectric polarization

- It is defined as product of charge Q and distance between the two charges. It is a vector since length is in vector, length vector is directed from negative to positive charge, therefore dipole moment is from negative to positive charge.

$$P = Q.l$$

- The elastic shifting of charged clouds in an atom of dielectric material when it is subjected to an electric field is called polarization. It is defined as movement of dipole.

$$P = \frac{m}{V}$$

- If there are n dipoles the volume then total dipole moment is

$$m = m_1 + m_2 + \ldots + m_n \Delta V$$

$$m = \sum_{i=1}^{n\Delta V} m_i$$

- polarization = total displacement / volume

$$= \frac{\sum_{i=1}^{n\Delta V} m_i}{V}$$

- Consider a dielectric material cutting the form of a slab of permittivity ? as shown and placed in uniform electric field.

- The effect of field due to polarize the dielectric inducing atomic dipole through out the volume of specimen in the alignment with the electric field.

- Consequently neutralization of equal and opposite charge inside the dielectric charges reside on the slab and form dipole.

The internal field $E_i = E_a + E^1$

- here Ea is applied field, E1 is field induced in the slab which is opposite to that of applied field.

$$E^1 = -\frac{\sigma_p}{\epsilon_0} u_l$$

$$= -\frac{P}{\epsilon_0} u_l$$

$$E_i = E_a - \frac{P}{\epsilon_0}$$

$$E_a = E_i + \frac{P}{\epsilon_0}$$

$$\epsilon_0 E_a = \epsilon_0 E_i + P$$

$$D = \epsilon_0 E_i + P \quad (1)$$

$$P \, \alpha \, E_i$$

$$P = \epsilon_0 \psi_p E_i$$

$$D = \epsilon_0 E_i + \epsilon_0 \psi_p E_i$$

$$D = \epsilon_0 E_i (1 + \psi_p)$$

$$D = \epsilon_0 \epsilon_r E_i$$

Electric field in multiple dielectrics, Boundary conditions

- When the flux lines are flow through single medium they are continuous. If they go through boundary formed by two dielectrics they get reflected.
- First boundary condition deals with electric field intensity. E1 and E2 are electric field in medium 1 and 2 respectively. Construct a rectangular path ABCDA as shown and apply conservative property for the rectangular loop ABCDA.

$\int E.dl = 0$

$\int_{AB} E.dl + \int_{BC} E.dl + \int_{CD} E.dl + \int_{DA} E.dl = 0$

$E_{t1}\, \Delta l - E_{n1} \frac{\Delta h}{2} - E_{n2} \frac{\Delta h}{2} - E_{t2}\, \Delta l + E_{n2} \frac{\Delta h}{2} + E_{n1} \frac{\Delta h}{2} = 0$

$E_{t1} = E_{t2}$ (1)

At the boundary the tangent along components of electric field vectors are equal.

$\sin\theta_1 = \frac{Et1}{E_1}$

$E_{t1} = E_1 \sin\theta_1$ (2)

$E_{t2} = E_2 \sin\theta_2$ (3)

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ (4)

Second boundary equations:

- D_{n1} and D_{n2} are normal components of flux density vectors in medium 1 and 2 respectively. An infinite sheet with charge density W C/m2 is at the boundary. Second boundary condition deals with flux density. Construct the pill box at the boundary as shown. Apply Gauss's law

Flux enter the pill box $= D_{n2}\, dS$

Flux leave the pill box $= D_{n1}\, dS$

Net flux in the pill box $= D_{n2}\, dS - D_{n1}\, dS = \sigma\, dS$

$D_{n2} - D_{n1} = \sigma$

If the charge sheet is not present then

$D_{n2} - D_{n1} = 0$

$D_{n2} = D_{n1}$ (5)

This is known as second boundary condition.

$\cos\theta_1 = \frac{D_{n1}}{D_1}$

$D_{n1} = D_1 \cos\theta_1$ (6)

$\cos\theta_2 = \frac{D_{n2}}{D_2}$

$D_{n2} = D_2 \cos\theta_2$ (7)

$D_1 \cos\theta_1 = D_2 \cos\theta_2$ (8)

$\frac{E_1 sin\theta_1}{D_1 cos\theta_1} = \frac{E_2 sin\theta_2}{D_2 cos\theta_2}$

$D_1 = \epsilon_0 \epsilon_{r1} E_1$

$D_2 = \epsilon_0 \epsilon_{r2} E_2$

$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_{r2}}{\epsilon_{r1}}$

θ_1 is angle of emergence.

θ_2 is angle of incidence.

This is relation between two dielectric surfaces.

Video Content / Details of website for further learning:

- http://gvpcew.ac.in/Material%203%20Units/2%20EEE%20EMF.pdf

Important Books/Journals for further learning including the page nos.: 150-153

- Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011

Course Faculty

LECTURE HANDOUTS | **L15**

EEE | **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The work per unit of charge required to move a charge from a reference point to a specified point, measured in joules per coulomb or volts. The static electric field is the negative of the gradient of the electric potential

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Coulomb's Law

Problem 1

Given a field $E = \frac{-6y}{x^2}\bar{a}_x + \frac{6}{x}\bar{a}_y + 5\bar{a}_z$ *V/m, find the potential difference* V_{AB} *between A (– 7, 2, 1) and B (4, 1, 2).*

Solution:

$$E = \frac{-6y}{x^2}\bar{a}_x + \frac{6}{x}\bar{a}_y + 5\bar{a}_z$$

$$\text{Potential difference } V_{AB} = -\int_B^A E \cdot dl$$

$$= -\int_4^{-7} -\frac{6y}{x^2}\,dx - \int_1^2 \frac{6}{x}\,dy - \int_2^1 5\,dx$$

$$= 6y\left[\frac{-1}{x}\right]_4^{-7} - \frac{6}{x}\left[y\right]_1^2 - 5\left[x\right]_2^1$$

$$= \frac{66}{28}y - \frac{6}{x} + 5$$

$$x = 4, \quad y = 1$$

$$V_{AB} = 2.357 - 1.5 + 5 = \mathbf{5.857\ volts}$$

Problem 2

In a rectangular co-ordinate referenced system, the electric potential is, $V = \dfrac{20}{x^2 + y^2 + z^2}$

Determine the electric vector that is the gradient of potential.

☺***Solution:***

$$V = \frac{20}{x^2 + y^2 + z^2}$$

$$E = -\nabla \cdot \bar{V} = -\left(\frac{\partial}{\partial x} \frac{20}{x^2+y^2+z^2} \bar{a}_x + \frac{\partial}{\partial y} \frac{20}{x^2+y^2+z^2} \bar{a}_y + \frac{\partial}{\partial z} \frac{20}{x^2+y^2+z^2} \bar{a}_z \right)$$

$$= \frac{40}{(x^2+y^2+z^2)^2} [x\,\bar{a}_x + y\,\bar{a}_y + z\,\bar{a}_z] \text{ V/m}$$

Problem 3

If $V = 2x^2 y + 20z - \dfrac{4}{x^2+y^2}$ *volts, find* $\bar{E}$ *and* $\bar{D}$ *at* $\rho(6, -2.5, 3)$.

☺***Solution:*** $E = -\nabla V$

$$= -\left[\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z} \right] \left(2x^2 y + 20z - \frac{4}{x^2+y^2} \right)$$

$$= -\left[\bar{a}_x \left(4xy + \frac{8x}{(x^2+y^2)^2} \right) + \bar{a}_y \left(2x^2 + \frac{8y}{(x^2+y^2)^2} \right) + \bar{a}_z\, 20 \right]$$

$$E(6, -2.5, 3) = -[(-60 + 0.0268)\bar{a}_x + (72 - 0.012)\bar{a}_y + 20\,\bar{a}_z]$$

$$= 59.97\,\bar{a}_x - 71.99\,\bar{a}_y - 20\,\bar{a}_z \text{ V/m}$$

Video Content / Details of website for further learning :

- https://en.wikipedia.org/wiki/Electric_potential

Important Books/Journals for further learning including the page nos.: 85-87

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L16

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Poisson's and Laplace's equations

Introduction :

- The concept of electric field due to a continuous charge distribution. ... Coulomb's law gives the expression of on elctrostatic force on a charge due to other charge in it's vicinity.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Poisson's and Laplace's equations

- Laplace's and Poisson's Equations. A useful approach to the calculation of electric potentials is to relate that potential to the charge density which gives rise to it.
- Poisson's equation. This is an example of a very famous type of partial differential equation known as Poisson's equation. Poisson's equation has this property because it is linear in both the potential and the source term
- From the Gauss law we know that

$$\int D.ds = Q \quad (1)$$

- A body containing a charge density ρ uniformly distributed over the body. Then charge of that body is given by

$$Q = \int \rho \, dv \quad (2)$$

$$\int D.ds = \int \rho \, dv \quad (3)$$

- This is integral form of Gauss law. As per the divergence theorem

$$\int D.ds = \int \nabla.\mathbf{D}\ dv \quad (4)$$

$$\nabla.\mathbf{D} = \rho \quad (5)$$

- This is known as point form or vector form or polar form. This is also known as Maxwell's first equation

$$D = \varepsilon E \quad (6)$$

$$\nabla.\ \varepsilon E = \rho$$

$$\nabla.\ E = \rho/\varepsilon \quad (7)$$

- We know that E is negative potential medium

$$E = -\nabla V \quad (8)$$

- From equations 7 and 8

$$\nabla.\ (-\nabla V) = \rho/\varepsilon$$

$$\nabla^2 V = -\ \rho/\varepsilon \quad (9)$$

- It is known as Poisson's equation in static electric field.

- Consider a charge free region (insulator) the value of $\rho = 0$, since there is no free charges in dielectrics or insulators.

$$\nabla^2 V = 0$$

- This is known as Laplace's equation.

Video Content / Details of website for further learning :

- http://gvpcew.ac.in/Material%203%20Units/2%20EEE%20EMF.pdf

Important Books/Journals for further learning including the page nos.: 1-14

Mohammad Mirhosseini, A. Kaveh and Hossein Rahami, "Analytical Solution of Laplace and Poisson Equations Using Conformal Mapping and Kronecker Products", International Journal of Civil Engineering, 14(6) · June 2016

Course Faculty

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS

L17

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : II - Static Electric Field **Date of Lecture:**

Topic of Lecture: Capacitance- Energy density and Dielectric strength

Introduction :

- It is the property of capacitor which stores electrical energy in electric field. It is the property of capacitor which oppose the sudden change in voltage. It is the ratio of charge on one of its conductors to the potential difference between them

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Capacitance of Parallel plate capacitor

- Consider a parallel plate capacitor shown in fig. The space between 2 plates is filled with dielectric. The charge of each plate is Q and area of each plate is A and distance between 2 plates is d. We know that electric flux density

$D = Q/A \quad (1)$

$E = D/\varepsilon \quad (2)$

$E = Q/(A\varepsilon)$

$E = V/d \quad (3)$

Equating equations 2 and 3

$V/d = Q/(A\varepsilon)$

$C = \varepsilon A/d$

- Capacitance is directly proportional to area and inversely proportional to distance between plates.

Energy stored in a capacitor

- A capacitor is charged from a dc source as shown. The voltage across the capacitor is V in the closed loop. The V across the capacitor opposes the supply voltage, the workdone by the source is against the opposition of V is converted into energy. We know that potential is the workdone by the charge.

- Let dw is the workdone to establish a charge dq by the deficition of potential

$$V = \frac{dw}{dq}$$

$$dw = Vdq$$

$$= \frac{q}{c} dq$$

- Then the total workdone to establish a charge +Q can be obtained by integration

$$W = \int_0^Q \frac{q}{c}\, dq$$

$$= \frac{1}{2} \frac{Q^2}{c}$$

$$= \frac{1}{2} CV^2$$

$$= \frac{1}{2} QV$$

Energy Density

- The energy stored in a capacitor is

$$W = \frac{1}{2} CV^2$$

We know that $C = \frac{\epsilon A}{d}$

$$W = \frac{1}{2} V^2 \frac{\epsilon A}{d} = \frac{1}{2} \epsilon (\frac{V}{d})^2 \text{ Ad}$$

$$W = \frac{1}{2} \epsilon \text{ } E^2 \text{ Volume}$$

$$\frac{Energy\ stored}{volume} = \frac{1}{2} \epsilon E^2$$

$$\text{Energy density} = \frac{1}{2} \epsilon E^2 = \frac{1}{2} DE$$

$$\text{Energy stored} = \int_v \frac{1}{2} DE \text{ dv}$$

Energy density due to n charges:

$$W = \frac{1}{2} QV$$

$$W = \frac{1}{2} \{Q_1V_1 + Q_2V_{2+ \ldots\ldots +} Q_nV_n\}$$

Dielectric strength

- Dielectric Strength reflects the **electric strength of insulating materials** at various power frequencies.
- It can be defined as the measure of dielectric breakdown resistance of a material under an

applied voltage and is expressed as Volts per unit thickness. It is an indicator of how good an insulator a material.

- In other words, it is the **voltage per unit thickness** at which a material will conduct electricity. The higher the value, the more electrically insulating a material is. It is an important property sought for materials used in applications where electrical field is present and is a vital parameter for electrical industry applications.

Video Content / Details of website for further learning :

- http://gvpcew.ac.in/Material%203%20Units/2%20EEE%20EMF.pdf

Important Books/Journals for further learning including the page nos.: 157-160 & 164-167
Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS

L18

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The property of capacitor which stores electrical energy in electric field. It is the property of capacitor which oppose the sudden change in voltage. It is the ratio of charge on one of its conductors to the potential difference between them.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Capacitance

Problem 1:

Determine the capacitance of a parallel plate capacitor composed of tin-foil sheets, 25 cm square for plates separated through a glass-dielectric 0.5 cm. thick with relative permittivity 6.

Solution. Use relation

$$C = \varepsilon_0 \varepsilon_r \frac{A}{t}$$

where

$$\varepsilon_0 = \frac{1}{36\pi\, 10^9}; \quad \varepsilon_r = 6$$

$$A = 0.20 \times 0.25\,\text{m}^2$$

$$t = 0.5 \times 10^{-2}\,\text{m}.$$

Substituting in the expression for C,

$$C = \frac{6}{36\pi\, 10^9} \times \frac{0.0625}{0.005}$$

$$= (663) \times 10^{-12}\,\text{farad}$$

$$= 663\ \text{micro-micro-farad}(\mu\mu\,\text{F})$$

$$\mathbf{C = 663\,pF.}$$

Problem 2:

Deduce an expression for the capacitance of a parallel plate capacitor having two dielectric media.

A parallel plate capacitor has a plate separation t. The capacitance with air only between the plates is C. When a slab of thickness t' and relative permittivity ε_r is placed on one of the plates, the capacitance is C'. Show that

$$\frac{C'}{C} = \frac{\varepsilon_r t}{t' + \varepsilon_r (t - t')}$$

Solution.

