L1

LECTURE HANDOUTS

EEE **II/IV**

Course Name with Code: 19EEC06 & CONTROL SYSTEMS

Course Teacher : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Basic elements in control systems, Open and closed loop systems.

Introduction : (Maximum 5 sentences)

- A control system manages commands, directs or regulates the behavior of other devices or systems using control loops.
- It can range from a single home heating controller using a thermostat controlling a domestic boiler to large Industrial control systems which are used for controlling processes or machines.
- A control system is a system, which provides the desired response by controlling the output.
- In recent years, control systems have gained an increasingly importance in the development and advancement of the modern civilization and technology.
- Disregard the complexity of the system; it consists of an input (objective), the control system and its output (result).

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals and Systems
- Amplifier

Detailed content of the Lecture:

Concept of control system :

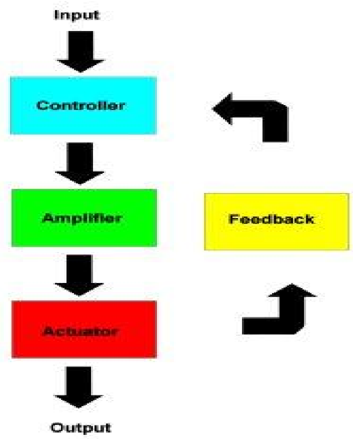
- A control system manages commands, directs or regulates the behavior of other devices or systems using control loops.
- It can range from a single home heating controller using a thermostat controlling a domestic boiler to large Industrial control systems which are used for controlling processes or machines.
- A control system is a system, which provides the desired response by controlling the output. The following figure shows the simple block diagram of a control system.

- Examples − Traffic lights control system, washing machine Traffic lights control system is an example of control system.
- Here, a sequence of input signal is applied to this control system and the output is one of the three lights that will be on for some duration of time.

- During this time, the other two lights will be off. Based on the traffic study at a particular junction, the on and off times of the lights can be determined.
- Accordingly, the input signal controls the output. So, the traffic lights control system operates on time basis.

Basic elements in control systems:

There are four basic elements of a typical motion control system. These are the controller, amplifier, actuator, and feedback.



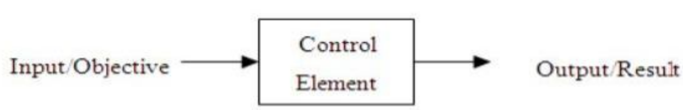
Controller:
- Controller section of the AC drive is the brain if the system.
- It typically consists of a microprocessor based CPU and memory that is used to process data once it is collected and stored.
- This controller section of the AC drive will process information received from the inputs of the drive and also from the feedback signals that will usually be a representation of the position or speed of the actuator.

Amplifier:
- The amplifier section of the drive receives the commands from the control section.
- The amplifier then generates the power signal necessary for the actuator to drive the load with the correct speedand direction.

Actuator:
- The actuator portion of the system will most often be an induction AC motor or permanent magnet AC motor with windings and insulation that are specially designed to handle the heat and stress generated by a pulse width modulated output.

Feedback:
- The feedback element of the motion control system may be handled by the system in a number of ways depending on the information needed.
- Encoders or resolvers can be used to provide feedback signals from the actuator in a closed loop control or Hall Effect sensors can provide feedback from the output of the AC drive in an open loop control.

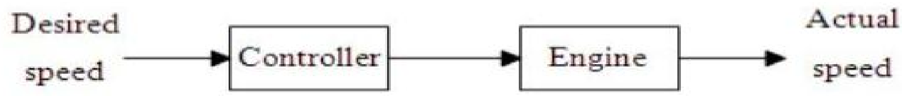
There are two main branches of control systems:
1) Open-loop systems
2) Closed-loop systems.



Basic Components of Control System

1.Open-loop systems:
- The open-loop system is also called the non-feedback system. This is the simpler of the two systems.
- In this open-loop system, there is no way to ensure the actual speed is close to the desired

speed automatically.
- The actual speed might be way off the desired speed because of the wind speed and / or road conditions, such as uphill or downhill etc.



Basic Open Loop System

Practical Examples of Open Loop Control System:

1. Electric Hand Drier - Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.
2. Automatic Washing Machine - This machine runs according to the pre-set time irrespective of washing is completed or not.

Advantages of Open Loop Control System:

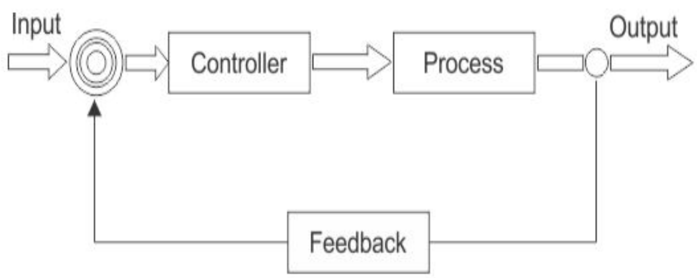
1. Simple in construction and design.
2. Economical.
3. Easy to maintain.
4. Generally stable.
5. Convenient to use as output is difficult to measure.

Disadvantages of Open Loop Control System:

1. They are inaccurate.
2. They are unreliable.
3. Any change in output cannot be corrected automatically

2. Closed-loop systems:

- The closed-loop system is also called the feedback system. A simple closed system is shown in Figure.
- It has a mechanism to ensure the actual speed is close to the desired speed automatically.



Closed loop system

Practical Examples of Closed Loop Control System:

1. Automatic Electric Iron - Heating elements are controlled by output temperature of the iron.
2. Servo Voltage Stabilizer - Voltage controller operates depending upon output voltage of the system.
3. Water Level Controller - Input water is controlled by water level of the reservoir.

Advantages of Closed Loop Control System

1. Closed loop control systems are more accurate even in the presence of nonlinearity.
2. Highly accurate as any error arising is corrected due to presence of feedback signal.
3. Bandwidth range is large.
4. Facilitates automation.

Disadvantages of Closed Loop Control System

1. They are costlier.
2. They are complicated to design.
3. Required more maintenance.

Video Content / Details of website for further learning (if any):
➢ https://www.youtube.com/watch?v=u1pgaJHiiew
Important Books/Journals for further learning including the page nos.:
➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:2-3)

Course Faculty

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L2

LECTURE HANDOUTS

EEE

II/IV

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture: Electrical analogy of mechanical systems, Electrical analogy of thermal systems
Introduction : (Maximum 5 sentences) Two systems are said to be analogous to each other if the following two conditions are satisfied. • The two systems are physically different • Differential equation modeling of these two systems are same
Prerequisite knowledge for Complete understanding and learning of Topic: • Force, Newton's second law, Torque • Laplace transform • Differential equation

Detailed content of the Lecture:

- Electrical systems and mechanical systems are two physically different systems.
- There are two types of electrical analogies of translational mechanical systems.
- Those are force voltage analogy and force current analogy.

Force Voltage Analogy:

In force voltage analogy, the mathematical equations of translational mechanical system are compared with mesh equations of the electrical system.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1c)(1c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy:

In force current analogy, the mathematical equations of the translational mechanical system are compared with the nodal equations of the electrical system.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance(1R)(1R)
Spring constant(K)	Reciprocal of Inductance(1L)(1L)
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Torque Voltage Analogy:

In this analogy, the mathematical equations of rotational mechanical system are compared with mesh equations of the electrical system.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1c)(1c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Torque Current Analogy:

In this analogy, the mathematical equations of the rotational mechanical system are compared with the nodal mesh equations of the electrical system.

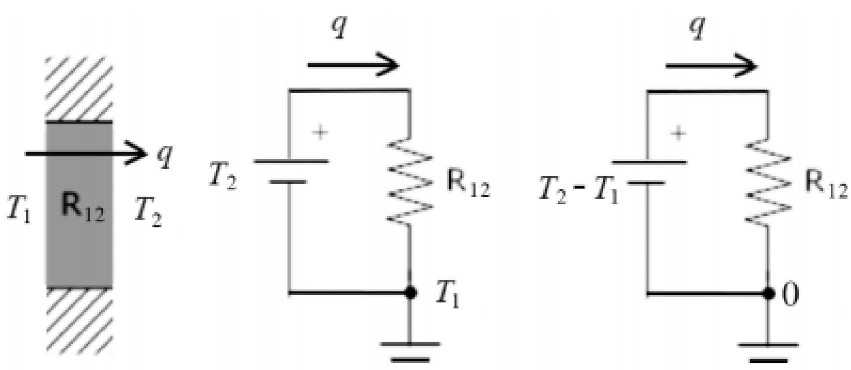
Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance(1R)(1R)
Torsion spring constant(K)	Reciprocal of Inductance(1L)(1L)
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

- There are two fundamental physical elements that make up thermal networks, thermal resistances and thermal capacitance.

- There are also three sources of heat, a power source, a temperature source, and fluid flow.
- In practice temperature when we discuss temperature we will use degrees Celsius (°C), while SI unit for temperature is to use Kelvins (0°K = -273.15°C).
- However, we will generally be interested in temperature differences, not absolute temperatures (much as electrical circuits deal with voltage differences).
- Therefore, we will generally take a reference temperature (which we will label T1), and measure all temperatures relative to this reference.
- We will also assume that the reference temperature is constant.
- Thus, if T1 is =25°C, and the temperature of interest is Ti=32°C, we will say that Ti=7° above reference.
- Note: this is consistent with electrical systems in which we assign one voltage to be ground (and assume that it is constant) and assign it the value of zero volts. We then measure all voltages relative to ground.

Thermal resistance:

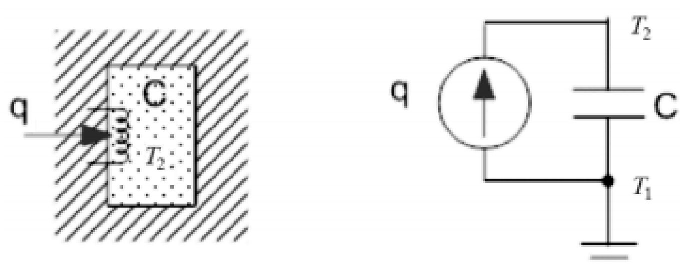
- Consider the situation in which there is a wall, one side of which is at a temperature T1, with the other side at temperature T2. The wall has a thermal resistance of R12.



Thermal-electrical analogy of conduction heat transfer.

Thermal capacitance:

- In addition to thermal resistance, objects can also have thermal capacitance (also called thermal mass).
- The thermal capacitance of an object is a measure of how much heat it can store.
- If an object has thermal capacitance its temperature will rise as heat flows into the object, and the temperature will lower as heat flows out.
- To understand this, envision a rock in the sun.
- During the day heat goes in to the rock from the sunlight, and the temperature of the rock increases as energy is stored in the rock as an increased temperature.
- At night energy is released, and the rock cools down.



$$C \frac{dT_2}{dt} = q$$

Thermal-electrical analogy of thermal capacitance.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=vqI791VCjjA

Important Books/Journals for further learning including the page nos.: P4
M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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LECTURE HANDOUTS | **L3**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**
:

Topic of Lecture:
Transfer function. Synchros

Introduction :

- The transfer function of a system is defined as the ratio of Laplace transform of output to the Laplace transform of input where all the initial conditions are zero.
- That is, the transfer function of the system multiplied by the input function gives the output function of the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Laplace transform
- Differential equation

Detailed content of the Lecture:

- A **transfer function** represents the relationship between the output signal of a control system and the input signal, for all possible input values.
- A block diagram is a visualization of the control system which uses blocks to represent the transfer function, and arrows which represent the various input and output signals.
- For any control system, there exists a reference input known as excitation or cause which operates through a transfer operation (i.e. the transfer function) to produce an effect resulting in controlled output or response.
- Thus the cause and effect relationship between the output and input is related to each other through a **transfer function**.

- In a Laplace Transform, if the input is represented by R(s) and the output is represented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$$

That is, the transfer function of the system multiplied by the input function gives the output function of the system.

Procedure for determining the transfer function of a control system are as follows:

- We form the equations for the system.
- Now we take Laplace transform of the system equations, assuming initial conditions as zero.
- Specify system output and input.
- Lastly we take the ratio of the Laplace transform of the output and the Laplace transform of the input which is the required transfer function.

Methods of Obtaining a Transfer Function:

There are major two ways of obtaining a transfer function for the control system. The ways are:

- Block Diagram Method: It is not convenient to derive a complete transfer function for a complex control system. Therefore the transfer function of each element of a control system is represented by a block diagram. Block diagram reduction techniques are applied to obtain the desired transfer function.
- Signal Flow Graphs: The modified form of a block diagram is a signal flow graph. Block diagram gives a pictorial representation of a control system. Signal flow graph further shortens the representation of a control system.
- The Synchro is a type of transducer which transforms the angular position of the shaft into an electric signal. It is used as an error detector and as a rotary position sensor.

Synchros System Types:

The synchro system is of two types. They are

- Control Type Synchro.
- Torque Transmission Type Synchro.

Torque Transmission Type Synchros:

- This type of synchros has small output torque, and hence they are used for running the very light load like a pointer.
- The control type Synchro is used for driving the large loads.

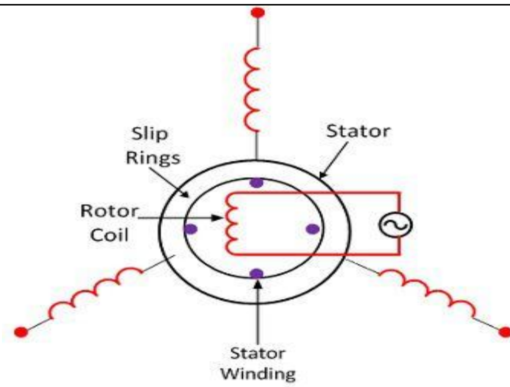
Control Type Synchros System:

The controls synchros is used for error detection in positional control systems. Their systems consist two units. They are

- Synchro Transmitter
- Synchro receiver
- The synchro always works with these two parts. The detail explanation of synchros transmitter and receiver is given below.

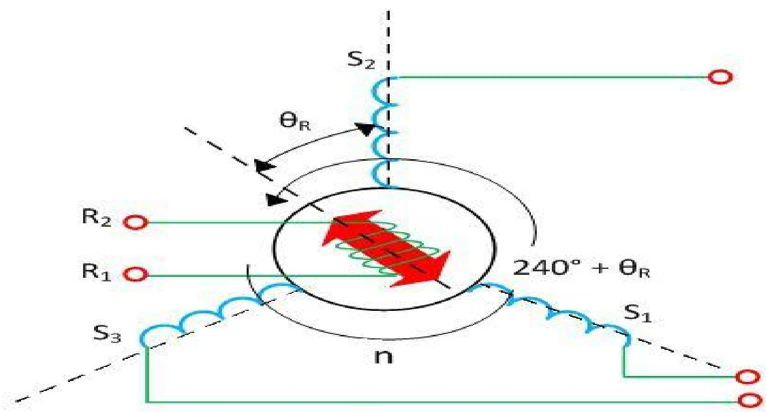
Synchros Transmitter :

- Their construction is similar to the three phase alternator. The stator of the synchros is made of steel for reducing the iron losses.
- The stator is slotted for housing the three phase windings. The axis of the stator winding is kept 120^{o} apart from each other.



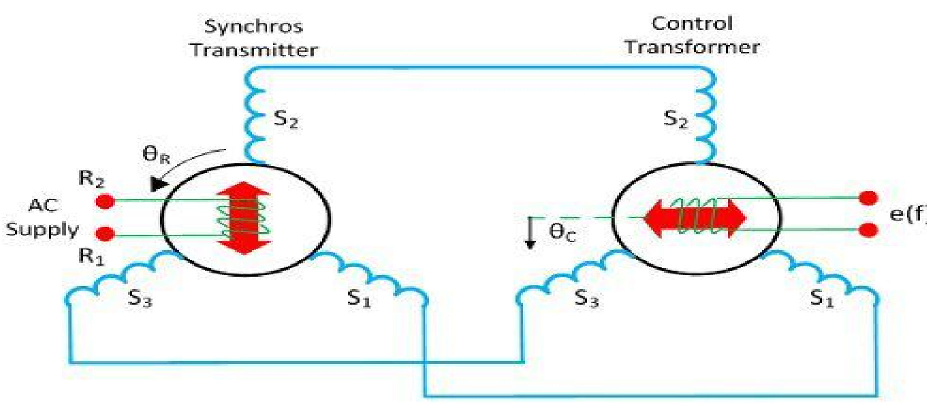
Constructional Feature of Synchros Transmitter

- The coils of the stator windings are connected in star. The rotor of the synchros is a dumbbell in shape, and a concentric coil is wound on it.
- The AC voltage is applied to the rotor with the help of slip rings.
- The constructional feature of the synchros isshown in the figure below.



Sychro Transmitter

- Consider the voltage is applied to the rotor of the transmitter as shown in the figure above.



Syncho Error Dectector

- The voltage applied to the rotor induces the magnetizing current and an alternating flux along its axis.
- The voltage is induced in the stator winding because of the mutual induction between the rotor and stator flux.
- The flux linked in the stator winding is equal to the cosine of the angle between the rotor and stator.
- The voltage is induced in the stator winding.

- Let V_{s1}, V_{s2}, V_{s3} be the voltages generated in the stator windings S_1, S_2, and S_3 respectively.
- The figure below shows the rotor position of the synchro transmitter. The rotor axis makes an angle θr concerning the stator windings S2.

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 120°)$$

$$V_{s2n} = kV_r Sin\omega_c t cos\theta_R$$

$$V_{s1n} = kV_r Sin\omega_c t cos(\theta_R + 240°)$$

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Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=ZOnolyI634k

Important Books/Journals for further learning including the page nos.: P45

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

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LECTURE HANDOUTS | **L4**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:

AC and DC servomotors

Introduction :

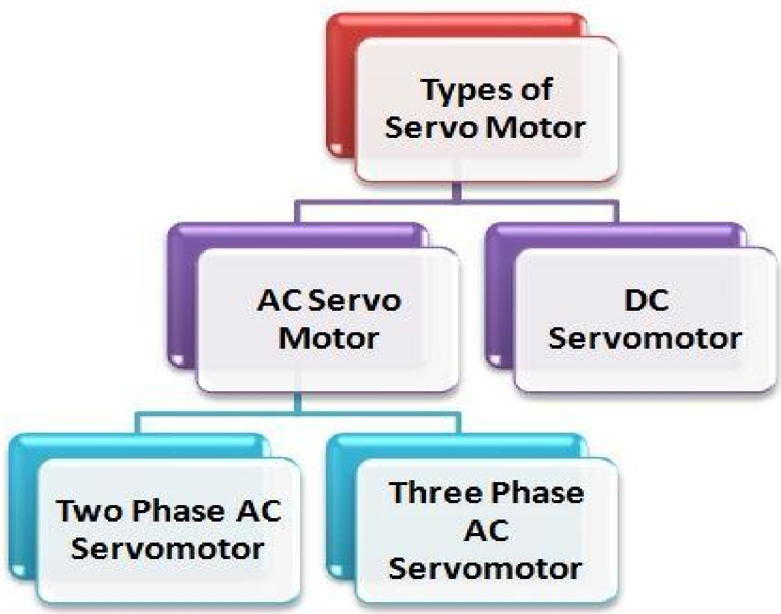
- Servo Motor are also called Control motors. They are used in feedback control systems as output actuators and does not use for continuous energy conversion.
- The principle of the Servomotor is similar to that of the other electromagnetic motor, but the construction and the operation are different.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Faradays law
- Mutual inductance

Detailed content of the Lecture:

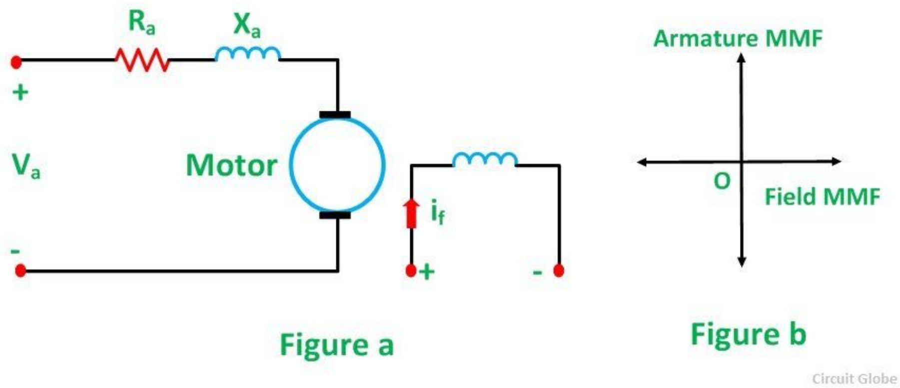
- They are classified as AC and DC Servo Motor. The AC servomotor is further divided into two types.
  1. Two Phase AC Servo Motor
  2. Three Phase AC Servo Motor



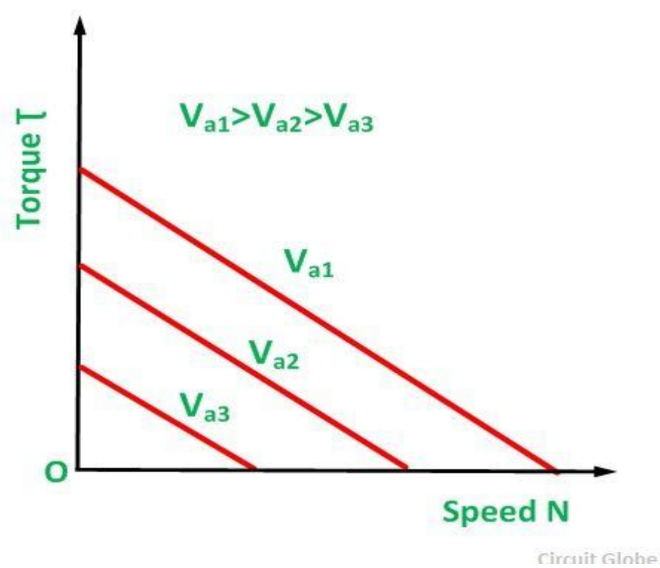
DC Servo Motor:

- DC Servo Motors are separately excited DC motor or permanent magnet DC motors.

- The figure (a) shows the connection of Separately Excited DC Servo motor and the figure (b) shows the armature MMF and the excitation field MMF in quadrature in a DC machine.



- This provides a fast torque response because torque and flux are decoupled.
- Therefore, a small change in the armature voltage or current brings a significant shift in the position or speed of the rotor.
- Most of the high power servo motors are mainly DC.
- The Torque-Speed Characteristics of the Motor is shown below.



- As from the above characteristics, it is seen that the slope is negative.
- Thus, a negative slope provides viscous damping for the servo drive system.

AC Servo Motor:

- The AC Servo Motors are divided into two types 2 and 3 Phase AC servomotors.
- Most of the AC servomotors are of the two-phase squirrel cage induction motor type.
- They are used for low power applications.
- The three phase squirrel cage induction motor is now utilized for the applications where high power system is required.

Applications of the Servo Motor:

- The power rating of the servo motor may vary from the fraction of watts to few hundreds of watts.

- The rotor of servo motor has low inertia strength, and therefore they have a high speed of inertia.
- The Applications of the Servomotor are as follows:-
    - They are used in Radar system and process controller.
    - Servomotors are used in computers and robotics.
    - They are also used in machine tools.
    - Tracking and guidance systems.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=ditS0a28Sko

Important Books/Journals for further learning including the page nos.: P52

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Block diagram reduction techniques

Introduction :

- A control system may consist of a number of components.
- A block diagram of a system is a pictorial representation of the function performed by each component and of the flow of signals.
- Such a diagram depicts the inter-relationships which exists between the various components.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signal system
- Transfer function

Detailed content of the Lecture:

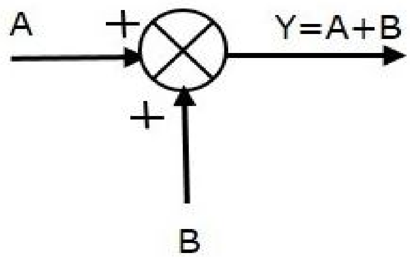
- The basic elements of block diagram such as
  1. Block
  2. Summing point
  3. Branch point

Block:

- The transfer function of a component is represented by a block.
- Block has single input and single output.
- The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

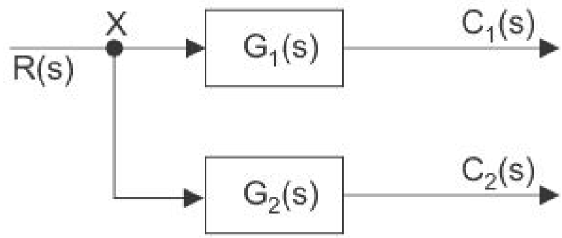
Summing Point:

- The summing point is represented with a circle having cross (X) inside it.
- It has two or more inputs and single output.
- It produces the algebraic sum of the inputs.
- It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs.
- Let us see these three operations one by one.
- The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign.
- So, the summing point produces the output, Y as sum of A and B. i.e.,Y = A + B.



