MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L1

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.PUNITHA**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Signals - Classification of Signals – Continuous time and Discrete Time Signals

Introduction :

A signal is a function of one or more independent variables which contain some information.

Eg: Radio signal, TV signal, Telephone signal etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Engineering Mathematics I & II, Partial Differential Equation

Signals

A signal is a function of one or more independent variables which contain some information.

Classification of Signals

- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

CT & DT Signals

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by x(t).Eg: AC waveform, ECG etc.

Discrete time signals are defined at discrete instances of time. It is represented by x(n).

Eg: Amount deposited in a bank per month.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 1.1 -1.6, 1.53-1.84)

Course Faculty

Verified by HOD

L2

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Deterministic and random signal, even and odd signals, periodic and aperiodic signals, energy and power signals

Introduction :

- A signal is said to be periodic signal if it repeats at equal intervals. Aperiodic signals do not repeat at regular intervals.(CT & DT)

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT & DT

Deterministic and random signal

- Signals which can be defined exactly by a mathematical formula are known as deterministic signals. There is uncertainty with respect to its value at some instant of time.
- Non- deterministic signals are random in nature hence they are called random signals.

Even and Odd signals

- A CT signal is $x(t)$ is said to be an even signal if $x(t)=x(-t)$ and an odd signalif $x(-t)=-x(t)$.
- A DT signal $x(n)$ is said to be an even signal if $x(-n)=x(n)$ and an odd signalif $x(-n)=-x(n)$.

Periodic and Aperiodic signals.

- A signal is said to be periodic signal if it repeats at equal intervals. Aperiodic signals do not repeat at regular intervals.
- A CT signal which satisfies the equation $x(t) = x(t+T0)$ is said to be periodic and a DT signal which satisfies the equation $x(n) = x(n+N)$ is said to be periodic.

Energy and Power Signals.

The signal $x(t)$ is said to be power signal, if and only if the normalized average power p is finite and non-zero. ie. $0<p<4$

A signal $x(t)$ is said to be energy signal if and only if the total normalized energy is finite and non-zero. ie. $0<E<4$

Video Content / Details of website for further learning (if any): • https://nptel.ac.in/courses/117101055/ • http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html
Important Books/Journals for further learning including the page nos.: Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008. (Page no : 1.53–1.84)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L3

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Problems in Classification of signals
Introduction :   • A signal is said to be periodic signal if it repeats at equal intervals. Aperiodic signals do not repeat at regular intervals.(CT & DT)
Prerequisite knowledge for Complete understanding and learning of Topic: • Signals, CT & DT

Problems in Classification of signals

Ex.1 What is the periodicity of x(t) = $e^{j100\Pi t}$ + 30^{o}?

Here x(t) = $e^{j100\Pi t} + 30^{o}$

Comparing above equation with $e^{j\omega t+\Phi}$, we get ω = 100 Π. Therefore period T is given as,

$T=2\Pi/\omega = 2\Pi/100\Pi = 1/50 = 0.02$ sec.

Ex.2 What is the period T of the signal x(t) = 2cos (n/4)?

Here, x (n) = 2cos (n/4).

Compare x(n) with Acos (2∏ fn). This gives,

2∏fn = n/4 =>F =1/8 ∏. Which is not rational. Hence this is not periodic signal.

Ex.3 What is the periodicity of x(t) = $e^{j100\Pi t}$ + 30^{o} ?

Here x(t) = $e^{j100\Pi t} + 30^{o}$

Comparing above equation with $e^{j\omega t+\Phi}$, we get ω = 100 Π. Therefore period T is given as,

$T=2\Pi/\omega = 2\Pi/100\Pi = 1/50 = 0.02$ sec.

Ex.4 Find the fundamental period of the signal x(n)= 3 $e^{j3\Pi(n+1/2)}$

$X(n) = 3/5\, e^{j3\Pi n}. e^{j3\Pi/2}$

$= -j3/5\, e^{j3\Pi n}$

Here, ω=3Π, hence, f=3/2=k/N. Thus the fundamental period is N = 2.

Ex.5Find the fundamental period of the signal x(n) = 3 $e^{j3\Pi(n+1/2)}$

$X(n) = 3/5\, e^{j3\Pi n}. e^{j3\Pi/2}$

$= -j3/5\, e^{j3\Pi n}$

Here, ω=3Π, hence, f=3/2=k/N. Thus the fundamental period is N = 2.

Ex.6 Find the fundamental period T of the following signal.

Here the three frequency components are,

$2f_1\, n = n\Pi/2 \Rightarrow f_1 = 1/4$ therefore $N_1 = 4$

$2\Pi f_2 n = n\Pi/8 \Rightarrow f_2 = 1/16$ therefore $N_2 = 16$

$2\Pi f_3 n = n\Pi/4 \Rightarrow f_3 = 1/8$ therefore $N_3 = 4$

Here f_1, f_2 and f_3 are rational, hence individual signals are periodic.

The overall signal will also be periodic since $N_1/N_2 = 4/16 = 1/4$ and $N_2 / N_3 = 16/8 = 2$. The period of the signal will be least common multiple of N_1, N_2 and N_3 which is 16. Thus fundamental period, N = 16.

Ex.7 Find whether the following signal is periodic or not. x(n) = 5cos(6Πn)

Compare the give signal with, x(n) = A cos (2Πfn). We get, $2\Pi fn = 6\Pi n \Rightarrow f = 3$, which is rational. Hence this signal is periodic. Its period is given as, $F = k/N = 3/1 \Rightarrow N = 1$.

Ex.8 What is the periodicity of the signal x(t) = sin 100πt+ cos150πt?

Compare the given signal with,

$X(t) = \sin 2\pi f_1 t + \cos 2\pi f_2 t$

$\therefore \quad 2\pi f_1 t = 100\pi t \Rightarrow f_1 = 50 \therefore T_1 = \frac{1}{f1} = \frac{1}{50}$

$\therefore \quad 2\pi f_2 t = 150\pi t \Rightarrow f_2 = 75 \therefore T_2 = \frac{1}{f2} = \frac{1}{75}$

Since $\frac{T1}{T2} = \frac{1/50}{1/75} = \frac{3}{2}$ i.e. rational, the signal is periodic. The fundamental period will be, $T=2T_1 = 3T_2$, i.e. least common multiple of T_1 and T_2. Here $T=2T_1=3T_2=1/25$.

Ex.9 Determine energy of the discrete time signal.

$$x(n) = \begin{cases} (\frac{1}{2})^n, & n \geq 0 \\ 3^n, & n < 0 \end{cases}$$

Energy of the signal is given as,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{-1} 3^n + \sum_{n=0}^{\infty} (\frac{1}{2})^n$$

$$= \sum_{n=1}^{\infty} 3^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^n$$

Here $3^{-n} = (3^{-1})^n = (1/3)^n$. Hence above equation will be,

$$E = \sum_{n=1}^{\infty} (\frac{1}{3})^n + \sum_{n=0}^{\infty} (\frac{1}{2})^n$$

Here let us use $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$, for a < 1 for the second summation. i.e.,

$$E = \sum_{n=1}^{\infty} (\frac{1}{3})^n + \frac{1}{1-1/2}$$

$$= [\frac{1}{3} + (\frac{1}{3})^2 + (\frac{1}{3})^3 + (\frac{1}{3})^4 + \ldots\ldots] + 2$$

$$= 1/3[1 + (\frac{1}{3}) + (\frac{1}{3})^2 + (\frac{1}{3})^3 + \ldots\ldots] + 2$$

The term inside the brackets is geometric series which can be written as,

$1+a+a^2+a^3+\ldots \frac{1}{1-a}$, $|a| < 1$. Thus,

$$E = 1/3, \frac{1}{1-1/3} + 2$$

$$= ½ + 2 = 2.5$$

Ex.10 Determine whether the following signals are energy or power signals and evaluate their

normalized energy or power?

$i)\ x(n) = \left(\frac{1}{2}\right)^n u(n) \qquad ii)\ x(t) = rect\left(\frac{t}{T_o}\right)$

i) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

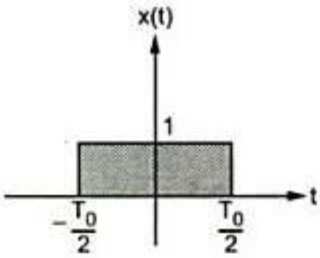
$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n\right]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \qquad \text{since } u(n) = 1 \text{ for } n = 0 \text{ to } \infty$$

Here use, $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ for $|a| < 1$. The above equation will be

$$E = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

ii) $x(t) = rect\left(\frac{t}{T_o}\right)$



$$rect\left(\frac{t}{T_o}\right) = \begin{cases} 1 & \text{for } -\frac{T_o}{2} \le t \le \frac{T_o}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} (1)^2\, dt$$

$$= [t]_{-T_o/2}^{T_o/2} = T_o$$

The energy is finite and non-zero. It is energy signal with $E = T_o$.

Ex.11 Consider a continuous time signal x(t) = δ(t+2)-δ(t-2)
Calculate the value of E_α for the signals.

$$\mathbf{y(t)} = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{t} [\delta(t+2) - \delta(t-2)])\, d\tau$$

Since $\int \delta(\tau) d(\tau)$ = u(t), above equation becomes,
y(t) = u(t+2) – u(t-2)

$\int_{-\infty}^{\infty} x \int_{-\infty}^{t} x(\tau)$
= 1 for $-2 \le t \le 2$
0 elsewhere.