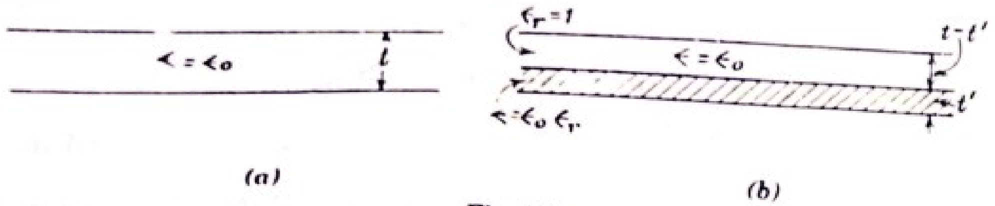


Fig. 4.7.

With air alone as dielectric,

$$C = \varepsilon_0 \frac{A}{t} \text{ (as } \varepsilon_r = 1) \qquad \ldots(4.30)$$

Now, with the introduction of a slab of thickness t', the thickness of air-film is reduced to $(t - t')$ (See Fig. 4.7). Let E_1 and E_2 be the field intensities in the air film and slab respectively. V_1 and V_2 the potential differences across them.

Video Content / Details of website for further learning :

- http://gvpcew.ac.in/Material%203%20Units/2%20EEE%20EMF.pdf

Important Books/Journals for further learning including the page nos.: 161-162

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

L19

LECTURE HANDOUTS

EEE | **II/III**

Course Name with Code: 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Lorentz Law of force & Magnetic field intensity

Introduction :

- In physics (Specifically in electromagnetism) the Lorentz force (or electromagnetic force) is the combination of electric and magnetic force on a point charge due to electromagnetic fields

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Studied in the field of electrical sciences and physics

Lorentz Law of force

- We have been concerned with electrostatics the forces generated by and acting upon charges at rest. We now begin to consider how things change when charges are in motion. A simple apparatus demonstrates that something weird happens when charges are in motion: If we run currents next to one another in parallel,

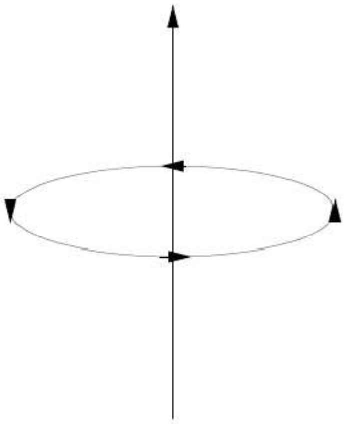
- We find that they are attracted when the currents run in the same direction; they are repulsed when the currents run in opposite directions. This is despite the fact the wires are completely neutral: if we put a stationary test charge near the wires, it feels no force.

- Furthermore, experiments show that the force is proportional to the currents — double the current in one of the wires, and you double the force. Double the current in both wires, and you quadruple the force

- This all indicates a force that is proportional to the velocity of a moving charge; and, that points in a direction perpendicular to the velocity. These conditions are screaming for a force that depends on a cross product.

- What we say is that some kind of field B the “magnetic field” arises from the current. The direction of this field is kind of odd: it wraps around the current in a circular fashion, with a direction that is defined by the right-hand rule: We point our right thumb in the direction of the current, and our fingers curl in the same sense as the magnetic field.

- With this sense of the magnetic field defined, the force that arises when a charge moves through this field is given by



$$\vec{F} = q\frac{\vec{v}}{c} \times \vec{B} \,,$$

- Where c is the speed of light. The appearance of c in this force law is a hint that special relativity plays an important role in these discussions. If we have both electric and magnetic fields, the total force that acts on a charge is of course given by

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) \,.$$

This combined force law is known as the Lorentz force.

Magnetic field intensity

- Magnetic field intensity is the magnetomotive force per unit length of a magnetic path. H= Magnetic field intensity (Wb/A-t m) Fm = magnetomotive force (A-t) l = average length of the path (m) N = number of turns I = current (A)

  H = Fm/l (or)
  H = NI/l

- Magnetic field intensity represents the effort that a given current must put into establishing a certain flux density in a material
- If a material is permeable, then a greater flux density will occur for a given magnetic field intensity. The relation between B (flux density) and H (the effort to establish the field) is

  B = μH
- μ = permeability (Wb/A-t m). H= Magnetic field intensity (Wb/A-t m). This relation between B and H is valid up to saturation, when further increase in H has no affect on B

Video Content / Details of website for further learning :

- http://web.mit.edu/sahughes/www/8.022/lec10.pdf
- http://www.csd.nutn.edu.tw/EC/Ch-07.pdf

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L20

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Biot Savart Law, Ampere's Law and Magnetic field due to straight conductors

Introduction :

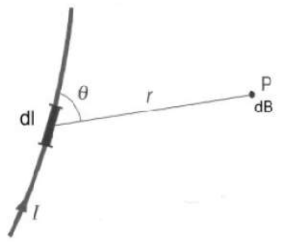
- Magnetic field strength is directly proportional to the magnitude of current flowing in the conductor. i.e. B∝I, greater the current in the conductor, stronger will be the magnetic field produced.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Biot Savart Law

- Let a certain conductor be carrying a current 'I' in a direction on in the figure. Let 'P' be the point where the magnetic field due to the wire is to be studied. Let a small portion be considered which is of length 'dl'. Let the line joining 'dl' and point 'P' from an angle θ with the tangent to 'dl'.



- Since the portion considered is very small, the magnetic field given by it at point P will also be small. By experimental observations and empirically also, dB is found to depend on several factors.
- i) Here dB is the measurement of magnetic energy which arises from the electrical energy represented by I which act respectively as output and input. Therefore they should have direct dependency. i.e. dB I
- ii) In a certain length of a conductor, certain amount of charge is present at a moment and the magnetic effect it can produce depend on the total number of charges, which in turn depend on the length consider, i.e. dB dl
- iii) Any force or phenomena which spread out spherically have inverse proportionality to the square of the distance between the source and point of observation. dB 2 r 1

- iv) Similarly the magnetic field is found to be least when the angle between r and dl is the smallest (Zero degree) and it is largest when the angle is 90o . So, dB sinθ

$$dB \propto \frac{Idl\sin\theta}{r^2} \qquad \text{In SI units, the value of } k = \frac{\mu_o}{4\pi} = 10^{-7}\ Hm^{-1}$$

$$\text{Or,} \quad dB = k\frac{Idl\sin\theta}{r^2} \qquad \text{Or,} \quad dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$$

- This expression is called as Biot Savart Law or Laplace Law. It is the basic formula to find the magnetic field due to any structures for which all the dB's along the length of the structure have to be added to find out the total B.

Ampere's Law

- The force that a magnetic field exerts on charges and current; but, we have not yet said anything about where this field comes from. I will now give, without any proof or motivation, a few key results that allow us to determine the magnetic field in many situations, The main result we need is Ampere's law:

$$\oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{\rm encl}$$

- In words, if we take the line integral of the magnetic field around a closed path, it equals 4π/c times the current enclosed by the path.
- Ampere's law plays a role for magnetic fields that is similar to that played by Gauss's law for electric fields. In particular, we can use it to calculate the magnetic field in situations that are sufficiently symmetric. An important example is the magnetic field of a long, straight wire: In this situation, the magnetic field must be constant on any circular path around the wire. The amount of current enclosed by this path is just I, the current flowing in the wire:

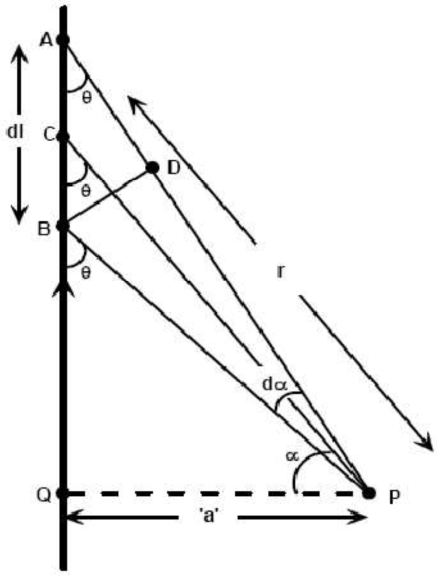
$$\oint \vec{B} \cdot d\vec{s} = B(r)2\pi r = \frac{4\pi}{c} I$$

$$\rightarrow B(r) = \frac{2I}{cr} . \qquad \vec{B} = \frac{2I}{cr}\hat{\phi}$$

- The magnetic field from a current thus falls off as 1/r. Recall that we saw a similar 1/r law not so long ago — the electric field of a long line charge also falls off as 1/r. As we'll see fairly soon, this is not a coincidence. The direction of this field is in a circulation sense — the B~ field winds around the wire according to the right-hand rule2. This direction is often written φˆ, the direction of increasing polar angle φ. The full vector magnetic field is thus written

Magnetic field due to straight conductors,

- Let a straight wire be considered whose magnetic field is to be determined at a certain point P which is nearby the conductor at a distance 'a'. Let a small portion of length dl be considered whose distance from P is 'r'. Therefore the magnetic field at point P due to this small length is given by,



$$dB = \frac{kIdl\sin\theta}{r^2}$$

- This magnetic field due to the whole wire is found by adding all the magnetic fields due to all these dB's of the whole wire for which the expression has to be changed to integrable form. For this, the point 'P' is joined to A, B & C. Similarly a perpendicular BD is drawn to AP at D. Let ∠CPQ =α . Then ∠APB is the small variation in dα due to the consideration of the angles at the two ends of small length dl.
- Since dl is very small, points A and C lie very close to each other. Therefore ∠BAD = ∠BCP = θ.
  So in triangle ABD, sin∠BAD = BD/AB

Or, $\sin\theta = \frac{BD}{AB}$ Or, $AB\sin\theta = BD$

Or, $dl\sin\theta = BD$............(i)

Similarly in triangle BDP, $\sin\angle BPD = \frac{BD}{BP}$

Or, $\sin d\alpha = \frac{BD}{BP}$

Since dl is very small, B & C also lie close together. So BP = CP = r. Similarly the angle dα is also very small. So, $\sin d\alpha = d\alpha$.

So, $d\alpha = \frac{BD}{r}$

Therefore,

$r\,d\alpha = BD$.................. (ii)

Equations (i) and (ii) give $dl\sin\theta = r\,d\alpha$

The expression for dB becomes, $dB = \frac{kIdl\sin\theta}{r^2}$ or, $dB = \frac{kIrd\alpha}{r^2}$

Or, $dB = \frac{kId\alpha}{r}$

In triangle CPQ, $\cos\angle CPQ = \frac{PQ}{CP}$ or, $\cos\alpha = \frac{PQ}{CP}$

or, $\cos\alpha = \frac{a}{r}$ or, $\frac{\cos\alpha}{a} = \frac{1}{r}$

which means $dB = \frac{kI\cos\alpha}{a}d\alpha$

The total magnetic field is given by summing up all these small dB's throughout the whole length of the conductor.

Total magnetic field (B) $= \int dB d\alpha = \int \frac{kI\cos\alpha}{a}d\alpha$

Here the variable α varies within certain given values α_1 and α_2, where α_1 is the angle formed by the lower tip of the conductor at P and α_2 is by the upper tip. But when the angle goes below PQ, its value becomes negative, since $\alpha_1 < 0^o$.

$$B = \frac{kI}{a}\int_{\alpha_1}^{\alpha_2}\cos\alpha d\alpha = \frac{kI}{a}[\sin\alpha]_{\alpha_1}^{\alpha_2}$$

$$\text{Or, } B = \frac{kI}{a}[\sin\alpha_2 - \sin\alpha_1]$$

This is the expression for the magnetic field at a certain point due to a straight conductor of finite length at a distance 'a' such that the angles formed at the two ends α_1 and α_2.

Special case:
In most cases the wires are very long compared to the distance of the point of observation from the wire in such cases, the angles will be 90° at both sides.

$$B = \frac{kI}{a}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{kI}{a}\left[1 + \sin\left(\frac{\pi}{2}\right)\right] = \frac{kI}{a}[1+1]$$

$$\text{Or, } B = \frac{2kI}{a}$$

Using the value of k as $\frac{\mu_o}{4\pi}$ gives $B = \frac{2\mu_o}{4\pi}\frac{I}{a}$

$$\therefore B = \frac{\mu_o I}{2\pi a}$$

Video Content / Details of website for further learning :

- http://web.mit.edu/sahughes/www/8.022/lec10.pdf

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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Estd. 2000

LECTURE HANDOUTS

L21

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Magnetic field strength is directly proportional to the magnitude of current flowing in the conductor. i.e. B∝I, greater the current in the conductor, stronger will be the magnetic field produced.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Basics of Magnetic field intensity / Magnetic flux density

Magnetic field intensity

Problem 1: Around copper conductor is carrying a current of 250 A. determine the magnetizing force and flux density at a distance of 10cm from the conductor

: $I = 250\ A,\quad r = 10\ cm = 0.1\ m$

Flux density $B = \frac{\mu_0 N I}{2\pi r}$ [N = 1 turn]

$$B = \frac{4\pi \times 10^{-7} \times 1 \times 250}{2\pi \times 0.1}$$

$$B = 0.5 \times 10^{-3}\ Wb/m^2$$

$$B = 0.5\ mWb/m^2$$

Magnetic force $H = \frac{B}{\mu_0}$

$$H = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7}} = 397.88\ AT/m$$

Problem 2 - Calculate the magnetic flux density due to circular coil of 100 ampere turn and area of 70cm2 on the axis of the coil at a distance of 10cm from the centre

Solution:

$$N I = 100 \text{ AT}, \quad d = 0.10 \text{ m}$$

$$\pi a^2 = 70 \text{ cm}^2$$

$$\therefore \quad a^2 = 22.28 \text{ cm}^2 = 22.28 \times 10^{-4} \text{ m}$$

$$\text{Magnetic flux density } B = \frac{\mu_0 N I a^2}{2 (a^2 + d^2)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 100 \times 22.28 \times 10^{-4}}{2 (22.28 \times 10^{-4} + 0.01)^{3/2}}$$

$$= 103.7 \times 10^{-6} \text{ Tesla} = \mathbf{103.7 \ \mu \ Tesla}$$

Problem 3 - A single phase circuit comprises of two parallel conductors A and B 1cm diameter and spaced 1 metere apart. The conductor A and B carry currents of 10A and -10A respectively. Determine the field intensity at the surface of each conductor and also in the middle of A and B

$$d_1 = 1 \text{ m}, \quad d_2 = 0.5 \times 10^{-2} \text{ m}$$

$$H = \frac{I}{2\pi d_1} + \frac{I}{2\pi d_2}$$

$$H = \frac{I}{2\pi} + \frac{I}{2\pi \times 0.5 \times 10^{-2}}$$

$$= \frac{10}{2\pi} \left[1 + \frac{10^2}{0.5} \right]$$

$$= \frac{10^3}{\pi} = 318.3 \text{ A/m}$$

This is same as the magnetic field intensity at the surface of A due to A and B.