Branch point:

- The take-off point is a point from which the same input signal can be passed through more than one branch.
- That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.
- In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



Block Diagram Reduction Rules:

- Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

  Rule 1 − Check for the blocks connected in series and simplify.

  Rule 2 − Check for the blocks connected in parallel and simplify.

  Rule 3 − Check for the blocks connected in feedback loop and simplify.

  Rule 4 − If there is difficulty with take-off point while simplifying, shift it towards right.

  Rule 5 − If there is difficulty with summing point while simplifying, shift it towards left.

  Rule 6 − Repeat the above steps till you get the simplified form, i.e., single block.

- Note − The transfer function present in this single block is the transfer function of the overall block diagram

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=4KUTNL9dA8E

Important Books/Journals for further learning including the page nos.: P52

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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Course Name with Code: 19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Signal flow graphs

Introduction :

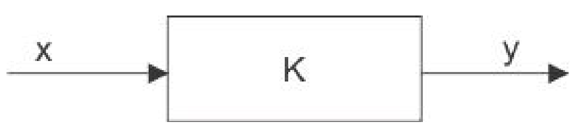
- The block diagram reduction process takes more time for complicated systems.
- Because, we have to draw the (partially simplified) block diagram after each step.
- So, to overcome this drawback, use signal flow graphs (representation).

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signal system
- Transfer function

Detailed content of the Lecture:

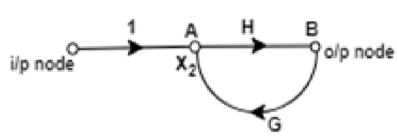
- Signal flow graph of control system is further simplification of block diagram of control system.
- Here, the blocks of transfer function, summing symbols and take off points are eliminated by branches and nodes.
- The transfer function is referred as transmittance in signal flow graph.
- Let us take an example of equation y = Kx.
- This equation can be represented with block diagram as below.



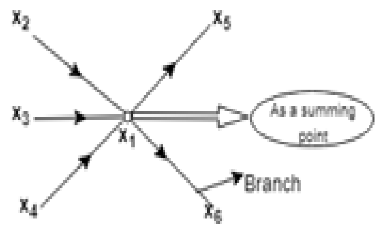
- The same equation can be represented by signal flow graph, where x is input variable node, y is output variable node and the transmittance of the branch connecting directly these two nodes.

Key Definitions:

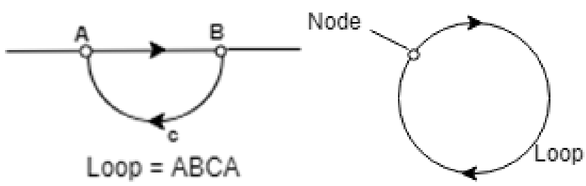
- **Node**: It represents the system variable which equals to the sum of all signals. Outgoing signal from the node does not affect the value of node variables.



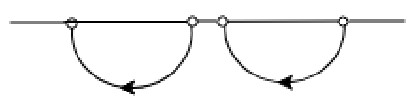
- **Branch:** Branch is defined as a path from one node to another node, in the direction indicated by the branch arrow.



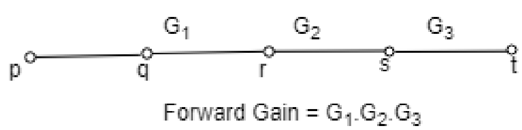
- **Input node or source**: It is the node which have only outgoing branches.
- **Output node or sink**: It is a node which has only incoming branches.
- Forward Path**: It is a path from an input node to an output node in the direction of branch arrow.**
- **Loop**: It is a path that starts and ends at the same node.



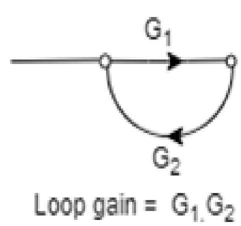
- **Non-touching loop**: Loop is said to be non-touching if they do not have any common node.



- **Forward path gain**: A product of all branches gain along the forward path is called Forward path gain.



- **Loop Gain**: Loop gain is the product of branch gain which travels in the loop.



Mason's gain formula is

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,
C(s) is the output node
R(s) is the input node
T is the transfer function or gain between R(s)R(s) and C(s)C(s)
Pi is the ith forward path gain

- Δ=1−(sum of all individual loop gains) +(sum of gain products of all possible two nontouching loops) −(sum of gain products of all possible three nontouching loops)+... Δi is obtained from Δ by removing the loops which are touching the ith forward path.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=-jyP7J3iDMI

Important Books/Journals for further learning including the page nos.: P45

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Electrical analogy of mechanical systems (Tutorial - I)

Introduction :
Two systems are said to be analogous to each other if the following two conditions are satisfied.

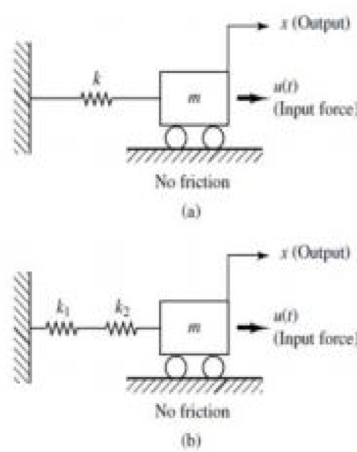
- The two systems are physically different
- Differential equation modeling of these two systems are same

Prerequisite knowledge for Complete understanding and learning of Topic:

- Force, Newton's second law, Torque
- Laplace transform
- Differential equation

Detailed content of the Lecture:

1. **Obtain mathematical models ($\frac{X(s)}{U(s)}$) of the mechanical systems shown below.**



Solution

a)

Here, the input is u(t) and the output is the displacement x as shown in the figure.

$$u(t) = m\ddot{x} + kx$$

- the rollers under the mass means there is no friction.

Converting the above equation to Laplace domain.

$$U(s) = ms^2X(s) + kX(s)$$

Taking X(s) out of the brackets:

$$U(s) = (ms^2 + k)X(s)$$

Then the transfer function (T.F) equal to:

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + k}$$

b)

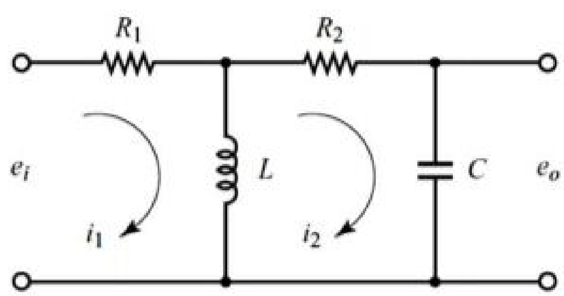
The solution will be the same procedure, but the spring factor will be:

$$k_T = \frac{k_1 k_2}{k_1 + k_2}$$

Then the T.F would be:

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + k_T}$$

2. **Obtain the transfer function $Eo(s)/Ei(s)$ of the electrical circuit shown in.**



The equations for the given circuit are as follow:

$$R_1 L_1 + L\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = e_i$$
$$R_2 i_2 + \frac{1}{c}\int i_2 dt + L\left(\frac{di_2}{dt} - \frac{di_1}{dt}\right) = 0$$
$$\frac{1}{c}\int i_2 dt = e_0$$

The Laplace transforms of these three equations, with zero initial conditions, are

$$R_1 I_1(s) + L[sI_1(s) - sI_2(s)] = E_i(s)$$
$$R_2 I_2(s) + \frac{1}{Cs} I_2(s) + L[sI_2 - sI_1(s)] = 0$$
$$\frac{1}{Cs} I_2(s) = E_0(s)$$

From Equation (2) we obtain.

$$\left(R_2 + \frac{1}{Cs} + Ls\right) I_2(s) = LsI_1(s)$$

or

$$I_2(s) = \frac{LCs^2}{LCs^2+R_2Cs+1} I_1(s) \qquad (4)$$

Or

Substituting equation (4) into equation (1), we get

$$(R_1 + Ls - Ls\frac{LCs^2}{LCs^2+R_2Cs+1})I_1(s) = E_i(s)$$

Or

$$\frac{LC(R_1+R_2)s^2+(R_1R_2C+L)s+R_1}{LCs^s+R_2Cs+1} I_1(s) = E_1(s) \qquad (5)$$

From Equation (3) and (4), we get

$$\frac{Ls}{LCs^2+R_2Cs+1} I_1(s) = E_0(s) \qquad (6)$$

From equation (5) and (6), we obtain

$$\frac{E_0(s)}{E_i(s)} = \frac{Ls}{LC(R_1+R_2)s^2+(R_1R_2C+L)s+R_1}$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=-jyP7J3iDMI

Important Books/Journals for further learning including the page nos.: P45

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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LECTURE HANDOUTS **L8**

EEE **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Electrical analogy of mechanical systems (Tutorial - II)

Introduction :
Two systems are said to be analogous to each other if the following two conditions are satisfied.

- The two systems are physically different
- Differential equation modeling of these two systems are same

Prerequisite knowledge for Complete understanding and learning of Topic:

- Force, Newton's second law, Torque
- Laplace transform
- Differential equation

Detailed content of the Lecture:

Write the differential equations governing the mechanical rotational system shown n fig 1.12.1. Draw the torque-voltage and torque-current electrical analogous circuits nd verify by writing mesh and node equations.

OLUTION

The given mechanical rotational stem has two nodes (moment of inertia of asses). The differential equations governing mechanical rotational system are given torque balance equations at these nodes.

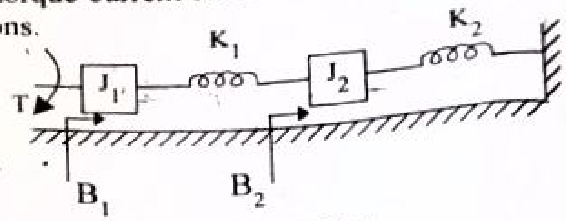
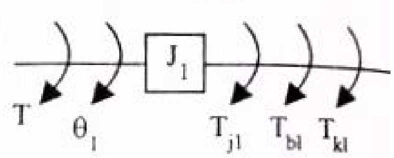


Fig 1.12.1

Let the angular displacements of J_1 and J_2 be θ_1 and θ_2 respectively. The responding angular velocities be ω_1 and ω_2. The free body diagram of J_1 is shown in 1.12.2. The opposing torques are marked as T_{j1}, T_{b1} and T_{k1}.

$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2} \; ; \; T_{b1} = B_1 \frac{d\theta_1}{dt} \quad \text{and}$$

$$T_{k1} = K_1(\theta_1 - \theta_2)$$



By Newton's second law, $T_{j1} + T_{b1} + T_{k1} = T$

Fig 1.12.2

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T \qquad .....(1.12.1)$$

The free body diagram of J_2 is shown in fig 1.12.3. The opposing torques ar rked as T_{j2}, T_{b2}, T_{k2} and T_{k1}.

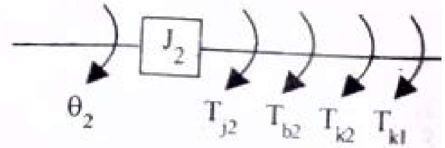


Fig 1.12.3

$$T_{j2} = J_2 \frac{d^2\theta_2}{dt^2} \; ; \qquad T_{b2} = B_2 \frac{d\theta_2}{dt}$$

$$T_{k2} = K_2\theta_2 \quad \text{and } T_{k1} = K_1(\theta_2 - \theta_1)$$

By Newton's second law, $T_{j2} + T_{b2} + T_{k2} + T_{k1} = 0$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1) = 0 \qquad .....(1.12.2)$$

On replacing the angular displacements by angular velocity in the differential ations (1.12.1) and (1.12.2) governing the mechanical rotational system we get,

$$J_1 \frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1 \int (\omega_1 - \omega_2)\, dt = T$$

$$J_2 \frac{d\omega_2}{dt} + B_2(\omega_2 - \omega_3) + B_1(\omega_2 - \omega_1) + K_1 \int (\omega_2 - \omega_1)\, dt = 0$$

$$J_3 \frac{d\omega_3}{dt} + B_2(\omega_3 - \omega_2) + K_3 \int \omega_3 dt = 0$$

VOLTAGE ANALOGOUS CIRCUIT

ie electrical analogous elements for the elements of mechanical rotational system n below.

$\rightarrow e(t)$	$\omega_1 \rightarrow i_1$	$J_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$
	$\omega_2 \rightarrow i_2$	$J_2 \rightarrow L_2$	$B_2 \rightarrow R_2$	$K_3 \rightarrow 1/C_3$
	$\omega_3 \rightarrow i_3$	$J_3 \rightarrow L_3$		

he mesh basis equations using Kirchoff's voltage law for the circuit shown in fig re given below *[Refer fig (1.13.6), (1.13.7) and (1.13.8)]*.

$$L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i_2)dt = e(t) \qquad .....(1.13.7)$$

$$L_2 \frac{di_2}{dt} + R_2(i_2 - i_3) + R_1(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)\, dt = 0 \qquad .....(1.13.8)$$

$$L_3\frac{di_3}{dt}+R_2(i_3-i_2)+\frac{1}{C_3}\int i_3dt=0$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=-jyP7J3iDMI

Important Books/Journals for further learning including the page nos.: P45

M. Gopal, 'Control Systems, Principles and Design', 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **I-** Systems and Their Representation **Date of Lecture:**

Topic of Lecture:
Block diagram reduction techniques(Tutorial - III)

Introduction :

- A control system may consist of a number of components.
- A block diagram of a system is a pictorial representation of the function performed by each component and of the flow of signals.
- Such a diagram depicts the inter-relationships which exists between the various components.

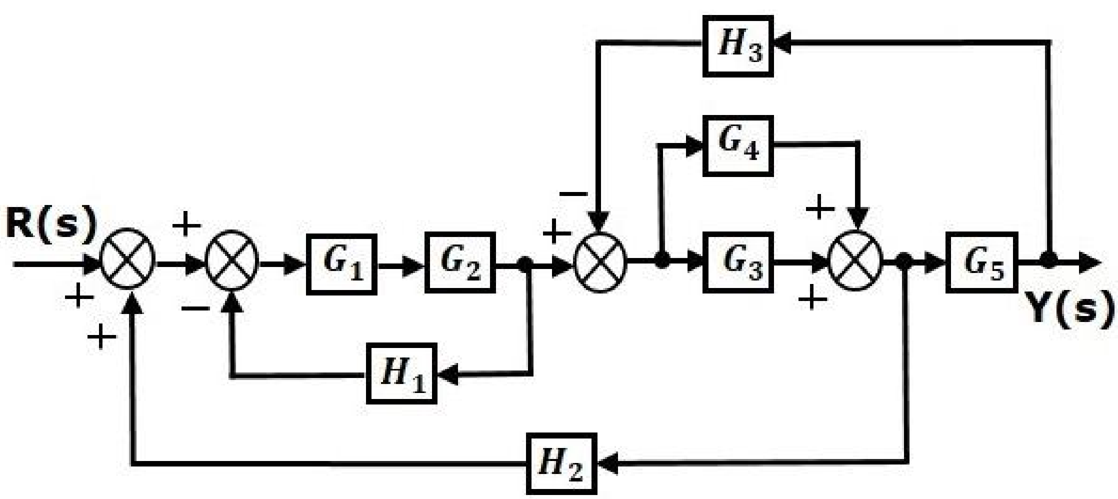
Prerequisite knowledge for Complete understanding and learning of Topic:

- Signal system
- Transfer function

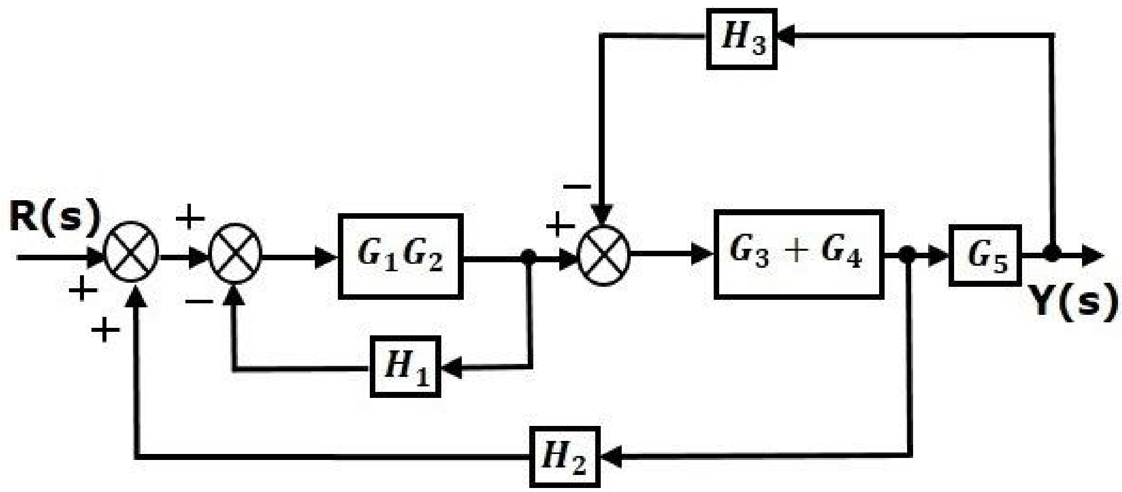
Detailed content of the Lecture:

Example:

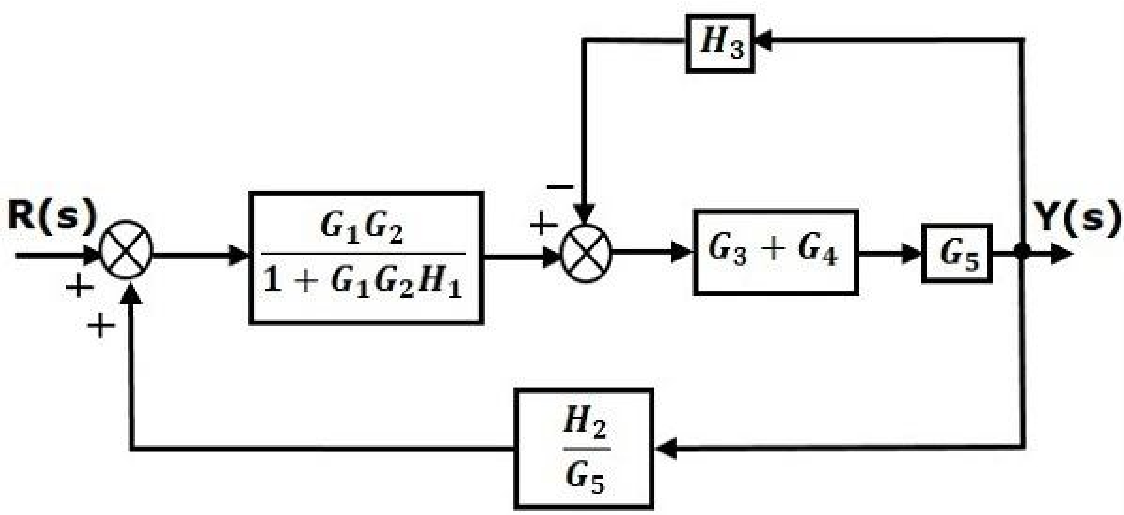
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



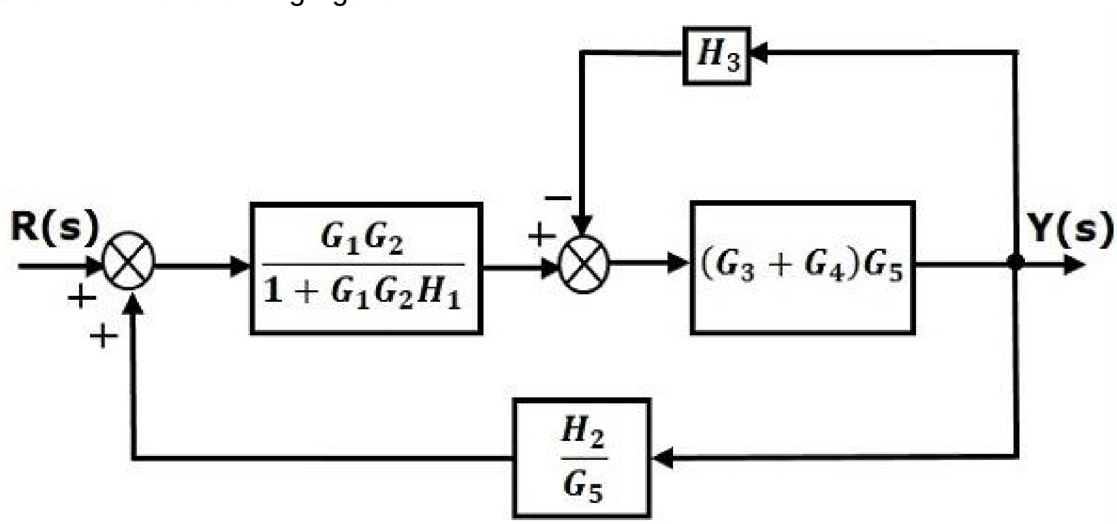
Step 1 − Use Rule 1 for blocks G1G1 and G2G2. Use Rule 2 for blocks G3G3 and G4G4. The modified block diagram is shown in the following figure.



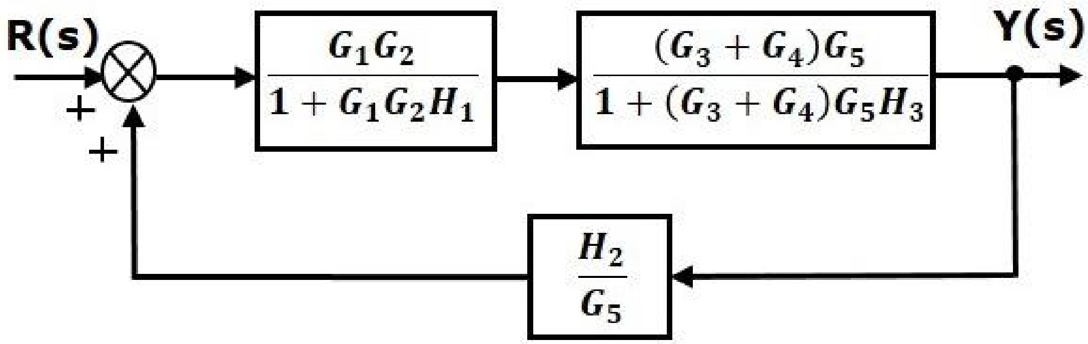
Step 2 − Use Rule 3 for blocks G1G2G1G2 and H1H1. Use Rule 4 for shifting take-off point after the block G5G5. The modified block diagram is shown in the following figure.



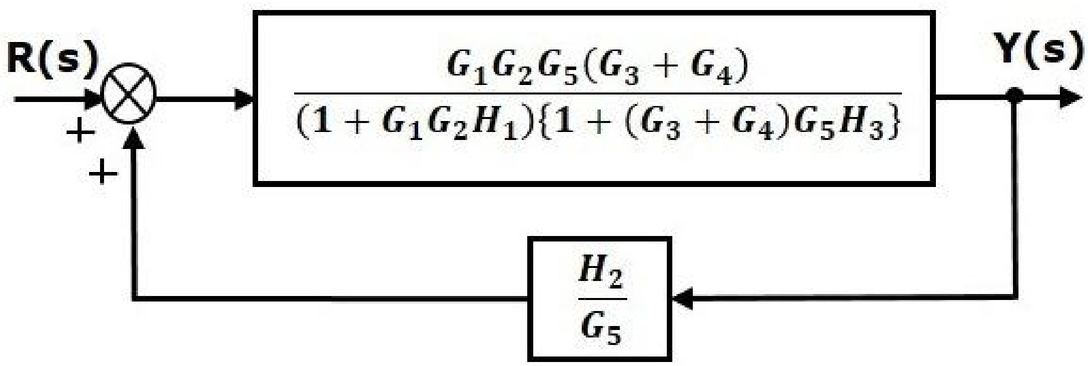
Step 3 − Use Rule 1 for blocks (G3+G4)(G3+G4) and G5G5. The modified block diagram is shown in the following figure.