E = $\int_{-\infty}^{\infty} x^2$ (t) dt

$= \int_{-\infty}^{\infty} y^2(t)\,dt = \int_{-2}^{2} 1^2 dt = 4$

Ex.12 *Find and sketch the even and odd components of the following :*

i) $x(n) = e^{-(n/4)} \ u(n)$

ii) $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

iii) $x(t) = \cos^2\left(\frac{\pi t}{2}\right)$

iv) $x(n) = \text{Im}[e^{jn\pi/4}]$

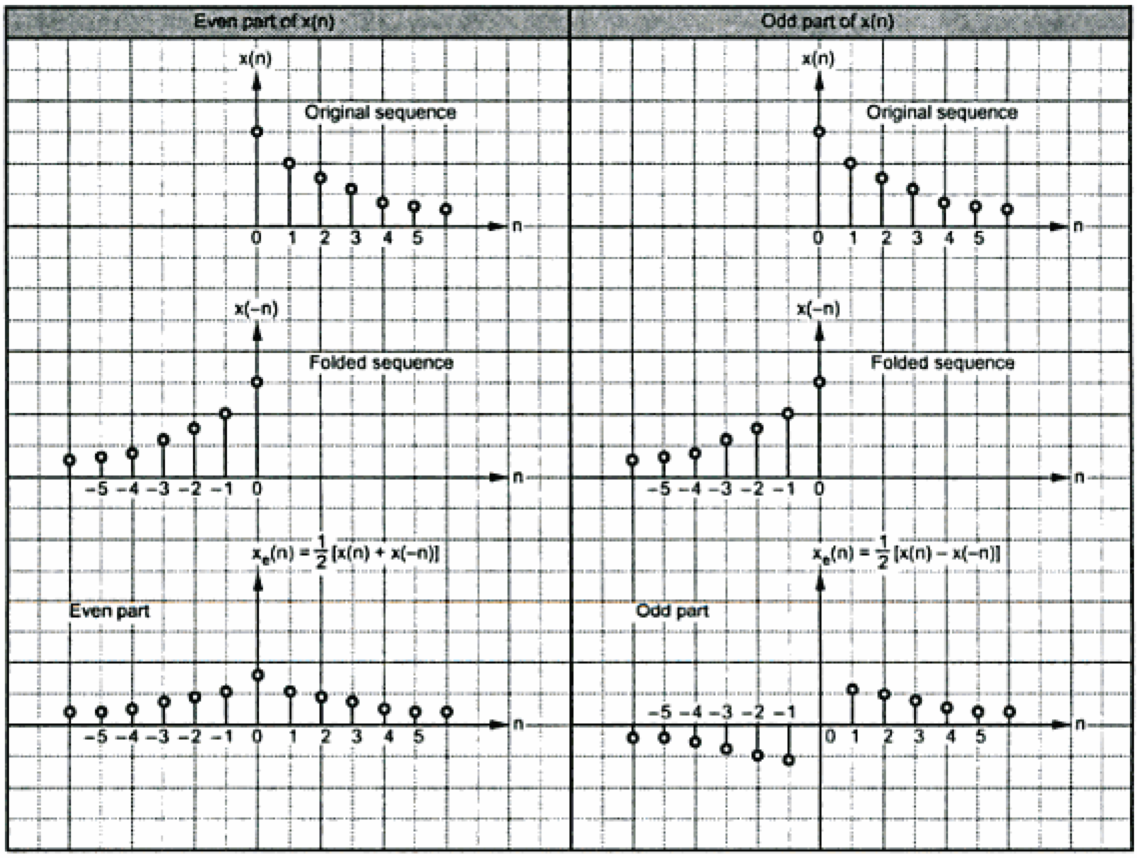
v) $x(t) = e^{jt}$ **(Jan./Feb.-2004, 6 Marks)**

Solution :i) $x(n) = e^{-(n/4)} \ u(n)$

Even and odd parts of the sequence $x(n)$ are given by equation (1.2.10) as,

Even part, $x_e(n) = \frac{1}{2}\{x(n) + x(-n)\}$ and

Odd part, $x_0(n) = \frac{1}{2}\{x(n) - x(-n)\}$



Ex.13 *Determine the energy or power as applicable for the following signals.*

i) $x(n) = e^{j\left(\frac{\pi n}{2}+\frac{\pi}{6}\right)}$ ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Ans. : i) $x(n) = e^{j\left(\frac{n\pi}{2}+\frac{\pi}{6}\right)}$

$$= \cos\left(\frac{n\pi}{2}+\frac{\pi}{6}\right)+j\sin\left(\frac{n\pi}{2}+\frac{\pi}{6}\right)$$

$$= \cos(2\pi f_1+\theta)+j\sin(2\pi f_2+\theta)$$

Here $f_1 = f_2 = \frac{1}{4}$ or $N = 4$

For the periodic signals power is given as,

$$P = \frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^2$$

Here $|x(n)| = \sqrt{\cos^2\left(\frac{n\pi}{2}+\frac{\pi}{6}\right)+\sin^2\left(\frac{n\pi}{2}+\frac{\pi}{6}\right)} = 1$

$\therefore$ $$P = \frac{1}{2\times 4+1}\sum_{n=-4}^{4}1$$

$$= 1$$

ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$

This is energy signal. Energy is given as,

$$E = \sum_{n=-\infty}^{\infty}|x(n)|^2$$

$$= \sum_{n=0}^{\infty}\left|\left(\frac{1}{3}\right)^n\right|^2 = \sum_{n=0}^{\infty}\left[\left(\frac{1}{3}\right)^2\right]^n = \sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 1.53 - 1.84)

Course Faculty

Verified by HOD

L4

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Basic Continuous –time and Discrete time signals- step, impulse, Ramp, Exponential, sinusoidal ,Exponentially damped sinusoidal signals, Pulse

Introduction :

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by x(t).

Eg: AC waveform, ECG etc.

Discrete time signals are defined at discrete instances of time. It is represented by x(n).

Eg: Amount deposited in a bank per month.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT & DT

Basic Continuous –time and Discrete time signals

Continuous time signals are defined for all values of time. It is also called as an analog signal and is represented by x(t).Eg: AC waveform, ECG etc.

Discrete time signals are defined at discrete instances of time. It is represented by x(n). Eg: Amount deposited in a bank per month.

Unit Step, Impulse, Ramp

Unit step function is defined as

$U(t) = 1$ for $t >= 0$

0 otherwise

Unit ramp function is defined as

$r(t) = t$ for $t>=0$

0 for $t<0$

Unit delta function is defined as

$\delta(t)= 1$ for $t=0$

Relation between Step, Ramp and Delta functions (CT).

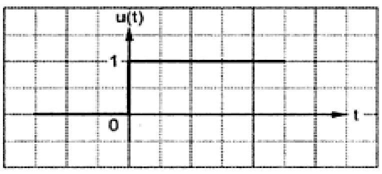
The relationship between unit step and unit delta function is

$\delta (t) = u(t)$

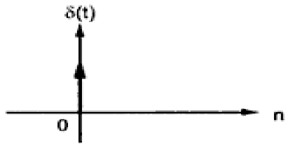
The relationship between delta and unit ramp function is

$\delta (t).dt = r(t)$

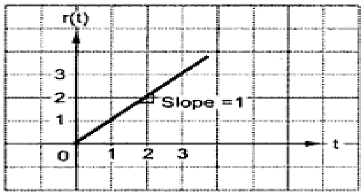
i. Unit Step signal



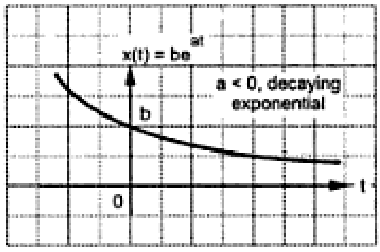
ii. Unit Impulse signal

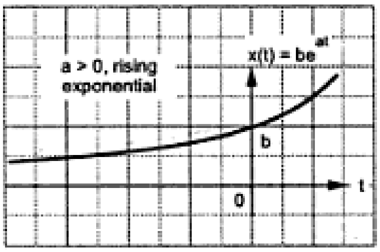


iii. Unit ramp Signal



iv. Complex Exponential Signal





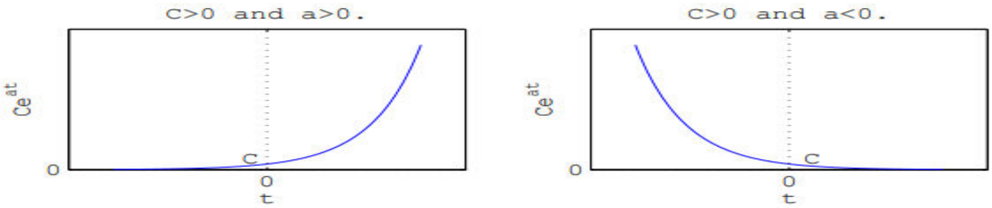
Exponential, sinusoidal ,Exponentially damped sinusoidal signals

Continuous-time complex exponential and sinusoidal signals:

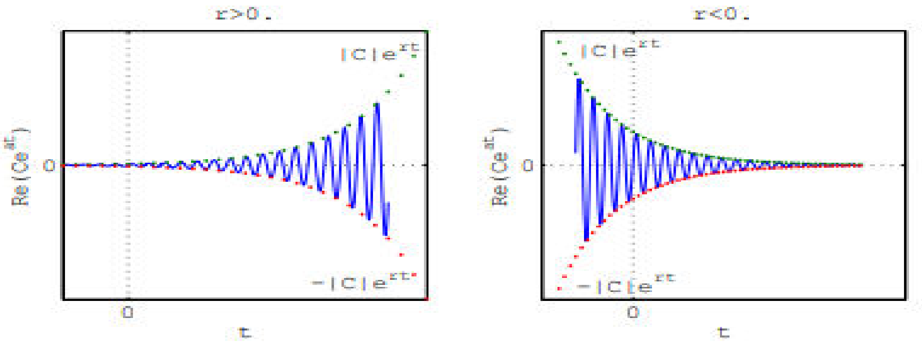
$$x(t) = Ce^{at}$$

where C and a are in general complex numbers.

Real exponential signals: C and a are reals.



- The case $a > 0$ represents exponential growth. Some signals in unstable systems exhibit exponential growth.
- The case $a < 0$ represents exponential decay. Some signals in stable systems exhibit exponential decay.



- If $r = 0$, the real and imaginary part are sinusoidals.
- If $r > 0$, the real and imaginary part are sinusoidals multiplied by a growing exponential. Such signals arise in unstable systems.
- If $r < 0$, the real and imaginary part are sinusoidals multiplied by a decaying exponential. Such signals arise in stable systems, for example, in RLC circuits, or in mass-spring-friction system, where the energy is dissipated due to the resistors, friction, etc.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no :1.7 – 1.28)

Course Faculty

Verified by HOD

L5

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Properties of Impulse Signal, Transformation of independent variables

Introduction :

Signals are two variable parameters in general:

- Amplitude
- Time

Prerequisite knowledge for Complete understanding and learning of Topic:

Signals &CT Signal , DT Signals

Properties of Impulse Signal

- $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$
- $\delta(t) = \delta(-t)$
- $\delta(t) = \frac{d}{dt}u(t)$, where $u(t)$ is the unit step.
- $f(t)\delta(t) = f(0)\delta(t)$

The last of these is especially important as it gives rise to the sifting property of the dirac delta function, which selects the value of a function at a specific time and is especially important in studying the relationship of an operation called convolution to time domain analysis of linear time invariant systems. The sifting property is shown and derived below.

Transformation of independent variables

- The resulting transformation of x(t) into y(t) is hence called an "affine transformation on the independent variable."
- All such transformations can be decomposed into just three fundamental types of signal transformations on the independent variable: time shift, time scaling, and time reversal.

Time Shifting

- Suppose that we have a signal x(t) and we define a new signal by adding/subtracting a finite time value to/from it. We now have a new signal, y(t).

- This means that the time-shifting operation results in the change of just the positioning of the signal without affecting its amplitude or span.

Time Scaling

- Basically, when we perform time scaling, we change the rate at which the signal is sampled. Changing the sampling rate of a signal is employed in the field of speech processing.
- A particular example of this would be a time-scaling-algorithm-based system developed to read text to the visually impaired.

Time Reversal

- Whenever the time in a signal gets multiplied by -1, the signal gets reversed.
- It produces its mirror image about Y or X-axis.
- This is known as Reversal of the signal. Reversal can be classified into two types based on the condition whether the time or the amplitude of the signal is multiplied by -1.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.