Magnetic field intensity at the mid-point of A and B.

$$H = \frac{I}{2\pi \times 0.5} + \frac{I}{2\pi \times 0.5}$$

$$= \frac{2 I}{2\pi \cdot (0.5)}$$

$$= \frac{2 \times 10}{\pi} = \mathbf{6.366 \ A/m}$$

Video Content / Details of website for further learning :

- https://www.youtube.com/watch?v=O5jJlz3C3ow

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L22

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Magnetic field due to circular loop & infinite sheet of current

Introduction :

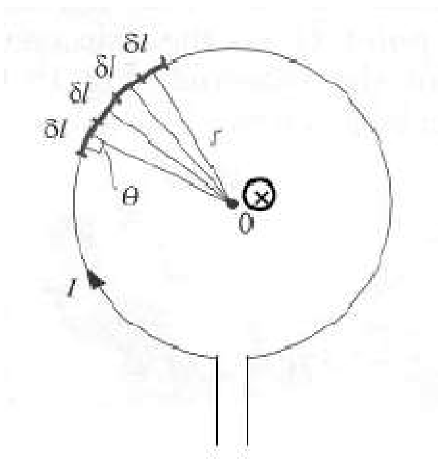
- Magnetic Field Pattern due to a Circular Loop Carrying Current. Each segment of circular loop carrying current produces magnetic field lines in the same direction with in the loop. The direction of magnetic field at the centre of circular coil is perpendicular to the place of the coil. i.e. along the axis of the coil

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic of Magnetic field intensity / Magnetic flux density
- Fleming's Right-Hand Thumb rule

Magnetic field due to circular loop

- Let a coil be considered which is bent in the form of almost complete circle. Let a current I be supplied in clockwise direction which will give the overall magnetic field away from the observer at the centre 'O' (according to Fleming's Right-Hand Thumb rule). Let a small portion 'dl' be considered somewhere and a radius be drawn from 'dl' to 'O'. Then according to Biot Savart Law, a small magnetic field 'dB' given by 'dl' can be expressed as:



$$dB = k\frac{Idl\sin\theta}{r^2},$$ where θ is the angle between dl and r.

$$\therefore \quad B = \frac{kI}{r^2}\int_0^{2\pi r} dl \qquad \text{Or,} \quad B = \frac{kI}{r^2}[l]_0^{2\pi r} \qquad \text{Or,} \quad B = \frac{kI}{r^2} 2\pi r$$

$$\text{Or,} \quad B = \frac{kI}{r}2\pi \qquad \text{Or,} \quad B = k\frac{2\pi I}{r} \qquad \text{Or,} \quad B = \frac{\mu_o}{4\pi}\frac{2\pi I}{r}$$

$$\text{Or,} \quad B = \frac{\mu_o I}{2r}, \text{ (r is the radius of the coil)}$$

If the number of coils is more than one, for example 'n', the magnetic field will be

$$B = \frac{\mu_o nI}{2r}$$

Here wherever 'dl' is considered, the angle between it and 'r' is always equal to 90^0. Therefore,

$$dB = k\frac{Idl\sin 90^0}{r^2} = \frac{kIdl}{r^2}$$

The overall magnetic field will be equal to the sum of all these small magnetic fields.

$$\text{i.e.} \quad B = \int dB \qquad \text{Or,} \quad B = \int \frac{kIdl}{r^2}$$

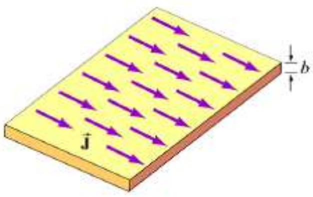
$$\text{Or,} \quad B = \frac{kI}{r^2}\int dl$$

Here the variable is 'l'. if all the small "dl's" are added one by one, "l" will extend from l = 0 to l = circumference (= $2\pi r$).

Magnetic field due to infinite sheet of current

- Consider an infinitely large sheet of thickness b lying in the xy plane with a uniform current density. Find the magnetic field everywhere.

$$I_{enc} = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J_0(b\ell)$$



Applying Ampere's law leads to

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B(2\ell) = \mu_0 I_{enc} = \mu_0(J_0 b\ell)$$

or $B = \mu_0 J_0 b/2$. Note that the magnetic field outside the sheet is constant, independent of the distance from the sheet. Next we find the magnetic field inside the sheet. The amount of current enclosed by path C_2 is

$$I_{enc} = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J_0(2|z|\ell)$$

Applying Ampere's law, we obtain

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B(2\ell) = \mu_0 I_{enc} = \mu_0 J_0(2|z|\ell)$$

or $B = \mu_0 J_0 |z|$. At $z = 0$, the magnetic field vanishes, as required by symmetry. The results can be summarized using the unit-vector notation as

$$\vec{\mathbf{B}} = \begin{cases} -\dfrac{\mu_0 J_0 b}{2}\hat{\mathbf{j}}, & z > b/2 \\ -\mu_0 J_0 z\,\hat{\mathbf{j}}, & -b/2 < z < b/2 \\ \dfrac{\mu_0 J_0 b}{2}\hat{\mathbf{j}}, & z < -b/2 \end{cases}$$

Let's now consider the limit where the sheet is infinitesimally thin, with $b \to 0$. In this case, instead of current density $\mathbf{J} = J_0\hat{\mathbf{i}}$, we have surface current $\mathbf{K} = K\hat{\mathbf{i}}$, where $K = J_0 b$. Note that the dimension of K is current/length. In this limit, the magnetic field becomes

$$\vec{\mathbf{B}} = \begin{cases} -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > 0 \\ \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < 0 \end{cases}$$

Video Content / Details of website for further learning :

- http://web.mit.edu/viz/EM/visualizations/coursenotes/modules/guide09.pdf
- http://sajitsir.tripod.com/Books/Biot_Savart_Law.pdf

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

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LECTURE HANDOUTS

L23

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Magnetic flux density (B) –B in free space, conductor

Introduction :

- Dielectric polarization is the term given to describe the behavior of a material when an external electric field is applied on it. A simple picture can be made using a capacitor

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- overview of Magnetic flux and Magnetic flux density

Magnetic flux density (B) –B in free space, conductor

In free space, magnetic flux density $\bar{B}$ is defined as:

$\bar{B} = \mu_o \bar{H}$ Webers/m^2 or Tesla (T), $\mu_o = 4\pi \times 10^{-7}$ H/m (henrys/meter)

μ_o is the permeability of free space,

The total magnetic flux through a surface is given by

$$\Phi = \int_s \bar{B} \cdot \bar{ds} \quad \text{Wb}$$

Applying the definition of the divergence,

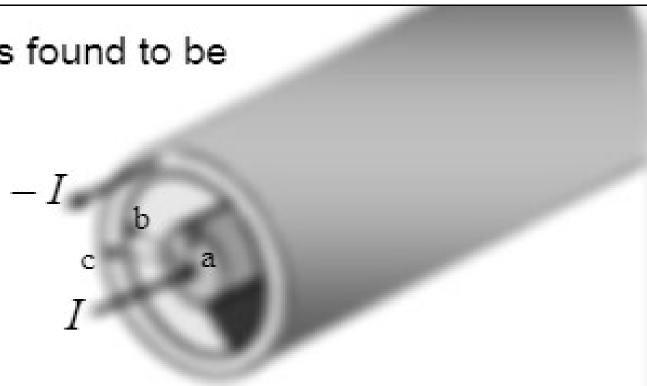
$$\nabla \cdot \bar{B} = \lim \frac{\int \bar{B} \cdot \bar{ds}}{\Delta v} = 0 \quad \rightarrow \quad \nabla \cdot \bar{B} = 0$$

Therefore lines $\bar{H}$ or $\bar{B}$ are closed lines ($\bar{E}$ or $\bar{D}$ are open lines).

Ex: Find the magnetic flux between the conductors of a coaxial line of length d .

The magnetic field intensity was found to be

$$H_\varphi = \frac{I}{2\pi\rho} \qquad a < \rho < b$$



Therefore the magnetic flux density is given by

$$B = \mu_o H = \frac{\mu_o I}{2\pi\rho}\hat{a}_\varphi$$

The magnetic flux contained between the conductors in a length d is the flux crossing any radial plane extending from $\rho = a$ to $\rho = b$ and from $z = 0$ to $z = d$.

$$\Phi = \int_s B \cdot ds = \int_0^d \int_a^b \frac{\mu_o I}{2\pi\rho}\hat{a}_\varphi \cdot d\rho\, dz\, \hat{a}_\varphi = \int_a^b \frac{\mu_o I}{2\pi}\frac{d\rho}{\rho}$$

$$\Phi = \frac{\mu_o I d}{2\pi}\ln\left(\frac{b}{a}\right)$$

Video Content / Details of website for further learning:

- https://www.coursehero.com/file/p439er6u/Magnetic-Flux-and-Magnetic-Flux-Density-In-free-space-magnetic-flux-density-B/

Important Books/Journals for further learning including the page nos.:

- Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011

Course Faculty

LECTURE HANDOUTS

L24

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Magnetic materials, Boundary conditions and Scalar and vector potential

Introduction :

- The study of magnetism has very different consequences as compared to the introduction of material media into the study of electrostatics. When we dealt with dielectric materials in electrostatics, their effect was always to reduce E G below what it would otherwise be, for a given amount of "free" electric charge.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Generalization of circuit theory
- Property of magnetic materials

Magnetic materials

In contrast, when we deal with magnetic materials, their effect can be one of the following:

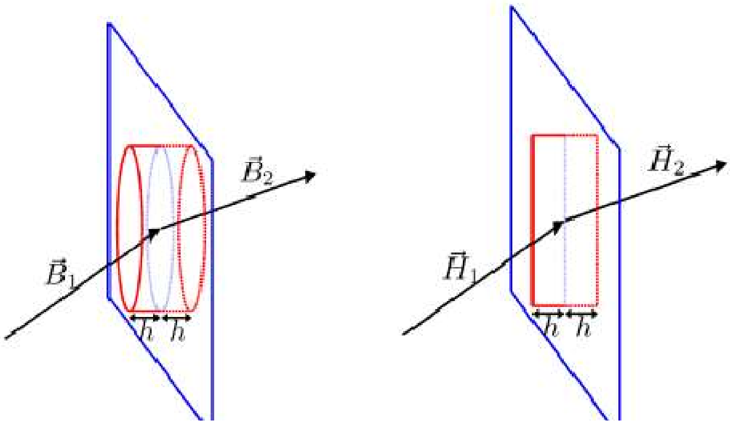
- **Paramagnetism** - The atoms or molecules comprising paramagnetic materials have a permanent magnetic dipole moment. Left to themselves, the permanent magnetic dipoles in a paramagnetic material never line up spontaneously. In the absence of any applied external magnetic field, they are randomly aligned. Thus, M = 0 G G and the average magnetic field BM G is also zero. However, when we place a paramagnetic material in an external field , the dipoles experience a torque B0 G = × 0 τ μ B G G G that tends to align μ G with B0 G , thereby producing a net magnetizationM parallel to G B0 G . Since BM G is parallel to B0 G , it will tend to enhance . The total magnetic field B is the sum of these two fields:
- Note how different this is than in the case of dielectric materials. In both cases, the torque on the dipoles causes alignment of the dipole vector parallel to the external field. However, in the paramagnetic case, that alignment enhances the external magnetic field, whereas in the dielectric case it reduces the external electric field. In most paramagnetic substances, the magnetization M G is not only in the same direction as , but also linearly proportional to . This is plausible because without the external field there would be no alignment of dipoles and hence no magnetization M
- **Diamagnetism -** In the case of magnetic materials where there are no permanent magnetic dipoles, the presence of an external field will induce magnetic dipole moments in the atoms or molecules. However, these induced magnetic dipoles are anti-parallel to , leading to a magnetization and average field B0 G B0 G M G BM G anti-parallel to B0 G , and therefore a reduction in the total magnetic field strength. For diamagnetic materials, we can still define the magnetic permeability, as in equation (8-5), although now 1 κ m < , or 0 χ m < , although χ m is usually on the order of 5

10− − to 9 10− − . Diamagnetic materials have $\mu_m < \mu_0$.

- **Ferromagnetism -** In ferromagnetic materials, there is a strong interaction between neighboring atomic dipole moments. Ferromagnetic materials are made up of small patches called domains, as illustrated in Figure (a). An externally applied field B0 G will tend to line up those magnetic dipoles parallel to the external field, as shown in Figure (b). The strong interaction between neighboring atomic dipole moments causes a much stronger alignment of the magnetic dipoles than in paramagnetic materials.

Boundary conditions

- Consider first a short cylinder or "pill box" that crosses the boundary between two media, with the flat ends of the cylinder parallel to the boundary, see Figure. Applying Gauss' theorem to Maxwell's equation (2) gives:



$$\int_V \nabla \cdot \vec{B}\, dV = \oint_{\partial V} \vec{B} \cdot d\vec{S} = 0,$$

- Where the boundary ∂V encloses the volume V within the cylinder. If we take the limit where the length of the cylinder (2h – see Fig. 9 (a)) approaches zero, then the only contributions to the surface integral come from the flat ends; if these have infinitesimal area dS, then since the orientations of these surfaces are in opposite directions on opposite sides of the boundary, and parallel to the normal component of the magnetic field, we find:

$$-B_{1\perp}\, dS + B_{2\perp}\, dS = 0,$$

- Where $B_{1\perp}$ and $B_{2\perp}$ are the normal components of the magnetic flux density on either side of the boundary. Hence:

$$B_{1\perp} = B_{2\perp}.$$

- In other words, the normal component of the magnetic flux density is continuous across a boundary. Applying the same argument, but starting from Maxwell's equation (1), we find:

$$D_{2\perp} - D_{1\perp} = \rho_s,$$

- Where $D_\perp$ is the normal component of the electric displacement, and ρ_s is the surface charge density (i.e. the charge per unit area, existing purely on the boundary). A third boundary condition, this time on the component of the magnetic field parallel to a boundary, can be obtained by applying Stokes' theorem to Maxwell's equation (3).
- In particular, we consider a surface S bounded by a loop ∂S that crosses the boundary of the material, see Figure. If we integrate both sides of Eq. (3) over that surface, and apply Stokes' theorem (7), we find:

$$\int_S \nabla \times \vec{H} \cdot d\vec{S} = \oint_{\partial S} \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} + \frac{\partial}{\partial t}\int_S \vec{D} \cdot d\vec{S},$$

- Where I is the total current flowing through the surface S. Now, let the surface S take the form of a thin strip, with the short ends perpendicular to the boundary, and the long ends parallel to the boundary. In the limit that the length of the short ends goes to zero, the area of S goes to zero: the electric displacement integrated over S becomes zero.