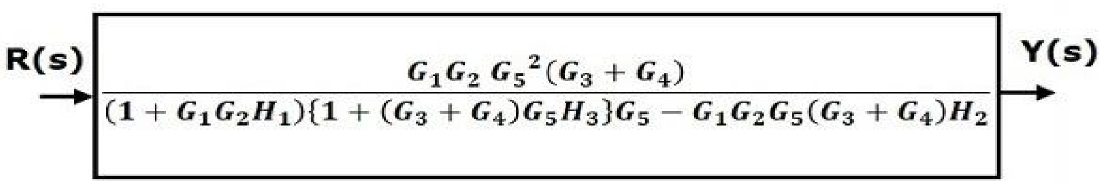
Step 4 − Use Rule 3 for blocks (G3+G4)G5(G3+G4)G5 and H3H3. The modified block diagram is shown in the following figure.



Step 5 − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



Therefore, the transfer function of the system is

$$Y(s)R(s)=G_1G_2G_{25}(G_3+G_4)(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=4KUTNL9dA8E

Important Books/Journals for further learning including the page nos.: P52

M. Gopal, ‘Control Systems, Principles and Design’, 4th Edition, Tata McGraw Hill, New Delhi, 2012

Course Faculty

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LECTURE HANDOUTS | **L10**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Time response, Time domain Specifications.

Introduction :

- If the output of control system for an input varies with respect to time, then it is called the time response of the control system. The time response consists of two parts.
  - Transient response
  - Steady state response

Prerequisite knowledge for Complete understanding and learning of Topic:

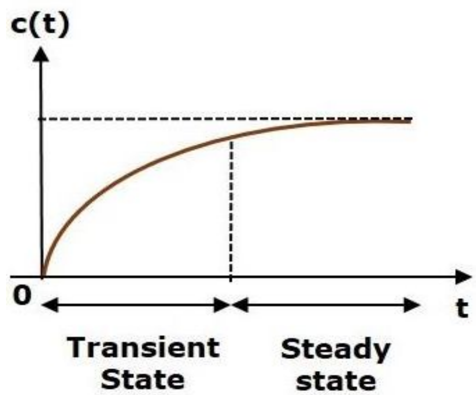
1.Time domain

2.Frequency domain

Detailed content of the Lecture:

Transient Response:

- After applying input to the control system, output takes certain time to reach steady state.
- So, the output will be in transient state till it goes to a steady state.
- Therefore, the response of the control system during the transient state is known as transient response.



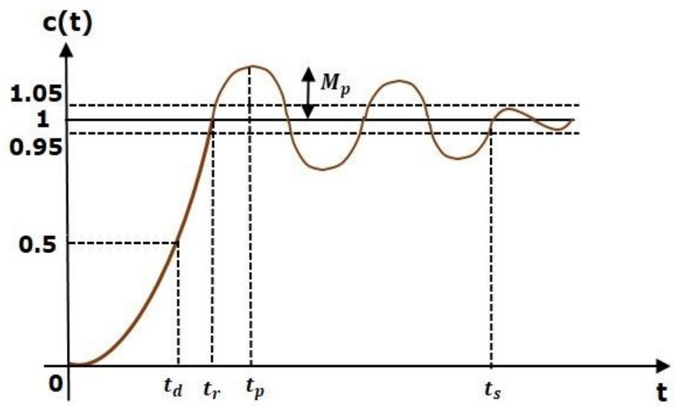
- The transient response will be zero for large values of ‘t’. Ideally, this value of ‘t’ is infinity and practically, it is five times constant.
- Mathematically, we can write it as

$$\lim_{t\to\infty} c_{tr}(t) = 0$$

Steady state Response:

- The part of the time response that remains even after the transient response has zero value for large values of 't' is known as steady state response.
- This means, the transient response will be zero even during the steady state.
- Let us find the transient and steady state terms of the time response of the control system c(t)=10+5e−t.
- Here, the second term 5e−t5e−t will be zero as t denotes infinity. So, this is the transient term.
- The first term 10 remains even as t approaches infinity. So, this is the steady state term.

The step response of the second order system for the underdamped case is shown in the following figure.



Delay Time:

- It is the time required for the response to reach half of its final value from the zero instant. It is denoted by t_d.

Rise Time:

- It is the time required for the response to rise from 0% to 100% of its final value. This is applicable for the under-damped systems.

Peak Time:

- It is the time required for the response to reach the peak value for the first time.

Peak Overshoot:

- Peak overshoot Mp is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum overshoot.

Settling Time:

- It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value.

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=uIdc_kaI5xI
Important Books/Journals for further learning including the page nos.:
➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:257)

Course Faculty

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LECTURE HANDOUTS | **L11**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Types of test input, I order& II nd system response

Introduction :

- The order of the system is defined by the number of independent energy storage elements in the system, and intuitively by the highest order of the linear differential equation that describes the system.
- First order of system is defined as first derivative with respect to time.

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Time domain

2.Frequency domain

Detailed content of the Lecture:

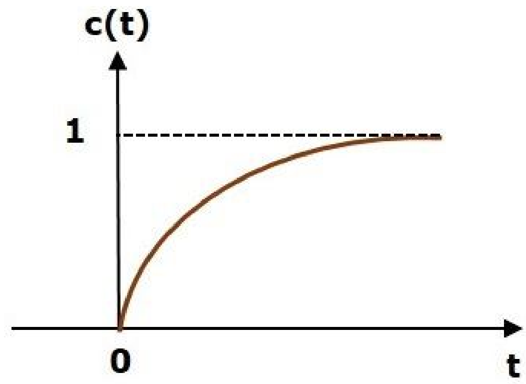
- A first order differential equation contains a first order derivative but no derivative higher than first order – the order of a differential equation is the order of the highestorder derivative present in the equation.
- First order control system tell us the speed of the response that what duration it reaches to the steady state.
- If the input is unit step, R(s) = 1/s so the output is step response C(s). The general equation of 1st order control system is $C(s) = R(s)G(s)$, i.e $C(s) = a/s(s+a)\ and\ G(s)$is transfer function.
- There are two poles, one is input pole at the origin s = 0 and other is system pole at s = -a, this pole is at negative axis of pole plot.

First order system step response:

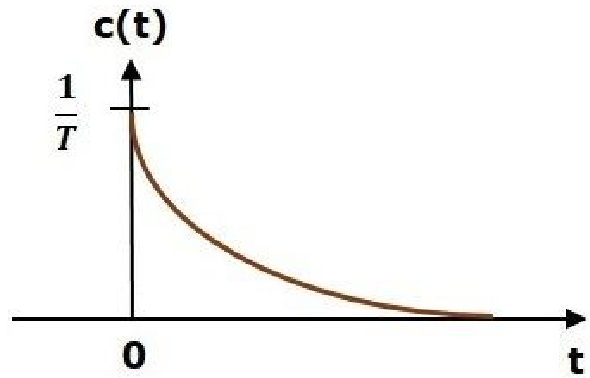
- The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the first order system.
- We can re-write the above equation as

$$C(s)=(1sT+1)R(s)$$

C(s) is the Laplace transform of the output signal c(t),
R(s) is the Laplace transform of the input signal r(t), and
T is the time constant



The unit impulse response is shown in the following figure.



The unit impulse response, c(t) is an exponential decaying signal for positive values of ‘t’ and it is zero for negative values of ‘t’.

Unit ramp response:

The unit ramp response**c**(t) has both the transient and the steady state terms.

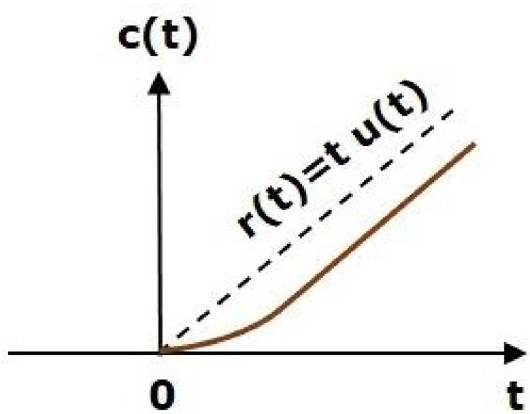
The transient term in the unit ramp response is -

$$c_{tr}(t)=Te^{-(\frac{t}{T})}u(t)ctr(t)=Te-(tT)u(t)$$

The steady state term in the unit ramp response is -

$$c_{ss}(t)=(t-T)u(t)css(t)=(t-T)u(t)$$

The following figure shows the unit ramp response.



- The unit ramp response, c(t) follows the unit ramp input signal for all positive values of t. But, there is a deviation of T units from the input signal.

Time Constant:

- It can be defined as the time it takes for the step response to rise up to 63% or 0.63 of it's final value.
- We call it as t = 1/a. If we take reciprocal of time constant, its unit is 1/seconds, or frequency.

Rise Time:

- Rise time is defined as the time for waveform to go from 0.1 to 0.9 or 10% to 90% of its final value.

Settling Time:

- It is defined as the time for the response to reach and stay within 2% of its final value. We can limit the percentage up to 5% of its final value.
- The equation of settling time is given by: Ts = 4/a.

Conclusion of First Order Control System:

- A pole of the input function generates the form of the forced response. It is because of pole at the origin which generates a step function at output.
- A pole of the transfer function generates the natural response. It the pole of the system.
- A pole on the real axis generates an exponential frequency of the form e-at. Thus, the farther the pole to the origin, the faster the exponential transient response will decay to zero.
- Using poles and zeros, we can speed up the performance of system and get the desired output.
- The order of a control system is determined by the power of 's' in the denominator of its transfer function.
- If the power of s in the denominator of the transfer function of a control system is 2, then the system is said to be second order control system.
- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as essess. We can find steady state error using the final value theorem as follows.

  $e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)$

  Where,E(s) is the Laplace transform of the error signal, e(t).

- When the reference input is applied to the given system then the information given about the level of desired output is observed.
- The actual output is feed back to the input side and it is compared with the input signal.

The error signal in s-domain ,E(s)can be expressed as a product of two s-domain functions

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$E(s)= F(s)R(s)$$

Where e(t)= error signal in time domain
f(t)=Inverse laplace transform of F(s)
r(t)=input signal in time domain

- The response that remain after the transient response has died out is called steady state response.

- The steady sate response is important to find the accuracy of the output.
- The difference between steady state response and desired response gives the steady state error.
- The control system has following steady state errors for change in positions, velocity and acceleration.

  Kp = Positional error constant

  Kv = Velocity error constant

  Ka = Acceleration error constant

- These constants are called static error coefficient. They have the ability to minimize the steady error.
- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as essess. We can find steady state error using the final value theorem as follows.

  $e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)$

  Where,E(s) is the Laplace transform of the error signal, e(t).

- When the reference input is applied to the given system then the information given about the level of desired output is observed.
- The actual output is feed back to the input side and it is compared with the input signal.

The error signal in s-domain ,E(s)can be expressed as a product of two s-domain functions

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$E(s)= F(s)R(s)$$

Where e(t)= error signal in time domain

f(t)=Inverse laplace transform of F(s)

r(t)=input signal in time domain

- The response that remain after the transient response has died out is called steady state response.

- The steady sate response is important to find the accuracy of the output.
- The difference between steady state response and desired response gives the steady state error.
- The control system has following steady state errors for change in positions, velocity and acceleration.

  Kp = Positional error constant

  Kv = Velocity error constant

  Ka = Acceleration error constant

- These constants are called static error coefficient. They have the ability to minimize the steady error.

Video Content / Details of website for further learning (if any): https://www.youtube.com/watch?v=T8ATyg_Fo6Y https://www.youtube.com/watch?v=AnB6VR-g6PI
Important Books/Journals for further learning including the page nos.: ➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:33)

Course Faculty

Verified by HOD

LECTURE HANDOUTS | **L12**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Error coefficients, Generalized error series

Introduction :

- Steady-state error is defined as the difference between the input (command) and the output of a system in the limit as time goes to infinity (i.e. when the response has reached steady state).

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Steady state

2.Transient

Detailed content of the Lecture:

- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as essess. We can find steady state error using the final value theorem as follows.

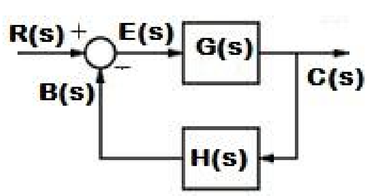
  ess=limt→∞e(t)=lims→0sE(s)

  Where,E(s) is the Laplace transform of the error signal, e(t).

- When the reference input is applied to the given system then the information given about the level of desired output is observed.
- The actual output is feed back to the input side and it is compared with the input signal.
- Thus steady state error can also be defined as the difference between the reference input and the feedback signal.
- These constants are called static error coefficient. They have the ability to minimize the steady error.

Derivation of Steady state error:

- Consider a simple closed loop system as shown in figure below:



- Different notation used

R(s)= Laplace transformation of input, r(t)
B(s)= Laplace transformation of feedback signal, b(t)
E(s)= Laplace transformation of error signal, e(t)
H(s)= Laplace transformation of feedback element,
C(s)= Laplace transformation of output signal, c(t)

- The steady state error is denoted by ess and it is given by,

Steady State Error= lim (t→∞) e(t)

Effect of Input on Steady state error:

Type of Input- R(s)
Type of system- G(s) H(s)

The Steady state error is calculated for three types of input : step, ramp, parabolic. The equation obtained of ess is valid for any input R(s), hence it will be used for these inputs.

- The order of a control system is determined by the power of ‘s’ in the denominator of its transfer function.
- If the power of s in the denominator of the transfer function of a control system is 2, then the system is said to be second order control system.
- The general expression of the transfer function of a second order control system is given as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Here, ζ and ωn are the damping ratio and natural frequency of the system, respectively (we will learn about these two terms in detail later on).
- Rearranging the formula above, the output of the system is given a

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Critical Damping Time Response of Control System:

- The time response expression of a second order control system subject to unit step input is

$$c(t) - 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left\{\left(\omega_n\sqrt{1-\zeta^2}\right)t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right\}$$

- The reciprocal of constant of negative power of exponential term in the error part of the output signal is actually responsible for damping of the output response. Here in this equation it is ζωn.
- The reciprocal of constant of negative power of exponential term in error signal is known as time constant.

Under damped response:

- when the value of ζ (also know as damping ratio) is less than unity, the oscillation of the response decays exponentially with a time constant 1/ζωn. This is called under damped response.

Over damped response:

- On the other hand. when ζ is greater than unity, the response of the unit step input given to the system, does not exhibit oscillating part in it. This is called **over damped response.**

Critical Damping:

- When damping ratio is unity that is ζ = 1. In that situation the damping of the response is governed by the natural frequency ωn only. The actual damping at that condition is known as **critical damping** of the response.

Damping ratio:

- The ratio of time constant of critical damping to that of actual damping is known as damping ratio. As the time constant of time response of control system is 1/ζωn when ζ≠ 1 and time constant is 1/ωn when ζ = 1.

$$\frac{Time\ Constant\ of\ critical\ damping}{Time\ Constant\ of\ actual\ damping} = \frac{\frac{1}{\omega_n}}{\frac{1}{\zeta\omega_n}} = \zeta$$

Second Order System Transfer Function:

- The general equation for the transfer function of a second order control system is given as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=_p6w7oztrwQ
https://www.youtube.com/watch?v=_p6w7oztrwQ

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:284)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Steady state error

Introduction :

- In feedback control system a controller may be introduced to modify the error signal and to achieve better control action.
- The introduction of controllers will modify the transient response and steady state error of the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Controller

2.Gain

Detailed content of the Lecture:

Proportional:

- The motor current is set in proportion to the existing error.
- However, this method fails if, for instance, the arm has to lift different weights: a greater weight needs a greater force applied for a same error on the down side, but a smaller force if the error is on the upside.

Integral:

- An integral term increases action in relation not only to the error but also the time for which it has persisted.
- So, if applied force is not enough to bring the error to zero, this force will be increased as time passes.
- A pure "I" controller could bring the error to zero, but it would be both slow reacting at the start (because action would be small at the beginning, needing time to get significant) and brutal (the action increases as long as the error is positive, even if the error has started to approach zero).

Derivative:

- A derivative term does not consider the error (meaning it cannot bring it to zero: a pure D controller cannot bring the system to its setpoint), but the rate of change of error, trying to bring this rate to zero.

Effects of proportional controller:

- The proportional controller produces an output signal which is proportional to error signal.
- The transfer function of proportional controller is K_p. The term K_p is called the gain of the cotroller.
- Hence the proportional controller amplifies the error signal and increases the loop gain of the system.
- The following aspects of system bahaviour are improved by increasing loop gain.

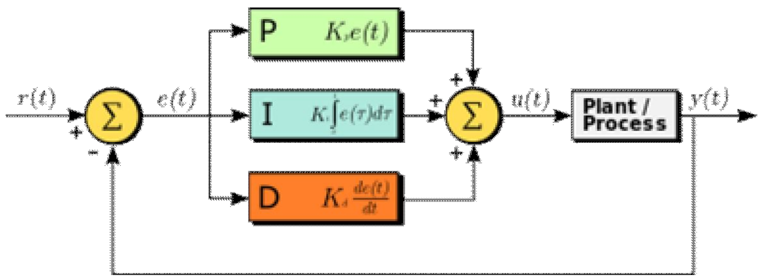
  i)Steady state tracking accuracy

  ii)Disturbance signal rejection

  iii)Relative Stability

Proportional–integral–derivative controller:

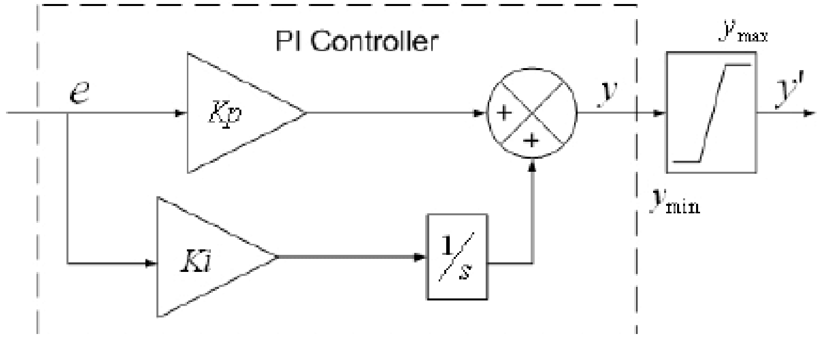
- A proportional–integral–derivative controller (PID controller. or three-term controller) is a control loop mechanism employing feedback that is widely used in industrial control systems and a variety of other applications requiring continuously modulated control.



- The distinguishing feature of the PID controller is the ability to use the three control terms of proportional, integral and derivative influence on the controller output to apply accurate and optimal control.
- The block diagram on the right shows the principles of how these terms are generated and applied.
- Term P is proportional to the current value of the SP − PV error e(t).
- Term I accounts for past values of the SP − PV error and integrates them over time to produce the I term.
- Term D is a best estimate of the future trend of the SP − PV error, based on its current rate of change.

P.I Controller:

- A P.I Controller is a feedback control loop that calculates an error signal by taking the difference between the output of a system, which in this case is the power being drawn from the battery, and the set point.



- The integral mode of the controller is the last term of the equation. Its function is to integrate or continually sum the controller error, e(t), over time.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=lVQxEwxVqgk

https://www.youtube.com/watch?v=Z0BcL8UVNBI

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:310)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Root Locus Construction

Introduction :

- Time-domain and frequency-domain analysis commands let you compute and visualize SISO and MIMO system responses such as Bode plots, Nichols plots, step responses, and impulse responses.
- You can also extract system characteristics such as rise time and settling time, overshoot, and stability margins. Most linear analysis commands can either return response data or generate response plots.

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Time domain

2.Frequency domain

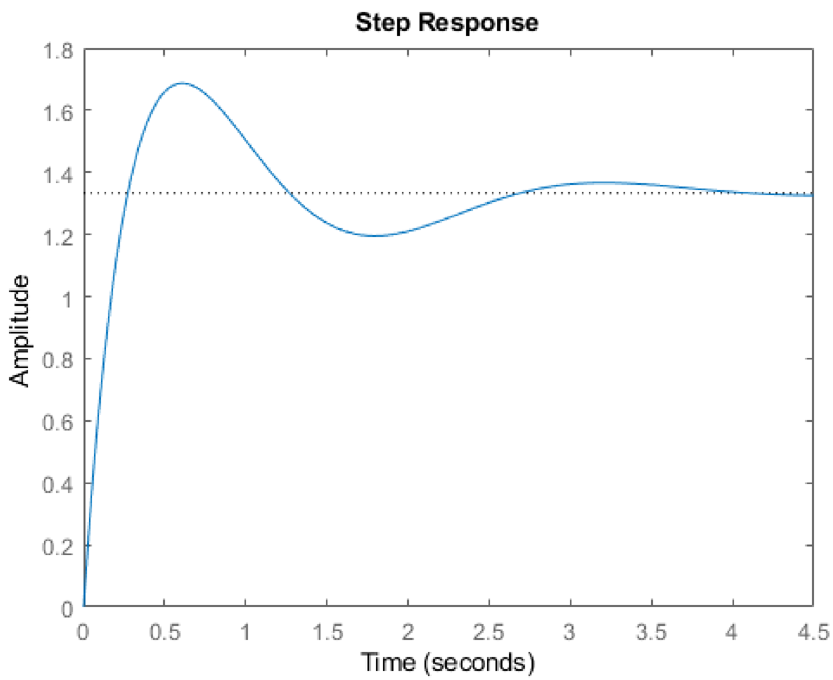
Detailed content of the Lecture:

Time domain analysis:

- Stability is a standard requirement for control systems to avoid loss of control and damage to equipment.
- For linear feedback systems, stability can be assessed by looking at the poles of the closed-loop transfer function.
- Gain and phase margins measure how much gain or phase variation at the gain crossover frequency will cause a loss of stability.
- Together, these two quantities give an estimate of the *safety margin* for closed-loop stability. The smaller the stability margins, the more fragile stability is.

step	Step response plot of dynamic system; step response data
stepinfo	Rise time, settling time, and other step-response characteristics
impulse	Impulse response plot of dynamic system; impulse response data
initial	Initial condition response of state-space model
lsim	Simulate time response of dynamic system to arbitrary inputs

lsiminfo	Compute linear response characteristics
gensig	Generate test input signals for lsim
covar	Output and state covariance of system driven by white noise
stepDataOptions	Options set for step



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=z-yqMQTCIYI
https://www.youtube.com/watch?v=JmSWrw2hDHA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:260)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Effects of
P, PI, PID modes of feedback control &Time response analysis (Only simulation)

Introduction :

- The coefficient of error is a standard statistical value that is used extensively in the stereological literature.
- The definition of the CE is rather simple. It is defined as the standard error of the mean of repeated estimates divided by the mean.

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Steady state

2.Transient

Detailed content of the Lecture:

- The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as essess. We can find steady state error using the final value theorem as follows.

  $e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to0}sE(s)$

  Where,E(s) is the Laplace transform of the error signal, e(t).

- When the reference input is applied to the given system then the information given about the level of desired output is observed.
- The actual output is feed back to the input side and it is compared with the input signal.
- Thus steady state error can also be defined as the difference between the reference input and the feedback signal.

There are two types of error coefficient.

1) static error coefficient

2) Dynamic error coefficient

Dynamic error coefficient:

- Use to express dynamic error
- Provides error signal as function of time
- Used for determining any type of input
- The dynamic error coefficient provides a simple way of estimating error signal to arbitrary

inputs and the steady sate error without solving the system differential equation.