(Page no : 1.29-1.52)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 6

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Basic operations on signals-amplitude scaling

Introduction :

Signals are two variable parameters in general:

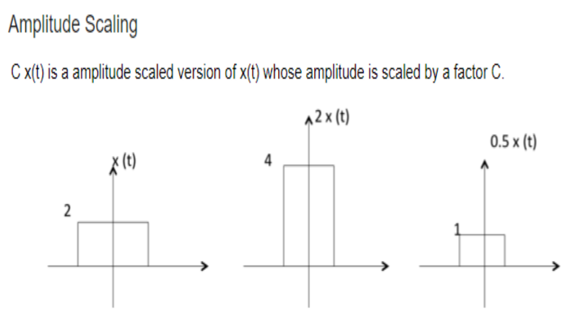
- Amplitude
- Time

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT, DT& Types of Signal

Basic operations on signals-amplitude scaling

Amplitude scaling is a very basic operation performed on signals to vary its strength. It can be mathematically represented as Y(t) = α X(t). α<1 → signal is attenuated. α>1 → signal is amplified.



Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html
- https://www.tutorialspoint.com/signals_and_systems/signals_basic_operations.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 1.37 -1.38)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L 7

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Problems in Scaling

Introduction :

Signals are two variable parameters in general:

- Amplitude
- Time

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT, DT& Types of Signal

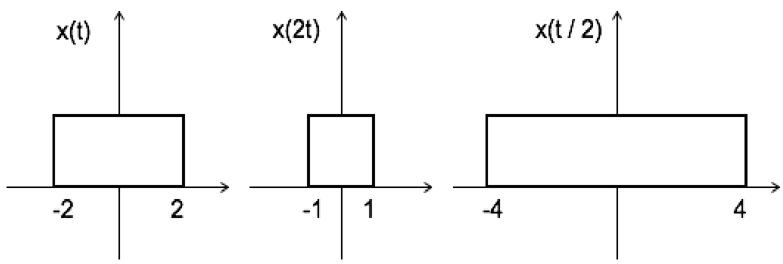
Problems in Scaling

Time Scaling

x(At) is time scaled version of the signal x(t). where A is always positive.

|A| > 1 [Math Processing Error] Compression of the signal

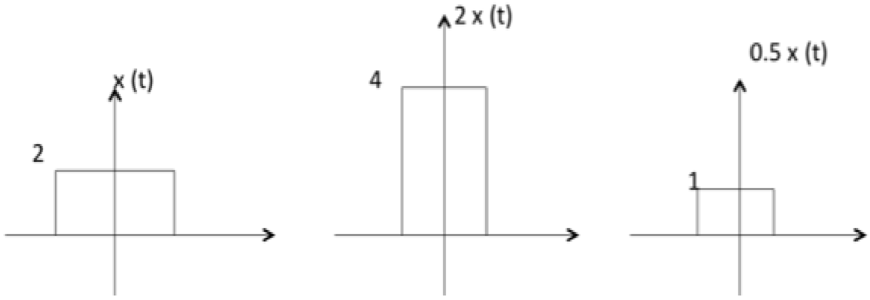
|A| < 1 [Math Processing Error] Expansion of the signal



Note: u(at) = u(t) time scaling is not applicable for unit step function.

Amplitude Scaling

C x(t) is a amplitude scaled version of x(t) whose amplitude is scaled by a factor C.



Video Content / Details of website for further learning (if any): • https://nptel.ac.in/courses/117101055/ • http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html • https://www.tutorialspoint.com/signals_and_systems/signals_basic_operations.htm
Important Books/Journals for further learning including the page nos.: Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008. (Page no : 1.37 -1.40)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L8

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Multiplication, Differentiation and Integration

Introduction :

Signals are two variable parameters in general:

- Amplitude
- Time

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT, DT& Types of Signal

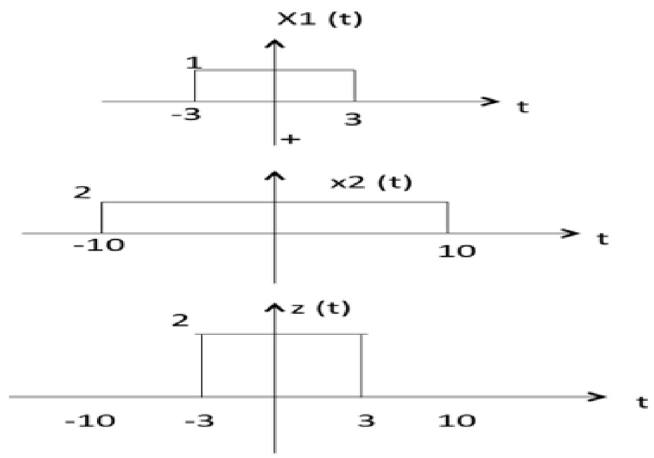
Multiplication, Differentiation and Integration

Multiplication operation performed over two discrete-time signals. ... Thus, we can conclude that the multiplication operation results in the generation of a signal whose values can be obtained by multiplying the corresponding values of the original signals

Integration is the counterpart of **differentiation**. If we **integrate** a **signal** x(t), the result y(t) is represented as ∫x(t) ∫ x (t) . Graphically, the act of **integration** computes the area under the curve of the original **signal**.

Multiplication

Multiplication of two signals is nothing but multiplication of their corresponding amplitudes. This can be best explained by the following example:



As seen from the diagram above,

-10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

-3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2

3 < t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

Video Content / Details of website for further learning (if any): • https://nptel.ac.in/courses/117101055/ • http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html • https://www.tutorialspoint.com/signals_and_systems/signals_basic_operations.htm
Important Books/Journals for further learning including the page nos.: Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008. (Page no :1.29-1.52)

Course Faculty

Verified by HOD

LECTURE HANDOUTS

L8

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Multiplication, Differentiation and Integration

Introduction :

Signals are two variable parameters in general:

- Amplitude
- Time

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, CT, DT& Types of Signal

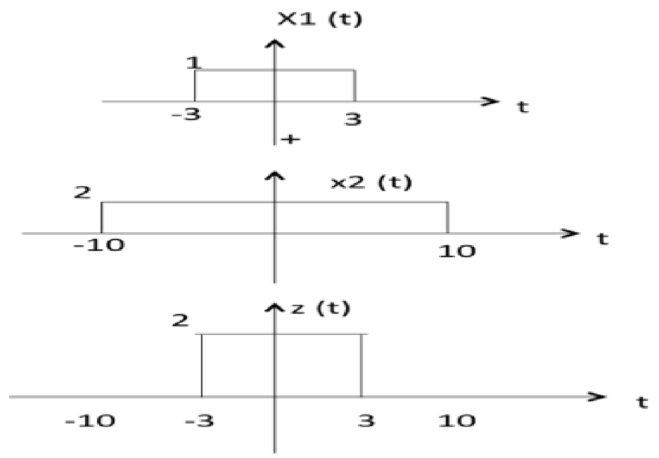
Multiplication, Differentiation and Integration

Multiplication operation performed over two discrete-time signals. ... Thus, we can conclude that the multiplication operation results in the generation of a signal whose values can be obtained by multiplying the corresponding values of the original signals

Integration is the counterpart of **differentiation**. If we **integrate** a **signal** x(t), the result y(t) is represented as ∫x(t) ∫ x (t) . Graphically, the act of **integration** computes the area under the curve of the original **signal**.

Multiplication

Multiplication of two signals is nothing but multiplication of their corresponding amplitudes. This can be best explained by the following example:



As seen from the diagram above,

$-10 < t < -3$ amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

$-3 < t < 3$ amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2

$3 < t < 10$ amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

Video Content / Details of website for further learning (if any): • https://nptel.ac.in/courses/117101055/ • http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html • https://www.tutorialspoint.com/signals_and_systems/signals_basic_operations.htm
Important Books/Journals for further learning including the page nos.: Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008. (Page no :1.29-1.52)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L9

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : I – Signals and Systems

Date of Lecture:

Topic of Lecture: Systems- Classification of systems

Introduction: A system is a set of elements or functional block that is connectedtogether and produces an output in response to an input signal.Eg: An audio amplifier, attenuator, TV set etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Systems- Classification of systems

There are many classifications of systems based on parameter used to classify them. They are

Linear, nonlinear systems -Time variant, time invariant systems -Stable, unstable systems Causal, non-causal systems - Continuous time, discrete time systems- Invertible and noninvertible systems- Dynamic and static systems

Linear, nonlinear systems

A linear system is one which satisfies the principle of **superposition** and **homogeneity or scaling**.Consider a linear system Ϭ characterized by the transformation operator T []. Let x_1, x_2 are the inputs applied to it and y_1, y_2 are the outputs.

$y_1 = T[x_1]$, $y2 = T[x_2]$

Principle of homogeneity: $T[a*x_1] = a*y_1$, $T[b*x_2] = b*y_2$

Principle of superposition: $T[x_1] + T[x_2] = a*y_1+b*y_2$

Linearity: $T[a*x_1] + T[b*x_2] = a*y_1+b*y_2$

Where a, b are constants.

Linearity ensures regeneration of input frequencies at output. Nonlinearity leads to generation of new frequencies in the output different from input frequencies. Most of the control theory is devoted to explore linear systems.

Time variant, time invariant systems

A system is said to be time variant system if its **response varies with time.** If the system response to an input signal does not change with time such system is termed as time invariant system. The behavior and characteristics of time variant system are fixed over time.In time invariant systems if input is delayed by time t_0 the output will also gets delayed by t_0. Mathematically it is specified as follows

$$y(t-t_0) = T[x(t-t_0)]$$

For a discrete time invariant system the condition for time invariance can be formulated mathematically by replacing t as n*Ts is given as

$$y(n-n_0) = T[x(n-n_0)]$$

Where n_0 is the time delay. Time invariance minimizes the complexity involved in the analysis of systems. Most of the systems in practice are time invariant systems.

Stable, unstable systems

Most of the control system theory involves estimation of stability of systems. Stability is an

important parameter which determines its applicability. Stability of a system is formulated in bounded input bounded output sense i.e. a **system is stable if its response is bounded for a bounded input** (bounded means finite).
An unstable system is one in which the **output of the system is unbounded for a bounded inpu**t. The response of an unstable system diverges to infinity.

Causal, non-causal systems
The principle of causality states that the output of a system always succeeds input. A system for which the principle of causality holds is defined as causal system. If an input is applied to a system at time t=0 s then the **output of a causal system is zero for t<0**. If the output depends on present and past inputs then system is casual otherwise non casual.

Continuous time, discrete time systems
A **system which deals with continuous time signals** is known as continuous time system. For such a system the outputs and inputs are continuous time signals.
Discrete time system **deals with discrete time signals**. For such a system the outputs and inputs are discrete time signals.

Invertible and non-invertible systems
A system is said to be invertible ***if distinct inputs lead to distinct outputs***. For such a system there exists an inverse transformation (inverse system) denoted by $T^{-1}[]$ which maps the outputs of original systems to the inputs applied. Accordingly we can write

$$TT^{-1} = T^{-1}T = I$$

Where I = 1 one for single input and single output systems.
A non-invertible system is one in **which distinct inputs leads to same outputs**. For such a system an inverse system will not exist.