- In principle, there may be some "surface current", with density (i.e. current per unit length) J~ s: this contribution to the right hand side of Equation remains non-zero in the limit that the lengths of the short sides of the loop go to zero. In particular, note that we are interested in the component of J~ s that is perpendicular to the component of H~ parallel to the surface. We denote this component of the surface current density Js⊥. Then, we find from Equation (taking the limit of zero length for the short sides of the loop):

$$H_{2\|} - H_{1\|} = -J_{s\perp},$$

- where H1k is the component of the magnetic intensity parallel to the boundary at a point on one side of the boundary, and H2k is the component of the magnetic intensity parallel to the boundary at a nearby point on the other side of the boundary. A final boundary condition can be obtained using the same argument that led to Equation, but starting from Maxwell's equation (3). The result is:

$$E_{2\|} = E_{1\|},$$

the tangential component of the electric field E~ is continuous across any boundary

Scalar and vector potential

We can define a magnetic field using the following requirements.

$$\nabla \times \nabla V = 0$$

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

• Just as $\vec{E} = \nabla V$, we can define a magnetic scalar potential Vm related to **H** when the current density is zero as

$$\vec{H} - -\nabla V_m, \vec{J} - 0$$

$$\vec{J} = \nabla \times \vec{H} = \nabla \times (-\nabla V_m) = 0$$

$$\nabla^2 V_m = 0, \vec{J} = 0$$

• The requirement for a solenoidal field (and Maxwell's 4th law of electrostatics) stipulates

$$\nabla \cdot \vec{B} = 0$$

• And we can therefore define a magnetic vector potential, **A**, as

$$\vec{B} = \nabla \times \vec{A}$$

• Just as we defined the Electric Potential as $V = \int \frac{dQ}{4\pi\varepsilon_0 r}$, We can define the Magnetic Vector Potential as,

$$\vec{A} = \int_L \frac{\mu_0 I d\vec{l}}{4\pi R} \quad \text{for Line Current}$$

$$\vec{A} = \int_L \frac{\mu_0 \vec{K} dS}{4\pi R} \quad \text{for Surface Current}$$

$$\vec{A} = \int_L \frac{\mu_0 \vec{J} dv}{4\pi R} \quad \text{for Volume Current}$$

Video Content / Details of website for further learning :

- http://web.mit.edu/viz/EM/visualizations/coursenotes/modules/guide09.pdf

Important Books/Journals for further learning including the page nos.:

- Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

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L25

LECTURE HANDOUTS

EEE | **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The study of magnetism has very different consequences as compared to the introduction of material media into the study of electrostatics. When we dealt with dielectric materials in electrostatics, their effect was always to reduce E G below what it would otherwise be, for a given amount of "free" electric charge.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Self and mutual inductance

Problem 1 – calculate the inductance of a solenoid of 200 turns wound on a cylindrical tube 6cms diameter. The length of the tube is 60cm and the medium is air

Solution:

$$N = 200 \text{ turns}$$

$$r = 3\text{ cm} = 0.03\text{ m}$$

$$l = 60\text{ cm} = 0.6\text{ m}$$

$$\mu_r = 1$$

$$A = \pi r^2$$

Inductance of the solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (200)^2 \times \pi \times (0.3)^2}{0.6}$$

$$L = 0.2368 \text{ mH} \quad \text{Ans.}$$

Problem 2 – A solenoid consisting of 1000 turns of wire wound on a wire of length 100cm and diameter 3cm is placed coaxially within another solenoid of the same length and number of turns but with a diameter of 6cm. find the mutual inductance and the coupling coefficient of the arrangement.

Solution

Solenoid 1

$$N_1 = 1000$$
$$l_1 = 1 \text{ m}$$
$$d_1 = 3 \times 10^{-2} \text{ m}$$
$$\text{Area } A_1 = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 3 \times 3 \times 10^{-4}$$
$$= 7.07 \times 10^{-4} \text{ m}^2$$
$$L_1 = \frac{\mu_0 N_1^2 A_1}{l_1}$$

Solenoid 2

$$N_2 = 1000$$
$$l_2 = 1 \text{ m}$$
$$d_2 = 6 \times 10^{-2} \text{ m}$$
$$A_2 = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 6 \times 6 \times 10^{-4}$$
$$= 28.27 \times 10^{-4} \text{ m}^2$$
$$L_2 = \frac{\mu_0 N_2^2 A_2}{l_2}$$

$$L_1 = \frac{4\pi \times 10^{-7} \times (1000)^2 \times 7.07 \times 10^{-4}}{1} = 888.4 \times 10^{-6} \text{ Henry}$$

$$L_2 = \frac{4\pi \times 10^{-7} \times (1000)^2 \times 28.27 \times 10^{-4}}{1} = 3.55 \times 10^{-3} \text{ Henry}$$

[Alternatively

$$L_2 = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{A_2}{A_1}\right)^2 ; \; L_1 = \left(\frac{1000}{1000}\right)^2 \left(\frac{28.27 \times 10^{-4}}{7.07 \times 10^{-4}}\right) L_1 = 4 L_1 \,]$$

Since the inner solenoid has half of the diameter as that of outer solenoid, the only 50% of flux produced by outer coil links with the inner coil.

Coupling coefficient $K = \mathbf{0.5}$ **Ans.**

$$M = K\sqrt{L_1 L_2}$$
$$= 0.5\sqrt{888.4 \times 4 \times 10^{-6} \times 3.55 \times 10^{-3}}$$
$$= \mathbf{0.888 \times 10^{-3} \text{ Wb}} \quad \textbf{Ans.}$$

☺☺

Video Content / Details of website for further learning :

- https://www.toppr.com/guides/physics/electromagnetic-induction/inductance/

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS **L26**

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Magnetic force, Torque, Inductance, Energy density & Magnetic circuits

Introduction :

- The magnetic field must be generated somehow. What if it was generated by field produced from current passing through a second current element nearby. This means that currents in neighboring wires generate magnetic fields that generate forces on each other.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic of scalar and vector
- Studied in the field of electrical sciences and physics

Force

- One can also use field calculations to determine the force acting on a current element, I**dl**=**K**dS=**J**dv, due to an applied external magnetic field **B**.
- Assume that a copper wire carries a current density, **J**= ρ_v**u**.
- We know:

$$Id\vec{l} = \vec{J}dv = \rho_v u = dQu$$

- And that:

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$$
$$where \quad \vec{E} \rightarrow 0$$
$$\vec{F} = q(\vec{u} \times \vec{B})$$

Thus we can solve for the force on the first wire:

$$d\vec{F} = dq(\vec{u} \times \vec{B}) = Id\vec{l} \times \vec{B}$$

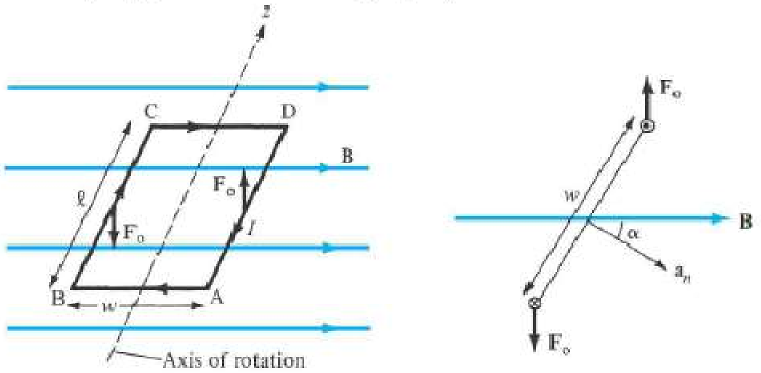
$$\vec{F} = \oint_L Id\vec{l} \times \vec{B}$$

Likewise

$$\vec{F} = \oint_L \vec{K}dS \times \vec{B} = \vec{F} = \oint_L \vec{J}dv \times \vec{B}$$

Torque

• Let's examine the Torque applied to a current carrying loop.



• Torque, **T**, on the loop is the vector product of the force, **F**, and the moment arm, **r**.

$$\vec{T} = \vec{r} \times \vec{F}$$

$$|\vec{T}| = |\vec{r}||\vec{F}|\sin\alpha$$

And for a uniform magnetic field,

$$|\vec{F}_0| = IBl$$

$$|\vec{T}| = IBlw \sin\alpha$$

But, $lw = S$,so

$$|\vec{T}| = IBS \sin\alpha$$

Where we can now define a quantity **m** as the magnetic dipole moment with units A/m2 which is the product of the current and area of the loop in the direction normal the surface area defined by the loop

$$\vec{m} = IS\hat{a}_n$$

$$\vec{T} = \vec{m} \times \vec{B}$$

Inductors and Inductance

• We now know that closed magnetic circuit carrying current I produces a magnetic field with flux

$$\Psi = \int \vec{B} \cdot d\vec{S}$$

• We define the flux linkage between a circuit with N identical turns as,

$$\lambda = N\Psi$$

• As long as the medium the flux passes through is linear (isotropic) then then flux linkage is proportional to the current I producing it and can be written as,

$$\lambda = LI$$

Where L is a constant of proportionality called the inductance of the circuit. A circuit that contains inductance is said to be an inductor.

• One can equate the inductance to the magnetic flux of the circuit as

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

where L is measured in units of Henrys (H) = Wb/A.

• The magnetic energy (in Joules) stored by the inductor is expressed as,

$$W_m = \frac{1}{2}LI^2$$

Magnetic energy

We can derive a similar term as derived for electric energy, for magnetic energy using the relation for energy as a function of inductance.

$$W_m = \frac{1}{2}LI^2$$

$$\Delta L = \frac{\Delta\Psi}{\Delta I} = \frac{\mu H \Delta x \Delta y}{\Delta I}$$

$$where, \Delta I = H\Delta z$$

$$\Delta L = \frac{\mu H \Delta x \Delta y}{\Delta I} = \frac{\mu \Delta x \Delta y}{\Delta z}$$

$$\Delta W_m = \frac{1}{2} \Delta L \Delta I^2 = \frac{1}{2} \mu H^2 \Delta x \Delta y \Delta z$$

$$\Delta W_m = \frac{1}{2} \mu H^2 \Delta v$$

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} = \frac{1}{2} \mu H^2 = \frac{1}{2} \left(\bar{H} \cdot \bar{B} \right) = \frac{B^2}{2\mu}$$

$$W_m = \int w_m dv = \int \frac{1}{2} \left(\vec{H} \cdot \vec{B} \right) dv = \int \frac{1}{2} \mu H^2 dv$$

Magnetic circuits

•The following relations allow one to solve magnetic field problems in a manner similar to that of electronic circuits. It provides a clear means of designing transformers, motors, generators, and relays using a lumped circuit model. The analogy between electronic and magnetic circuits is provided below.

Electric	Magnetic
Conductivity σ	Permeability μ
Field intensity E	Field intensity H
Current $I = \int \mathbf{J} \cdot d\mathbf{S}$	Magnetic flux $\Psi = \int \mathbf{B} \cdot d\mathbf{S}$
Current density $J = \frac{I}{S} = \sigma E$	Flux density $B = \frac{\Psi}{S} = \mu H$
Electromotive force (emf) V	Magnetomotive force (mmf) $\mathcal{F}$
Resistance R	Reluctance $\mathfrak{R}$
Conductance $G = \frac{1}{R}$	Permeance $\mathcal{P} = \frac{1}{\mathfrak{R}}$
Ohm's law $R = \frac{V}{I} = \frac{\ell}{\sigma S}$ or $V = E\ell = IR$	Ohm's law $\mathfrak{R} = \frac{\mathcal{F}}{\Psi} = \frac{\ell}{\mu S}$ or $\mathcal{F} = H\ell = \Psi\mathfrak{R} = NI$
Kirchhoff's laws: $\Sigma I = 0$, $\Sigma V - \Sigma RI = 0$	Kirchhoff's laws: $\Sigma \Psi = 0$, $\Sigma \mathcal{F} - \Sigma \mathfrak{R} \Psi = 0$

Video Content / Details of website for further learning :

- http://www.vssut.ac.in/lecture_notes/lecture1423903135.pdf

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : III – Static Magnetic Field **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Due to energy conservation, the energy needed to drive the original current must have an outlet. For an inductor, that outlet is the magnetic field the energy stored by an inductor is equal to the work needed to produce a current through the inductor.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Generalization of circuit theory

Inductance / Flux density

Problem 1: - An iron ring with across sectional area of 3cm2 and a mean circumference of 15cm is wound with 250 turns wire carrying a current of 0.3A. The relative permeability of the ring is 1500. Calculate the flux established in the ring

Solution: Area, $A = 3\ cm^2 = 3 \times 10^{-4}\ m$

Mean circumference $2\pi r = 15\ cm = 0.15\ m$

$N = 250, \quad \mu_r = 1500, \quad A = 3\ cm^2 = 3 \times 10^{-4}\ m^2$

$I = 0.3\ A$

$$\text{The flux established in the coil} = \frac{\mu_0 \mu_r N A I}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times 1500 \times 250 \times 3 \times 10^{-4} \times 0.3}{0.15}$$

$$= 282.743\ \mu\ \text{webers}$$

Problem 2: - A round copper conductor is carrying a current of 250A. Determine the magnetizing force and flux density at a distance of 10cm from the conductor

©**Solution:** $I = 250$ A, $r = 10$ cm $= 0.1$ m

Flux density $B = \frac{\mu_0 N I}{2\pi r}$ [N = 1 turn]

$$B = \frac{4\pi \times 10^{-7} \times 1 \times 250}{2\pi \times 0.1}$$

$$B = 0.5 \times 10^{-3} \text{ Wb/m}^2$$

$$\mathbf{B = 0.5 \text{ mWb/m}^2}$$

Magnetic force $H = \frac{B}{\mu_0}$

$$\mathbf{H = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7}} = 397.88 \text{ AT/m}}$$

Problem 3: - calculate the inductance of a solenoid of 200 turns wound on a cylindrical tube 6cm diameter. The length of the tube is 60cm and the medium is air

is air.

© ***Solution:***

$N = 200$ turns

$r = 3$ cm $= 0.03$ m

$l = 60$ cm $= 0.6$ m

$\mu_r = 1$

$A = \pi r^2$

Inductance of the solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (200)^2 \times \pi \times (0.3)^2}{0.6}$$

$$\mathbf{L = 0.2368 \text{ mH}} \quad \textbf{Ans.}$$

Video Content / Details of website for further learning :

- http://www.vssut.ac.in/lecture_notes/lecture1423903135.pdf

Important Books/Journals for further learning including the page nos.: 161-162

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L28

LECTURE HANDOUTS

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Faraday's law

Introduction :

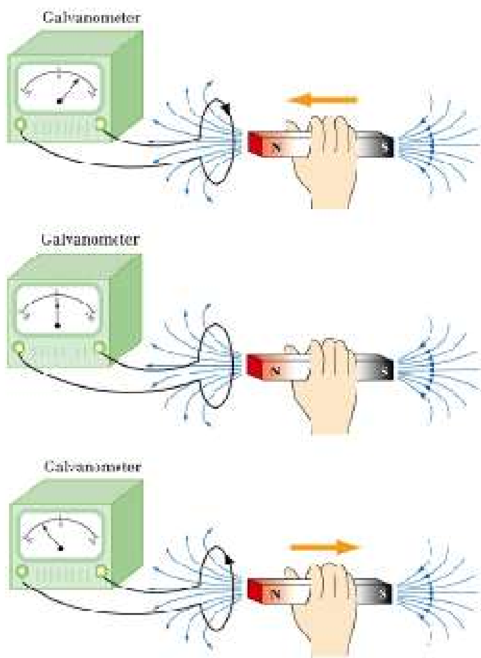
- The electric fields and magnetic fields considered up to now have been produced by stationary charges and moving charges (currents), respectively. Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field

Prerequisite knowledge for Complete understanding and learning of Topic:

- Lenz's law
- Basics of Electric field and magnetic field

Faraday's law

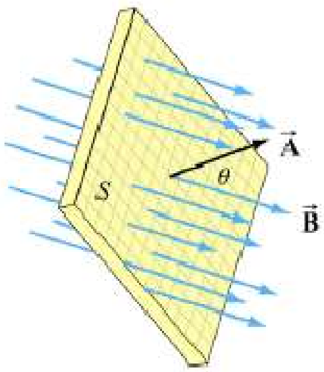
- Michael Faraday discovered that, by varying magnetic field with time, an electric field could be generated. The phenomenon is known as electromagnetic induction. Figure. illustrates one of Faraday's experiments.