Static error coefficient:

- The response that remain after the transient response has died out is called steady state response.
- The steady sate response is important to find the accuracy of the output.
- The difference between steady state response and desired response gives the steady state error.
- The control system has following steady state errors for change in positions, velocity and acceleration.

Kp = Positional error constant
Kv = Velocity error constant
Ka = Acceleration error constant

- These constants are called static error coefficient. They have the ability to minimize the steady error.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=_p6w7oztrwQ

https://www.youtube.com/watch?v=_p6w7oztrwQ

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:165)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Time response(Tutorial – I).

Introduction :

- If the output of control system for an input varies with respect to time, then it is called the time response of the control system. The time response consists of two parts.
  - Transient response
  - Steady state response

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Time domain

2.Frequency domain

Detailed content of the Lecture:

Figures 5–58(a) shows a mechanical vibratory system. When 2 lb of force (step input) is applied to the system, the mass oscillates, as shown in Figure 5–58(b). Determine m, b, and k of the system from this response curve. The displacement x is measured from the equilibrium position.

Solution. The transfer function of this system is

$$\frac{X(s)}{P(s)} = \frac{1}{ms^2 + bs + k}$$

Since

$$P(s) = \frac{2}{s}$$

we obtain

$$X(s) = \frac{2}{s(ms^2 + bs + k)}$$

It follows that the steady-state value of x is

$$x(\infty) = \lim_{s \to 0} sX(s) = \frac{2}{k} = 0.1 \text{ ft}$$

Hence

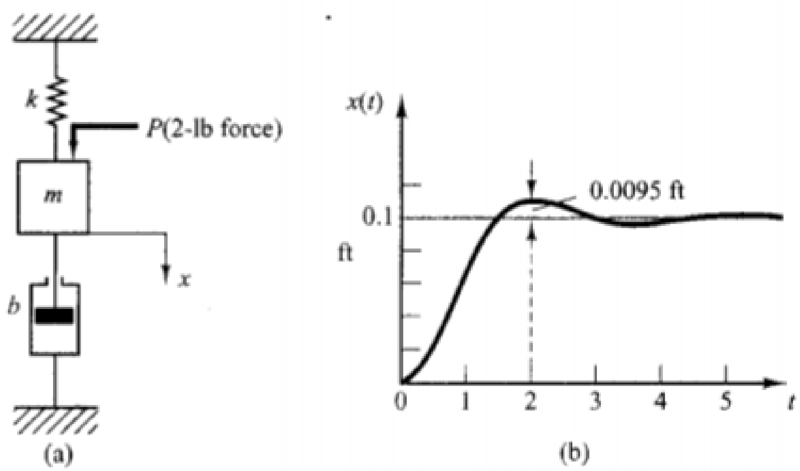
$$k = 20 \text{ lb}_f/\text{ft}$$

Note that $M_p = 9.5\%$ corresponds to $\zeta = 0.6$. The peak time t_p is given by

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.8\omega_n}$$

The experimental curve shows that $t_p = 2$ sec. Therefore,

$$\omega_n = \frac{3.14}{2 \times 0.8} = 1.96 \text{ rad/sec}$$



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=uIdc_kaI5xI

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:257)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Time domain Specifications(Tutorial - II)

Introduction :

The response up to the settling time is known as transient response and the response after the settling time is known as steady state response.

- The different time domains are
  - Delay time
  - Rise time
  - Peak time
  - Peak overshoot
  - Settling time

Prerequisite knowledge for Complete understanding and learning of Topic:

1.Transient Response

2.Steady state response

Detailed content of the Lecture:

Determine the values of K and k of the closed-loop system shown in Figure 5–57 so that the maximum overshoot in unit-step response is 25% and the peak time is 2 sec. Assume that $J = 1$ kg-m^2.

Solution. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Kks + K}$$

By substituting $J = 1$ kg-m^2 into this last equation, we have

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + Kks + K}$$

Note that in this problem

$$\omega_n = \sqrt{K}, \qquad 2\zeta\omega_n = Kk$$

The maximum overshoot M_p is

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

which is specified as 25%. Hence

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.25$$

from which

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.386$$

or

$$\zeta = 0.404$$

The peak time t_p is specified as 2 sec. And so

$$t_p = \frac{\pi}{\omega_d} = 2$$

or

$$\omega_d = 1.57$$

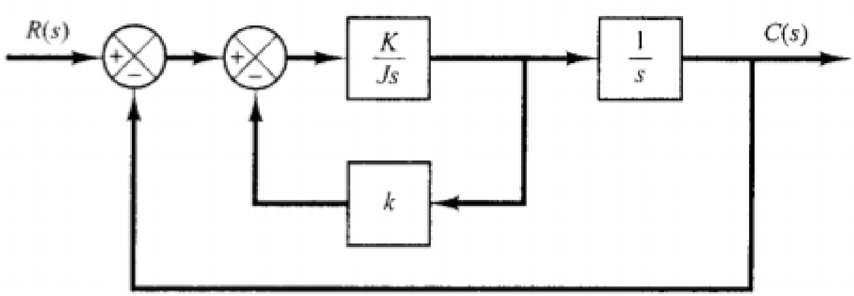
Then the undamped natural frequency ω_n is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{1.57}{\sqrt{1-0.404^2}} = 1.72$$

Therefore, we obtain

$$K = \omega_n^2 = 1.72^2 = 2.95 \text{ N-m}$$

$$k = \frac{2\zeta\omega_n}{K} = \frac{2 \times 0.404 \times 1.72}{2.95} = 0.471 \text{ sec}$$



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=MzrgBc4s-jk

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:258)

Course Faculty

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MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

DESIGNING YOUR FUTURE
Estd. 2000

LECTURE HANDOUTS | **L18**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **II-** TIME RESPONSE ANALYSIS **Date of Lecture:**

Topic of Lecture: Type of test unit-first order system(Tutorial – III)

Introduction :

- The order of the system is defined by the number of independent energy storage elements in the system, and intuitively by the highest order of the linear differential equation that describes the system.
- First order of system is defined as first derivative with respect to time.

Prerequisite knowledge for Complete understanding and learning of Topic:

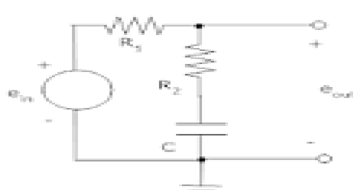
1.Time domain

2.Frequency domain

Detailed content of the Lecture:

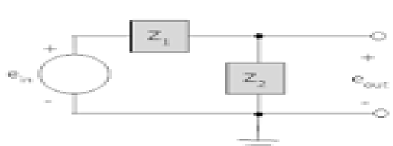
Example: Step response of first order system (3)

If the input voltage, $e_{in}(t)$, of the following system is a unit step, find $e_{out}(t)$.



Solution:

First we find the transfer function. We note that the circuit is a voltage divider with two impedances



$$\frac{E_{out}(s)}{E_{in}(s)} = H(s) = \frac{Z_2}{Z_1 + Z_2}$$

where Z_1 is R_1 and Z_2 is R_2 in series with C.

$$Z_1 = R_1$$

$$Z_2 = Z_{R2} + Z_C = R_2 + \frac{1}{sC}$$

$$H(s) = \frac{E_{out}(s)}{E_{in}(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

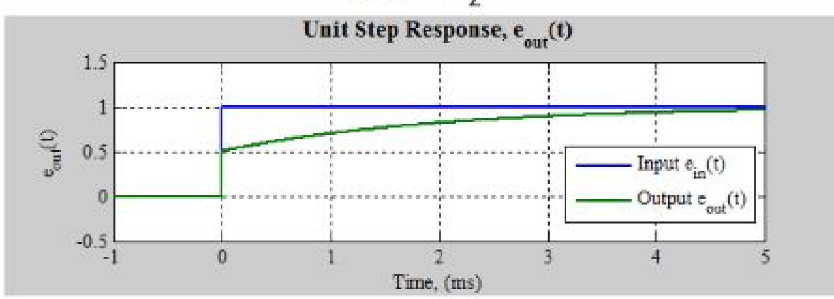
$$= \frac{sCR_2 + 1}{sC(R_1 + R_2) + 1} = \frac{1}{C(R_1 + R_2)} \frac{sCR_2 + 1}{s + \frac{1}{C(R_1 + R_2)}} = \frac{CR_2}{C(R_1 + R_2)} \frac{s + \frac{1}{CR_2}}{s + \frac{1}{C(R_1 + R_2)}}$$

$$= \frac{R_2}{(R_1 + R_2)} \frac{s + \frac{1}{CR_2}}{s + \frac{1}{C(R_1 + R_2)}}$$

To find the unit step response, multiply the transfer function by the unit step (1/s) and the inverse Laplace transform using Partial Fraction Expansion..

With $R_1 = R_2 = 1$kOhm and $C = 1\mu F$, we get

$$e_{out}(t) = 1 - \frac{1}{2} e^{-t/0.002}$$



Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=T8ATyg_Fo6Y

https://www.youtube.com/watch?v=AnB6VR-g6PI

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:33)

Course Faculty

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS | **L19**

EEE | **II/1V**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Frequency Response

Introduction : (Maximum 5 sentences)

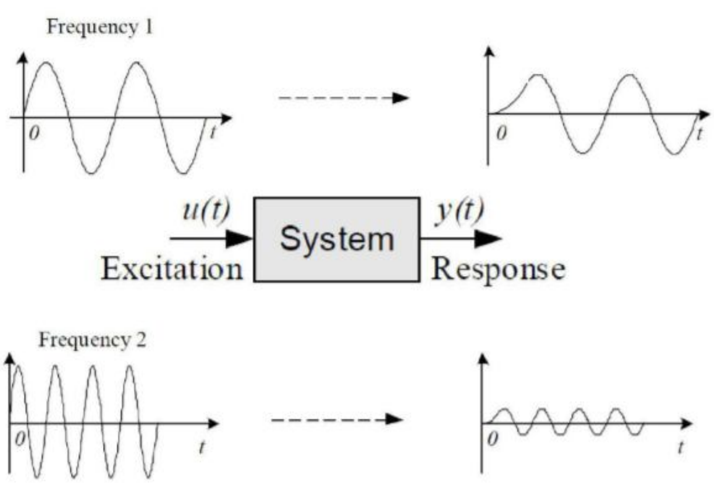
- The frequency response of a system is a frequency dependent function which expresses how a sinusoidal signal of a given frequency on the system input is transferred through the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals and Systems
- Amplifier

Detailed content of the Lecture:

- Time-varying signals at least periodical signals —which excite systems, as the reference (set point) signal or a disturbance in a control system or measurement signals which are inputs signals to signal filters, can be regarded as consisting of a sum of frequency components.



- Each frequency component is a sinusoidal signal having certain amplitude and a certain frequency. (The Fourier series expansion or the Fourier transform can be used to express these frequency components quantitatively.)

- The frequency response expresses how each of these frequency components is transferred through the system.

- Some components may be amplified, others may be attenuated, and there will be some phase lag through the system.
- The frequency response is an important tool for analysis and design of signal filters (as low pass filters and high pass filters), and for analysis, and to some extent, design, of control systems.
-

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=BsF0EkfEAxc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No:470-472)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:Bode plot

Introduction :

- Plots of the magnitude and phase characteristics are used to fully describe the frequency response
- A Bode plot is a (semilog) plot of the transfer function magnitude and phase angle as a function of frequency. The gain magnitude is many times expressed in terms of decibels (dB)

$$db = 20 \log 10 A$$

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals and Systems

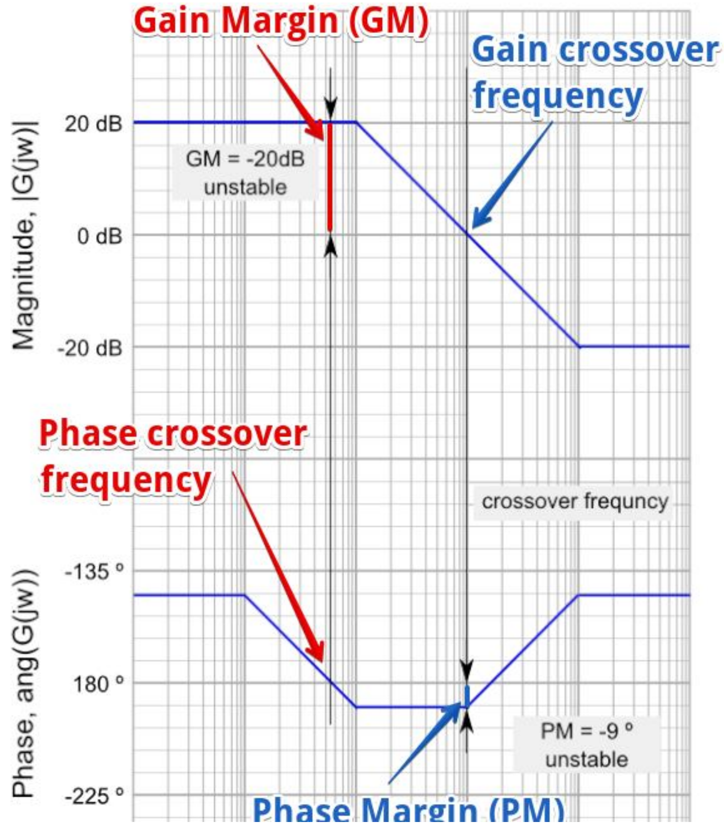
Detailed content of the Lecture:

- A **Bode plot** is a graph commonly used in control system engineering to determine the stability of a control system.
- A Bode plot maps the frequency response of the system through two graphs – the Bode magnitude plot (expressing the magnitude in decibels) and the Bode phase plot (expressing the phase shift in degrees)
- Bode plots offer a relatively simple method to calculate system stability, they can not handle transfer functions with right half plane singularities (unlike Nyquist stability criterion).

Gain Margin

- The greater the Gain Margin (GM), the greater the stability of the system. The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.

- We can usually read the gain margin directly from the Bode plot (as shown in the diagram below). This is done by calculating the vertical distance between the magnitude curve (on the Bode magnitude plot) and the x-axis at the frequency where the Bode phase plot = 180°. This point is known as the phase crossover frequency.



Phase Margin

- The greater the Phase Margin (PM), the greater will be the stability of the system. The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.
- We can usually read the phase margin directly from the Bode plot (as shown in the diagram above).
- This is done by calculating the vertical distance between the phase curve (on the Bode phase plot) and the x-axis at the frequency where the Bode magnitude plot = 0 dB. This point is known as the gain crossover frequency.
-

Bode Plot Stability

- Below are a summarised list of criterion relevant to drawing Bode plots (and calculating their stability):
- **Gain Margin:** Greater will the gain margin greater will be the stability of the system. It refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed in dB.
- **Phase Margin**: Greater will the **phase margin** greater will be the stability of the system. It refers to the phase which can be increased or decreased without making the system unstable. It is usually expressed in phase.
- **Gain Crossover Frequency**: It refers to the frequency at which magnitude curve cuts the zero dB axis in the bode plot.
- **Phase Crossover Frequency**: It refers to the frequency at which phase curve cuts the negative times the 180o axis in this plot.

- **Corner Frequency:** The frequency at which the two asymptotes cuts or meet each other is known as break frequency or corner frequency.
- **Resonant Frequency:** The value of frequency at which the modulus of G (jω) has a peak value is known as the resonant frequency.
- **Factors:** Every loop transfer function {i.e. G(s) × H(s)} product of various factors like constant term K, Integral factors (jω), first-order factors (1 + jωT)(± n) where n is an integer, second order or quadratic factors.
- **Slope:** There is a slope corresponding to each factor and slope for each factor is expressed in the dB per decade.
- **Angle:** There is an angle corresponding to each factor and angle for each factor is expressed in the degrees.

How to Draw Bode Plot

- Keeping all the above points in mind, we are able to draw a Bode plot for any kind of control system. Now let us discuss the procedure of drawing a Bode plot:
- Substitute the s = jω in the open loop transfer function G(s) × H(s).
- Find the corresponding corner frequencies and tabulate them.
- Now we are required one semi-log graph chooses a frequency range such that the plot should start with the frequency which is lower than the lowest corner frequency.
- Mark angular frequencies on the x-axis, mark slopes on the left hand side of the y-axis by marking a zero slope in the middle and on the right hand side mark phase angle by taking -180o in the middle.
- Calculate the gain factor and the type or order of the system.
- Now calculate slope corresponding to each factor.

For drawing the Bode magnitude plot:

- Mark the corner frequency on the semi-log graph paper.
- Tabulate these factors moving from top to bottom in the given sequence.
  1. Constant term K.
  2. Integral factor $\frac{1}{j\omega^n}$
  3. First order factor $\frac{1}{1+j\omega T}$
  4. First order factor (1+jωT).
  5. Second order or quadratic factor: $\left[\frac{1}{1+(2\zeta/\omega)} \times (j\omega) + \left(\frac{1}{\omega^2}\right) \times (j\omega)^2\right]$
- Now sketch the line with the help of the corresponding slope of the given factor. Change the slope at every corner frequency by adding the slope of the next factor. You will get the magnitude plot.
- Calculate the gain margin.

For drawing the Bode phase plot:

- Calculate the phase function adding all the phases of factors.
- Substitute various values to the above function in order to find out the phase at different points and plot a curve. You will get a phase curve.
- Calculate the phase margin.

Bode Stability Criterion

- Stability conditions are given below:
- For Stable System: Both the margins should be positive or phase margin should be greater than the gain margin.
- For Marginal Stable System: Both the margins should be zero or phase margin should be equal to the gain margin.
- For Unstable System: If any of them is negative or **phase margin** should be less than the gain margin.

Advantages of Bode Plot

- It is based on the asymptotic approximation, which provides a simple method to plot the logarithmic magnitude curve.
- The multiplication of various magnitude appears in the transfer function can be treated as an addition, while division can be treated as subtraction as we are using a logarithmic scale.
- With the help of this plot only we can directly comment on the stability of the system without doing any calculations.
- **Bode plots** provides relative stability in terms of **gain margin** and **phase margin**.
- It also covers from low frequency to high frequency range.

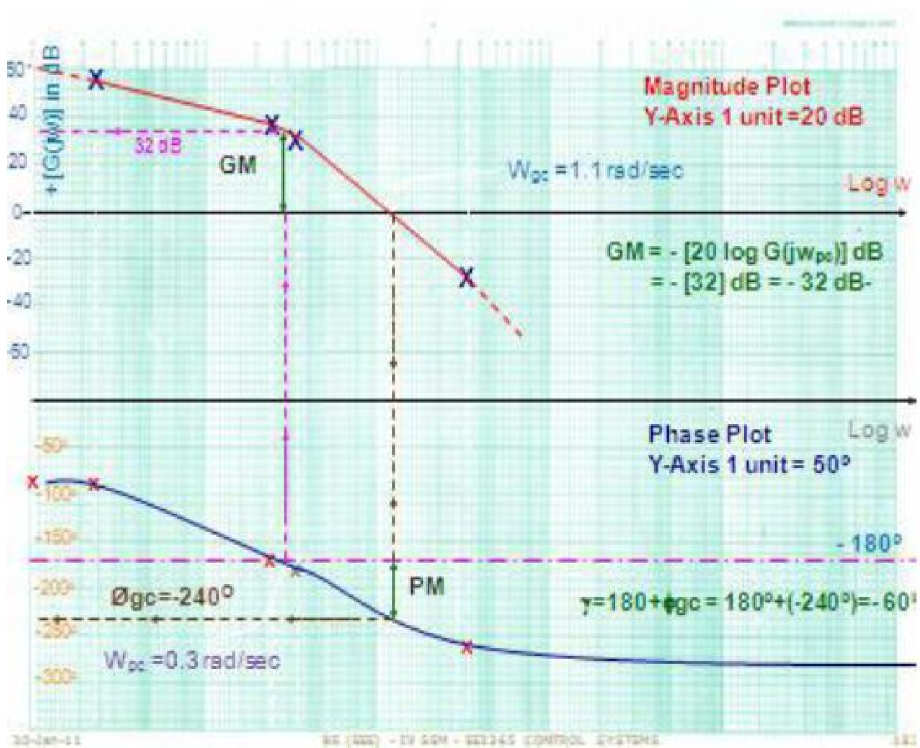
Example For the following T.F draw the Bode plot and obtain Gain cross over frequency (wgc) , Phase cross over frequency , Gain Margin and Phase Margin.

G(s) = 20 / [s (1+3s) (1+4s)]
Solution: The sinusoidal T.F of G(s) is obtained by replacing s by jw in the given T.F
G(jw) = 20 / [jw (1+j3w) (1+j4w)]
Corner frequencies: wc1= 1/4 = 0.25 rad /sec ;
wc2 = 1/3 = 0.33 rad /sec
Choose a lower corner frequency and a higher Corner frequency
wl= 0.025 rad/sec ;
wh = 3.3 rad / sec
Calculation of Gain (A) (MAGNITUDE PLOT)
A @ wl ; A= 20 log [20 / 0.025] = 58 .06 dB
A @ wc1 ; A = [Slope from wl to wc1 x log (wc1 / wl] + Gain (A)@wl = - 20 log [0.25 / 0.025] + 58.06 = 38.06 dB
A @ wc2 ; A = [Slope from wc1 to wc2 x log (wc2 / wc1] + Gain (A)@ wc1 = - 40 log [0.33 / 0.25] + 38 = 33 dB
A @ wh ; A = [Slope from wc2 to wh x log (wh / wc2] + Gain (A) @ wc2 = - 60 log [3.3 / 0.33] + 33 =-27 dB

Calculation of Phase angle for different values of frequencies [PHASE PLOT]

Ø = -90O- tan -1 3w – tan -1 4w
When

Frequency in rad / sec	Phase angles in Degree
w=0	Ø= -90 0
w = 0.025	Ø= -990
w = 0.25	Ø= -1720
w = 0.33	Ø= -1880
w =3.3	Ø= -2590
w =∞	Ø= -2700



Calculations of Gain cross over frequency

The frequency at which the dB magnitude is Zero wgc = 1.1 rad / sec

Calculations of Phase cross over frequency

The frequency at which the Phase of the system is - 180o wpc = 0.3 rad / sec

Gain Margin

The gain margin in dB is given by the negative of dB magnitude of G(jw) at phase cross over frequency GM = - { 20 log [G(jwpc)] = - { 32 } = -32 dB

Phase Margin

Γ = 1800+ Øgc= 1800 + (- 2400o) = -600

Conclusion

For this system GM and PM are negative in values. Therefore the system is unstable in nature.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=_eh1conN6YM

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No:478-494)

Course Faculty

Verified by HOD

LECTURE HANDOUTS | **L21**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:Polar plot

Introduction :

- Polar plot is a plot which can be drawn between magnitude and phase. Here, the magnitudes are represented by normal values only.
- **The Polar plot** is a plot, which can be drawn between the magnitude and the phase angle of G(jω)H(jω) by varying ω from zero to ∞.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals and Systems

Detailed content of the Lecture:

PROCEDURE

- Express the given expression of OLTF in (1+sT) form.
- Substitute s = jω in the expression for G(s)H(s) and get G(jω)H(jω).
- Get the expressions for | G(jω)H(jω)| & angle G(jω)H(jω).
- Tabulate various values of magnitude and phase angles for different values of ω ranging from 0 to ∞.
- Usually the choice of frequencies will be the corner frequency and around corner frequencies
- Choose proper scale for the magnitude circles.
- Fix all the points in the polar graph sheet and join the points by a smooth curve.
- Write the frequency corresponding to each of the point of the plot.

MINIMUM PHASE SYSTEMS:

- Systems with all poles & zeros in the Left half of the s-plane – Minimum Phase Systems.
- For Minimum Phase Systems with only poles
- Type No. determines at what quadrant the polar plot starts.
- Order determines at what quadrant the polar plot ends.
- Type No. → No. of poles lying at the origin
- Order → Max power of_s' in the denominator polynomial of the transfer function.

GAIN MARGIN

- Gain Margin is defined as ―the factor by which the system gain can be increased to drive the system to the verge of instability‖.

For stable systems, ωgc< ωpc Magnitude of G(j)H(j) at ω=ωpc < 1

GM = in positive dB

More positive the GM, more stable is the system.