Dynamic and static systems
In static system the **outputs at present instant depends only on present inputs**. These systems are also called as memory less systems as the system output at give time is dependent only on the inputs at that same time.
Dynamic systems are those in which the **output at present instant depends on past inputs and past outputs**. These are also called as systems with memory as the system output needs to store information regarding the past inputs or outputs.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- https://nptel.ac.in/courses/108104100/
- http://www.ee.nchu.edu.tw/pic/courseitem/1438_chapter1.pdf
- http://ecetutorials.com/signals-systems/classification-of-systems/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html
- https://www.tutorialspoint.com/signals_and_systems/systems_classification.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 2.1 – 2.46)

Course Faculty

Verified by HOD

L10

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Static & Dynamic, Linear & Nonlinear

Introduction: A system is a set of elements or functional block that is connectedtogether and produces an output in response to an input signal.Eg: An audio amplifier, attenuator, TV set etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Static & Dynamic, Linear & Nonlinear

Dynamic and static systems

In static system the **outputs at present instant depends only on present inputs**. These systems are also called as memory less systems as the system output at give time is dependent only on the inputs at that same time.

Dynamic systems are those in which the **output at present instant depends on past inputs and past outputs**. These are also called as systems with memory as the system output needs to store information regarding the past inputs or outputs.

Linear, nonlinear systems

A linear system is one which satisfies the principle of **superposition** and **homogeneity or scaling**.Consider a linear system Ϭ characterized by the transformation operator T []. Let x_1, x_2 are the inputs applied to it and y_1, y_2 are the outputs.

$y_1 = T[x_1]$, $y2 = T[x_2]$

Principle of homogeneity: $T [a*x_1] = a*y_1$, $T [b*x_2] = b*y_2$

Principle of superposition: $T [x_1] + T [x_2] = a*y_1+b*y_2$

Linearity: $T [a*x_1] + T [b*x_2] = a*y_1+b*y_2$

Where a, b are constants.

Linearity ensures regeneration of input frequencies at output. Nonlinearity leads to generation of new frequencies in the output different from input frequencies. Most of the control theory is devoted to explore linear systems.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- https://nptel.ac.in/courses/108104100/
- http://www.ee.nchu.edu.tw/pic/courseitem/1438_chapter1.pdf
- http://ecetutorials.com/signals-systems/classification-of-systems/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-

ramesh.html

- https://www.tutorialspoint.com/signals_and_systems/systems_classification.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.

(Page no : 2.1 – 2.11)

Course Faculty

Verified by HOD

L11

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture: Time-variant & Time-invariant

Introduction: A system is a set of elements or functional block that is connectedtogether and produces an output in response to an input signal.Eg: An audio amplifier, attenuator, TV set etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Time-variant & Time-invariant

A system is said to be time variant system if its **response varies with time.** If the system response to an input signal does not change with time such system is termed as time invariant system. The behavior and characteristics of time variant system are fixed over time.In time invariant systems if input is delayed by time t_0 the output will also gets delayed by t_0. Mathematically it is specified as follows

$$y(t-t_0) = T[x(t-t_0)]$$

For a discrete time invariant system the condition for time invariance can be formulated mathematically by replacing t as n*Ts is given as

$$y(n-n_0) = T[x(n-n_0)]$$

Where n_0 is the time delay. Time invariance minimizes the complexity involved in the analysis of systems. Most of the systems in practice are time invariant systems.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- https://nptel.ac.in/courses/108104100/
- http://www.ee.nchu.edu.tw/pic/courseitem/1438_chapter1.pdf
- http://ecetutorials.com/signals-systems/classification-of-systems/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html
- https://www.tutorialspoint.com/signals_and_systems/systems_classification.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 2.12 – 2.17)

Course Faculty

Verified by HOD

L12

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS/ 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **I – Signals and Systems**

Date of Lecture:

Topic of Lecture :Causal & Non causal, Stable & Unstable

Introduction: A system is a set of elements or functional block that is connected together and produces an output in response to an input signal.Eg: An audio amplifier, attenuator, TV set etc.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Causal, non-causal systems

The principle of causality states that the output of a system always succeeds input. A system for which the principle of causality holds is defined as causal system. If an input is applied to a system at time t=0 s then the **output of a causal system is zero for t<0**. If the output depends on present and past inputs then system is casual otherwise non casual.

Stable, unstable systems

Most of the control system theory involves estimation of stability of systems. Stability is an important parameter which determines its applicability. Stability of a system is formulated in bounded input bounded output sense i.e. a **system is stable if its response is bounded for a bounded input** (bounded means finite).

An unstable system is one in which the **output of the system is unbounded for a bounded input**. The response of an unstable system diverges to infinity.

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/courses/117101055/
- https://nptel.ac.in/courses/108104100/
- http://www.ee.nchu.edu.tw/pic/courseitem/1438_chapter1.pdf
- http://ecetutorials.com/signals-systems/classification-of-systems/
- http://ecerelatedbooks.blogspot.com/2018/03/signals-and-system-full-book-by-ramesh.html
- https://www.tutorialspoint.com/signals_and_systems/systems_classification.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 2.3 – 2.6, 2.17-2.19)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Fourier Series Analysis- Trigonometric Fourier Series

Introduction: Signals can be represented using complex exponentials – continuous-time and discrete-time Fourier series and transform. · If the input to an LTI system is expressed as a linear combination of periodic complex exponentials or sinusoids, the output can also be expressed in this form.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Fourier Series Analysis- Trigonometric Fourier Series

The theory derived for LTI convolution, used the concept that any input signal can represented as a linear combination of shifted impulses (for either DT or CT signals) . These are known as CT-FS The bases are scaled and shifted sinusoidal signals, which can be represented as complex exponentials.

Periodic Signals & Fourier Series:

A periodic signal has the property x(t) = x(t+T), T is the fundamental period, w_0 = 2p/T is the fundamental frequency. Two periodic signals include:

$$x(t) = \cos(\omega_0 t)$$
$$x(t) = e^{j\omega_0 t}$$

For each periodic signal, the Fourier basis the set of harmonically related complex exponentials:

$$\phi_k(t) = e^{jk\omega_0 t} = e^{jk(2\pi/T)t} \qquad k = 0, \pm1, \pm2, \ldots$$

Thus the Fourier series is of the form: $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$

k=0 is a constant

k=+/-1 are the fundamental/first harmonic components

k=+/-N are the N^{th} harmonic components

Fourier Series Representation of a CT Periodic Signal:

Given that a signal has a Fourier series representation, we have to find $\{a_k\}_k$. Multiplying through by $e^{-jn\omega_0 t}$

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} e^{-jn\omega_0 t}$$

$$\int_0^T x(t)e^{-jn\omega_0 t}dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t}dt$$

Using Euler's formula for the complex exponential integral

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \int_0^T \cos((k-n)\omega_0 t) dt + j\int_0^T \sin((k-n)\omega_0 t) dt$$

It can be shown that

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \begin{cases} T & k = n \\ 0 & k \neq n \end{cases}$$

Therefore

$$a_n = \tfrac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$$

which allows us to determine the coefficients. Also note that this result is the same if we integrate over any interval of length T (not just [0,T]), denoted by

$$a_n = \tfrac{1}{T}\int_T x(t) e^{-jn\omega_0 t} dt$$

$$\int_T |x(t)| dt < \infty$$

To summarise, if x(t) has a Fourier series representation, then the pair of equations that defines the Fourier series of a periodic, continuous-time signal:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$$

$$a_k = \tfrac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt = \tfrac{1}{T}\int_T x(t) e^{-jk(2\pi/T)t} dt$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- http://www.gvpcew.ac.in/Material%203%20Units/2%20ECE%20SS.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no :5.1-5.3)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L14

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Polar Fourier Series Representation

Introduction: Signals can be represented using complex exponentials – continuous-time and discrete-time Fourier series and transform. · If the input to an LTI system is expressed as a linear combination of periodic complex exponentials or sinusoids, the output can also be expressed in this form.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals

Polar Fourier Series Representation

Example 1: Fourier Series $\sin(w_0t)$

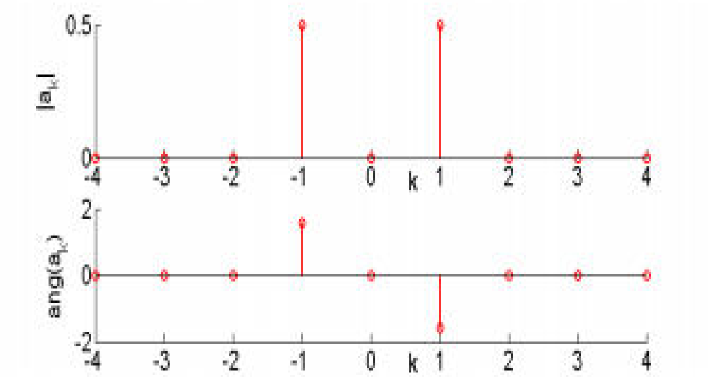
The fundamental period of $\sin(w_0t)$ is w_0

By inspection we can write:

$$\sin(\omega_0 t) = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

So $a_1 = 1/2j$, $a_{-1} = -1/2j$ and $a_k = 0$ otherwise

The magnitude and angle of the Fourier coefficients are:



Convergence of Fourier Series:

Not every periodic signal can be represented as an infinite Fourier series, however just about all interesting signals can be (note that the step signal is discontinuous)

The **Dirichlet conditions** are necessary and sufficient conditions on the signal.

1. Over any period, x(t) must be absolutely integrable. $\int_T |x(t)|dt < \infty$

2. In any finite interval, x(t) is of bounded variation; that is there is no more than a finite number of maxima and minima during any single period of the signal
3. In any finite interval of time, there are only a finite number of discontinuities. Further, each of these discontinuities are finite.

Complex exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}, \; t_0 \le t \le t_0 + T_0$$

$$\text{where } X_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}$$

Trignometic Form:

The complex exponential Fourier series can be arranged as follows

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$= X_0 + \sum_{n=1}^{\infty}\left[X_n e^{jn\omega_0 t} + X_{-n} e^{-jn\omega_0 t}\right]$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- http://www.gvpcew.ac.in/Material%203%20Units/2%20ECE%20SS.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no :5.4-5.5)

Course Faculty

Verified by HOD

L15

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Exponential Form of Fourier Series

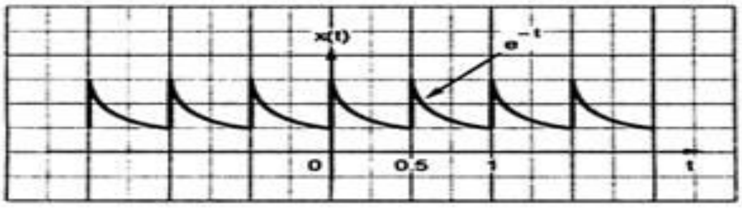
Introduction: The trigonometric Fourier series is a periodic function of period T0 = 2π/ω0. If the function g(t) is periodic with period T0, then a Fourier series representing g(t) over an interval T0 will also represent g(t) for all t

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series

Analysis of Continuous Time Signals Using TFS

Find trigonometric Fourier series for the periodic signal shown in



Solution : Here period $T = 0.5$ and $x(t) = e^{-t}$ over one period.