- Faraday showed that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop. However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop. In particular, the galvanometer deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away.

- Faraday's experiment demonstrates that an electric current is induced in the loop by changing the magnetic field. The coil behaves as if it were connected to an emf source. Experimentally it is found that the induced emf depends on the rate of change of magnetic flux through the coil.

- Magnetic Flux Consider a uniform magnetic field passing through a surface S, as shown in Figure below:



- Let the area vector be $\vec{\mathbf{A}} = A\hat{\mathbf{n}}$, where A is the area of the surface and $\hat{\mathbf{n}}$ its unit normal. The magnetic flux through the surface is given by

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta$$

- where θ is the angle between $\vec{\mathbf{B}}$ and $\hat{\mathbf{n}}$. If the field is non-uniform, Φ_B then becomes

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

- The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Video Content / Details of website for further learning :

- http://web.mit.edu/viz/EM/visualizations/notes/modules/guide10.pdf

Important Books/Journals for further learning including the page nos.: 165

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS **L29**

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Induced EMF, Static EMF and Dynamic EMF

Introduction :

- The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change.

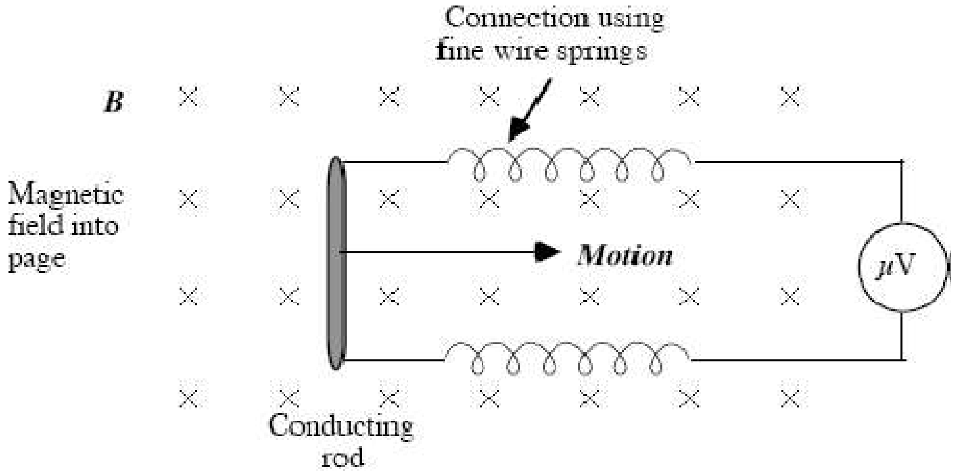
Prerequisite knowledge for Complete understanding and learning of Topic:

- Generalization of electromagnetic induction, EMF, Magnetic flux and Transformer
- Studied in the field of electrical sciences and physics.

Induced EMF

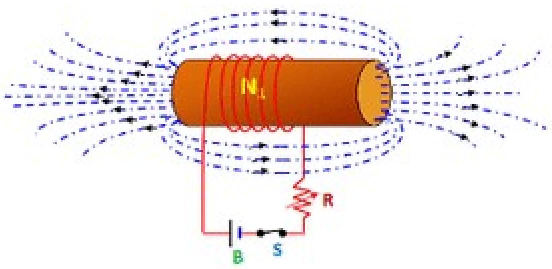
- Electromagnetic induction is a term used to describe the production of EMFs by two apparently quite different mechanisms: (1) the movement of a conductor through a region of space where there is a magnetic field and (2) the existence of a changing magnetic field in some region of space.
- In the first case, charged particles within a moving conductor experience magnetic forces which produce a charge separation which in turn creates a potential difference. In the second mechanism, a changing magnetic field creates an electric field - even in empty space. This induced electric field is not an electrostatic field because its field lines don't start and end on charges - but if some conducting matter is brought into the space the induced electric field can produce a charge separation and a measurable potential difference.
- The value of the EMF is defined to be the energy supplied per charge. When the separated charges are in equilibrium and there is no current in the system, the potential difference produce by the separated charges is equal to the EMF. It turns out that both of the effects called electromagnetic induction can be described by the same mathematical law, which we call Faraday's law after the person who probably did most to elucidate electromagnetic induction experimentally.
- A brass rod is driven at a steady speed between the poles of a strong magnet. The rod, the magnetic field and the direction of motion are all perpendicular to each other. When the rod is moved through the magnetic field there is an induced voltage; when the motion stops the voltage disappears.
- When the velocity is reversed the same voltage, in the opposite sense, appears. When the conductor is then replaced with one of half the length, and it is driven through the magnetic

field at the same velocity as before, the voltage registered is half the previous value. Careful measurements confirm that for the same speed and magnetic field the induced voltage is proportional to the length of the conductor.



Static induced EMF

- Statically induced e.m.f is two types which are Mutually induced e.m.f. and Self-induced e.m.f.
- Self-induced e.m.f is the e.m.f which is produced in the coil due to the change of its own flux linked with it. If the current of the coil is changed, then the flux linked with its own turns will also change which will produce an e.m.f that is called self-induced e.m.f.
- The property of self-inductance is a particular form of electromagnetic induction. Self inductance is defined as the induction of a voltage in a current-carrying wire when the current in the wire itself is changing. In the case of self-inductance, the magnetic field created by a changing current in the circuit itself induces a voltage in the same circuit. Therefore, the voltage is self-induced.
- The term inductor is used to describe a circuit element possessing the property of inductance and a coil of wire is a very common inductor. In circuit diagrams, a coil or wire is usually used to indicate an inductive component. Taking a closer look at a coil will help understand the reason that a voltage is induced in a wire carrying a changing current.
- The alternating current running through the coil creates a magnetic field in and around the coil that is increasing and decreasing as the current changes. The magnetic field forms concentric loops that surround the wire and join to form larger loops that surround the coil as shown in the image below.



- When the current increases in one loop the expanding magnetic field will cut across some or all of the neighboring loops of wire, inducing a voltage in these loops. This causes a voltage to be induced in the coil when the current is changing.
- By studying this image of a coil, it can be seen that the number of turns in the coil will have an effect on the amount of voltage that is induced into the circuit. Increasing the number of turns or the rate of change of magnetic flux increases the amount of induced voltage. Therefore, Faraday's Law must be modified for a coil of wire and becomes the following.

$$V_L = N\frac{d\phi}{dt}$$

- Where: VL = induced voltage in volts N = number of turns in the coil, dø/dt = rate of change of magnetic flux in webers/second

- The equation simply states that the amount of induced voltage (VL) is proportional to the number of turns in the coil and the rate of change of the magnetic flux (dø/dt). In other words, when the frequency of the flux is increased or the number of turns in the coil is increased, the amount of induced voltage will also increase.
- In a circuit, it is much easier to measure current than it is to measure magnetic flux, so the following equation can be used to determine the induced voltage if the inductance and frequency of the current are known. This equation can also be reorganized to allow the inductance to be calculated when the amount of inducted voltage can be determined and the current frequency is known.

$$V_L = L\frac{di}{dt}$$

- Where: VL = the induced voltage in volts L = the value of inductance in henries di/dt = the rate of change of current in amperes per second

Video Content / Details of website for further learning :

- http://www.physics.usyd.edu.au/super/life_sciences/E/E7.pdf

Important Books/Journals for further learning including the page nos.: 173

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

L30

LECTURE HANDOUTS

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Amperes law
- Generalization of field intensity and flux density

Electrodynamic fields

Problem 1 - In free space, $H = 0.2 \cos(\omega t - \beta x)\, a_x$ A/m. **Find the total power passing through a circular disc of radius 5cm**

Solution

Given:

$$H = 0.2 \cos(\omega t - \beta x)\, \vec{a}_x$$

$$\eta = 120\pi$$

$$\text{Area} = \pi r^2 = \pi \times (0.05)^2$$

$$E = 120\pi \times 0.2 = 75.4 \text{ N/m}$$

$$\text{Average power } P_{av} = \frac{1}{2} E \cdot H = \frac{1}{2} \times 75.4 \times 0.2$$

$$P_{av} = 7.54 \text{ W/m}^2$$

$$\text{Total power } P = P_{av} \times \text{Area} = 7.54 \times \pi \times (0.05)^2$$

$$= \mathbf{0.059 \text{ W or } 59 \text{ mW}} \quad \textbf{Ans.}$$

Problem 2 – Determine the magnetic flux density both inside and outside on infinite long, straight conductor with a circular cross section of radius x cm carrying a steady current I.plot the variation of the flux density with radius distance

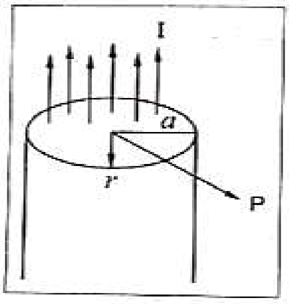
Solution : consider a solid cylindrical conductor of radius 'a', infinite length, carries a steady state current of I

The current density of the conductor in J

$$J = \frac{I}{\pi a^2} \text{ A/m}$$

Consider any point P at a radial distance r inside the conductor where magnetic flux density is to be determined.

For inside the conductor ($r \le a$).



Apply Ampere's law,

$$\oint H \cdot dl = I = J \cdot A$$

$$H \cdot 2\pi r = \frac{I}{\pi a^2} \cdot \pi r^2$$

$$H = \frac{I r}{2\pi a^2}$$

Magnetic flux density $B = \mu H$

$$B = \frac{\mu I r}{2\pi a^2}$$

At the surface of the conductor, $r = a$

$$B = \frac{\mu I}{2\pi a}$$

For outside the conductor ($r > a$).

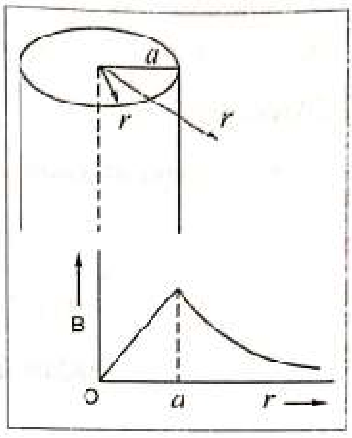
According to Ampere's law,

$$\oint H \cdot dl = I$$

$$H \cdot 2\pi r = I$$

$$H = \frac{I}{2\pi r}$$

$$B = \frac{\mu I}{2\pi r}$$



Video Content / Details of website for further learning : 178

- https://www.eolss.net/Sample-Chapters/C02/E6-04-04-01.pdf

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L31

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Introduction and applications of Maxwell's equation

Introduction :

- The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc. Maxwell's equations describe how electric and magnetic fields are generated by charges, currents, and changes of the fields.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Overview of Magnetic flux and Magnetic flux density

Introduction to Maxwell's equation

- Maxwell's equations are a set of coupled partial differential equations that, together with the Lorentz force law, form the foundation of classical electromagnetism, classical optics, and electric circuits. The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc.
- They describe how electric and magnetic fields are generated by charges, currents, and changes of the fields. The equations are named after the physicist and mathematician James Clerk Maxwell, who, between 1861 and 1862, published an early form of the equations that included the Lorentz force law. Maxwell first used the equations to propose that light is an electromagnetic phenomenon.
- An important consequence of Maxwell's equations is that they demonstrate how fluctuating electric and magnetic fields propagate at a constant speed (c) in a vacuum. Known as electromagnetic radiation, these waves may occur at various wavelengths to produce a spectrum of light from radio waves to gamma rays.

- The equations have two major variants. The microscopic Maxwell equations have universal applicability but are unwieldy for common calculations. They relate the electric and magnetic fields to total charge and total current, including the complicated charges and currents in materials at the atomic scale. The "macroscopic" Maxwell equations define two new auxiliary fields that describe the large-scale behaviour of matter without having to consider atomic scale charges and quantum phenomena like spins. However, their use requires experimentally determined parameters for a phenomenological description of the electromagnetic response of materials.

Applications of Maxwell's equation

- A number of problems on the laser radiation propagation and absorption are stated on the Maxwell's equation base for the simulation of laser treatment of materials, namely cutting, welding, drilling of metals, selective laser melting and sintering of powders.
- The algorithm of numerical solution of the Maxwell's equations by the finite difference time domain method is employed with parallelizing elements; the peculiarities of setting of some boundary conditions for the problems of laser interaction for isotropic media are analyzed.
- The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc

Video Content / Details of website for further learning:

- https://study.com/academy/lesson/maxwells-equations-definition-application.html

Important Books/Journals for further learning including the page nos.: 182

- Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011

Course Faculty

LECTURE HANDOUTS **L32**

EEE **II/III**

Course Name with Code: 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Maxwell's equations 1 & 2

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862. Individually, the four equations are named Gauss' law, Gauss' law for magnetism, Faraday's law and Ampere's law.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Overview of Magnetic flux and Magnetic flux density

Maxwell's equations 1 & 2

- Gauss' law relates the distribution of electric charge to the field that charge creates. If you know the shape of the object and, therefore, how the charge is distributed, you can use Gauss' law to figure out an expression for the electric field. This is generally used when there's a degree of symmetry, making the equation simpler.
- Gauss' law for magnetism says that magnetic monopoles do not exist. It's really more of a statement than something we might use to derive expressions. Charges exist as positive or negative. But in magnetism, whenever you have a south pole, you also have a north pole - there are no single, or monopoles, as yet discovered.

Gauss' Law for electric field

- Gauss' Law is the first of Maxwell's Equations which dictates how the Electric Field behaves around electric charges. Gauss' Law can be written in terms of the Electric Flux Density and the Electric Charge Density as:

$$\nabla \cdot \mathbf{D} = \rho_V$$

- Equation is known as Gauss' Law in point form. That is, Equation is true at any point in space. That is, if there exists electric charge somewhere, then the divergence of D at that point is nonzero, otherwise it is equal to zero.