For marginally stable systems,

ωgc = ωpc

magnitude of G(j)H(j) at ω=ωpc = 1

GM = 0 dB

For Unstable systems, ωgc> ωpc , magnitude of G(j)H(j) at ω=ωpc > 1

GM = in negative dB

Gain is to be reduced to make the system stable

PHASE MARGIN

- Phase Margin is defined as ― the additional phase lag that can be introduced before the system becomes unstable.
- A' be the point of intersection of G(j)H(j) plot and a unit circle centered at the origin.
- Draw a line connecting the points ‗O' & ‗A' and measure the phase angle between the line OA and +ve real axis.
- This angle is the phase angle of the system at the gain cross over frequency.

  Angle of G(jwgc)H(jw gc) =θ gc

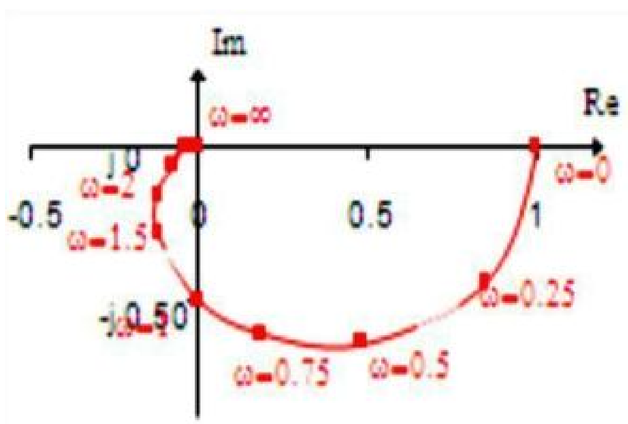
If an additional phase lag of θ PM is introduced at this frequency, then the phase angle G(jwgc)H(jw gc) will become 180 and the point ‗A‗ coincides with (-1+j0) driving the system to the verge of instability.

This additional phase lag is known as the Phase Margin. γ= 1800+ angle of G(jwgc)H(jw gc)

γ= 1800 + θ gc

[Since θ gc is measured in CW direction, it is taken as negative] For a stable system, the phase margin is positive.

A Phase margin close to zero corresponds to highly oscillatory system



A polar plot may be constructed from experimental data or from a system transfer function

- If the values of w are marked along the contour, a polar plot has the same information as a bode plot.
- Usually, the shape of a polar plot is of most interest.

Advantage

- It can capture the system behavior over the entire frequency range in a single plot

Disadvantage

- It can't tell the impact of individual component of open loop transfer function.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=OKo3GSeB3hU

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:496)

Course Faculty

Verified by HOD

LECTURE HANDOUTS **L22**

EEE **III/V**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Determination of closed loop response from open loopresponse

Introduction :
The correlation between time and frequency response has an explicit from only for first and second order systems.

Prerequisite knowledge for Complete understanding and learning of Topic:
- Signals and Systems

Detailed content of the Lecture:

time-domain specifications

(i) Delay time, td
(ii) Rise time, tr
(iii) Peak time, tp
(iv) Peak overshoot, Mp
(v) Settling time For unity step input,

(i)Delay time, td: It is the time required to reach 50% of output.
(ii)Rise time, tr : The time taken for response to raise from 0% to 100% for the very first time
(iii)Peak time, tp:The time required by the system response to reach the first maximum value.
(iv)Peak overshoot, Mp: It is the time required to reach 50% of output.
(iv)Settling time, ts: It is the time taken by the system response to settle down and stay with in $\pm$ 2% or $\pm$5% its final value.

frequency domain specifications

i. Resonant peak (Mr)
ii. Resonant frequency (ω_r)
iii. Cut-off frequency(ω_c)
iv. Band-width(ω_b)
v. Phase cross-over frequency
vi. Gain margin (GM)
vii. Gain cross-over frequency
viii. Phase margin (PM)

(i)Resonant peak: Maximum value of M(jω) when$\omega$is varied from 0 to Mr
(ii)Resonant peak : The frequency at which Mr occurs
(iii)Cut-off frequency: The frequency at which M(jω) has a value of $1/\sqrt{2}$. It is the frequency at which the magnitude is 3dB below its zero frequency value

(iv) Band-width: It is the range of frequencies in which the magnitude of a closed-loop system is $1/\sqrt{2}$times of Mr
(v) Phase cross-over frequency: The frequency at which phase plot crosses -180^0
(vi)Gain margin : It is the increase in open-loop gain in dB required to drive the closed-loop system to the verge of instability
(vii) Gain cross-over frequency: The frequency at which gain or magnitude plot crosses 0dB line
(viii) Phase margin : It is the increase in open-loop phase shift in degree required to drive the closed-loop system to the verge of instability
Correlation between time and frequency response
For a second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Putting $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega}$$

$$\Rightarrow \frac{C(j\omega)}{R(j\omega)} = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j2\zeta\left(\frac{\omega}{\omega_n}\right)}$$

Let, $u = \frac{\omega}{\omega_n}$, then

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{\left(1 - u^2\right) + j2\zeta u}$$

Now,

$$M(j\omega) = |M(j\omega)| \angle M(j\omega)$$

Where,

$$|M(j\omega)| = \frac{1}{\sqrt{\left(1 - u^2\right)^2 + (2\zeta u)^2}}$$

$$\theta = -\tan^{-1}\left(\frac{2\zeta u}{1 - u^2}\right)$$

Now,

$$M_r = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$$

$$\omega_b = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$PM = -180^0 + \varphi$$

Where, $\varphi = \tan^{-1} \dfrac{2\zeta}{\sqrt{\sqrt{4\zeta^2 + 1} - 2\zeta^2}}$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=s6PsiMA4qm4

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:578-585)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Correlation between frequency domain and time domain specifications

Introduction :
The compensator may be electrical , mechanical, hydraulic, pneumatic or other type of device or network. Usually an electric network serves as compensator in many control systems
There are three types of compensators

- Lag compensators.
- Lead compensators
- Lag-lead compensators.

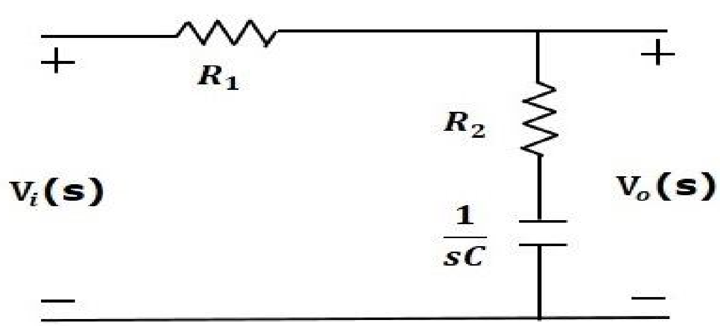
Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals and Systems

Detailed content of the Lecture:

Lag Compensator

- The Lag Compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied. The lag compensator circuit in the 's' domain is shown in the following figure.



Here, the capacitor is in series with the resistor R2 and the output is measured across this combination.

The transfer function of this lag compensator is -

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}\right)$$

$$\tau = R_2 C$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

From the above equation, α is always greater than one.

- From the transfer function, we can conclude that the lag compensator has one pole at $s=-1/\alpha\tau$ and one zero at $s=-1/\tau$. This means, the pole will be nearer to origin in the pole-zero configuration of the lag compensator.Substitute, $s=j\omega$ in the transfer function.

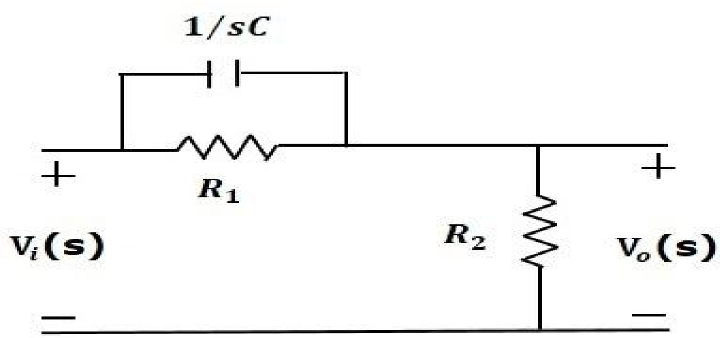
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{\alpha}\left(\frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}}\right)$$

Phase angle $\phi = \tan^{-1}\omega\tau - tan^{-1}\alpha\omega\tau$

- We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function.
- So, in order to produce the phase lag at the output of this compensator, the phase angle of the transfer function should be negative. This will happen when $\alpha>1$.

Lead Compensator

- The lead compensator is an electrical network which produces a sinusoidal output having phase lead when a sinusoidal input is applied. The lead compensator circuit in the 's' domain is shown in the following figure.



- Here, the capacitor is parallel to the resistor R1 and the output is measured across resistor R2.

The transfer function of this lead compensator is

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau+1}{\beta s\tau+1}\right)$$

Where

$$\tau = R_1 C$$

$$\beta = \frac{R_2}{R_1+R_2}$$

From the transfer function, we can conclude that the lead compensator has pole at s=−1/β=−1β and zero at s=−1βτ.Substitute, s=jω in the transfer function.

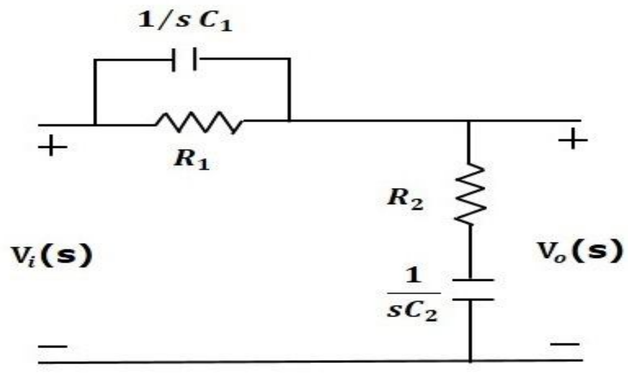
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \beta\left(\frac{j\omega\tau+1}{\beta j\omega\tau+1}\right)$$

Phase angle $\phi = tan^{-1}\omega\tau - tan^{-1}\beta\omega\tau$

- We know that, the phase of the output sinusoidal signal is equal to the sum of the phase angles of input sinusoidal signal and the transfer function.
- So, in order to produce the phase lead at the output of this compensator, the phase angle of the transfer function should be positive. This will happen when 0<β<1. Therefore, zero will be nearer to origin in pole-zero configuration of the lead compensator.

Lag-Lead Compensator

- Lag-Lead compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region. It is a combination of both the lag and the lead compensators. The lag-lead compensator circuit in the 's' domain is shown in the following figure.



- This circuit looks like both the compensators are cascaded. So, the transfer function of this circuit will be the product of transfer functions of the lead and the lag compensators.

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau_1+1}{\beta s\tau_1+1}\right)\frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau_2}}{s+\frac{1}{\alpha\tau_2}}\right)$$

We know $\alpha\beta = 1$.

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \left(\frac{s+\frac{1}{\tau_1}}{s+\frac{1}{\beta\tau_1}}\right)\left(\frac{s+\frac{1}{\tau_2}}{s+\frac{1}{\alpha\tau_2}}\right)$$

Where

$$\tau_1 = R_1 C_1$$

$$\tau_2 = R_2 C_2$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=d-iPshIZS_I

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:578-585)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Analysis Using MATLAB

Introduction :
The compensator may be electrical , mechanical, hydraulic, pneumatic or other type of device or network. Usually an electric network serves as compensator in many control systems
There are three types of compensators

- Lag compensators.
- Lead compensators
- Lag-lead compensators.

The frequency domain design can be carried out using Nyquist plot, Bode plot, or Nichols chart. But bode plot are popularly used for design because they are easier to draw and modify.

Prerequisite knowledge for Complete understanding and learning of Topic:
- Signals and Systems

Detailed content of the Lecture:

Lag Compensator

- Lead compensators: increase the stability and tune the steady-state error by increasing the phase at the crossover frequency
- Impact lag compensator = leadcompensator, but different approach!
- By decreasing the gain, the gain crossover frequency comes down to a frequency at which the corresponding phase is higher
- Large difference between lead and lag: their effect on the bandwidth of the system and hence on its speed of response
- A lead compensator increases the bandwidth/speed of response Good if you want the system to react fast. A lag compensator decreases the bandwidth/speed of response Good if your model is bad at high frequencies. Good to reduce the impact of (mostly high-frequency) noise

$$C(s) = K\frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}} \text{ with } \beta > 1$$

- Design process: we use
  - one degree of freedom to have a sufficient drop in gain
  - one degree of freedom to push the drop in the phase to lower frequencies (that way we can

use $\angle P(s)$ as an approximation of $\angle P(s)C(s)$ reliably to some extent

- one more degree of freedom to tune the steady state error
- Increase of phase margin ⇒ decrease of the magnitude at some higher frequencies (wrt DC)
- Decrease of the steady state error ⇒ increase of the magnitude at DC

⇒ A lag compensator can realize both conditions

At DC value, the gain becomes:

$$\lim_{s \to 0} K \frac{s+1/\tau}{s+1/\beta\tau} = K\beta$$

At high frequencies, the gain becomes:

$$\lim_{s \to \infty} K \frac{s+1/\tau}{s+1/\beta\tau} = K$$

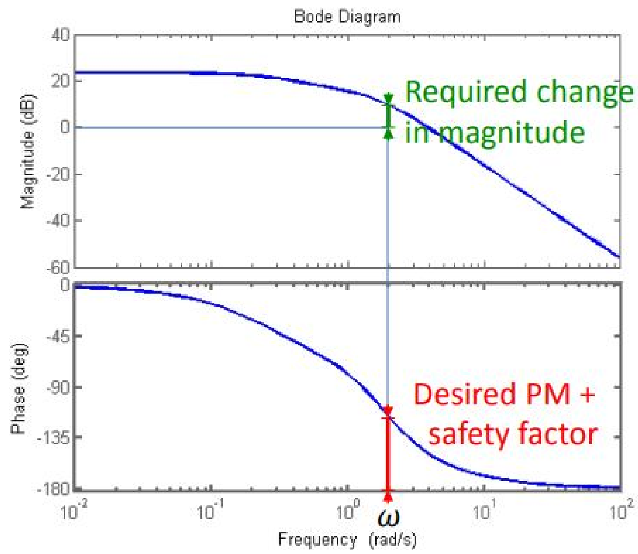
K has to be such that the drop in magnitude is sufficient, the value of β has make the steady state error decrease enough and the value of τ has to be such that the transfer between from $K\ \beta$ to K occurs at the right frequency.

- **<u>Determination of K</u>**

  - Easily read from Bode plot
  - Find the frequency (ω) with desired phase margin (+ safety factor), then find the magnitude at that frequency; which is equal to the required change in magnitude = Q
  - K is then simply equal to $1/Q$
  - Safety factor of about 10°:

    1. the drop in magnitude will not be complete (this is very marginal)

    2. the lag compensator influences the phase plot



- **<u>Determination of β</u>**

  - We can find k β in a similar way as we found $K\alpha$ for lead compensators
  - Translate steady state error requirement in a requirement on

Kp (= lim $s\to 0$ $P(s)C(s)$), Kv (= lim $s\to 0$ $(sP(s)C(s))$), Ka (= lim $s\to 0$ ($s^2P(s)C(s)$), or another error constant

  - With this $Kp/v/a/\ldots$ and lim$s\to 0$ $P(s)$ we can determine lim $s\to 0$ $C(s)= K\beta$

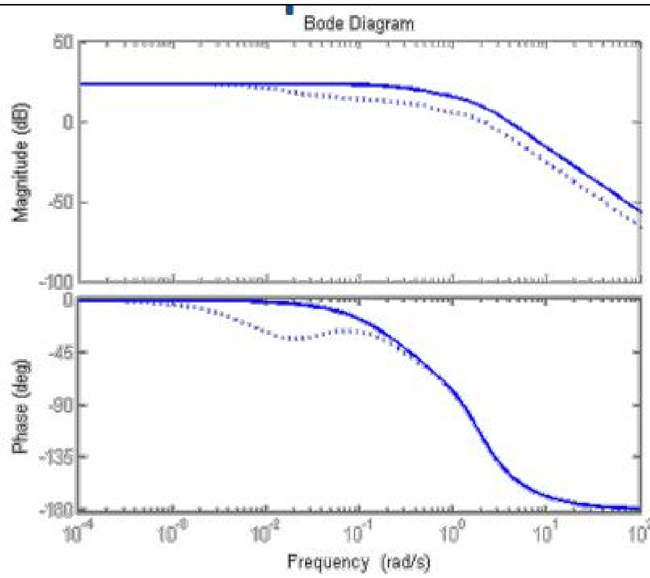
- **<u>Determination of τ</u>**

  - Take τ large enough such that the magnitude is almost entirely dropped, and the phase drop has almost disappeared

  - Take the zero one decade smaller than the frequency (ω) at which $P(s)$ had the desired phase ($-180°$ + the desired phase margin + a safety factor of 10°)
  - Verify the effect of a single zero at a frequency one decade smaller than ω

    - The drop in magnitude is as good as complete

    - The drop in phase cannot be more than $-5.7°$



- **<u>Let's translate all of this into a recipe again:</u>**

1. Translate your steady-state requirement into a requirement on lim $s \to 0$ $C(s) = K\beta$ and verify whether a proportional controller with gain $K\beta$ would suffice

2. Read ω, the frequency at which the phase margin equals −180° + your desired phase margin + 10°, off the Bode diagram; this allows us to compute $\tau = 10/\omega$

3. Read Q, the magnitude at ω off the Bode plot and determine $K = 1/Q$

4. Determine $\beta = K\beta/K$

5. Verify the behavior of the resulting system

<u>Lead compensators:</u>

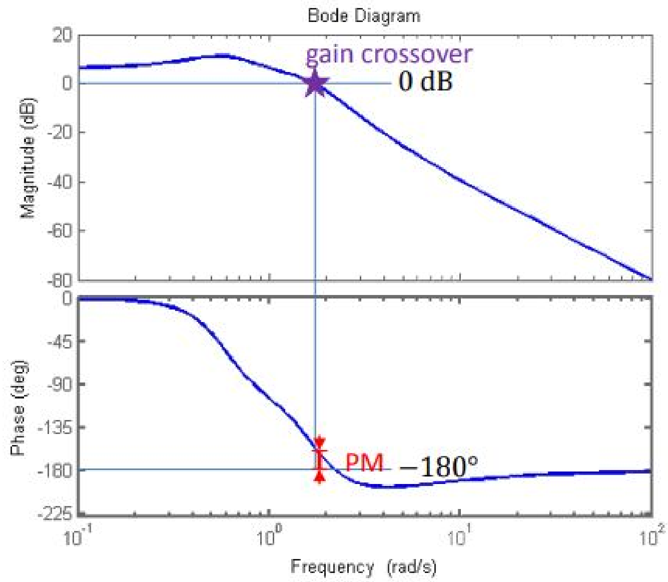
<u>impact</u>

- They push the poles of the closed loop system to the left

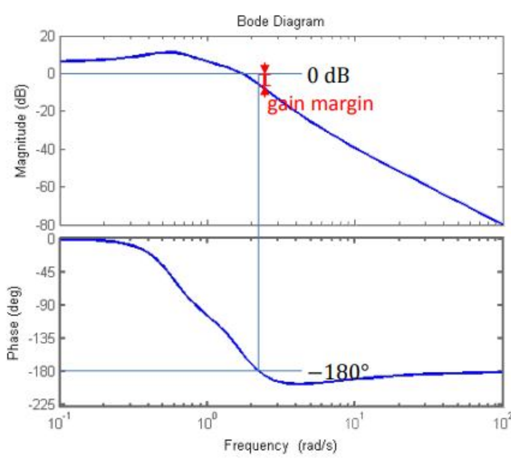
  Stabilization of system

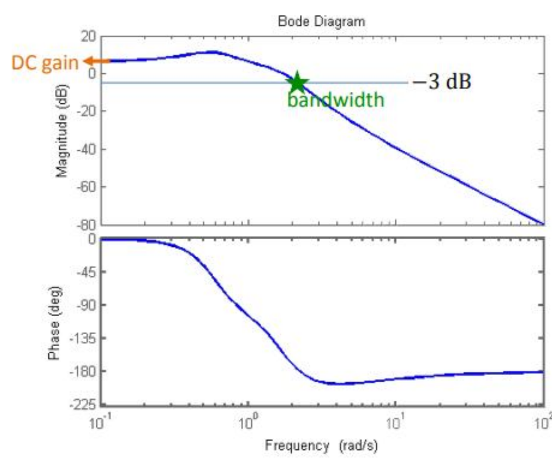
  Fast response

- Increase phase margin
- Focus: design lead compensators to tune the phase margin (PM)



➢ Other design characteristics are also possible





- ➢ Design process: tuning of the phase margin, with as a surplus (because we will have one extra degree of freedom) the tuning of the steady state error
- ➢ Compensate for the excessive phase lag that is a result of the components of $P(s)$
- ➢ Increase in phase at gain crossover frequency (GCF) if GCF is around pole and zero of the lead compensator
- ➢ Gain is impacted by the lead compensator $\Rightarrow G\mathrm{CF}_{\mathrm{p}(s)C(s)}) \neq G\mathrm{CF}_{\mathrm{p}(s)}$

How can we mathematically design a good lead compensator?

- ➢ Required increase in phase gain: ϕ
- ➢ To compensate for increase GCF due to $C(s) \Rightarrow \phi_m = \phi + 5°$ This will determine α and τ

➢ K will be used to tune the steady state error
Video Content / Details of website for further learning (if any): ➢ https://www.youtube.com/watch?v=rH44ttR3G4Q
Important Books/Journals for further learning including the page nos.: ➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:465)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Tutorial – I

Introduction :

The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Prerequisite knowledge for Complete understanding and learning of Topic:

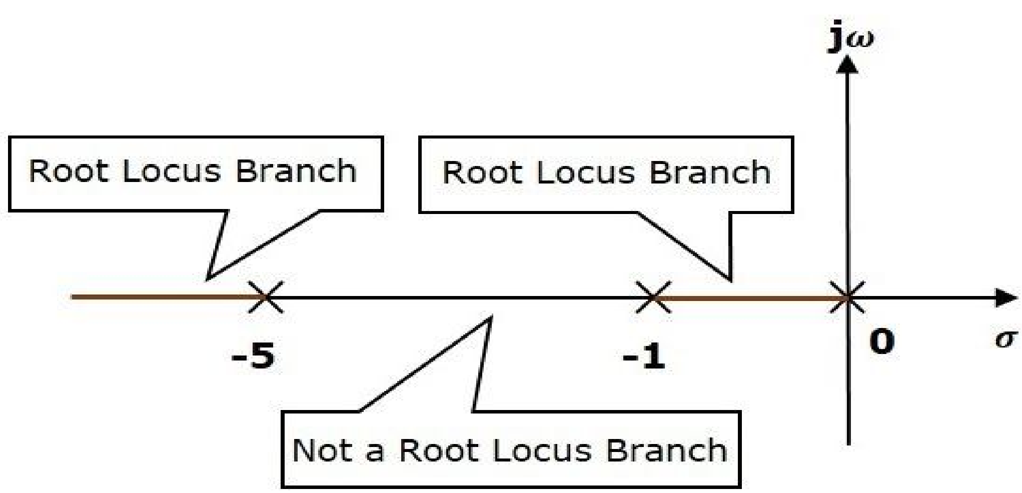
- Characteristics equation

Detailed content of the Lecture:

Let us now draw the root locus of the control system having open loop transfer function,
$G(s)H(s)=\frac{K}{s(s+1)(s+5)}$

Step 1 − The given open loop transfer function has three poles at $s=0, s=-1$ and $s=-5$. It doesn't have any zero. Therefore, the number of root locus branches is equal to the number of poles of the open loop transfer function.

$$N=P=3$$

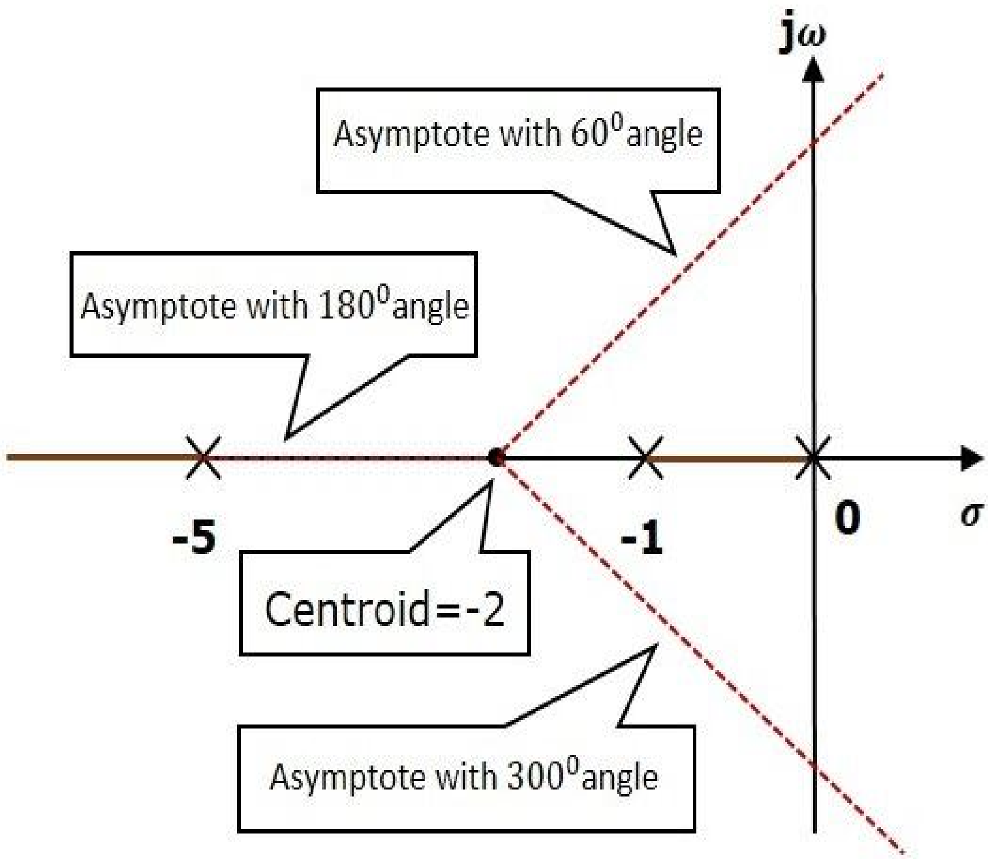


The three poles are located are shown in the above figure. The line segment between $s=-1s=-1$ and $s=0s=0$ is one branch of root locus on real axis. And the other branch of the root locus on the real axis is the line segment to the left of $s=-5s=-5$.