Step 1 : To calculate a(0).

$$a(0) = \frac{1}{T}\int_{<T>} x(t)\,dt, \text{ By equation (3.2.1)}$$

$$= \frac{1}{0.5}\int_0^{0.5} e^{-t}\,dt$$

$$= \frac{1}{0.5}[-e^{-t}]_0^{0.5} = 0.7869$$

Step 2 : To calculate *a(k)*

$$a(k) = \frac{2}{T}\int_{<T>} x(t)\cos k\,\omega_0\,t\,dt \text{ By equation (3.2.1)}$$

Here $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$

Hence, $a(k) = \frac{2}{0.5}\int_0^{0.5} x(t)\cos(k\cdot 4\pi t)\,dt$

$$= 4\int_0^{0.5} e^{-t}\cos(4\pi k t)\,dt$$

Here we will use $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$ with $a = -1$ and $b = 4\pi k$.

Then above equation will be,

$$a(k) = 4\left\{\frac{e^{-t}}{1+(4\pi k)^2}[(-1)\cos(4\pi k)t+(4\pi k)\sin(4\pi k)t]\right\}_0^{0.5}$$

$$= 4\left\{\frac{e^{-0.5}}{1+(4\pi k)^2}[-\cos(4\pi k)0.5+4\pi k\sin(4\pi k)0.5]\right.$$

$$\left.-\frac{e^{0}}{1+(4\pi k)^2}[-\cos(4\pi k)0+4\pi k\sin(4\pi k)\cdot 0]\right\}$$

$$= \frac{4}{1+(4\pi k)^2}\{0.606[-\cos(2\pi k)+4\pi k\sin(2\pi k)]$$

$$-[-\cos(0)+4\pi k\sin(0)]\}$$

$$= \frac{4}{1+(4\pi k)^2}\{-0.606+0+1+0\}$$

$$= \frac{1.576}{1+(4\pi k)^2}$$

Step 3 : To calculate *b(k)*.

$$b(k) = \frac{2}{T}\int_{<T>} x(t)\sin k\omega_0 t\,dt \quad \text{By equation (3.2.1)}$$

$$= \frac{2}{0.5}\int_0^{0.5} e^{-t}\sin(k\cdot 4\pi t)\,dt$$

$$= 4\int_0^{0.5} e^{-t}\sin(4\pi k t)\,dt$$

$$= \frac{6.32\pi k}{1+(4\pi k)^2}$$

Step 4 : To obtain Fourier series.

Putting the expressions for *a(0)*, *a(k)* and *b(k)* in equation (3.2.1),

$$x(t) = 0.7869+\sum_{k=1}^{\infty}\frac{1.576}{1+(4\pi k)^2}\cos k\,\omega_0\,t+\sum_{k=1}^{\infty}\frac{6.32\pi k}{1+(4\pi k)^2}\sin k\,\omega_0\,t$$

Cosine Representation of x(t)

Half range cosine series (or) cosine series	Half range sine series (or) sine series
Don't check for even or odd function, just subtitute the following formulae	
$b_n = 0$	$a_0 = a_n = 0$
$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\ell}$ where $a_0 = \frac{2}{\ell}\int_0^{\ell} f(x)dx$, $a_n = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\frac{n\pi x}{\ell}dx$	$f(x) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{\ell}$ where $b_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\frac{n\pi x}{\ell}dx$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- https://www.tutorialspoint.com/signals_and_systems/fourier_series_types.htm
- http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/Fourier-series-tutorial.pdf
- https://rmd.ac.in/dept/eie/notes/3/TPDE/unit2.pdf
- http://www.gvpcew.ac.in/Material%203%20Units/2%20ECE%20SS.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu“Signals And System”, Scitec, Fourth Edition, 2008. (Page no :5.6 – 5.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L16

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

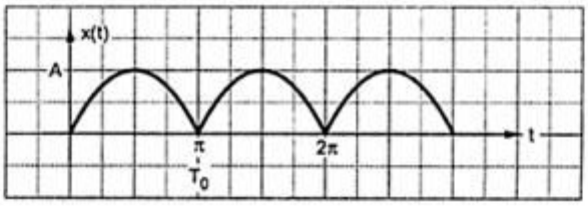
Topic of Lecture: Spectrum of Continuous Time (CT Signal)

Introduction: Cosine Fourier Series ,Exponential Fourier Series

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series,TFS

Spectrum of Continuous Time (CT Signal)
Determine cosine Fourier series of FWR



Solution :

Step 1 : Mathematical representation of waveform.

$$x(t) = A \sin t \quad \text{for } 0 \le t \le \pi$$

And $T_0 = T = \pi$ Therefore $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

Step 2 : To obtain $a(0)$

$$a(0) = \frac{1}{T} \int_{<T>} x(t)\, dt$$

$$= \frac{1}{\pi} \int_0^{\pi} A \sin t \, dt$$

$$= \frac{A}{\pi} \int_0^{\pi} \sin t \, dt$$

$$= \frac{A}{\pi} \cdot [-\cos t]_0^{\pi} = \frac{2A}{\pi}$$

Step 3 : To calculate *a(k)*.

$$a(k) = \frac{2}{T}\int_{<T>} x(t)\cos k\,\omega_0\, t\, dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\cos k\,2t\ dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\cos k\,2t\ dt$$

$$= \frac{2A}{\pi}\int_0^{\pi} \frac{\sin(t-2kt)+\sin(t+2kt)}{2}\, dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \sin(1-2k)t\, dt + \int_0^{\pi} \sin(1+2k)t\ dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{-\cos(1-2k)t}{1-2k}\right]_0^{\pi} + \left[\frac{-\cos(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{1-\cos(1-2k)\pi}{1-2k} + \frac{1-\cos(1+2k)\pi}{1+2k}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{2}{1-2k} + \frac{2}{1+2k}\right\}$$

$$= \frac{4A}{\pi(1-4k^2)}$$

Step 4 : To calculate *b(k)*.

$$b(k) = \frac{2}{T}\int_{<T>} x(t)\sin k\,\omega_0\, t\, dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\sin k\,2t\, dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\sin 2kt\, dt = \frac{2A}{\pi}\int_0^{\pi} \frac{\cos(t-2kt)-\cos(t+2kt)}{2}\, dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \cos(1-2k)t\, dt - \int_0^{\pi} \cos(1+2k)t\ dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{\sin(1-2k)t}{1-2k}\right]_0^{\pi} - \left[\frac{\sin(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= 0$$

Step 5 : To express Fourier series.

$$x(t) = a(0) + \sum_{k=1}^{\infty} a(k)\cos k\,\omega_0\, t + \sum_{k=1}^{\infty} b(k)\sin k\,\omega_0\, t$$

Putting values in above equation,

$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)}\cos k\,\omega_0\, t + 0$$

Obtain exponential Fourier series for the signal of Fig. 3.3.1 plot the magnitude and phase spectrum

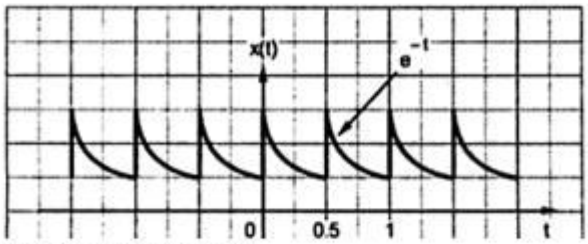


Fig.3.3.1

Solution : Step 1 : To obtain $X(k)$.

$$X(k) = \frac{1}{T}\int_{<T>} x(t) e^{-jk\omega_0 t}\, dt$$

Here $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$ and $x(t) = e^{-t}$ for 0 to 0.5 i.e.,

$$X(k) = \frac{1}{0.5}\int_0^{0.5} e^{-t} e^{-jk4\pi t}\, dt = 2\int_0^{t} e^{-(1+j4\pi k)t}\, dt$$

$$= 2\frac{1}{-(1+j4\pi k)}[e^{-(1+j4\pi k)t}]_0^{0.5} = -\frac{2}{1+j4\pi k}[e^{-(1+j4\pi k)0.5} - e^0]$$

$$= -\frac{2}{1+j4\pi k}[e^{-0.5}\cdot e^{-j2\pi k} - 1]$$

Here $e^{-j2\pi k} = \cos 2\pi k - j\sin 2\pi k = 1$ always.

Hence, $X(k) = -\frac{2}{1+j4\pi k}[0.606 - 1] = \frac{0.7869}{1+j4\pi k}$

Step 2 : To express exponential Fourier series.

Putting for $X(k)$ in synthesis equation of equation (3.2.3),

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{0.7869}{1+j4\pi k} e^{jk\omega_0 t}$$

Step 3 : To obtain magnitude and phase spectrum of $X(k)$.

We have $X(k) = \frac{0.7869}{1+j4\pi k}$ By equation (3.3.25)

$$= \frac{0.7869}{1+j4\pi k}\times\frac{1-j4\pi k}{1-j4\pi k} = \frac{0.7869(1-j4\pi k)}{1+(4\pi k)^2}$$

$$= \frac{0.7869}{1+(4\pi k)^2} - j\frac{0.7869\times 4\pi k}{1+(4\pi k)^2} \quad \ldots$$

$$\therefore \quad |X(k)| = \sqrt{\frac{(0.7869)^2}{[1+(4\pi k)^2]^2} + \frac{(0.7869\times 4\pi k)^2}{[1+(4\pi k)^2]^2}}$$

$$= \sqrt{\frac{(0.7869)^2 + (0.7869)^2(4\pi k)^2}{[1+(4\pi k)^2]^2}} = \sqrt{\frac{(0.7869)^2(1+(4\pi k)^2)}{[1+(4\pi k)^2]^2}}$$

$$\therefore \quad |X(k)| = \frac{0.7869}{\sqrt{1+(4\pi k)^2}}$$

And phase spectrum is given as,

$$\angle X(k) = \tan^{-1}\left[\frac{\text{Imaginary part of equation (3.3.26)}}{\text{Real part of equation (3.3.26)}}\right]$$

$$\therefore \quad \angle X(k) = -\tan^{-1}(4\pi k)$$

Following table lists the calculation of $(X(k))$ and $\angle X(k)$.

k	$\lvert X(k)\rvert = \frac{0.7869}{\sqrt{1+(4\pi k)^2}}$	$\angle X(k) = -\tan^{-1}(4\pi k)$ (in radians)
− 3	0.0208	1.5442
− 2	0.0312	1.5310
− 1	0.0624	1.491
0	0.7869	0
1	0.0624	− 1.491
2	0.0312	− 1.5310
3	0.0208	− 1.5442

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- https://www.tutorialspoint.com/signals_and_systems/fourier_series_types.htm
- http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/Fourier-series-tutorial.pdf
- http://www.gvpcew.ac.in/Material%203%20Units/2%20ECE%20SS.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no :5.27 – 5.29)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Properties of Fourier Series

Introduction: Fourier series using Analysis the Periodic Signal.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series,TFS

Properties of Fourier Series

i) The function x(t) should be within the interval T_o.

ii) The function x(t) should have finitie number of maxima and minima in the interval T_o.

iii) The function x(t) should have finite number of discontinuities in the interval T_0.