- To get some more intuition on Gauss' Law, let's look at Gauss' Law in integral form. To do this, we assume some arbitrary volume (we'll call it V) which has a boundary (which is written S). Then integrating Equation [1] over the volume V gives Gauss' Law in integral form:

$$\int_V (\nabla \cdot \mathbf{D}) dV = \int_V \rho_V dV \qquad \int_S \mathbf{D} \cdot \mathbf{dS} = Q_{enc}$$

Gauss' Law for Magnetic Fields

- Gauss' Law for Electric Fields. If that makes sense, then the second of Maxwell's Equations will be pretty easy. First, observe both of Gauss' Laws, written in Equation :

$$\nabla \cdot \mathbf{D} = \rho_V \qquad \nabla \cdot \mathbf{B} = 0$$

- You see that both of these equations specify the divergence of the field in question. For the top equation, we know that Gauss' Law for Electric Fields states that the divergence of the Electric Flux Density D is equal to the volume electric charge density. But the second equation, Gauss' Magnetism law states that the divergence of the Magnetic Flux Density (B) is zero.
- Why? Why isn't the divergence of B equal to the magnetic charge density?
- Well - it is. But it just so happens that no one has ever found magnetic charge - not in a laboratory or on the street or on the subway. And therefore, until this hypothetical magnetic charge is found, we set the right side of Gauss' Law for Magnetic Fields to zero:

$$\nabla \cdot \mathbf{B} = 0 \quad \text{(Magnetic Charge Does Not Exist)}$$

$$\nabla \cdot \mathbf{H} = 0 \quad \text{(also true since } \mathbf{B} = \mu \mathbf{H}\text{)}$$

- Since B and the Magnetic Field H are related by the permeability μ, we note in Equation [2] that the divergence of the magnetic field is also zero.
- Now, you may have played with magnets when you were little, and these magnetic objects attracted other magnets similar to how electric charges repel or attract like electric charges. However, there is something special about these magnets - they always have a positive and negative end. This means every magnetic object is a magnetic dipole, with a north and south pole. No matter how many times you break the magnetic in half, it will just form more magnetic dipoles. Gauss' Law for Magnetism states that magnetic monopoles do not exist - or at least we haven't found them yet.

Video Content / Details of website for further learning :

- https://study.com/academy/lesson/maxwells-equations-definition-application.html

Important Books/Journals for further learning including the page nos.: 185

- Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

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LECTURE HANDOUTS **L33**

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Maxwell's equations 3 & 4

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862. Individually, the four equations are named Gauss' law, Gauss' law for magnetism, Faraday's law and Ampere's law.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Studied in the field of electrical sciences and physics
- Lenz Law

Maxwell's equations 3 & 4

- On this page, we'll explain the meaning of the 3rd of Maxwell's Equations, Faraday's Law, which is given in Equation:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- Faraday was a scientist experimenting with circuits and magnetic coils way back in the 1830s. His experiment setup, which led to Farday's Law,
- When the switch was initially changed from open to closed, the magnetic flux within the magnetic core increased from zero to some maximum number (which was a constant value, versus time). When the flux was increasing, there existed an induced current on the opposite side.
- Similarly, when the switch was opened, the magnetic flux in the core would decrease from it's constant value back to zero. Hence, a decreasing flux within the core induced an opposite current on the right side.
- Faraday figured out that a changing Magnetic Flux within a circuit (or closed loop of wire) produced an induced EMF, or voltage within the circuit. He wrote this as:

$$EMF = -\frac{d\Phi}{dt}$$

- Φ is the Magnetic Flux within a circuit, and EMF is the electro-motive force, which is basically a voltage source. Equation then says that the induced voltage in a circuit is the opposite of the time-rate-of-change of the magnetic flux. For more information on derivatives, see the partial derivatives page.
- Equation is known as Lenz's Law. Lenz was the guy who figured out the minus sign. We know that an electric current gives rise to a magnetic field - but thanks to Farady we also know that a magnetic field within a loop gives rise to an electric current. The universe loves symmetry and Maxwell's Equations has a lot of it.
- Stokes figured out that integrating (averaging) of a field around a loop is exactly equivalent to integrating the curl of the field within the loop. This should have somewhat of an intuitive truth to you: the curl is the measure of the rotation of a field, so the curl of a vector field within a surface should be related to the integral of a field around a loop that encloses the surface. If it doesn't make sense, think about it more or just accept the following as truth (because it is true - not just for E fields but for any field):

$$\oint_{Circuit} \mathbf{E}\cdot\mathbf{dL} = \int_S \nabla\times\mathbf{E}\cdot\mathbf{dS} \quad \text{[Stokes' Theorem]}$$

- Now we are almost there. If we replace Farday's Law of Equation, with the terms we found in Equation, then we get:

$$\int_S \nabla\times\mathbf{E}\cdot\mathbf{dS} = -\frac{d}{dt}\int_S \mathbf{B}(t)\cdot\mathbf{dS} = \int_S \frac{-d\mathbf{B}(t)}{dt}\cdot\mathbf{dS}$$

$$EMF = -\frac{d\Phi}{dt} \qquad \Rightarrow \boxed{\nabla\times\mathbf{E} = \frac{-\partial\mathbf{B}(t)}{\partial t}}$$

- In Equation, we note that if we have two integrals over surfaces, and the surfaces can be however we choose, then the quantities we integrate must also be the same. And this is how we obtained Faraday's Law in final form, as listed on Maxwell's Equations

Video Content / Details of website for further learning :

- https://study.com/academy/lesson/maxwells-equations-definition-application.html

Important Books/Journals for further learning including the page nos.: 189

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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LECTURE HANDOUTS

L34

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Maxwell's equations
- Lenz Law

Maxwell's equations

Problems 1 – For 1A conductor current in copper wire, find the corresponding displacement current at 100 MHz. Assume for copper 5.8 x10^{7} ohm/m.

Solution

Conduction current $I_C = J_C A = 1 \text{ Amp}$

$$J_C = \frac{I}{A} = \sigma E$$

$$E = \frac{J_C}{\sigma} = \frac{I/A}{\sigma} = \frac{0.172 \times 10^{-7}}{A} \text{ V/m}$$

Displacement current $I_D = \omega \varepsilon E \cdot A = \omega \varepsilon_0 \varepsilon_r E A$

For copper $\varepsilon_r = 1$, $I_D = 2\pi \times 100 \times 10^6 \times \frac{10^{-9}}{36\pi} \times \frac{0.172 \times 10^{-7}}{A} A$

$$I_D = 9.556 \times 10^{-11} \text{ A} \quad \text{Ans.}$$

Problems 2 – Write the Maxwell equation integral, point and phasor form and also indicate the associative laws

Maxwell's equations: ***Point form:***

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

Integral form:

$$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds = \iint \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) ds$$

$$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t} ds = -\mu \iint \frac{\partial H}{\partial t} ds$$

$$\oiint_s D \cdot ds = \iiint \rho_v \cdot dv$$

$$\oiint_s B \cdot ds = 0$$

Phasor form:

Point form

$$\nabla \times H = (\sigma + j\omega\varepsilon) E$$

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

Integral form

$$\oint H \cdot dl = \iint (\sigma + j\omega\varepsilon) E \cdot ds$$

$$\oint E \cdot dl = -\mu \iint j\omega H \cdot ds$$

$$\oint D \cdot ds = \iiint \rho_v \, dv$$

$$\oint B \cdot ds = 0$$

Video Content / Details of website for further learning :

- https://study.com/academy/lesson/maxwells-equations-definition-application.html

Important Books/Journals for further learning including the page nos.: 198

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

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L35

LECTURE HANDOUTS

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : IV – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Displacement current, Relation between field theory and circuit theory

Introduction :

- In electromagnetism, displacement current is a quantity appearing in Maxwell's equations that is defined in terms of the rate of change of electric displacement field. Displacement current has the units of electric current density, and it has an associated magnetic field just as actual currents do

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basic of capacitor
- Amperes law

Displacement current

- The displacement current is defined as Id= Where phi is the electric flux linked between the plates of the capacitor at any instant. Therefore Ampere-Maxwell circuital law may be expressed as. Thus modified Ampere-circuital law involves both conduction current Ic and displacement current Id.

- When a capacitor starts charging there is no conduction of charge between the plates. However, because of change in charge accumulation with time above the plates, the electric field changes causing the displacement current

- The electric displacement field is defined as:

  $$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

  where:

  ε0 is the permittivity of free space

  E is the electric field intensity

  P is the polarization of the medium

- Differentiating this equation with respect to time defines the displacement current density, which therefore has two components in a dielectric:

$$\boldsymbol{J_D} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{P}}{\partial t}$$

- The first term on the right hand side is present in material media and in free space. It doesn't necessarily come from any actual movement of charge, but it does have an associated magnetic field, just as a current does due to charge motion. Some authors apply the name displacement current to the first term by itself.

- The second term on the right hand side, called polarization current density, comes from the change in polarization of the individual molecules of the dielectric material. Polarization results when, under the influence of an applied electric field, the charges in molecules have moved from a position of exact cancellation.

- The positive and negative charges in molecules separate, causing an increase in the state of polarization P. A changing state of polarization corresponds to charge movement and so is equivalent to a current, hence the term "polarization current".

$$\boldsymbol{I_D} = \iint_S \boldsymbol{J_D} \cdot \mathrm{d}\boldsymbol{S} = \iint_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{S} = \frac{\partial}{\partial t} \iint_S \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = \frac{\partial \Phi_D}{\partial t}$$

- This polarization is the displacement current as it was originally conceived by Maxwell. Maxwell made no special treatment of the vacuum, treating it as a material medium. For Maxwell, the effect of P was simply to change the relative permittivity εr in the relation $D = \varepsilon_r \varepsilon_0 E$.

- According to Maxwell, an electric field sets up a current and hence a magnetic field. Such a current is called displacement current. Hence the behavior of the electric and magnetic field is symmetric.

Circuit theory.

- Signals must be routed from one point to another using trans-mission lines, modelled using transmission line theory. If the component dimensions be comparable to the wavelength then accurate understanding and prediction of behaviour may require modelling using electromagnetic field and circuit theory.

- As the name suggests, circuit theory deals with electrical circuit. An engineer can predict the performance of complicated electrical networks with the help of circuit theory. But this theory has certain limitations like :

  It cannot be applied in free space.

  It is useful only at low frequencies.

- This theory is unsuccessful in explaining the radiation of electromagnetic waves into space in radio communications.
- It cannot be used to analyse or design a complete communication system. Example: Radio Communication System.

  This analysis is originated by its own.

  Applicable only for portion of radiofrequency range.

  It is dependent and independent parameter, I and V are directly obtained from the given circuit.

  Parameters of medium are not involved.

  Laplace Transform is employed.

  Z, Y and H parameters are used. and Low power is involved.

  Simple to understand.

  Dimensional analysis.

  Frequency is used for reference.

  Lumped components are used.

Field Theory

- Although electromagnetic Field Theory (EMFT) is complex in comparison with circuit theory but EMFT is simplified by using appropriate mathematics. This theory deals with E and H vectors, whereas circuit theory deals with voltages and currents.

  This theory has following advantages in comparison to circuit theory:

  It is also applicable in free space.

  It is useful at all frequencies, particularly at high frequencies,

- The radiation effect can be considered.
- This theory can be used to analyse or design a complete communication system. Example: Wireless Communication, Radio Communication.

  Evolved from transmission ratio.

Video Content / Details of website for further learning :

- https://byjus.com/physics/displacement-current/

Important Books/Journals for further learning including the page nos.: 201

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

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LECTURE HANDOUTS

L36

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Time varying fields and Maxwell's equations **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Maxwell's equations
- Lenz Law

Maxwell's equations / Displacement current

Problem 1: - The magnetic field intensity in free space is given as $H = H_0 \sin\theta \, \bar{a}_y$ *A/m, where* $\theta = \omega t - \beta z$ *and* β *is* **a constant quantity. Determine the displacement current density.**

Solution:

$$H = H_0 \sin\theta \; \bar{a}_y \quad \text{A/m}$$

$$H = H_0 \sin(\omega t - \beta z)\, \bar{a}_y \quad \text{A/m}$$

The displacement current in free space is given by

$$\nabla \times H = J$$

$$\nabla \times H = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_0 \sin\theta & 0 \end{vmatrix}$$

$$\nabla \times H = \bar{a}_x \left[0 - \frac{\partial}{\partial z}(H_0 \sin\theta) \right] + 0 + \bar{a}_z \left[\frac{\partial}{\partial x}(H_0 \sin\theta) - 0 \right]$$

$$= -\bar{a}_x H_0 \frac{\partial}{\partial z}[\sin(\omega t - \beta z)] + \bar{a}_y H_0 \frac{\partial}{\partial x}[\sin(\omega t - \beta z)]$$

$$= -H_0 \beta \cos(\omega t - \beta z)\ \bar{a}_x$$

$$\mathbf{J} = -\beta\, \mathbf{H}_0 \cos(\omega t - \beta z)\, \bar{a}_x \quad \textbf{Ans.}$$

☺☺

Problem 2: - A conductor 1cm in length is parallel to Z-axis and rotates at radius of 25 cm at 1200 pm. Find induced voltage, if the radial field is given by B=0.5 ar T.

☺ ***Solution:*** Let $E = v \times B$

In one minute, there are 1200 revolutions which corresponds to 20 revolution in one second. In one revolution, distance travelled is $2\pi r$ meters.

Hence in 20 revolutions, the distance travelled in one second is $40\pi r$ meters.

$$v = (40\pi r)\, a_\phi = 40\pi\,(25 \times 10^{-2})\, a_\phi$$

$$v = 31.416\, a_\phi \quad \text{m/s}$$

$$E = v \times B = 31.416\, a_\phi \times 0.5\, a_r$$

$$E = 15.708\,(-a_z)$$

The induced voltage is given by

$$e = \oint E \cdot dl$$

$$dl = (dz)\, a_z$$

As the conductor is parallel to z axis

$$e = \int_{z=0}^{0.01} 15.708\,(-a_z)\,(dz)\, a_z = -15.708\,[z]_0^{0.01}$$

$$e = -157.08 \text{ mV} \quad \textbf{Ans.}$$

Video Content / Details of website for further learning :

- https://study.com/academy/lesson/maxwells-equations-definition-application.html

Important Books/Journals for further learning including the page nos.: 205

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Electromagnetic wave generation and equations

Introduction :

- The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Electromagnetic wave generation and equations

- Starting with Maxwell's equations. We have to start some- where, and Maxwell's equations govern all of (classical) electricity and magnetism. There are four of these equations, although when Maxwell first wrote them down, there were 22 of them. But they were gradually rewritten in a more compact form over the years. Maxwell's equations in vacuum in SI units are (in perhaps overly-general form):
- Our goal is to derive the wave equation for the E and B fields in vacuum. Since there are no charges of any kind in vacuum, we'll set ρE and JE = 0 from here on. And we'll only need the differential form of the equations, which are now

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \end{aligned}$$

- Then use Equation to get ride of B, we obtain

$$\begin{aligned}\nabla \times (\nabla \times \mathbf{E}) &= -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \\ &= -\frac{\partial(\nabla \times \mathbf{B})}{\partial t} \\ &= -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \\ &= -\mu_0\epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}\,.\end{aligned}$$

See Problem [to be added] for a derivation of this. This formula holds even if we have differential operators (such as ∇) instead of normal vectors, but we have to be careful to keep the ordering of the letters the same (this is evident if you go through the calculation in Problem [to be added]). Since both $\mathbf{A}$ and $\mathbf{B}$ are equal to ∇ in the present application, the ordering of $\mathbf{A}$ and $\mathbf{B}$ in the $\mathbf{B}(\mathbf{A}\cdot\mathbf{C})$ term doesn't matter. But the $\mathbf{C}(\mathbf{A}\cdot\mathbf{B})$ must correctly be written as $(\mathbf{A}\cdot\mathbf{B})\mathbf{C}$. The lefthand side of Eq. (12) then becomes

$$\begin{aligned}\nabla \times (\nabla \times \mathbf{E}) &= \nabla(\nabla\cdot\mathbf{E}) - (\nabla\cdot\nabla)\mathbf{E} \\ &= 0 - \nabla^2\mathbf{E},\end{aligned}$$

$$-\nabla^2\mathbf{E} = -\mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \quad\Longrightarrow\quad \boxed{\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{1}{\mu_0\epsilon_0}\nabla^2\mathbf{E}}$$

Note that we didn't need to use the second of Maxwell's equations to derive this.