Step 2 − We will get the values of the centroid and the angle of asymptotes by using the given formulae.

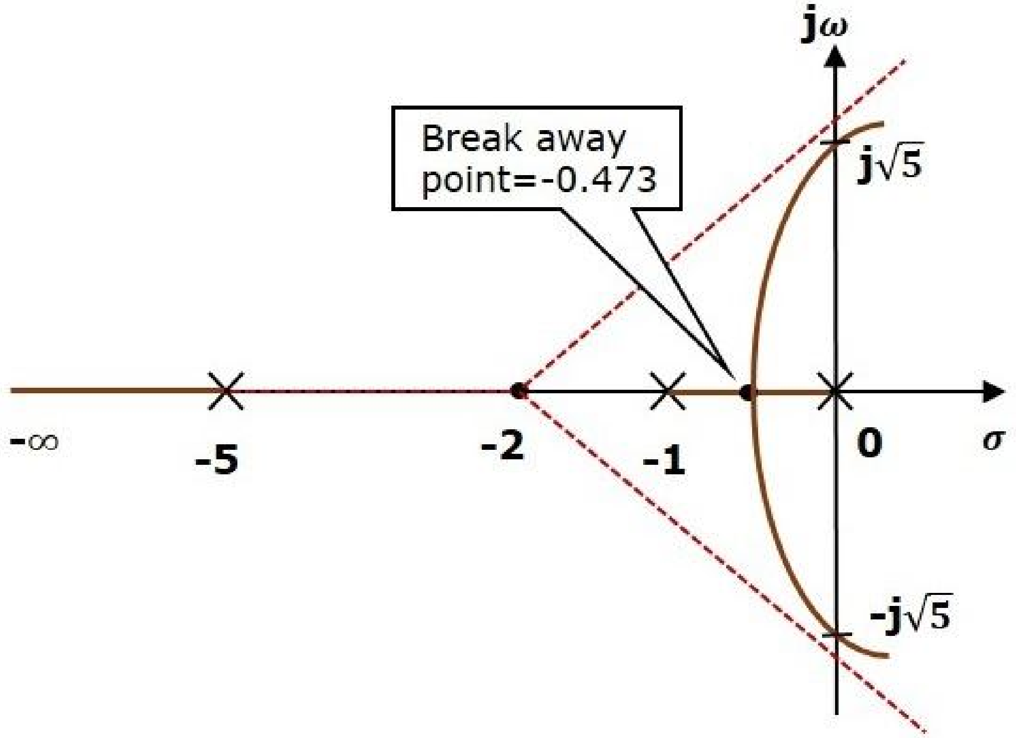
Centroid $\alpha=-2\alpha=-2$
The angle of asymptotes are $\theta=60_0,180_0\theta=600,1800$ and 300_03000.

The centroid and three asymptotes are shown in the following figure.



Step 3 –
Since two asymptotes have the angles of 60_0600 and 300_03000, two root locus branches intersect the imaginary axis. By using the Routh array method and special case(ii), the root locus branches intersects the imaginary axis at $j5–\sqrt{}j5$ and $-j5–\sqrt{}-j5$.
There will be one break-away point on the real axis root locus branch between the poles $s=-1s=-1$ and $s=0s=0$. By following the procedure given for the calculation of break-away point, we will get it as $s=-0.473s=-0.473$.

The root locus diagram for the given control system is shown in the following figure.



In this way, you can draw the root locus diagram of any control system and observe the movement of poles of the closed loop transfer function.

From the root locus diagrams, we can know the range of K values for different types of damping.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=CRvVDoQJjYI.

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:369)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS

L26

EEE

III/V

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Root locus construction (Tutorial – II)

Introduction :

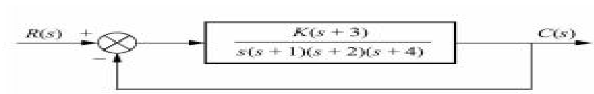
The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Characteristics equation

Detailed content of the Lecture:

- Consider the unity feedback system



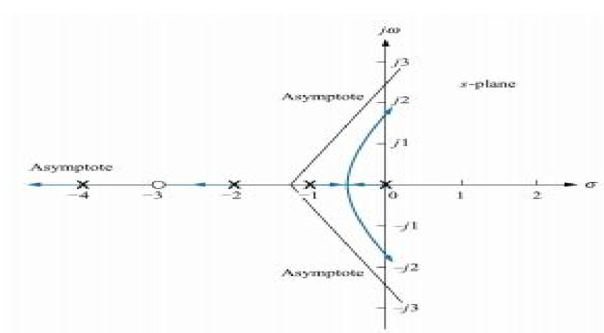
- The real axis intercept for the asymptotes:

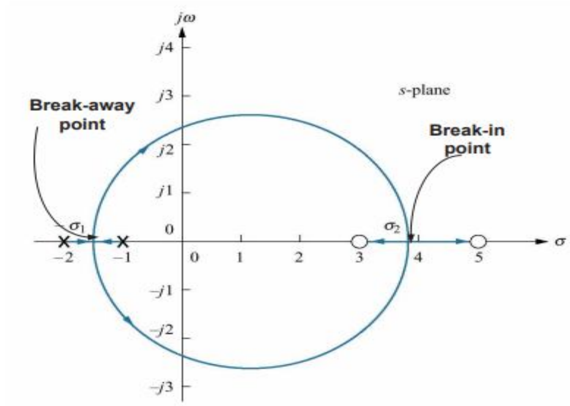
$$\sigma_a = \frac{(-1-2-4)-(-3)}{4-1} = -\frac{4}{3}$$

- The angles

$$\theta_a = \frac{(2m+1)\pi}{3}$$

which yields $\frac{\pi}{3}$, π and $\frac{5\pi}{3}$





- Break-away point: The point where root-locus leaves the real axis.
- Break-in point: The point where root locus enters the real axis.
- Variation of K as a function of σ

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=CRvVDoQJjYI.

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:369)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **III-** Frequency Response Analysis **Date of Lecture:**

Topic of Lecture:
Root locus construction (Tutorial – III)

Introduction :

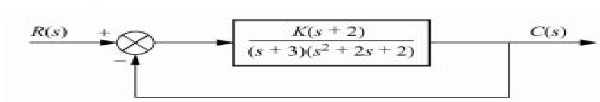
The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Prerequisite knowledge for Complete understanding and learning of Topic:

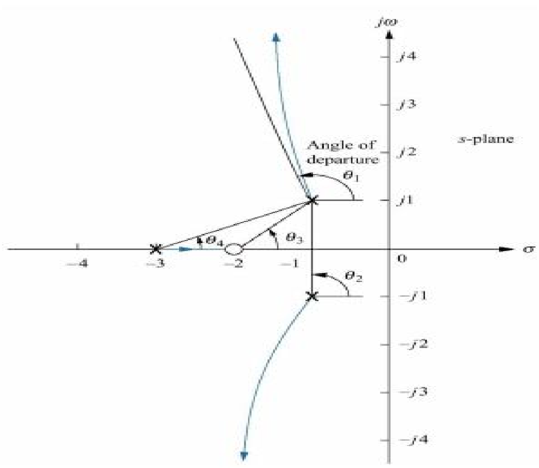
- Characteristics equation

Detailed content of the Lecture:

Consider the system



The root locus for this system

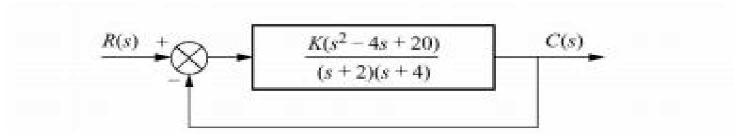


from this figure

$$-\theta_1-\theta_2+\theta_3-\theta_4 = -\theta_1-90^o+\arctan(\frac{1}{1})-\arctan(\frac{1}{2}) = 180^o$$

from which we obtain $\theta_1 = -108.4^o$.

Problem: Sketch the Root-Locus of the system



- The number of branches: 2
- Open Loop Poles: $-2, -4$ (starting points)
- Open Loop Zeros: $2 + j4$, $2 - j4$ (ending points)
- Real Axis segments: $[-4, -2]$.
- Number of finite poles $=$ Number of Finite Zeros $\Rightarrow$ No Asymptotes
- Break-away point: Take the derivative of $K = -\frac{1}{\sigma}$

  $$\frac{dK}{d\sigma} = -\frac{d}{d\sigma}\frac{(\sigma+2)(\sigma+4)}{\sigma^2 - 4\sigma + 20} = \frac{-10s^2 + 24s + 152}{(\sigma^2 - 4\sigma + 20)^2}$$

  equating to zero we obtain $\sigma_b = -2.87$ and $K = 0.0248$.
- $j\omega$ axis crossing occurs for $K = 1.5$ and at $\pm j3.9$.
- The root locus crosses $\zeta = 0.45$ line for $K = 0.417$ at $3.4\angle 116.7^o$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=CRvVDoQJjYI.

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:369)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Characteristics equation.

Introduction : (Maximum 5 sentences)

- The stability of a system relates to its response to inputs or disturbances. A system which remains in a constant state unless affected by an external action and which returns to a constant state when the external action is removed can be considered to be stable.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Partial differential equation

Detailed content of the Lecture:

- The local behaviour of a system of differential equations,

$$\frac{dX_i}{dt} = f_i(X_1,\ldots,X_n) \qquad (i=1,\ldots,n)$$

near an equilibrium point depends on the roots (eigenvalues) of the characteristic equation

$$|A - \lambda I| = 0 \qquad (4.1)$$

where $A = (a_{ij})$ is the matrix of first partial derivatives $\frac{\partial f_i}{\partial X_j}$ evaluated at the equilibrium point.

- If the real parts of all the roots are negative, the system returns to equilibrium after a small perturbation. If the real parts of all the roots are positive, the system moves away from equilibrium (is locally unstable).

- If some roots have positive and some negative real parts, the behaviour of the system depends on how it is perturbed; it sometimes returns to equilibrium but for other displacements moves away.

- In biological systems we usually assume the perturbations to be unconstrained so that eventually the system will be displaced in a direction which allows the positive root to lead the system away from equilibrium.
- A single zero real part gives a neutral or passive equilibrium, but multiple zero roots can give unbounded solutions (unstable equilibrium).
- If a root is complex the system oscillates at a frequency given by the imaginary part while the amplitude behaves according to the real part of the root.

Video Content / Details of website for further learning (if any):

-

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:683)

Course Teacher

Verified by HOD

LECTURE HANDOUTS

L29

EEE

II/IV

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Routh Hurwitz criterion.

Introduction : (Maximum 5 sentences)
Any pole of the system lies on the right hand side of the origin of the s plane, it makes the system unstable. On the basis of this condition A. Hurwitz and E.J. Routh started investigating the necessary and sufficient conditions of stability of a system.

Prerequisite knowledge for Complete understanding and learning of Topic:
- Partial differential equation

Detailed content of the Lecture:

Routh-Hurwitz Stability Criterion
- Routh-Hurwitz stability criterion is having one necessary condition and one sufficient condition for stability. If any control system doesn't satisfy the necessary condition, then we can say that the control system is unstable.
- But, if the control system satisfies the necessary condition, then it may or may not be stable. So, the sufficient condition is helpful for knowing whether the control system is stable or not.

Necessary Condition for Routh-Hurwitz Stability
- The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts.
- Consider the characteristic equation of the order 'n' is -

a0sn+a1sn−1+a2sn−2+...+an−1s1+ans0=0a0sn+a1sn−1+a2sn−2+...+an−1s1+ans0=0

- Note that, there should not be any term missing in the **n^th^** order characteristic equation. This means that the **n^th^** order characteristic equation should not have any coefficient that is of zero value.

Sufficient Condition for Routh-Hurwitz Stability
- The sufficient condition is that all the elements of the first column of the Routh array should have the same sign. This means that all the elements of the first column of the Routh array should be either positive or negative.

Routh Array Method

- If all the roots of the characteristic equation exist to the left half of the 's' plane, then the control system is stable. If at least one root of the characteristic equation exists to the right half of the 's' plane, then the control system is unstable.
- So, we have to find the roots of the characteristic equation to know whether the control system is stable or unstable. But, it is difficult to find the roots of the characteristic equation as order increases.
- So, to overcome this problem there we have the Routh array method. In this method, there is no need to calculate the roots of the characteristic equation.
- First formulate the Routh table and find the number of the sign changes in the first column of the Routh table.
- The number of sign changes in the first column of the Routh table gives the number of roots of characteristic equation that exist in the right half of the 's' plane and the control system is unstable.

Follow this procedure for forming the Routh table.

- Fill the first two rows of the Routh array with the coefficients of the characteristic polynomial as mentioned in the table below. Start with the coefficient of s^nsn and continue up to the coefficient of s^0s0.
- Fill the remaining rows of the Routh array with the elements as mentioned in the table below. Continue this process till you get the first column element of **row** s^0s0 is $a_n$an. Here, $a_n$an is the coefficient of s^0s0 in the characteristic polynomial.

Note − If any row elements of the Routh table have some common factor, then you can divide the row elements with that factor for the simplification will be easy.

The following table shows the Routh array of the n^{th} order characteristic polynomial.

$$a_0s^n+a_1s^{n-1}+a_2s^{n-2}+...+a_{n-1}s^1+a_ns^0$$

- A stability test for time invariant linear systems can also be derived in the frequency domain. It is known as Nyquist stability criterion. It is based on the complex analysis result known as Cauchy's principle of argument.
- Note that the system transfer function is a complex function. By applying Cauchy's principle of argument to the open-loop system transfer function, we will get information about stability of the closed-loop system transfer function and arrive at the Nyquist stability criterion (Nyquist, 1932).
- The importance of Nyquist stability lies in the fact that it can also be used to determine the relative degree of system stability by producing the so-called phase and gain stability margins. These stability margins are needed for frequency domain controller design techniques.
- We present only the essence of the Nyquist stability criterion and define the phase and gain stability margins. The Nyquist method is used for studying the stability of linear systems with pure time delay.
- For a SISO feedback system the closed-loop transfer function is given by where represents the system and is the feedback element.

$$M(s) = \frac{G(s)}{1 + H(s)G(s)}$$

- Since the system poles are determined as those values at which its transfer function becomes infinity, it follows that the closed-loop system poles are obtained by solving the following equation which, in fact, represents the system characteristic equation.

$$1 + H(s)G(s) = 0 = \Delta(s)$$

- In the following we consider the complex function whose zeros are the closed-loop poles of the transfer function.

$$D(s) = 1 + H(s)G(s)$$

- In addition, it is easy to see that the poles of are the zeros of . At the same time the poles of are the open-loop control system poles since they are contributed by the poles of , which can be considered as the open-loop control system transfer function—obtained when the feedback loop is open at some point.
- The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function . First, we state Cauchy's principle of argument.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=WBCZBOB3LCA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:223)

Course Teacher

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Nyquist stability criterion.

Introduction :

The Nyquist stability criterion works on the **principle of argument**. It states that if there are P poles and Z zeros are enclosed by the 's' plane closed path, then the corresponding G(s)H(s)G(s)H(s) plane must encircle the origin P−ZP−Z times.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Partial differential equation

Detailed content of the Lecture:

Nyquist Stability Criterion:

The Nyquist stability criterion works on the **principle of argument**. It states that if there are P poles and Z zeros are enclosed by the 's' plane closed path, then the corresponding G(s)H(s)G(s)H(s) plane must encircle the origin P−ZP−Z times. So, we can write the number of encirclements N as,

$$N=P-Z \quad N=P-Z$$

- If the enclosed 's' plane closed path contains only poles, then the direction of the encirclement in the G(s)H(s)G(s)H(s) plane will be opposite to the direction of the enclosed closed path in the 's' plane.
- If the enclosed 's' plane closed path contains only zeros, then the direction of the encirclement in the G(s)H(s)G(s)H(s) plane will be in the same direction as that of the enclosed closed path in the 's' plane.

Let us now apply the principle of argument to the entire right half of the 's' plane by selecting it as a closed path. This selected path is called the **Nyquist** contour.

We know that the closed loop control system is stable if all the poles of the closed loop transfer function are in the left half of the 's' plane. So, the poles of the closed loop transfer function are nothing but the roots of the characteristic equation. As the order of the characteristic equation increases, it is difficult to find the roots. So, let us correlate these roots of the characteristic equation as follows.

- The Poles of the characteristic equation are same as that of the poles of the open loop transfer function.
- The zeros of the characteristic equation are same as that of the poles of the closed loop

transfer function.

We know that the open loop control system is stable if there is no open loop pole in the the right half of the 's' plane.

i.e.,$P=0 \Rightarrow N=-Z$ $P=0 \Rightarrow N=-Z$

We know that the closed loop control system is stable if there is no closed loop pole in the right half of the 's' plane.

i.e.,$Z=0 \Rightarrow N=P$ $Z=0 \Rightarrow N=P$

Nyquist stability criterion states the number of encirclements about the critical point (1+j0) must be equal to the poles of characteristic equation, which is nothing but the poles of the open loop transfer function in the right half of the 's' plane. The shift in origin to (1+j0) gives the characteristic equation plane.

Rules for Drawing Nyquist Plots:

Follow these rules for plotting the Nyquist plots.

- Locate the poles and zeros of open loop transfer function G(s)H(s)G(s)H(s) in 's' plane.
- Draw the polar plot by varying $\omega\omega$ from zero to infinity. If pole or zero present at s = 0, then varying $\omega\omega$ from 0+ to infinity for drawing polar plot.
- Draw the mirror image of above polar plot for values of $\omega\omega$ ranging from $-\infty$ to zero (0^- if any pole or zero present at s=0).
- The number of infinite radius half circles will be equal to the number of poles or zeros at origin. The infinite radius half circle will start at the point where the mirror image of the polar plot ends. And this infinite radius half circle will end at the point where the polar plot starts.

After drawing the Nyquist plot, we can find the stability of the closed loop control system using the Nyquist stability criterion. If the critical point (-1+j0) lies outside the encirclement, then the closed loop control system is absolutely stable.

Stability Analysis using Nyquist Plots:

From the Nyquist plots, we can identify whether the control system is stable, marginally stable or unstable based on the values of these parameters.

- Gain cross over frequency and phase cross over frequency
- Gain margin and phase margin

Phase Cross over Frequency:

The frequency at which the Nyquist plot intersects the negative real axis (phase angle is 180^0) is known as the **phase cross over frequency**. It is denoted by $\omega_{pc}\omega_{pc}$.

Gain Cross over Frequency:

The frequency at which the Nyquist plot is having the magnitude of one is known as the **gain cross over frequency**. It is denoted by $\omega_{gc}\omega_{gc}$.

The stability of the control system based on the relation between phase cross over frequency and gain cross over frequency is listed below.

- If the phase cross over frequency $\omega_{pc}\omega_{pc}$ is greater than the gain cross over frequency ω_{gc}, then the control system is **stable**.

- If the phase cross over frequency ω_{pc} is equal to the gain cross over frequency ω_{gc}, then the control system is **marginally stable**.
- If phase cross over frequency ω_{pc} is less than gain cross over frequency ω_{gc}, then the control system is **unstable**.

Gain Margin:

The gain margin GM is equal to the reciprocal of the magnitude of the Nyquist plot at the phase cross over frequency.

$$GM = \frac{1}{M_{pc}}$$

Where, M_{pc} is the magnitude in normal scale at the phase cross over frequency.

Phase Margin:

The phase margin PM is equal to the sum of 180^0 and the phase angle at the gain cross over frequency.

$$PM = 180^0 + \phi_{gc}$$

Where, ϕ_{gc} is the phase angle at the gain cross over frequency.

The stability of the control system based on the relation between the gain margin and the phase margin is listed below.

- If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is **stable**.
- If the gain margin GM is equal to one and the phase margin PM is zero degrees, then the control system is **marginally stable**.
- If the gain margin GM is less than one and / or the phase margin PM is negative, then the control system is **unstable**.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=sof3meN96MA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:463)

Course Teacher

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Lag, Lead networks

Introduction :

Addition of poles to the transfer function has the **effect** of pulling the root locus to the right, making the system less stable. **Addition** of **zeros** to the transfer function has the **effect** of pulling the root locus to the left, making the system more stable

Prerequisite knowledge for Complete understanding and learning of Topic:

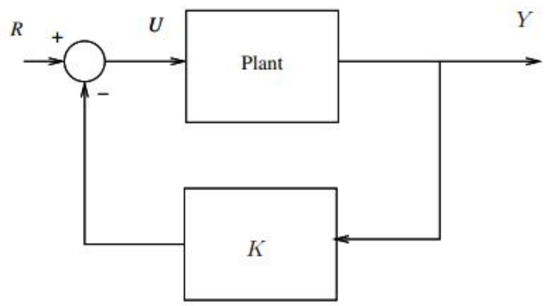
- Characteristics equation

Detailed content of the Lecture:

Dominantly first order systems. Effects of additional poles and zeros on performance of a first order control system:

Step response of a first order system

- Consider a first order system of the form $\dot{x} = -ax + r$, or, equivalently, $X(s) = \frac{1}{s+a} R(s)$. Let $r(t)$ be a unit step input, i.e, $R(s) = \frac{1}{s}$. Then, assuming $x(0) = 0$, $X(s) = \frac{1}{s(s+a)} = \frac{1/a}{s} - \frac{1/a}{s+a}$, and $x(t) = \frac{1}{a}(1 - e^{-at})$, and $x(t) \to \frac{1}{a}$ as $t \to \infty$.
- The output of the system monotonically approaches to the final value of $1/a$; the rate of convergence is exponential and is determined by the pole of the system at $s = -a$.
- Note that the time constant of the transient is $\tau = 1/a$ is also related to the location of the pole. To achieve a desired rate of convergence, the designer must place the pole at a corresponding location; e.g., using the state or output feedback; see Figure 1.
- Example Suppose the plant is quite slow, its time constant is $\tau_{orig} = 100$ sec, $\dot{x} = -0.01x + u$, or, equivalently, $X(s) = \frac{1}{s+0.01} U(s)$. To speed up it to $\tau_{desired} = 1$ sec, use the feedback controller
  $u = -kx + r$.
- Then, the closed loop system is $\dot{x} = -(0.01 + k)x + r$, or, equivalently, $X(s) = \frac{1}{s+0.01+k} R(s)$. To achieve the desired time constant of 1 sec, the pole of the closed loop system must be placed at $s = -1/\tau_{desired} = -1$, i.e., we need $a = (0.01 + k) = 1$, which is achieved using the gain of $k = 0.99$.