iv) The function should be absolutely integrable.

i.e., $\int_{<T0>} |x(t)dt < \infty$

Properties of FS

Property	Periodic function $x(t)$ with period $T = 2\pi/\Omega$	Fourier series C_k
Time shifting	$x(t \pm t_0)$	$C_k e^{\pm jk\Omega t_0}$
Time scaling	$x(\alpha t),\ \alpha > 0$	C_k with period $\frac{T}{\alpha}$
Differentiation	$\frac{d}{dt}x(t)$	$jk\Omega C_k$
Integration	$\int_{-\infty}^{t} x(t)dt < \infty$	$\frac{1}{jk\Omega}C_k$
Linearity	$\sum_i \alpha_i x_i(t)$	$\sum_i \alpha_i C_{ik}$
Conjugation	$x^*(t)$	C^*_{-k}
Time reversal	$x(-t)$	C_{-k}
Modulation	$x(t)e^{jK\Omega t}$	C_{k-K}
Multiplication	$x(t)y(t)$	$\sum_{i=-\infty}^{\infty} C_{xi}C_{y(k-i)}$
Periodic convolution	$\int_T x(\theta)y(t-\theta)d\theta$	$TC_{xk}C_{yk}$
Symmetry	$x(t) = x^*(t)$ real	$\begin{cases} C_k = C^*_{-k},\ \lvert C_k\rvert = \lvert C_{-k}\rvert, \\ \mathrm{Re}\, C_k = \mathrm{Re}\, C_{-k}, \\ \mathrm{Im}\, C_k = -\mathrm{Im}\, C_{-k}, \\ \arg C_k = -\arg C_{-k} \end{cases}$
	$x(t) = x^*(t) = x(-t)$ real and even	$\begin{cases} C_k = C_{-k},\ C_k = C^*_k, \\ \text{real and even} \end{cases}$
	$x(t) = x^*(t) = -x(-t)$ real and odd	$\begin{cases} C_k = -C_{-k},\ C_k = -C^*_k, \\ \text{imaginary and odd} \end{cases}$
Parseval's theorem	$\frac{1}{T}\int_{-T/2}^{T/2} \lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert C_k\rvert^2$	

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- https://www.tutorialspoint.com/signals_and_systems/fourier_series_types.htm
- http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/Fourier-series-tutorial.pdf
- http://www.gvpcew.ac.in/Material%203%20Units/2%20ECE%20SS.pdf
- https://link.springer.com/content/pdf/bbm%3A978-1-4020-4818-0%2F1.pdf
- https://www.tutorialspoint.com/signals_and_systems/fourier_series_properties.htm

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 5.23-5.27)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE
(An Autonomous Institution)
(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L18

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Problems in Fourier Series

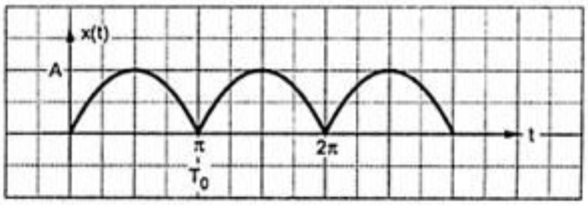
Introduction: $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$$X(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}\, d\omega$$

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series, Fourier Transform

Problems in Fourier Series



Solution :

Step 1 : Mathematical representation of waveform.

$x(t) = A \sin t \quad$ for $0 \le t \le \pi$

And $T_0 = T = \pi \quad$ Therefore $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

Step 2 : To obtain $a(0)$

$$a(0) = \frac{1}{T}\int_{<T>} x(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A \sin t\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin t\, dt$$

$$= \frac{A}{\pi}\cdot[-\cos t]_0^{\pi} = \frac{2A}{\pi}$$

Step 3 : To calculate *a(k)*.

$$a(k) = \frac{2}{T}\int_{<T>} x(t)\cos k\omega_0 t\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\cos k2t\;dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\cos k2t\;dt$$

$$= \frac{2A}{\pi}\int_0^{\pi} \frac{\sin(t-2kt)+\sin(t+2kt)}{2}\,dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \sin(1-2k)t\,dt + \int_0^{\pi} \sin(1+2k)t\;dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{-\cos(1-2k)t}{1-2k}\right]_0^{\pi} + \left[\frac{-\cos(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{1-\cos(1-2k)\pi}{1-2k} + \frac{1-\cos(1+2k)\pi}{1+2k}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{2}{1-2k} + \frac{2}{1+2k}\right\}$$

$$= \frac{4A}{\pi(1-4k^2)}$$

Step 4 : To calculate *b(k)*.

$$b(k) = \frac{2}{T}\int_{<T>} x(t)\sin k\omega_0 t\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\sin k2t\,dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\sin 2kt\,dt = \frac{2A}{\pi}\int_0^{\pi} \frac{\cos(t-2kt)-\cos(t+2kt)}{2}\,dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \cos(1-2k)t\,dt - \int_0^{\pi} \cos(1+2k)t\;dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{\sin(1-2k)t}{1-2k}\right]_0^{\pi} - \left[\frac{\sin(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= 0$$

Step 5 : To express Fourier series.

$$x(t) = a(0) + \sum_{k=1}^{\infty} a(k)\cos k\omega_0 t + \sum_{k=1}^{\infty} b(k)\sin k\omega_0 t$$

Putting values in above equation,

$$x(t) = \frac{2A}{\pi} + \sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)}\cos k\omega_0 t + 0$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- http://www.thefouriertransform.com/
- https://lpsa.swarthmore.edu/Fourier/Xforms/FTPairsProps.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.

(Page no : 5.34-5.37)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE
(An Autonomous Institution)
(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L19

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

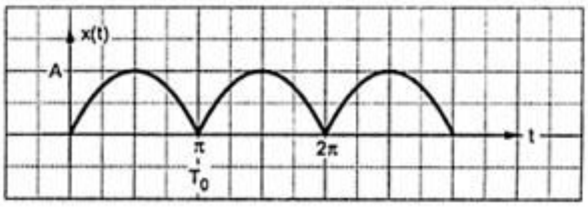
Topic of Lecture:Fourier Transform in CT Signal Analysis

Introduction: Cosine Fourier Series ,Exponential Fourier Series

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series,TFS

Fourier Transform in CT Signal Analysis



Solution :

Step 1 : Mathematical representation of waveform.

$$x(t) = A \sin t \quad \text{for } 0 \le t \le \pi$$

And $T_0 = T = \pi$ Therefore $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$

Step 2 : To obtain $a(0)$

$$a(0) = \frac{1}{T}\int_{<T>} x(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A \sin t\,dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \sin t\,dt$$

$$= \frac{A}{\pi}\cdot[-\cos t]_0^{\pi} = \frac{2A}{\pi}$$

Step 3 : To calculate *a(k)*.

$$a(k) = \frac{2}{T}\int_{<T>} x(t)\cos k\,\omega_0\,t\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\cos k\,2t\;dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\cos k\,2t\;dt$$

$$= \frac{2A}{\pi}\int_0^{\pi} \frac{\sin(t-2kt)+\sin(t+2kt)}{2}\,dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \sin(1-2k)t\,dt+\int_0^{\pi} \sin(1+2k)t\;dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{-\cos(1-2k)t}{1-2k}\right]_0^{\pi}+\left[\frac{-\cos(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{1-\cos(1-2k)\pi}{1-2k}+\frac{1-\cos(1+2k)\pi}{1+2k}\right\}$$

$$= \frac{A}{\pi}\left\{\frac{2}{1-2k}+\frac{2}{1+2k}\right\}$$

$$= \frac{4A}{\pi(1-4k^2)}$$

Step 4 : To calculate *b(k)*.

$$b(k) = \frac{2}{T}\int_{<T>} x(t)\sin k\,\omega_0\,t\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi} A\sin t\sin k\,2t\,dt \quad \text{since } \omega_0 = 2$$

$$= \frac{2A}{\pi}\int_0^{\pi} \sin t\sin 2kt\,dt = \frac{2A}{\pi}\int_0^{\pi} \frac{\cos(t-2kt)-\cos(t+2kt)}{2}\,dt$$

$$= \frac{A}{\pi}\left[\int_0^{\pi} \cos(1-2k)t\,dt-\int_0^{\pi} \cos(1+2k)t\;dt\right]$$

$$= \frac{A}{\pi}\left\{\left[\frac{\sin(1-2k)t}{1-2k}\right]_0^{\pi}-\left[\frac{\sin(1+2k)t}{1+2k}\right]_0^{\pi}\right\}$$

$$= 0$$

Step 5 : To express Fourier series.

$$x(t) = a(0)+\sum_{k=1}^{\infty} a(k)\cos k\,\omega_0\,t+\sum_{k=1}^{\infty} b(k)\sin k\,\omega_0\,t$$

Putting values in above equation,

$$x(t) = \frac{2A}{\pi}+\sum_{k=1}^{\infty} \frac{4A}{\pi(1-4k^2)}\cos k\,\omega_0\,t+0$$

Obtain exponential Fourier series for the signal of Fig. 3.3.1 plot the magnitude and phase spectrum

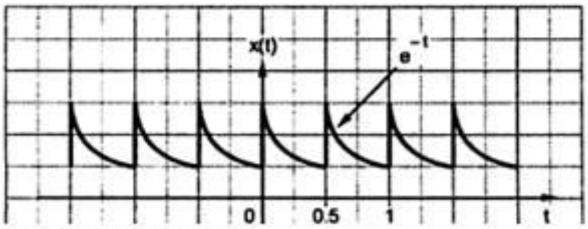


Fig.3.3.1

Solution : Step 1 : To obtain $X(k)$.

$$X(k) = \frac{1}{T}\int_{<T>} x(t) e^{-jk\omega_0 t}\, dt$$

Here $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi$ and $x(t) = e^{-t}$ for 0 to 0.5 i.e.,

$$X(k) = \frac{1}{0.5}\int_0^{0.5} e^{-t} e^{-jk4\pi t}\, dt = 2\int_0^{t} e^{-(1+j4\pi k)t}\, dt$$

$$= 2\frac{1}{-(1+j4\pi k)}[e^{-(1+j4\pi k)t}]_0^{0.5} = -\frac{2}{1+j4\pi k}[e^{-(1+j4\pi k)0.5} - e^0]$$

$$= -\frac{2}{1+j4\pi k}[e^{-0.5}\cdot e^{-j2\pi k} - 1]$$

Here $e^{-j2\pi k} = \cos 2\pi k - j\sin 2\pi k = 1$ always.

Hence, $\mathrm{X(k)} = -\frac{2}{1+j4\pi k}[0.606-1] \qquad = \frac{0.7869}{1+j4\pi k}$

Step 2 : To express exponential Fourier series.

Putting for $X(k)$ in synthesis equation of equation (3.2.3),

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{0.7869}{1+j4\pi k} e^{jk\omega_0 t}$$

Step 3 : To obtain magnitude and phase spectrum of $X(k)$.