In the above derivation, we could have instead eliminated $\mathbf{E}$ in favor of $\mathbf{B}$. The same steps hold; the minus signs end up canceling again, as you should check, and the first equation is now not needed. So we end up with exactly the same wave equation for $\mathbf{B}$:

$$\boxed{\frac{\partial^2\mathbf{B}}{\partial t^2} = \frac{1}{\mu_0\epsilon_0}\nabla^2\mathbf{B}} \qquad (\textit{wave equation})$$

The speed of the waves (both E and B) is given by the square root of the coefficient on the right hand side of the wave equation. The speed is therefore

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \approx 3\cdot 10^8\ \mathrm{m/s}.$$

Video Content / Details of website for further learning :

- https://scholar.harvard.edu/files/david-morin/files/waves_electromagnetic.pdf

Important Books/Journals for further learning including the page nos.: 216

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

L38

LECTURE HANDOUTS

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Wave parameters; velocity, intrinsic impedance

Introduction :

- A vibration can cause a disturbance to travel through a medium, transporting energy without transporting matter. This is what a wave is. But, how do we properly talk about waves? How do we compare them to one another? Can we measure the size and speed of a wave? How do we know how much energy it carries?

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Wave parameters, velocity and intrinsic impedance

- **E/m wave speed**: Radio waves travel at the same speed as light. For most practical purposes the speed is taken to be 300 000 000 metres per second although a more exact value is 299 792 500 metres per second. Although exceedingly fast, they still take a finite time to travel over a given distance. With modern radio techniques, the time for a signal to propagate over a certain distance needs to be taken into account.
- Radar for example uses the fact that signals take a certain time to travel to determine the distance of a target. Other applications such as mobile phones also need to take account of the time taken for signals to travel to ensure that the critical timings in the system are not disrupted and that signals do not overlap.
- **E/m wave wavelength**: This is the distance between a given point on one cycle and the same point on the next cycle as shown. The easiest points to choose are the peaks as these are the easiest to locate. The wavelength was used in the early days of radio or wireless to determine the position of a signal on the dial of a set.
- Although it is not used for this purpose today, it is nevertheless an important feature of any radio signal or for that matter any electromagnetic wave. The position of a signal on the dial of a radio set or its position within the radio spectrum is now determined by its frequency as this provides a more accurate and convenient method for determining the properties of the signal.
- **Frequency**: This is the number of times a particular point on the wave moves up and down in a given time (normally a second). The unit of frequency is the Hertz and it is equal to one cycle

per second. This unit is named after the German scientist who discovered radio waves.

- The frequencies used in radio are usually very high. Accordingly the prefixes kilo, Mega, and Giga are often seen. 1 kHz is 1000 Hz, 1 MHz is a million Hertz, and 1 GHz is a thousand million Hertz i.e. 1000 MHz. Originally the unit of frequency was not given a name and cycles per second (c/s) were used. Some older books may show these units together with their prefixes: kc/s; Mc/s etc. for higher frequencies.

- Wave velocity, distance traversed by a periodic, or cyclic, motion per unit time (in any direction). Wave velocity in common usage refers to speed, although, properly, velocity implies both speed and direction. The velocity of a wave is equal to the product of its wavelength and frequency (number of vibrations per second) and is independent of its intensity.

- Intrinsic Impedance refers to the impedance of a plane TEM (Transverse Electro-Magnetic wave) travelling through a homogeneous medium. The impedance of the wave everywhere in space is equal to the intrinsic impedance. It can also be defined as the ratio of the electric field to the magnetic flux density.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_R}{\varepsilon_0 \varepsilon_R}} = 376.73 \times \sqrt{\frac{\mu_R}{\varepsilon_R}}\ Ohms$$

$$\mu_0 = permeability\ of\ freespace = 4\pi \times 10^{-7}\ Henries/meter$$

$$\varepsilon_0 = permittivity\ of\ freespace = 8.854 \times 10^{-12} =\sim (1/36\pi) \times 1E-9\ Farads/meter$$

Video Content / Details of website for further learning :

- https://www.electronics-notes.com/articles/antennas-propagation/electromagnetic-waves-em/em-waves-basics.php

Important Books/Journals for further learning including the page nos.: 218

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, pp., 2011.

Course Faculty

LECTURE HANDOUTS **L39**

EEE **II/III**

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Uniform plane waves - Phase and group velocity, attenuation

Introduction :

- A vibration can cause a disturbance to travel through a medium, transporting energy without transporting matter. This is what a wave is. But, how do we properly talk about waves? How do we compare them to one another? Can we measure the size and speed of a wave? How do we know how much energy it carries?

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Uniform plane waves

- The uniform plane wave is defined as **the magnitude of the electric and magnetic fields**. They are the same at all points in the direction of propagation. The electric and magnetic fields are orthogonal to the direction of propagation. ... Waves do not have mass but contain energy, momentum, and velocity.

Phase and group velocity

- Waves can be in the group and such groups are called wave packets, so the velocity with a wave packet travels is called group velocity. The velocity with which the phase of a wave travels is called phase velocity. The relation between group velocity and phase velocity are **proportionate**.

- The phase velocity is the velocity of the wave with higher frequency. The group velocity is the **velocity of the wave with lower frequency**

- For most substances, therefore, the group velocity is smaller than the phase velocity. In such cases, it is mathematically possible that the group velocity may be **larger than** the phase velocity.

- By considering a material in which the tail has the opposite sign, it is shown that the group velocity can be **larger than c** without having any signal propagate faster than c.

Attenuation

- Attenuation is a general term that refers **to any reduction in the strength of a signal**. Attenuation occurs with any type of signal, whether digital or analog. Sometimes called loss, attenuation is a natural consequence of signal transmission over long distances.
- Attenuation is **the loss of signal strength in networking cables or connections**. This typically is measured in decibels (dB) or voltage and can occur due to a variety of factors. It may cause signals to become distorted or indiscernible.
- **The amplitude and intensity of ultrasound waves decrease as they travel through tissue**, a phenomenon known as attenuation. Given a fixed propagation distance, attenuation affects high frequency ultrasound waves to a greater degree than lower frequency waves.
- Attenuation is the reduction in power of the light signal as it is transmitted. Attenuation is caused by **passive media components, such as cables, cable splices, and connectors**. ... Dispersion is the spreading of the signal over time.

Video Content / Details of website for further learning :

- https://www.electronics-notes.com/articles/antennas-propagation/electromagnetic-waves-em/em-waves-basics.php

Important Books/Journals for further learning including the page nos.: 218

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, pp., 2011.

Course Faculty

LECTURE HANDOUTS

L40

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- An electromagnetic wave can be created by accelerating charges; moving charges back and forth will produce oscillating electric and magnetic fields, and these travel at the speed of light

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Maxwell's Equation

Problem 1 - Find the velocity of a plane wave in a lossless medium having a relative permittivity of 4 and relative permeability of 1.2.

☺ ***Solution:***

$$\varepsilon_r = 4$$

$$\mu_r = 1.2$$

$$v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

$$v = \frac{v_0}{\sqrt{\mu_r \varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{1.2 \times 4}} = \mathbf{1.37 \times 10^8 \text{ m/s}}$$

Problem 2 – find the characteristics impedance of the medium whose relative permittivity is 3 and relative permeability is1.

☺ *Solution:*

$$\varepsilon_r = 3$$

$$\mu_r = 1$$

Characteristic impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$

where $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\,\pi$

$$\eta = 120\,\pi\sqrt{\frac{1}{3}} = \mathbf{217.66\ ohms}\quad \textbf{Ans.}$$

Problem 3 – Find the depth of penetration of a plane wave in copper at a power frequency of 60 Hz and at microwave frequency 10^{10} Hz. Given $\alpha = 5.8 \times 10^7$ mho/m

☺ *Solution:* Depth of penetration $\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$

For $f = 60$ Hz, $\sigma = 5.8 \times 10^7$ mho/m, $\mu_r = 1$

$$\delta = \sqrt{\frac{2}{2\pi \times 60 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}}$$

$$= \mathbf{8.53 \times 10^{-3}\ m}\quad \textbf{Ans.}$$

For $f = 10^{10}$ Hz

$$\delta = \sqrt{\frac{2}{2\pi \times 10^{10} \times 4\pi \times 10^{-7} \times 5.8 \times 10^7}}$$

$$= \mathbf{0.66 \times 10^{-6}\ m}\quad \textbf{Ans.}$$

Video Content / Details of website for further learning :

- https://www.youtube.com/watch?v=mOEFTX9DAEw

Important Books/Journals for further learning including the page nos.: 240

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L41

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Propagation in good conductors / Propagation constant

Introduction :

- Propagation constant is a measure of changes in a sinusoidal electromagnetic wave in terms of amplitude and phase, while propagating through a medium. This can be a transmission line or free space. The Propagation constant is a dimensionless quantity.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Propagation constant

- The propagation constant, symbol {\displaystyle {\gamma }} , for a given system is defined by the ratio of the complex amplitude at the source of the wave to the complex amplitude at some distance x, such that,

$$\frac{A_0}{A_x} = e^{\gamma x}$$

- Since the propagation constant is a complex quantity we can write:

$$\gamma = \alpha + i\beta$$

- where

  α, the real part, is called the attenuation constant

  β, the imaginary part, is called the phase constant

- The complex constant (is defined as the propagation constant.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

- The real part of the propagation constant (") is defined as the attenuation constant while the imaginary part ($) is defined as the phase constant. The attenuation constant defines the rate at which the fields of the wave are attenuated as the wave propagates. An electromagnetic wave propagates in an ideal (lossless) media without attenuation ("= 0). The phase constant defines the rate at which the phase changes as the wave propagates.
- Separate but equivalent units are defined for the propagation, attenuation and phase constants in order to identify each quantity by its units [similar to complex power, with units of VA (complex power), W (real power) and VAR (reactive power)]

γ	propagation constant (m^{-1})
α	attenuation constant (Np/m)
β	phase constant (rad/m)

Given the properties of the medium (μ, ϵ, σ), we may determine equations for the attenuation and phase constants.

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) = (\alpha + j\beta)^2 = \alpha^2 + j2\alpha\beta - \beta^2$$

$$\left.\begin{aligned} Re\,\gamma^2 &= \alpha^2 - \beta^2 = -\omega^2\mu\epsilon \\ Im\,\gamma^2 &= 2\alpha\beta = \omega\mu\sigma \end{aligned}\right\} \text{Solve for } \alpha, \beta$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

Video Content / Details of website for further learning :

- https://en.wikipedia.org/wiki/Propagation_constant

Important Books/Journals for further learning including the page nos.: 220

Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011.

Course Faculty

LECTURE HANDOUTS

L42

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Waves in free space lossless dielectrics and conductors

Introduction :

- Propagation constant is a measure of changes in a sinusoidal electromagnetic wave in terms of amplitude and phase, while propagating through a medium. This can be a transmission line or free space. The Propagation constant is a dimensionless quantity.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Waves in free space

Air is typically very low loss (negligible attenuation) with little polarization or magnetization. Thus, we may model air as free space (vacuum) with $\sigma=0$, $\epsilon=\epsilon_o$, and $\mu=\mu_o$ ($\epsilon_r=1$, $\mu_r=1$). We may specialize the lossless medium equations for the case of free space.

$$\alpha = 0 \qquad \beta = \frac{\omega}{c}\sqrt{\mu_r\epsilon_r} = \frac{\omega}{c}$$

$$u = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r\epsilon_r}} = c \qquad \lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

$$\eta = \eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} \approx 377\,\Omega$$

-

Wave Propagation in Good Conductors ($\sigma \gg \omega\epsilon$)

In a good conductor, displacement current is negligible in comparison to conduction current.

$$\boldsymbol{J}_{total} = \boldsymbol{J}_{conduction} + \boldsymbol{J}_{displacement} = \sigma \boldsymbol{E} + j\omega\epsilon \boldsymbol{E}$$

$$|\boldsymbol{J}_{conduction}| > |\boldsymbol{J}_{displacement}| \qquad \text{if} \qquad (\sigma > \omega\epsilon)$$

Although this inequality is frequency dependent, most good conductors (such as copper and aluminum) have conductivities on the order of 10^7 ℧/m and negligible polarization ($\epsilon_r = 1$, $\epsilon = \epsilon_o = 8.854 \times 10^{-12}$ F/m) such that we never encounter the frequencies at which the displacement current becomes comparable to the displacement current. Given $\sigma \gg \omega\epsilon$, the propagation constant within a good conductor may be approximated by

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \approx \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma \angle 90^o} = \sqrt{\omega\mu\sigma} \angle 45^o$$

-

Waves in conductors

Waves are means of transporting energy or information. Typical examples of EM waves include radio waves, TV signals, radar beams, & light rays. All forms of EM energy share three fundamental characteristics as they all travel at high velocity in travelling, they assume the properties of waves & they radiate outward
from a source, without a benefit of any discernible physical vehicles.

- **Characteristics of EM medium**:
  - Free space: $\sigma = 0, \epsilon = \epsilon_0, \mu = \mu_0$
  - Perfect dielectric: $\sigma = 0, \epsilon$ and μ can have any value.
  - Good dielectrics: $\sigma \,\square\, 0, \epsilon = \epsilon_r \epsilon_0, \mu = \mu_r \mu_0 \; or \; \sigma << \omega \epsilon$.
  - Perfect conducting: $\sigma = \infty, \epsilon$ and μ have any value.
  - Good conductor: $\sigma \,\square\, \infty, \epsilon = \epsilon_0, \mu = \mu_r \mu_0 \; or \; \sigma >> \omega \epsilon$
- Where **σ** is Conductivity and ω is an Angular frequency of the wave.