Feedback Controller

Figure 1: Static feedback control system.

The Effect of an Additional Pole:

- A first order system often serves as an approximation of a system of higher order. Therefore, it is important to study the error of approximation.
- To this end, we examine the effect of additional poles and zeros on a first-order system.
- This will explain the difference between an original system and its first-order approximation. We now examine the step response of a system whose transfer function is given by

  Y (s) R(s) = p (s + 1)(s + p) = 1 (s + 1)(1 p s + 1)………………………..(1)

- That is, we are concerned with a 2nd order system which is approximated by the first order approximation 1 s+a , where a = 1.

Note on the form of the system.

- Eq. (1) is selected because its value at s = 0 equals 1 for any p. We will see that the step response of the system will approach the value of 1 for large values of t, independent of the value of p.
- This is convenient, since we wish to examine the step response for different values of p. However, the conclusions drawn from the example will apply to any second order system with two real poles.
- Let R(s) = 1/s and perform a partial-fraction expansion on the resulting step response transform Y (s):
  Y (s) = p s(s + 1)(s + p) = 1 s − p p−1 s + 1 + 1 p−1 s + p .
- The step response y(t) is given by y(t) = 1 − p p − 1 e −t + 1 p − 1 e −pt . ………..(2)
- We will consider this response as the sum of two terms. The first term is given by 1 − p p−1 e −t , and the second term is 1 p−1 e −pt . Let us examine the step response of Eq. (2) for different values of p:

The Effect of a Zero on a Dominantly First-order System :

- We have seen that an additional pole retards a dominantly first-order system as the pole moves along the negative real axis. We now examine the effect of moving a zero in along the negative real axis. The system to be studied is given by
- Y (s) /R(s) = 1 z s + 1 (s + 1)(1 10 s + 1).
- If we set z = ∞, then we obtain the system of the previous section in which p = 10. We have seen that for the example in the previous section, this value of the parameter p was sufficently large in order to say that the system was a dominantly first-order system.
- The zero-pole locus of the system (3) is shown in Figure 4. We are again interested in the step response, that is R(s) = 1/s,
- If we set z = ∞, then we obtain the system of the previous section in which p = 10. We have seen that for the example in the previous section, this value of the parameter p was sufficently large in order to say that the system was a dominantly first-order system.
- The zero-pole locus of the system (3) is shown in Figure 4. We are again interested in the

step response, that is R(s) = 1 / s,

Y (s) = 1 z s + 1 / s(s + 1)(1 10 s + 1).

Take the partial fraction expansion of Y (s):

Y (s) = 1 / s − 10 9 z−1/ z s + 1 + 1 9 z−10 z /s + 10 .

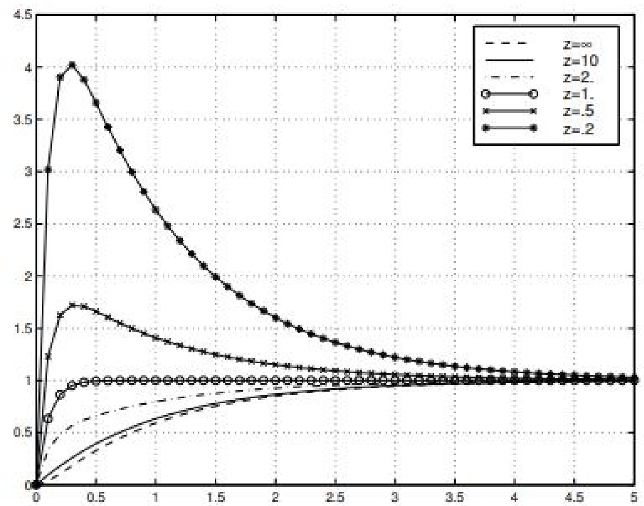


Figure 5: Step response of the dominantly first-order system with the additional zero at $s = -z$.

- Effects of the additional zero: z 10. We can see that if z 10, then 10 9 z−1 z ≈ 10/9, 1 9 z−10 z ≈ 1/9, and y(t) ≈ yd(t) = 1 − 10 9 e −t for sufficiently large t.
- Thus, the pole at s = −1 remains dominant. z = 10. This zero cancels the pole at s = −10. The system becomes a first-order system. 1 < z < 10.
- In this case, the additional zero speeds up the system; see Figure 5. z = 1. This cancels the pole at s = −1. z < 1. The additional zero becomes dominant. It speeds up the system and at the same time leads to occuring an overshoot; see Figure 5.
- From this analysis, one can see the general effect that the speed of the response increases as zero moves from +∞ to 0 along the negative real axis. When zero becomes dominant, an overshoot occurs.
-

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=emCLkMUjlnc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:470)

Course Faculty

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LECTURE HANDOUTS | L32

EEE | II/IV

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Lag, Lead Compensator design using Bode Plot

Introduction :
The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Characteristics equation

Detailed content of the Lecture:

Rules for Construction of Root Locus

Follow these rules for constructing a root locus.

Rule 1 − Locate the open loop poles and zeros in the 's' plane.

Rule 2 − Find the number of root locus branches.

We know that the root locus branches start at the open loop poles and end at open loop zeros. So, the number of root locus branches **N** is equal to the number of finite open loop poles **P** or the number of finite open loop zeros **Z**, whichever is greater.

Mathematically, we can write the number of root locus branches **N** as

N=PN=P if P≥ZP≥Z
N=ZN=Z if P<ZP<Z

Rule 3 − Identify and draw the real axis root locus branches.

If the angle of the open loop transfer function at a point is an odd multiple of 180^0, then that point is on the root locus. If odd number of the open loop poles and zeros exist to the left side of a point on the real axis, then that point is on the root locus branch. Therefore, the branch of points which satisfies this condition is the real axis of the root locus branch.

Rule 4 − Find the centroid and the angle of asymptotes.

- If P=ZP=Z, then all the root locus branches start at finite open loop poles and end at finite open loop zeros.
- If P>ZP>Z , then ZZ number of root locus branches start at finite open loop poles and end at finite open loop zeros and P−ZP−Z number of root locus branches start at finite

open loop poles and end at infinite open loop zeros.

- If $P<ZP<Z$, then P number of root locus branches start at finite open loop poles and end at finite open loop zeros and $Z-PZ-P$ number of root locus branches start at infinite open loop poles and end at finite open loop zeros.

So, some of the root locus branches approach infinity, when $P\neq ZP\neq Z$. Asymptotes give the direction of these root locus branches. The intersection point of asymptotes on the real axis is known as **centroid**.

We can calculate the **centroid α** by using this formula,

$\alpha=\sum Realpartoffiniteopenlooppoles-\sum RealpartoffiniteopenloopzerosP-Z\alpha=\sum Realpartoffiniteopenlooppoles-\sum RealpartoffiniteopenloopzerosP-Z$

The formula for the angle of **asymptotes θ** is

$$\theta=(2q+1)1800P-Z\theta=(2q+1)1800P-Z$$

Where,

$$q=0,1,2,....,(P-Z)-1q=0,1,2,....,(P-Z)-1$$

Rule 5 − Find the intersection points of root locus branches with an imaginary axis.

We can calculate the point at which the root locus branch intersects the imaginary axis and the value of **K** at that point by using the Routh array method and special **case (ii)**.

- If all elements of any row of the Routh array are zero, then the root locus branch intersects the imaginary axis and vice-versa.
- Identify the row in such a way that if we make the first element as zero, then the elements of the entire row are zero. Find the value of **K** for this combination.
- Substitute this **K** value in the auxiliary equation. You will get the intersection point of the root locus branch with an imaginary axis.

Rule 6 − Find Break-away and Break-in points.

- If there exists a real axis root locus branch between two open loop poles, then there will be a **break-away point** in between these two open loop poles.
- If there exists a real axis root locus branch between two open loop zeros, then there will be a **break-in point** in between these two open loop zeros.

Note − Break-away and break-in points exist only on the real axis root locus branches.

Follow these steps to find break-away and break-in points.

- Write KK in terms of ss from the characteristic equation $1+G(s)H(s)=01+G(s)H(s)=0$.
- Differentiate KK with respect to s and make it equal to zero. Substitute these values of ss in the above equation.
- The values of ss for which the KK value is positive are the **break points**.

Rule 7 − Find the angle of departure and the angle of arrival.

The Angle of departure and the angle of arrival can be calculated at complex conjugate open loop poles and complex conjugate open loop zeros respectively.

The formula for the **angle of departure** ϕdϕd is

$$\phi d=1800-\phi\phi d=1800-\phi$$

The formula for the **angle of arrival** ϕaϕa is

$$\phi a=1800+\phi\phi a=1800+\phi$$

Where,

$$\phi=\sum\phi P-\sum\phi Z$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=CRvVDoQJjYI.

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:369)

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LECTURE HANDOUTS | **L33**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Lag, Lead Compensator design using Bode Plot

Introduction : (Maximum 5 sentences)

The root locus is a graphical representation in s-domain and it is symmetrical about the real axis. Because the open loop poles and zeros exist in the s-domain having the values either as real or as complex conjugate pairs. In this chapter, let us discuss how to construct (draw) the root locus.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Characteristics equation

Detailed content of the Lecture:

- Root Locus is going out of favor as a practical tool because it gets really complicated by digital sampling models. As a conceptual thought model and as long as linear theory remains the paradigm of choice, the Root Locus does a really good job of visualizing a SISO system.

- Have ever used the imaginary plane to visualize anything? An example maybe an electrical circuit phase-plane plot (phasors). The location of the poles is one of the best known ways to understand second-order systems. The Root Locus falls naturally from that.

- When you introduce feedback to a system the solution of the differential equations of motion becomes a transfer function with a characteristic equation in the denominator. The locus of poles and zeros on the phase-plane plot that correspond to the solutions (roots) of the characteristic equation for various feedback gains are the Root Locus. Get it: 'locus.' 'Roots'. When gain is zero the system is open loop and the open loop poles and zeros make the start of the locus. As gain is increased the feedback effects become stronger and different natural responses result.

- In the end, you get a very good feel for the closed loop Eigenvalues as gain is increased. It's a mental thought model that will always be useful as long as linear theory is used, which will be for a long time.

- Practically speaking however, to represent a delay introduced by sampling you end up with a big mess and the visualization is difficult. I use the visualization without sampling effects.

- Root Locus analysis is useful for you to check the places where the poles of your linear system will be located on when you close its feedback loop with a gain factor. When you change the gain, the poles change their place through the loci.

  If you have a simple SISO LTI system with its Transfer Function:

  Y(s)=G(s)U(s)Y(s)=G(s)U(s)

  where,

  G(s)=N(s)D(s)G(s)=N(s)D(s),

  when you close the loop with a feedback gain, you'll get

  Y(s)=G(s)1+kG(s)U(s)=N(s)D(s)+kN(s)U(s)Y(s)=G(s)1+kG(s)U(s)=N(s)D(s)+kN(s)U(s).
- Without mathematical rigor, observe that if $k \to 0 k \to 0$, the poles of the two systems are almost equal. It means that the poles are near from the original ones. But if $k \to \infty \Rightarrow D(s) \ll kN(s) k \to \infty \Rightarrow D(s) \ll kN(s)$, and the poles approximate to the zeroes.
- The most important is that if the loci go through the right-half plane, the system can become unstable for some gain values - those which place poles there.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=CRvVDoQJjYI.

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:369)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Routh Hurwitz criterion. (Tutorial – I)

Introduction :

Any pole of the system lies on the right hand side of the origin of the s plane, it makes the system unstable. On the basis of this condition A. Hurwitz and E.J. Routh started investigating the necessary and sufficient conditions of stability of a system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Partial differential equation

Detailed content of the Lecture:

Let us find the stability of the control system having characteristic equation,

$$s^4+2s^3+s^2+2s+1=0 \quad s4+2s3+s2+2s+1=0$$

Step 1 − Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the characteristic polynomial, $s^4+2s^3+s^2+2s+1$ s4+2s3+s2+2s+1 are positive. So, the control system satisfied the necessary condition.

Step 2 − Form the Routh array for the given characteristic polynomial.

s^4 s4	11	11	11
s^3 s3	~~2~~ 1	~~2~~ 1	
s^2 s2	$\frac{(1\times1)-(1\times1)}{1}=0$ (1×1)−(1×1)1=0	$\frac{(1\times1)-(0\times1)}{1}=1$ (1×1)−(0×1)1=1	
s^1 s1			
s^0 s0			

The row s^3 s3 elements have 2 as the common factor. So, all these elements are divided by 2.

Special case (i) − Only the first element of row s^2 s2 is zero. So, replace it by ϵ ϵ and continue the process of completing the Routh table.

s^4 s4	1	1	1

s^3	1	1	
s^2	ϵ	1	
s^1	$\frac{(\epsilon\times 1)-(1\times 1)}{\epsilon}=\frac{\epsilon-1}{\epsilon}$		
s^0	1		

Step 3 − Verify the sufficient condition for the Routh-Hurwitz stability.

As ϵ tends to zero, the Routh table becomes like this.

s^4	1	1	1
s^3	1	1	
s^2	0	1	
s^1	$-\infty$		
s^0	1		

There are two sign changes in the first column of Routh table. Hence, the control system is unstable.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=WBCZBOB3LCA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:223)

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LECTURE HANDOUTS

L35

EEE

II/IV

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Routh Hurwitz criterion.(Tutorial – II)

Introduction : (Maximum 5 sentences)

Any pole of the system lies on the right hand side of the origin of the s plane, it makes the system unstable. On the basis of this condition A. Hurwitz and E.J. Routh started investigating the necessary and sufficient conditions of stability of a system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Partial differential equation

Detailed content of the Lecture:

Let us find the stability of the control system having characteristic equation,

$$s^5+3s^4+s^3+3s^2+s+3=0 \quad s5+3s4+s3+3s2+s+3=0$$

Step 1 − Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the given characteristic polynomial are positive. So, the control system satisfied the necessary condition.

Step 2 − Form the Routh array for the given characteristic polynomial.

s^5s5	1	1	1
s^4s4	~~3~~ 1	~~3~~ 1	~~3~~ 1
s^3s3	$\frac{(1\times1)-(1\times1)}{1}=0$ (1×1)−(1×1)1=0	$\frac{(1\times1)-(1\times1)}{1}=0$ (1×1)−(1×1)1=0	
s^2s2			
s^1s1			
s^0s0			

The row s^4s4 elements have the common factor of 3. So, all these elements are divided by 3.

Special case (ii) − All the elements of row s^3s3 are zero. So, write the auxiliary equation, A(s) of the row s^4s4.

$$A(s)=s^4+s^2+1 \quad A(s)=s4+s2+1$$

Differentiate the above equation with respect to s.

$$\frac{dA(s)}{ds}=4s^3+2s$$

Place these coefficients in row s^3.

s^5	1	1	1
s^4	1	1	1
s^3	4 2	~~2~~ 1	
s^2	$\frac{(2\times1)-(1\times1)}{2}=0.5$	$\frac{(2\times1)-(0\times1)}{2}=1$	
s^1	$\frac{(0.5\times1)-(1\times2)}{0.5}=\frac{-1.5}{0.5}=-3$		
s^0	1		

Step 3 − Verify the sufficient condition for the Routh-Hurwitz stability.

There are two sign changes in the first column of Routh table. Hence, the control system is unstable.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=WBCZBOB3LCA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:223)

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LECTURE HANDOUTS | **L36**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **IV-** Stability and Compensator **Date of Lecture:**

Topic of Lecture:
Routh Hurwitz criterion. (Tutorial – III)

Introduction : (Maximum 5 sentences)

Any pole of the system lies on the right hand side of the origin of the s plane, it makes the system unstable. On the basis of this condition A. Hurwitz and E.J. Routh started investigating the necessary and sufficient conditions of stability of a system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Partial differential equation

Detailed content of the Lecture:

Let us find the stability of the control system having characteristic equation,

$$s^4+3s^3+3s^2+2s+1=0 \quad s4+3s3+3s2+2s+1=0$$

Step 1 − Verify the necessary condition for the Routh-Hurwitz stability.

All the coefficients of the characteristic polynomial, $s^4+3s^3+3s^2+2s+1$ s4+3s3+3s2+2s+1 are positive. So, the control system satisfies the necessary condition.

Step 2 − Form the Routh array for the given characteristic polynomial.

s^4s4	11	33	11
s^3s3	33	22	
s^2s2	$\frac{(3\times3)-(2\times1)}{3}=\frac{7}{3}$(3×3)−(2×1)3=73	$\frac{(3\times1)-(0\times1)}{3}=\frac{3}{3}=1$(3×1)−(0×1)3=33=1	
s^1s1	$\frac{\left(\frac{7}{3}\times2\right)-(1\times3)}{\frac{7}{3}}=\frac{5}{7}$(73×2)−(1×3)73=57		
s^0s0	11		

Step 3 − Verify the sufficient condition for the Routh-Hurwitz stability.

All the elements of the first column of the Routh array are positive. There is no sign change in the first column of the Routh array. So, the control system is stable.

Video Content / Details of website for further learning (if any): ➢ https://www.youtube.com/watch?v=WBCZBOB3LCA
Important Books/Journals for further learning including the page nos.: ➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012.(Page No:223)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-** State Variable Analysis **Date of Lecture:**

Topic of Lecture:
Concept of state variables

Introduction :

- A state variable is one of the set of variables that are used to describe the mathematical "state" of a dynamical system.
- Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- State
- System

Detailed content of the Lecture:

- State variables are used to represent the states of a general system.
- The set of possible combinations of state variable values is called the state space of the system.
- The equations relating the current state of a system to its most recent input and past states are called the state equations, and the equations expressing the values of the output variables in terms of the state variables and inputs are called the output equations.

State space representation of Continuous Time systems:

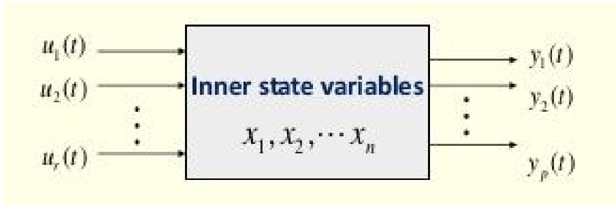
- The state variables may be totally independent of each other, leading to diagonal or normal form or they could be derived as the derivatives of the output.
- If them is no direct relationship between various states. We could use a suitable transformation to obtain the representation in diagonal form.

Phase Variable Representation:

- It is often convenient to consider the output of the system as one of the state variable and remaining state variable as derivatives of this state variable.
- The state variables thus obtained from one of the system variables and its (n-1) derivatives, are known as n-dimensional phase variables.
- In a third-order mechanical system, the output may be displacement x1, x1= x2= v and x2 = x3 = a in the case of motion of translation or angular displacement θ 1 = x1, x1= x2= w and x2 = x3 = α if the motion is rotational, Where v v,w,a, α respectively, are velocity, angular velocity acceleration, angular acceleration.

Physical Variable Representation:

- In this representation the state variables are real physical variables, which can be measured and used for manipulation or for control purposes.
- The approach generally adopted is to break the block diagram of the transfer function into subsystems in such a way that the physical variables can he identified.
- The governing equations for the subsystems can he used to identify the physical variables.



Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=BsF0EkfEAxc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 571-575)

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DESIGNING YOUR FUTURE
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LECTURE HANDOUTS

L38

EEE

II/IV

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-** State Variable Analysis **Date of Lecture:**

Topic of Lecture:
State models for linear and time invariant Systems

Introduction :

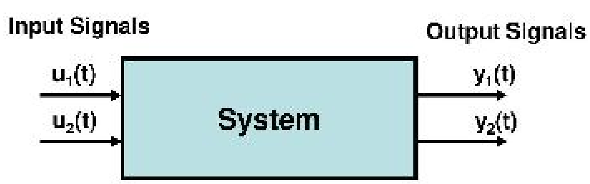
- A state variable is one of the set of variables that are used to describe the mathematical "state" of a dynamical system.
- Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system.

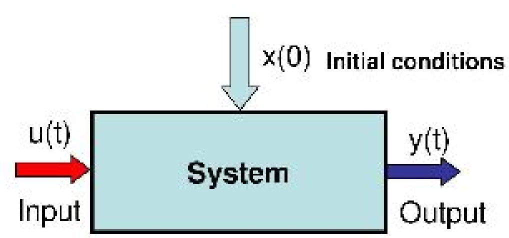
Prerequisite knowledge for Complete understanding and learning of Topic:

- State
- System

Detailed content of the Lecture:

- State variables are used to represent the states of a general system.





- State-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations or difference equations.
- State variables are variables whose values evolve through time in a way that depends on the values they have at any given time and also depends on the externally imposed values of input variables.
- Output variables' values depend on the values of the state variables.
- The internal state variables are the smallest possible subset of system variables that can represent the entire state of the system at any given time.
- If the system is represented in transfer function form, the minimum number of state variables is equal to the order of the transfer function's denominator after it has been reduced to a proper fraction.
- It is important to understand that converting a state-space realization to a transfer

function form may lose some internal information about the system, and may provide a description of a system which is stable, when the state-space realization is unstable at certain points.

- In electric circuits, the number of state variables is often, though not always, the same as the number of energy storage elements in the circuit such as capacitors and inductors.
- The state variables defined must be linearly independent, i.e., no state variable can be written as a linear combination of the other state variables or the system will not be able to be solved.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=ibhwKT6l_Os
- https://www.youtube.com/watch?v=g9G8b7FxEHc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No:575-577)

Course Faculty

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Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Solution of state and output equation controllable canonical form

Introduction :

- Linear time-invariant systems (LTI systems) are a class of systems used in signals and systems that are both linear and time-invariant.
- Time-invariant systems are systems where the output does not depend on when an input was applied.
- These properties make LTI systems easy to represent and understand graphically.

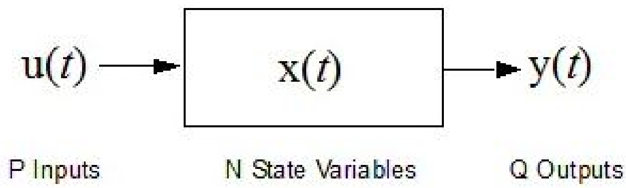
Prerequisite knowledge for Complete understanding and learning of Topic:

- Time variant
- Time Invariant

Detailed content of the Lecture:

Linear Time Invariant Systems:

- Linear Time Invariant Systems (LTI systems) are a class of systems used in signals and systems that are both linear and time invariant.
- Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs.
- Time invariant systems are systems where the output does not depend on when an input was applied. These properties make LTI systems easy to represent and understand graphically.
- LTI systems are superior to simple state machines for representation because they have more memory.
- LTI systems, unlike state machines, have memory of past states and have to ability predicted the future.
- LTI systems are used to predict long-term behavior in a system. So, they are often used to model systems like power plants.
- Time invariant systems are systems where the output for a particular input does not change depending on when that input was applied.