We have $X(k) = \frac{0.7869}{1+j4\pi k}$ By equation (3.3.25)

$$= \frac{0.7869}{1+j4\pi k} \times \frac{1-j4\pi k}{1-j4\pi k} = \frac{0.7869(1-j4\pi k)}{1+(4\pi k)^2}$$

$$= \frac{0.7869}{1+(4\pi k)^2} - j\frac{0.7869\times 4\pi k}{1+(4\pi k)^2} \quad \ldots$$

$$\therefore \quad |X(k)| = \sqrt{\frac{(0.7869)^2}{[1+(4\pi k)^2]^2} + \frac{(0.7869\times 4\pi k)^2}{[1+(4\pi k)^2]^2}}$$

$$= \sqrt{\frac{(0.7869)^2 + (0.7869)^2(4\pi k)^2}{[1+(4\pi k)^2]^2}} = \sqrt{\frac{(0.7869)^2(1+(4\pi k)^2)}{[1+(4\pi k)^2]^2}}$$

$$\therefore \quad |X(k)| = \frac{0.7869}{\sqrt{1+(4\pi k)^2}}$$

And phase spectrum is given as,

$$\angle X(k) = \tan^{-1}\left[\frac{\text{Imaginary part of equation (3.3.26)}}{\text{Real part of equation (3.3.26)}}\right]$$

$$\therefore \quad \angle X(k) = -\tan^{-1}(4\pi k)$$

Following table lists the calculation of $(X(k))$ and $\angle X(k)$.

k	$\lvert X(k)\rvert = \frac{0.7869}{\sqrt{1+(4\pi k)^2}}$	$\angle X(k) = -\tan^{-1}(4\pi k)$ (in radians)
− 3	0.0208	1.5442
− 2	0.0312	1.5310
− 1	0.0624	1.491
0	0.7869	0
1	0.0624	− 1.491
2	0.0312	− 1.5310
3	0.0208	− 1.5442

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 6.1-6.5)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L20

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Conditions for the Existence of Fourier Transform

Introduction:

If a function f (t) is not a periodic and is defined on an infinite interval, we cannot represent it by Fourier series. It may be possible, however, to consider the function to be periodic with an infinite period.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series

Conditions for the Existence of Fourier Transform

$x(t) \overset{FT}{\leftrightarrow} X(f)$ or $x(t) \overset{FT}{\leftrightarrow} X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{jwt} \, d\omega$$

Sl.No.	Fourier series	Fourier transform
1.	Fourier series is calculated for periodic signals.	Fourier transform is calculated for nonperiodic as well as periodic signals.
2.	Expands the signal in time domain.	Represents the signal in frequency domain.
3.	Three types of Fourier series such as trigonometric, polar and complex exponential.	Fourier transform has no such types.

x(t)	**X(ω)**
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega$ (Synthesis)	$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$ (Analysis)
$\delta(t)$ (impulse)	1 (constant)
$\Pi(t) = \begin{cases} 0, & \|t\| > 1/2 \\ 1, & \|t\| \leq 1/2 \end{cases}$ (unit rectangular pulse, width=1)	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$ (sinc)
1 (constant)	$2\pi\delta(\omega)$ (impulse)
$e^{j\omega_0 t}$ (complex exponential)	$2\pi\delta(\omega-\omega_0)$ (shifted impulse)
$e^{-\alpha t}\gamma(t)$ (causal exponential)	$\frac{1}{j\omega+\alpha}$ (same as Laplace w/ s=jω)
$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{t^2}{2\sigma^2}}$ (Gaussian)	$e^{-\frac{\sigma^2\omega^2}{2}}$ (Gaussian)

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.

(Page no : 6.2-6.2)

Course Faculty

Verified by HOD

L21

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Frequency Spectrum using Fourier Transform

Introduction:

If a function f (t) is not a periodic and is defined on an infinite interval, we cannot represent it by Fourier series. It may be possible, however, to consider the function to be periodic with an infinite period.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series

Frequency Spectrum using Fourier Transform-

$x(t) \overset{FT}{\leftrightarrow} X(f)$ or $x(t) \overset{FT}{\leftrightarrow} X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}\, d\omega$$

Sl.No.	Fourier series	Fourier transform
1.	Fourier series is calculated for periodic signals.	Fourier transform is calculated for nonperiodic as well as periodic signals.
2.	Expands the signal in time domain.	Represents the signal in frequency domain.
3.	Three types of Fourier series such as trigonometric, polar and complex exponential.	Fourier transform has no such types.

x(t)	**X(ω)**
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\omega)e^{j\omega t}d\omega$ (Synthesis)	$X(\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$ (Analysis)
$\delta(t)$ (impulse)	1 (constant)
$\Pi(t)=\begin{cases}0, & \lvert t\rvert>1/2\\ 1, & \lvert t\rvert\le 1/2\end{cases}$ (unit rectangular pulse, width=1)	$\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$ (sinc)
1 (constant)	$2\pi\delta(\omega)$ (impulse)
$e^{j\omega_0 t}$ (complex exponential)	$2\pi\delta(\omega-\omega_0)$ (shifted impulse)
$e^{-\alpha t}\gamma(t)$ (causal exponential)	$\frac{1}{j\omega+\alpha}$ (same as Laplace w/ s=jω)
$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{t^2}{2\sigma^2}}$ (Gaussian)	$e^{-\frac{\sigma^2\omega^2}{2}}$ (Gaussian)

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- https://link.springer.com/content/pdf/bbm%3A978-1-4020-4818-0%2F1.pdf
- http://www.thefouriertransform.com/
- https://lpsa.swarthmore.edu/Fourier/Xforms/FTPairsProps.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.

(Page no : 6.8 – 6.8)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L22

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Properties of Fourier Transform

Introduction: $x(t) \overset{FT}{\leftrightarrow} X(f)$ or $x(t) \overset{FT}{\leftrightarrow} X(\omega)$

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series, Fourier Transform

Properties of Fourier Transform

$x(t) \overset{FT}{\leftrightarrow} X(f)$ or $x(t) \overset{FT}{\leftrightarrow} X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}\, d\omega$$

- **properties of continuous-time Fourier transform**

1. Linearity
2. Time reversal
3. Time scaling
4. Conjugation.
5. Parseval's relation
6. Differentiation
7. Integration
8. Convolution
9. Multiplication.

Name	Time Domain	Frequency Domain
Linearity	$\alpha \cdot x_1(t) + \beta \cdot x_2(t)$	$\alpha \cdot X_1(\omega) + \beta \cdot X_2(\omega)$
Time Scaling	$x(t/a)$	$aX(\omega a)$
Time Delay (or advance)	$f(t-a)$	$X(\omega)e^{-j\omega a}$
Complex Shift	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time Reversal	$x(-t)$	$X(-\omega)$
Convolution	$x(t) * h(t)$	$X(\omega)H(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(\omega)$
Integration	$\int_{-\infty}^{t} x(\lambda) d\lambda$	$\frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Time multiplication	$t^n x(t)$	$j^n \frac{d}{d\omega^n} X(\omega)$
Parseval's Theorem	$\text{Energy} = \int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 dt$	$\text{Energy} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \lvert X(\omega)\rvert^2 d\omega$
Duality	$X(t)$	$2\pi x(-\omega)$
	$\frac{1}{2\pi} X(-t)$	$x(\omega)$

Symmetry Properties

x(t)	X(ω)
x(t) is real	$X(\omega) = X^*(-\omega)$ $\mathrm{Re}(X(\omega)) = \mathrm{Re}(X(-\omega))$ $\mathrm{Im}(X(\omega)) = -\mathrm{Im}(X(-\omega))$ $\lvert X(\omega)\rvert = \lvert X(-\omega)\rvert$ $\angle X(\omega) = -\angle X(-\omega)$ Real part of $X(\omega)$ is even, imaginary part is odd
x(t) real, even	$X(\omega) = X(-\omega)$ $\mathrm{Im}(X(\omega)) = 0$ $X(\omega)$ is real and even
x(t) real, odd	$\mathrm{Re}(X(\omega)) = 0$ $\mathrm{Im}(X(\omega)) = -\mathrm{Im}(X(-\omega))$ $X(\omega)$ is imaginary and odd

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-13.pdf
- https://link.springer.com/content/pdf/bbm%3A978-1-4020-4818-0%2F1.pdf
- http://www.thefouriertransform.com/
- https://lpsa.swarthmore.edu/Fourier/Xforms/FTPairsProps.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 6.18-6.30)

Course Faculty

Verified by HOD

L23

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Laplace Transform in CT Signal Analysis- Properties of Region of Convergence-

Introduction: The Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems. The Laplace transform is particularly useful in solving linear ordinary differential equations such as those arising in the analysis of electronic circuits.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series, Fourier Transform

Laplace Transform in CT Signal Analysis- Properties of Region of Convergence-

Laplase Transform

L{f(t)}=F(s)

Unilateral Laplace Transform

$$\mathcal{L}_t\left[f(t)\right](s) \equiv \int_0^{\infty} f(t)\, e^{-st}\, dt,$$

bilateral Laplace Transform

$$\mathcal{L}_t^{(2)}\left[f(t)\right](s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

Inverse Laplace Transform

f(t)=L−1{F(s)}

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, ds$$

Number	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u_s(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{(s+a)}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
8	$1-e^{-at}$	$\frac{a}{s(s+a)}$
9	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
10	$be^{-bt}-ae^{-at}$	$\frac{(b-a)s}{(s+a)(s+b)}$
11	$\sin at$	$\frac{a}{s^2+a^2}$
12	$\cos at$	$\frac{s}{s^2+a^2}$
13	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
14	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
15	$1-e^{-at}(\cos bt+\frac{a}{b}\sin bt)$	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$

Number	Time Function	Laplace Transform	Property
1	$\alpha f_1(t)+\beta f_2(t)$	$\alpha F_1(s)+\beta F_2(s)$	Superposition
2	$f(t-T)u_s(t-T)$	$F(s)e^{-sT};\ \ T\geq 0$	Time delay
3	$f(at)$	$\frac{1}{a}F(\frac{s}{a});\ \ a>0$	Time scaling
4	$e^{-at}f(t)$	$F(s+a)$	Shift in frequency
5	$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$	First-order differentiation
6	$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0^-)-f^{(1)}(0^-)$	Second-order differentiation
7	$f^n(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\ldots-f^{(n-1)}(0)$	n^{th}-order differentiation
6	$\int_{0^-}^{t}f(\zeta)d\zeta$	$\frac{1}{s}F(s)$	Integration
7	$f(0^+)$	$\lim_{s\to\infty}sF(s)$	Post-initial value theorem
8	$\lim_{t\to\infty}f(t)$	$\lim_{s\to 0}sF(s)$	Final value theorem
9	$tf(t)$	$-\frac{dF(s)}{ds}$	Multiplication by time

Video Content / Details of website for further learning (if any): • https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf • https://lpsa.swarthmore.edu/LaplaceZTable/LaplaceZFuncTable.html • http://web.mit.edu/2.737/www/handouts/LaplaceTransforms.pdf • https://www.emathhelp.net/notes/differential-equations/laplace-transform/inverse-laplace-transform/
Important Books/Journals for further learning including the page nos.: Ramesh Babu“Signals And System”, Scitec, Fourth Edition, 2008. (Page no :7.1-7.9)