Wave in lossy dielectric

A lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to the imperfect dielectric. A lossy dielectric is a partially conducting medium.

- **Vector wave equation or vector Helmholtz's equation**

(i) For field: $\nabla^2 \vec{E}_S - \gamma^2 \vec{E}_S = 0$

(ii) For field: $\nabla^2 \vec{H}_S - \gamma^2 \vec{H}_S = 0$

- Where **γ** is Propagation constant of medium and $\mathbf{E_S}$, $\mathbf{H_S}$ is Source electric and magnetic fields respectively.

$$\gamma = \alpha + j\beta \quad and \quad \gamma^2 = j\omega\mu(\sigma + j\omega \in)$$

- Where α is Attenuation constant (Neper/m) and β is Phase constant (rad/m).

$$\alpha = \omega\sqrt{\frac{\mu \in}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega \in}\right]^2} - 1\right]} \; ; \; \beta = \omega\sqrt{\frac{\mu \in}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega \in}\right]^2} + 1\right]}$$

- **Field equation of EM wave in s-domain**

$$E_S(z) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{i}_x$$
$$H_S(z) = H_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{i}_y$$

- **Field equation of EM wave in time domain**

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{i}_x$$
$$H(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{i}_y$$

Video Content / Details of website for further learning:

- https://en.wikipedia.org/wiki/Propagation_constant

Important Books/Journals for further learning including the page nos.: 225

- Gangadhar K A, Ramanathan, “Electromagnetic Field Theory”, Khanna Publishers, 2011

Course Faculty

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862. Individually, the four equations are named Gauss' law, Gauss' law for magnetism, Faraday's law and Ampere's law.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Maxwell's equations

Problem 1 – Find the conducting behavior of ground at 1 KHz, 10 MHz and 10 GHz. Given e_r = 10 and σ = 5 xx^{-3} mho/m

☺ *Solution:* The ratio of conduction current to displacement current is $\frac{\sigma}{\omega\varepsilon}$ which determines conducting behaviour.

For 1 kHz, $$\frac{\sigma}{\omega\varepsilon} = \frac{5\times10^{-3}}{2\pi\times1\times10^3\times10\times\frac{1}{36\pi\times10^9}} = 9\times10^3$$

$$\frac{\sigma}{\omega\varepsilon} >> 1$$

For 1 MHz, $$\frac{\sigma}{\omega\varepsilon} = \frac{5\times10^{-3}}{2\pi\times10\times10^6\times10\times\frac{1}{36\pi\times10^9}} = 0.9$$

$$\frac{\sigma}{\omega\varepsilon} \approx 1$$

For 10 GHz, $$\frac{\sigma}{\omega\varepsilon} = \frac{5\times10^{-3}}{2\pi\times10\times10^9\times10\times\frac{1}{36\pi\times10^9}} = 9\times10^{-4}$$

$$\frac{\sigma}{\omega\varepsilon} << 1$$

At 1 kHz, the ground acts like a good conductor and at 10 GHz, it acts like an insulator. For 10 MHz ground acts like quasi-conductor. ☺☺

Problem 2 –the electric flux intensity of plane wave travelling in a prefer dielectric medium is 10 cos (2π x 10^7 t – 0.1πz) V/m. Find the expression for magnetic field intensity if μ is unity

olution: $E_x = 10\cos(2\pi \times 10^7 t - 0.1\pi z)$ V/m

The general expression is $E_x = A\cos(\omega t - \beta z)$

then $\omega = 2\pi \times 10^7$

$$2\pi f = 2\pi \times 10^7$$

$$f = 10^7 \text{ Hz}$$

$$\beta = 0.1\pi$$

$$\frac{2\pi}{\lambda} = 0.1\pi$$

$$\lambda = 20 \text{ m}$$

Velocity of propagation in a medium $v = f\lambda = 10^7 \times 20$

$$= 2 \times 10^8 \text{ m/sec}$$

$$\frac{\text{Velocity in free space}}{\text{Velocity in a medium}} = \frac{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\mu_r \varepsilon_r}$$

$$\sqrt{\mu_r \varepsilon_r} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

$$\sqrt{\varepsilon_r} = 1.5 \qquad [\because \mu_r = 1]$$

$$\varepsilon_r = 2.25$$

impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$

$$= \frac{1}{\sqrt{\varepsilon_r}} \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\eta = \frac{120\pi}{1.5} = 80\pi$$

The magnetic field intensity in z-direction

$$H_z = \frac{E_x}{\eta} = \frac{10}{80\pi}\cos(2\pi \times 10^7 t - 0.1\pi z) \text{ A/m}$$

$$= \mathbf{0.04\cos(2\pi \times 10^7 t - 0.1\pi z)} \text{ A/m}$$ ☺☺

Problem 3 – The plane travelling electromagnetic wave H = 0.008 A/m in free space. Compare energy density and also the velocity of this wave in glass. Chose relative permittivity of glass is 3

☺ *Solution:* Energy density $= \frac{1}{2}\mu H^2 = \frac{1}{2}\mu_0 \mu_r H^2$

$$H = 0.008 \text{ A/m and } \mu_r = 1, \quad \varepsilon_r = 3$$

$$\text{Energy density} = \frac{1}{2} \times 4\pi \times 10^{-7} \times (0.008)^2$$

$$= \mathbf{4.02 \times 10^{-11} \text{ Joules/m}^3} \text{ Ans.}$$

$$\text{Velocity in a medium} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{\varepsilon_r}}$$

$$= \frac{3 \times 10^8}{\sqrt{3}} = \mathbf{1.732 \times 10^8 \text{ m/s}} \text{ Ans.}$$ ☺☺

Video Content / Details of website for further learning :

- https://www.khanacademy.org/science/physics/light-waves/introduction-to-light waves/v/electromagnetic-waves-and-the-electromagnetic-spectrum

Important Books/Journals for further learning including the page nos.: 241

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

MUTHAYAMMAL ENGINEERING COLLEGE

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

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LECTURE HANDOUTS

L44

EEE

II/III

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Skin depth

Introduction :

- Electromagnetic shield may be necessary to prevent waves from radiating out of the shielded volume or to prevent waves from penetrating into the shielded volume.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Skin depth

- A plane wave incident on a highly-conducting surface, the electric field (and thus current density) was found to be concentrated at the of the conductor. The same phenomenon occurs for a current carrying conductor such as a wire. The effect is frequency-dependent, just as it is in the incident plane wave example. This phenomenon is known as the *skin effect*.

$$\boldsymbol{E}_s = \frac{\boldsymbol{J}_s}{\sigma} \qquad \nabla \times \boldsymbol{H}_s = \boldsymbol{J}_s$$

$$\nabla \times \nabla \times \left(\frac{\boldsymbol{J}_s}{\sigma}\right) = -j\omega\mu \boldsymbol{J}_s$$

$$\nabla \times \nabla \times \boldsymbol{J}_s = -j\omega\mu\sigma \boldsymbol{J}_s$$

$$\nabla \times \nabla \times \boldsymbol{F} = \nabla(\nabla \cdot \boldsymbol{F}) - \nabla^2 \boldsymbol{F} \qquad \text{(vector identity)}$$

$$\nabla \times \nabla \times \boldsymbol{J} = \nabla(\nabla \cdot \boldsymbol{J}) - \nabla^2 \boldsymbol{J} = -\nabla^2 \boldsymbol{J}$$

$$\nabla^2 \boldsymbol{J}_s - j\omega\mu\sigma \boldsymbol{J}_s = 0$$

If we let $-j\omega\mu\sigma = T^2$, the governing equation for the conductor current density becomes

$$\nabla^2 \boldsymbol{J}_s + T^2 \boldsymbol{J}_s = 0 \qquad \begin{pmatrix}\text{vector wave} \\ \text{equation}\end{pmatrix} \qquad ③$$

The constant T in the vector wave equation may be written in terms of the skin depth of the conductor.

$$T = \sqrt{-j\omega\mu\sigma} = \sqrt{\frac{\omega\mu\sigma}{j}} = j^{-1/2}\sqrt{2\pi f\mu\sigma} = j^{-1/2}\frac{\sqrt{2}}{\delta}$$

Poynting theorem & Vector

Poynting's theorem is the fundamental energy-conservation theorem for electromagnetic fields. Using Poynting's theorem, we can identify all sources of energy related to electromagnetic fields in a given volume. The corresponding *Poynting vector* defines the vector power density (direction and density of power flow at a point). To derive Poynting's theorem, we start with the time-dependent Maxwell curl equations.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad ①$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad ②$$

The product of $\boldsymbol{E}$ and $\boldsymbol{H}$ gives units of W/m^2 (volume power density, analogous to volume current density). As shown for the uniform plane wave, the direction of $\boldsymbol{E} \times \boldsymbol{H}$ gives the direction of wave propagation (the direction of power flow). Thus, we seek a relationship defining the cross product of $\boldsymbol{E}$ and $\boldsymbol{H}$. Using the vector identity,

$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B}) \qquad ③$$

letting $A=E$ and $B=H$, we may obtain the necessary terms on the right hand side of ③ by dotting ① with H and ② with E.

$$\boldsymbol{H}\cdot(\nabla\times\boldsymbol{E}) = -\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} \quad ④$$

$$\boldsymbol{E}\cdot(\nabla\times\boldsymbol{H}) = \boldsymbol{E}\cdot\boldsymbol{J} + \boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} \quad ⑤$$

Inserting ④ and ⑤ into ③ yields

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) = -\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} - \boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} - \boldsymbol{E}\cdot\boldsymbol{J} \quad ⑥$$

The three terms on the right hand side of ⑥ may be rewritten as

$$\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t} = \mu\left(\boldsymbol{H}\cdot\frac{\partial\boldsymbol{H}}{\partial t}\right) = \mu\frac{1}{2}\frac{\partial}{\partial t}(\boldsymbol{H}\cdot\boldsymbol{H}) = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$$

$$\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} = \epsilon\left(\boldsymbol{E}\cdot\frac{\partial\boldsymbol{E}}{\partial t}\right) = \epsilon\frac{1}{2}\frac{\partial}{\partial t}(\boldsymbol{E}\cdot\boldsymbol{E}) = \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right)$$

$$\boldsymbol{E}\cdot\boldsymbol{J} = \sigma(\boldsymbol{E}\cdot\boldsymbol{E}) = \sigma E^2$$

which gives

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2 \quad ⑦$$

Integrating ⑦ over a given volume V (enclosed by a surface S) and applying the divergence theorem yields

$$\int_V \nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})\,dv = \int_V \left\{-\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2\right\}dv$$

$$\underbrace{\int_S (\boldsymbol{E}\times\boldsymbol{H})\cdot d\boldsymbol{s}}_{\text{net power flow out of } V} = \underbrace{-\frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dv}_{\text{decrease in the stored electric and magnetic energy within } V} \underbrace{- \int_V \sigma E^2\,dv}_{\text{ohmic losses within } V} \quad \text{(Poynting's theorem)}$$

Video Content / Details of website for further learning :

- https://gradeup.co/EM-Wave-Propagation-notes-topics-for-gate-ec-2019-i-ade2cda0-738b-11e7-9a49-9f4a78e2f6a9

Important Books/Journals for further learning including the page nos.: 235

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty

Course Name with Code : 19EEC01 / Electromagnetic Fields

Course Faculty : Ms V.Deepika

Unit : V – Electromagnetic Waves **Date of Lecture:**

Topic of Lecture: Tutorial Hour

Introduction :

- Maxwell's equations are a series of four partial differential equations that describe the force of electromagnetism. They were derived by mathematician James Clerk Maxwell, who first published them in 1861 and in 1862.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Basics of Electric field and magnetic field
- Maxwell's Equation

Maxwell's equations / Displacement current

Problem 1: - A free space conductor interface has H = 1 A/m on the free space side. The frequency is 31.8 MHz and the conductor constants are $e_r = \mu_r = 1$ and σ = 1.26 MS/m. Determine H and depth of penetration

Solution

$$f = 31.8 \text{ MHz}$$

$$H_i = 1 \text{ A/m}$$

$$\sigma = 1.26 \times 10^6 \text{ s/m}$$

$$\varepsilon_r = \mu_r = 1$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$= \sqrt{\frac{j2\pi \times 31.8 \times 10^6 \times 4\pi \times 10^{-7}}{1.26 \times 10^6 + j2\pi \times 31.8 \times 10^6 \times \frac{1}{36\pi \times 10^9}}}$$

$$= \sqrt{\frac{j251.083}{1.26 \times 10^6 + j1.766 \times 10^{-3}}}$$

$$\eta_2 = 1.414 \times 10^{-2} \angle 45^\circ \text{ ohms}$$

$$\frac{H_r}{H_i} = \frac{120\pi - 1.414 \times 10^{-2} \angle 45^\circ}{120\pi + 1.414 \times 10^{-2} \angle 45^\circ}$$

$$= \frac{120\pi - (0.01 + j0.01)}{120\pi + (0.01 + j0.01)} = \frac{376.98 - j0.01}{377 + j0.01}$$

$$= \frac{376.98 \angle 1.52 \times 10^{-3\circ}}{377 \angle 1.52 \times 10^{-3\circ}}$$

$$= 0.99995$$

$$H_r = 0.9995 \times H_i = \mathbf{0.99995 \ A/m \ \ Ans.}$$

$$H_t = \frac{2\eta_1}{\eta_1 + \eta_2} = \frac{2 \times 120\pi}{377 \angle 1.52 \times 10^{-3\circ}}$$

$$= 1.99995 \angle -0.00152$$

$$H_t = \mathbf{1.99995 \angle -0.00153^\circ \ A/m \ \ Ans.}$$

$$\text{Depth of penetration } \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$= \sqrt{\frac{2}{2\pi \times 31.8 \times 10^6 \times 4\pi \times 10^{-7} \times 1.26 \times 10^6}}$$

$$= \mathbf{7.05 \times 10^{-5} \ m \ \ Ans.}$$

☺☺

Problem 2: - A wave propagations from a dielectric medium to the interface with free space if the angle of incidence is the critical angle 20 degree find the relative permittivity

Solution

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\theta_1 = 20^\circ, \quad \varepsilon_2 = \varepsilon_0$$

$$\theta_2 = 90^\circ$$

$$\sqrt{\frac{\sin 20^\circ}{\sin 90^\circ}} = \sqrt{\frac{\varepsilon_0}{\varepsilon_0 \varepsilon_r}}$$

$$\sqrt{\varepsilon_r} = \frac{1}{0.342} = 2.924$$

$$\varepsilon_r = \mathbf{8.55 \ \ Ans.}$$

Video Content / Details of website for further learning :242

- https://www.youtube.com/watch?v=AaX1NJTGpbo

Important Books/Journals for further learning including the page nos.:

Gangadhar K A, Ramanathan, "Electromagnetic Field Theory", Khanna Publishers, 2011.

Course Faculty