Model of Linear Time-Invariant System

- In addition to linear and time invariant, LTI systems are also memory systems, invertible, causal, real, and stable.
- That means they have memory, they can be inverted, they depend only on current and past events, they have fully real inputs and outputs, and they produce bounded output for bounded input.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=5JT_Dm7BZlo

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 578-582)

Course Faculty

Verified by HOD

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Concepts of controllability

Introduction :

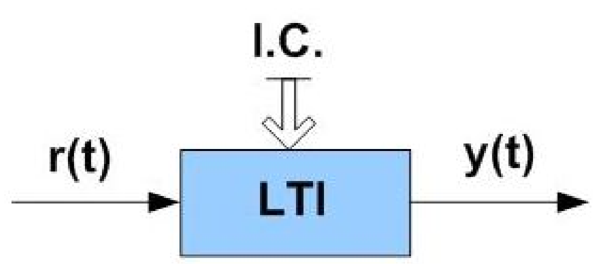
- Time invariant systems are systems where the output for a particular input does not change depending on when that input was applied.

Prerequisite knowledge for Complete understanding and learning of Topic:

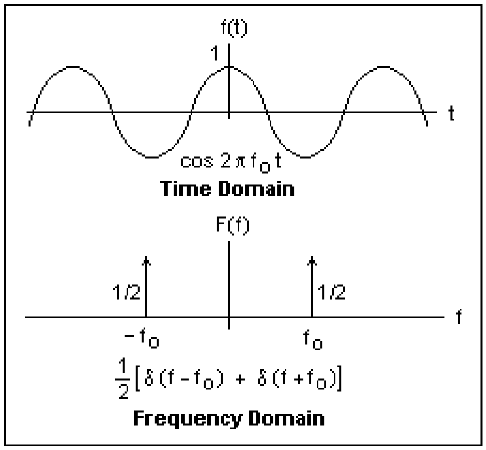
- Time variant
- Time Invariant

Detailed content of the Lecture:

- A time invariant systems that takes in signal and produces output will also, when excited by signal , produce the time shifted output .
- Thus, the entirety of an LTI system can be described by a single function called its impulse response. This function exists in the time domain of the system.
- For an arbitrary input, the output of an LTI system is the convolution of the input signal with the system's impulse response.
- Conversely, the LTI system can also be described by its transfer function. The transfer function is the Laplace transform of the impulse response.



- This transformation changes the function from the time domain to the frequency domain.
- This transformation is important because it turns differential equations into algebraic equations, and turns convolution into multiplication.
- In the frequency domain, the output is the product of the transfer function with the transformed input.



- State-Space Canonical Forms For any given system, there are essentially an infinite number of possible state space models that will give the identical input/output dynamics.
- Thus, it is desirable to have certain standardized state space model structures: these are the so-called canonical forms.
- Given a system transfer function, it is possible to obtain each of the canonical models. And, given any particular canonical form it is possible to transform it to another form.
- Consider the system defined by

$$(n)\ y + a1\ (n-1)\ y + \ldots + an-1\dot{y} + any = b0\ (n)\ u + b1\ (n-1)\ u + \ldots + bn-1\dot{u} + bnu$$

where u is the input, y is the output and (n) y represents the n th derivative of y with respect to time. Taking the Laplace transform of both sides we get:

$$Y(s)\ s\ n + a1s\ n-1 + \ldots + an-1s + an = U(s)\ b0s\ n + b1s\ n-1 + \ldots + bn-1s + bn$$

which yields the transfer function:
$Y(s)\ U(s) = b0s\ n + b1s\ n-1 + \ldots + bn-1s + bn\ s\ n + a1s\ n-1 + \ldots + an-1s + an$ (1)
Given the a system having transfer function as defined in (1) above, we will define the controllable canonical and observable canonical form.

Controllable Canonical Form:

$$\frac{Y(s)}{U(s)} = \frac{b_o s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

The following state-space representation is called a controllable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

Jordan canonical form:

- In this form the poles of the transfer function form a string along the main diagonal of the matrix.

$$G(s) = \frac{\beta_0 s^n + \beta_1 s^{n-1} + \dots\dots + \beta_n}{s^n + \alpha_n s^{n-1} + \dots\dots + \alpha_n}$$

- By long division , G(s) can be written as

$$G(s) = \beta_0 + \frac{\beta_1' s^{n-1} + \dots\dots + \beta_n'}{s^n + \alpha_1 s^{n-1} + \dots\dots + \alpha_n} = \beta_0 + G'(s)$$

or

$$G(s) = \frac{Y(s)}{U(s)} = \beta_0 + \frac{r1}{s-\lambda_1} + \frac{r2}{s-\lambda_2} + \dots\dots + \frac{r_n}{s-\lambda_n} \qquad (11)$$

- The coefficient r_i (i = 1,2,…….,n) are the residue of the transfer function G'(s) at the poles at s = λ_i (i = 1,2,…..,n).

- The controllable canonical form arranges the coefficients of the transfer function denominator across one row of the A matrix:

$$A = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix}$$

- The state model having minimum number of non-zero elements are called as canonical forms.
- So, we will find that in this particular form, the number of non-zero elements are minimum other elements are maximum 0's.

- The technique that is used to represent the mathematical entities or matrix in its standard form (or mathematical expression) is termed as canonical form.
- The triangular form, Jordan canonical form and row echelon form are some major canonical forms in Linear Algebra.

Controllable Canonical Form

- We consider the following state-space representation, being called a controllable canonical form, as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \qquad (3)$$

$$y = \begin{bmatrix} (b_n - a_n b_0) & (b_{n-1} - a_{n-1} b_0) & \dots & (b_1 - a_1 b_0) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u \qquad (4)$$

- Note that the controllable canonical form is important in discussing the pole-placement approach to the control system design.

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- Canonical form is a representation such that every object has a unique representation (with canonicalization being the process through which a representation is put into its canonical form).
- Canonical form can also mean a differential form that is defined in a natural (canonical) way.

Controllable Canonical Form (Example)

$$\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$$

- Let us Rewrite the given transfer function in following form

$$\frac{Y(s)}{U(s)} = \frac{0s^2+s+3}{s^2+3s+2}$$

$$a_2 = 2 \qquad a_1 = 3 \qquad b_2 = 3 \qquad b_1 = 1 \qquad b_o = 0$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=5JT_Dm7BZlo
Important Books/Journals for further learning including the page nos.:
➢ M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 582-589)

Course Faculty

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LECTURE HANDOUTS **L41**

EEE **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Effect of State Feedback

Introduction :

- The concept of controllability refers to the ability of a controller to arbitrarily alter the functionality of the system plant.
- The state-variable of a system, x, represents the internal workings of the system that can be separate from the regular input-output relationship of the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- State variable
- State functions

Detailed content of the Lecture:

Observability:

- The state-variables of a system might not be able to be measured for any of the following reasons: The location of the particular state variable might not be physically accessible (a capacitor or a spring, for instance).
- There are no appropriate instruments to measure the state variable, or the state-variable might be measured in units for which there does not exist any measurement device.
- The state-variable is a derived "dummy" variable that has no physical meaning
- A system with an initial state, {\displaystyle x(t_{0})} is observable if and only if the value of the initial state can be determined from the system output y(t) that has been observed through the time interval {\displaystyle t_{0}<t.
- If the initial state cannot be so determined, the system is unobservable.

Complete Observability:

- A system is said to be completely observable if all the possible initial states of the system can be observed. Systems that fail these criteria are said to be unobservable.

Detectability:

- A system is Detectable if all states that cannot be observed decay to zero asymptotically.

Constructability:

- A system is constructible if the present state of the system can be determined from the present and past outputs and inputs to the system.
- If a system is observable, then it is also constructible. The relationship does not work the other way around.
- A state x is unconstructable at a time t1 if for every finite time t < t1 the zero input response of the system is zero for all time t.
- A system is completely state constructible at time t1 if the only state x that is unconstructable at t0 is x = 0.

Observability Matrix:

- The observability of the system is dependent only on the system states and the system output, so we can simplify our state equations to remove the input terms.
- Therefore, we can show that the observability of the system is dependent only on the coefficient matrices A and C.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=FKSwcFOIjag

https://www.youtube.com/watch?v=S4_rIjCC70w

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 620-623)

Course Faculty

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LECTURE HANDOUTS

L42

EEE

III/V

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Effect of State Feedback

Introduction :

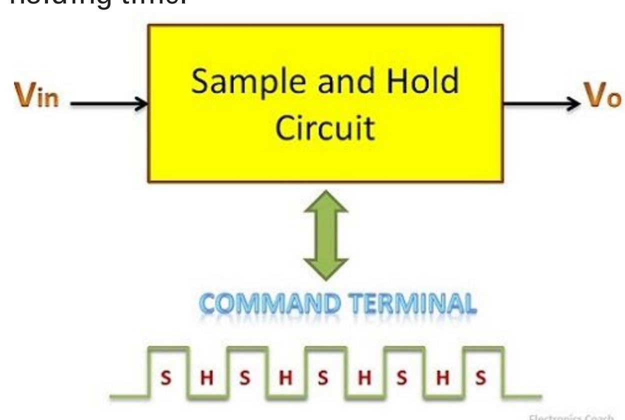
- The Sample and Hold circuit is an electronic circuit which creates the samples of voltage given to it as input, and after that, it holds these samples for the definite time.
- The time during which sample and hold circuit generates the sample of the input signal is called sampling time.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Sampling time
- Holding time

Detailed content of the Lecture:

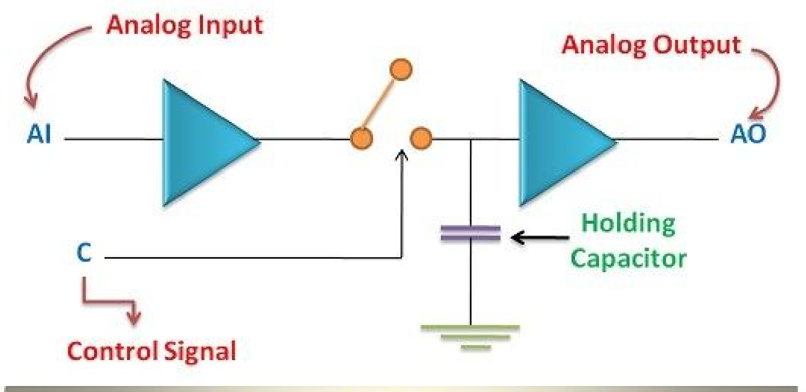
- The Sample and Hold **circui**t is an electronic circuit which creates the samples of voltage given to it as input, and after that, it holds these samples for the definite time.
- The time during which sample and hold circuit generates the sample of the input signal is called sampling time.
- Similarly, the time duration of the circuit during which it holds the sampled value is called holding time.



- Sampling time is generally between **1μs to 14 μs** while the holding time can assume any value as required in the application.
- It will not be wrong to say that capacitor is the heart of sample and hold circuit.
- This is because the capacitor present in it charges to its peak value when the switch is opened, i.e. during sampling and holds the sampled voltage when the switch is closed.

Circuit Diagram of Sample and Hold Circuit:

- The diagram below shows the circuit of the sample and hold circuit with the help of an Operational Amplifier.
- It is evident from the circuit diagram that two OP-AMPS are connected via a switch.
- When the switch is closed sampling process will come into the picture and when the switch is opened holding effect will be there.

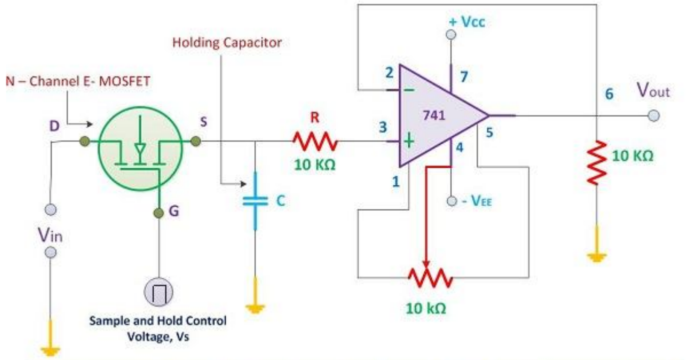


Circuit diagram of Sample and Hold Circuit

- The capacitor connected to the second operational amplifier is nothing but a holding capacitor.

Working of Sample and Hold Circuit:

- The working of sample and hold circuit can be easily understood with the help of working of its components.
- The main components which a sample and hold circuit involves is an N-channel Enhancement type MOSFET, a capacitor to store and hold the electric charge and a high precision operational amplifier.



Practical Circuit of Sample and Hold Circuit

- When the MOSFET acts as a closed switch, then the analogue signal applied to it through the drain terminal will be fed to the capacitor.
- The capacitor will then charge to its peak value. When the MOSFET switch is opened, then the capacitor stops charging.
- Due to the high impedance operational amplifier connected at the end of the circuit, the capacitor will experience high impedance due to this it cannot get discharged.
- This leads to the holding of the charge by the capacitor for the definite amount of time. This time can be referred as holding period. And the time in which samples of the input voltage is generated is called sampling period.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=za-Y2mT8zLg

https://www.youtube.com/watch?v=kXK3fpqeapU

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 669)

Course Faculty

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LECTURE HANDOUTS | **L43**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Solution of state and output equation controllable canonical form(Tutorial – I).

Introduction :

- The state model having minimum number of non-zero elements are called as canonical forms.
- So, we will find that in this particular form, the number of non-zero elements are minimum other elements are maximum 0's.

Prerequisite knowledge for Complete understanding and learning of Topic:

- State space
- Transfer function

Detailed content of the Lecture:

Controllable Canonical Form:

$$\frac{Y(s)}{U(s)} = \frac{b_o s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

The following state-space representation is called a controllable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

Example Consider the following state equations

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) + x_3(t) + 4u(t) ,\\ \dot{x}_2(t) &= -3x_1(t) + 2u(t) ,\\ \dot{x}_3(t) &= -5x_1(t) + x_2(t) + u(t) ,\\ y(t) &= x_1(t) ,\end{aligned}$$

and determine the observable canonical form.

Solution: Using the state equations (23), (24), (25), and (26), we write the following high order differential equation:

$$\frac{d^3}{dt^3}y(t) + \frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 3y(t) = 4\frac{d^2}{dt^2}u(t) + \frac{d}{dt}u(t) + 2u(t) .$$

We introduce $x_1(t) = y(t)$ in the equation, and collect all terms without differentiation on the right hand side, we get

$$\frac{d^3}{dt^3}x_1(t) + \frac{d^2}{dt^2}x_1(t) + \frac{d}{dt}x_1(t) - \frac{d^2}{dt^2}u(t) - \frac{d}{dt}u(t) = -3x_1(t) + 2u(t) ,$$

i.e.,

$$\frac{d}{dt}\left[\frac{d^2}{dt^2}x_1(t) + \frac{d}{dt}x_1(t) + x_1(t) - \frac{d}{dt}u(t) - u(t)\right] = -3x_1(t) + 2u(t) .$$

Now introduce the expression within the paranthesis as a new state variable

$$x_2(t) = \frac{d^2}{dt^2}x_1(t) + \frac{d}{dt}x_1(t) + 5x_1(t) - 4\frac{d}{dt}u(t) - u(t) ,$$

i.e.,

$$\dot{x}_2(t) = -3x_1(t) + 2u(t) .$$

Repeating this precedure yields

$$\frac{d}{dt}\left[\frac{d}{dt}x_1(t) + x_1(t) - 4u(t)\right] = x_2(t) - 5x_1(t) + u(t) ,$$

and we introduce

$$x_3(t) = \frac{d}{dt}x_1(t) + x_1(t) - 4u(t) ,$$

i.e.,

$$\dot{x}_1(t) = x_3(t) - x_1(t) + 4u(t) .$$

From (27), (28), and (29), we define the state-space form of

$$\dot{x}(t) = \begin{bmatrix}-1 & 0 & 1\\ -3 & 0 & 0\\ -5 & 1 & 0\end{bmatrix} x(t) + \begin{bmatrix}4\\ 2\\ 1\end{bmatrix} u(t) ,$$

$$y(t) = \begin{bmatrix}1 & 0 & 0\end{bmatrix} x(t) .$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=LKPErUCCnPA

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No: 617-618)

Course Faculty

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LECTURE HANDOUTS | **L44**

EEE | **III/V**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Concept of state variables(Tutorial - II)

Introduction :

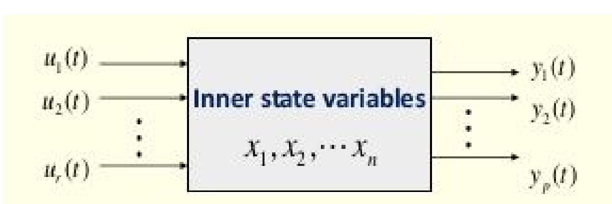
- A state variable is one of the set of variables that are used to describe the mathematical "state" of a dynamical system.
- Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- State
- System

Detailed content of the Lecture:

- State variables are used to represent the states of a general system.
- In this representation the state variables are real physical variables, which can be measured and used for manipulation or for control purposes.
- The approach generally adopted is to break the block diagram of the transfer function into subsystems in such a way that the physical variables can he identified.
- The governing equations for the subsystems can he used to identify the physical variables.



Determine the matrix exponential, and hence the state transition matrix, and the homogeneous response to the initial conditions $x_1(0) = 2$, $x_2(0) = 3$ of the system with state equations:

$$\begin{aligned} \dot{x}_1 &= -2x_1 + u \\ \dot{x}_2 &= x_1 - x_2. \end{aligned}$$

Solution: The system matrix is

$$\mathbf{A} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}.$$

From Eq. (9) the matrix exponential (and the state transition matrix) is

$$\begin{aligned} \mathbf{\Phi}(t) &= e^{\mathbf{A}t} \\ &= \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \ldots + \frac{\mathbf{A}^kt^k}{k!} + \ldots\right) \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} t + \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} \frac{t^2}{2!} \\ &\quad + \begin{bmatrix} -8 & 0 \\ 7 & -1 \end{bmatrix} \frac{t^3}{3!} + \ldots \\ &= \begin{bmatrix} 1 - 2t + \frac{4t^2}{2!} - \frac{8t^3}{3!} + \ldots & 0 \\ 0 + t - \frac{3t^2}{2!} + \frac{7t^3}{3!} + \ldots & 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \ldots \end{bmatrix}. \end{aligned}$$

The elements ϕ_{11} and ϕ_{22} are simply the series representation for e^{-2t} and e^{-t} respectively. The series for ϕ_{21} is not so easily recognized but is in fact the first four terms of the the expansion of $e^{-t} - e^{-2t}$. The state transition matrix is therefore

$$\mathbf{\Phi}(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix}$$

.

and the homogeneous response to initial conditions $x_1(0)$ and $x_2(0)$ is

$$\mathbf{x}_h(t) = \mathbf{\Phi}(t)\mathbf{x}(0)$$

or

$$\begin{aligned} x_1(t) &= x_1(0)e^{-2t} \\ x_2(t) &= x_1(0)\left(e^{-t} - e^{-2t}\right) + x_2(0)e^{-t}. \end{aligned}$$

With the given initial conditions the response is

$$\begin{aligned} x_1(t) &= 2e^{-2t} \\ x_2(t) &= 2\left(e^{-t} - e^{-2t}\right) + 3e^{-t} \\ &= 5e^{-t} - 2e^{-2t}. \end{aligned}$$

In general the recognition of the exponential components from the series for each element is difficult and is not normally used for finding a closed form for the state transition matrix.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=ibhwKT6l_Os

- https://www.youtube.com/watch?v=g9G8b7FxEHc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No:575-577)

Course Faculty

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LECTURE HANDOUTS | **L45**

EEE | **II/IV**

Course Name with Code:19EEC06 & CONTROL SYSTEMS

Course faculty : Dr. R. PRAKASH

Unit : **V-**STATE VARIABLE ANALYSIS **Date of Lecture:**

Topic of Lecture:
Concepts of controllability (Tutorial – III)

Introduction :

- The concept of controllability refers to the ability of a controller to arbitrarily alter the functionality of the system plant.
- The state-variable of a system, x, represents the internal workings of the system that can be separate from the regular input-output relationship of the system.

Prerequisite knowledge for Complete understanding and learning of Topic:

- State variable
- State functions

Detailed content of the Lecture:

Controllability Matrix :

- For LTI (linear time-invariant) systems, a system is reachable if and only if its controllability matrix, ζ, has a full row rank of p, where p is the dimension of the matrix A, and p × q is the dimension of matrix B.
- A system is controllable or "Controllable to the origin" when any state x1 can be driven to the zero state x = 0 in a finite number of steps.
- A system is controllable when the rank of the system matrix A is p, and the rank of the controllability matrix is equal.

Examples : Tank Problem: Model I.

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\alpha & 0 \\ \alpha & -\beta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$Q : [B : AB] = \begin{bmatrix} 1 & -\alpha \\ 0 & \alpha \end{bmatrix}$$

Rank $Q = 2 \quad \Longrightarrow$ System is controllable.

Model - 2 :

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\alpha & 0 \\ \alpha & -\beta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u.$$

$$Q = \begin{bmatrix} 0 & 0 \\ 1 & -\beta \end{bmatrix};$$

$\text{rank}(Q) = 1 \not< 2$

$\Longrightarrow$ System is not controllable.

Computation of Steering Control :

$$\begin{aligned} Cu &= w \\ CC^*v &= w \end{aligned}$$

where $u = C^*v$. The system is controllable iff

$$\begin{aligned} &C \text{ is onto.} \\ &\Longleftrightarrow C^* \text{is} 1-1. \\ &\Longleftrightarrow CC^* \text{is} 1-1. \\ &\Longleftrightarrow CC^* \text{is invertible.} \end{aligned}$$

If CC^* is invertible then

$$\begin{aligned} v &= (CC^*)^{-1}w \\ u &= C^*(CC^*)^{-1}w \end{aligned}$$

Example: If $A(t) = \begin{bmatrix} a\cos^2 t - 1 & 1 - \frac{a \sin 2t}{2} \\ -1 - \frac{a\sin 2t}{2} & a\sin^2 t - 1 \end{bmatrix}$

the solution $x(t) = \begin{bmatrix} e^{(a-1)t}\cos t & e^{-t}\sin t \\ e^{(a-1)t}\sin t & e^{-t}\cos t \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

with $x(0) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

It can be seen that $x(t) \to \infty$ as $t \to \infty$, if $1 < a < 2$; even though the eigenvalues of $A(t)$ are $\frac{a-2}{2} \pm \sqrt{(\frac{2-a}{2})^2 - (2-a)}$ which have negative real part.

We give an example to show that instability criteria of the linear time invarying systems do not apply to linear time varying systems.

Consider the system $\dot{x}(t) = A(t)x(t)$ with

$$A(t) = \begin{bmatrix} (-\frac{11}{2}) + (\frac{15}{2})\sin 12t & (\frac{15}{2})\cos 12t \\ (\frac{15}{2})\cos 12t & (-\frac{11}{2}) - (\frac{15}{2})\sin 12t \end{bmatrix}.$$

The eigenvalues of $A(t)$ are 2 and -13 for all t. The eigenvalue 2 has a positive real part. However the state transition matrix of $A(t)$ in (9) is [?]

$$X(t,0) = \begin{bmatrix} \frac{1}{2}e^{-t}(\cos 6t + 3\sin 6t) & \frac{1}{6}e^{-t}(\cos 6t + 3\sin 6t) \\ +\frac{1}{2}e^{-10t}(\cos 6t - 3\sin 6t) & +\frac{1}{6}e^{-10t}(\cos 6t - 3\sin 6t) \\ \frac{1}{2}e^{-t}(3\cos 6t - \sin 6t) & \frac{1}{6}e^{-t}(3\cos 6t - \sin 6t) \\ -\frac{1}{2}e^{-10t}(3\cos 6t + \sin 6t) & +\frac{1}{6}e^{-10t}(3\cos 6t + \sin 6t) \end{bmatrix}$$

Clearly $\|X(t,0)\| < \infty$ for all t and $\|X(t,0)\| \to 0$ as $t \to \infty$. So the system is asymptotically stable.

We conclude from above examples that stability and instability of linear time varying systems cannot be determined from the eigenvalues of their system matrix $A(t)$.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=ibhwKT6l_Os
- https://www.youtube.com/watch?v=g9G8b7FxEHc

Important Books/Journals for further learning including the page nos.:

- M. Gopal, Control Systems, Principles and Design, Tata McGraw Hill, 2012. (Page No:617-620)

Course Faculty

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