Course Faculty

Verified by HOD

L24

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **II – Analysis of Continuous Time Signals**

Date of Lecture:

Topic of Lecture: Properties of Laplace Transform

Introduction: The Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems. The Laplace transform is particularly useful in solving linear ordinary differential equations such as those arising in the analysis of electronic circuits.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Signals, Types of Signal, Classification of Signals, Fourier Series, Fourier Transform

Laplace Transform in CT Signal Analysis- Properties of Region of Convergence-

Laplase Transform

$$L\{f(t)\}=F(s)$$

Unilateral Laplace Transform

$$\mathcal{L}_t\left[f(t)\right](s) \equiv \int_0^\infty f(t)\, e^{-st}\, dt,$$

bilateral Laplace Transform

$$\mathcal{L}_t^{(2)}\left[f(t)\right](s) = \int_{-\infty}^\infty f(t)\, e^{-st}\, dt$$

Inverse Laplace Transform

$$f(t)=L^{-1}\{F(s)\}$$

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)\, e^{st}\, ds$$

Number	$f(t)$	$F(s)$
1	$\delta(t)$	1
2	$u_s(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	t^n	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{(s+a)}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
8	$1-e^{-at}$	$\frac{a}{s(s+a)}$
9	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
10	$be^{-bt}-ae^{-at}$	$\frac{(b-a)s}{(s+a)(s+b)}$
11	$\sin at$	$\frac{a}{s^2+a^2}$
12	$\cos at$	$\frac{s}{s^2+a^2}$
13	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
14	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
15	$1-e^{-at}(\cos bt+\frac{a}{b}\sin bt)$	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$

Number	Time Function	Laplace Transform	Property
1	$\alpha f_1(t)+\beta f_2(t)$	$\alpha F_1(s)+\beta F_2(s)$	Superposition
2	$f(t-T)u_s(t-T)$	$F(s)e^{-sT};\ \ T\geq 0$	Time delay
3	$f(at)$	$\frac{1}{a}F(\frac{s}{a});\ \ a>0$	Time scaling
4	$e^{-at}f(t)$	$F(s+a)$	Shift in frequency
5	$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$	First-order differentiation
6	$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0^-)-f^{(1)}(0^-)$	Second-order differentiation
7	$f^n(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\ldots-f^{(n-1)}(0)$	n^th^-order differentiation
6	$\int_{0^-}^{t} f(\zeta)d\zeta$	$\frac{1}{s}F(s)$	Integration
7	$f(0^+)$	$\lim_{s\to\infty} sF(s)$	Post-initial value theorem
8	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$	Final value theorem
9	$tf(t)$	$-\frac{dF(s)}{ds}$	Multiplication by time

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/nptel_data3/html/mhrd/ict/text/108104100/lec35.pdf
- https://lpsa.swarthmore.edu/LaplaceZTable/LaplaceZFuncTable.html
- http://web.mit.edu/2.737/www/handouts/LaplaceTransforms.pdf
- https://www.emathhelp.net/notes/differential-equations/laplace-transform/inverse-laplace-transform/

Important Books/Journals for further learning including the page nos.:

Ramesh Babu"Signals And System", Scitec, Fourth Edition, 2008. (Page no : 7.11-7.27)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 25

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : III– Linear Time-Invariant Continuous Time Systems

Date of Lecture:

Topic of Lecture: Differential Equation

Introduction: A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Fourier Transform And Series

1. Determine the complete response of the system

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\,y(t) = \frac{dx(t)}{dt}$$

$$\text{with } y(0)=0,\ \left.\frac{dy(t)}{dt}\right|_{t=0} = 1 \text{ and } x(t) = e^{-2t}\,u(t)$$

Solution

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\,y(t) = \frac{dx(t)}{dt}$$

The complete response will be given as,

$$y(t) = y^{(n)}(t) + y^{(p)}(t)$$

To determine $y^{(n)}(t)$ The given differential equation has order $N=2$.

$$r^2 + 5r + 4 = 0$$

Roots of this equation will be,

$$r_1 = -4 \text{ and } r_2 = -1$$

$$y^{(n)}(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$
$$= c_1 e^{-4t} + c_2 e^{-t}$$

To determine $y^{(p)}(t)$

The input is $x(t) = e^{-2t} u(t)$.

$$y^{(p)}(t) = k e^{-2t}$$

Hence putting $y(t) = y^{(p)}(t) = k e^{-2t}$

$$x(t) = e^{-2t} u(t)$$

$$\frac{d^2}{dt^2}\left(k e^{-2t}\right) + 5 \frac{d}{dt}\left(k e^{-2t}\right) + 4 \times k e^{-2t} = \frac{d}{dt}\left(e^{-2t}\right)$$

$$4 k e^{-2t} - 10 k e^{-2t} + 4 k e^{-2t} = -2 e^{-2t}$$
$$k = 1$$
$$y^{(p)}(t) = e^{-2t}$$

To determine $y(t)$

Putting $y^{(p)}(t)$ from above equation and $y^{(p)}(t)$

$$y(t) = c_1 e^{-4t} + c_2 e^{-t} + e^{-2t}$$

Now let us use initial conditions to determine the values of c1 and c2. Putting y(0)=o

$$0 = c_1 + c_2 + 1 \Rightarrow c_1 + c_2 = -1$$

$$\frac{dy(t)}{dt} = -4 c_1 e^{-4t} - c_2 e^{-t} - 2 e^{-2t}$$

Putting $\left.\frac{dy(t)}{dt}\right|_{t=0} = 1$ in above equation,

$$1 = -4 c_1 - c_2 - 2 \Rightarrow 4 c_1 + c_2 = -3$$

Solving for c1 and c2 , we get

$$c_1 = -\frac{2}{3} \text{ and } c_2 = -\frac{1}{3}$$

Hence the complete response becomes

$$y(t) = -\frac{2}{3}e^{-4t} - \frac{1}{3}e^{-t} + e^{-2t}$$

This is the required response of the system considering input as well as initial conditions.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=p_di4Zn4wz4
- https://byjus.com/maths/differential-equation/

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no :4.1 to 4.10)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 26

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : III– linear time-invariant continuous time systems

Date of Lecture:

Topic of Lecture: **Block Diagram Representation**

Introduction: A mathematical block diagram gives a graphically representation of a mathematical model. The block diagram in itself gives good information of the structure of the model

Prerequisite knowledge for Complete understanding and learning of Topic:

- Differential equation

?1 Cascade connection of two LTI systems The output $y(t)$ of the second system can be given as,

$$y(t) = y_1(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} y_1(\tau)\, h_2(t-\tau)\, d\tau \quad \text{... (3.2.20)}$$

The output of first system is $y_1(t)$. It can be given as,

$$y_1(\tau) = x(\tau) * h_1(\tau)$$

$$= \int_{-\infty}^{\infty} x(m)\ h_1(\tau - m)\, dm \quad \text{... (3.2.21)}$$

Here separate variables τ and m are used. Putting above equation for $y_1(\tau)$ in equation 3.2.20.

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(\tau - m) h_2(t-\tau) dm\, d\tau$$

Here put $\tau - m = n$, then we get

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(n) \cdot h_2(t-m-n)\, dm\, dn$$

$$= \int_{-\infty}^{\infty} x(m) \left[\int_{-\infty}^{\infty} h_1(n) \cdot h_2((t-m)-n)\, dn \right] dm \qquad \text{... (3.2.22)}$$

The integration in square brackets indicate convolution of $h_1(t)$ and $h_2(t)$ evaluated at t – m. i.e.,

$$\int_{-\infty}^{\infty} h_1(n)\, h_2\, ((t-m)-n)\, dn = h\,(t-m)$$

Putting this value in equation 3.2.22 we get,

$$y(t) = \int_{-\infty}^{\infty} x(m)\, h\,(t-m)\, dm$$

$$= x(t) * h(t)$$

Fig. 3.2.22 Equivalent of cascade connection of Fig. 3.2.21

Thus if the two systems are connected in cascade, the overall impulse response is equal to convolution of two impulse responses. This is shown in Fig. 3.2.22 .

We know that,

$$y_1(t) = x(t) * h_1(t)$$

and

$$y(t) = y_1(t) * h_2(t)$$

Putting for $y_1(t)$ in above equation,

$$y(t) = \left[x(t) * h_1(t)\right] * h_2(t) \qquad \text{... (3.2.23)}$$

And from Fig. 3.2.22 we can write,

$$y(t) = x(t) * h(t)$$

$$= x(t) * \left[h_1(t) * h_2(t)\right] \qquad \text{... (3.2.24)}$$

Thus equation 3.2.23 and above equation prove associative property. i.e.,

$$\left[x(t) * h_1(t)\right] * h_2(t) = x(t) * \left[h_1(t) * h_2(t)\right] \qquad \text{... (3.2.25)}$$

3. Distributive property of convolution :

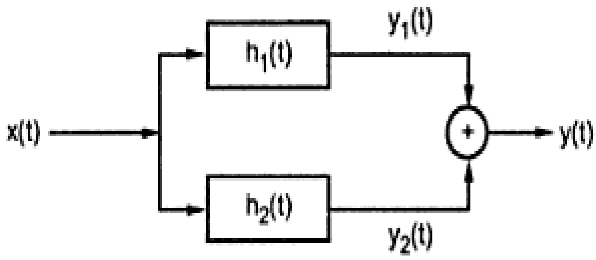


Fig. 3.2.23 Parallel connection of the systems

Consider the two systems connected in parallel as shown in Fig. 3.2.23.

The overall output is,

$$y(t) = y_1(t)+y_2(t)$$
$$= x(t)*h_1(t)+x(t)*h_2(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h_1(t-\tau)\, d\tau + \int_{-\infty}^{\infty} x(\tau)\, h_2(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)\left\{h_1(t-\tau)+h_2(t-\tau)\right\} d\tau = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$= x(t) * h(t)$$

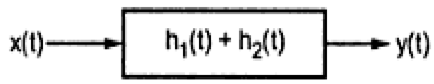


Fig. 3.2.24 Equivalent system of Fig. 3.2.23

Here $h(t)=h_1(t)+h_2(t)$. Thus impulse responses of the parallel connected systems are added. i.e.

This proves the distributive property which can be stated as,

$$x(t) *h_1(t)+x(t) *h_2(t) = x(t) *\left\{h_1(t)+h_2(t)\right\} \qquad \text{... (3.2.26)}$$

Video Content / Details of website for further learning (if any):

-

Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : :4.15 TO 4.16)

Course Faculty

Verified by HOD

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture: Impulse Response

Introduction:

An impulse response is the reaction of any dynamic system in response to some external change.

Prerequisite knowledge for Complete understanding and learning of Topic:

- IMPULSE SIGNAL

The computation of the convolution integral is complicated even in the situation when the input as well as the system are causal. We will see that the convolution computation is much easier by using the Laplace transform, even in the case of non-causal inputs or systems.

Graphically, the computation of the convolution integral (2.21) for a causal input ($x(t) = 0$, $t < 0$) and a causal system ($h(t) = 0$, $t < 0$), consists in:

1.Choosing a time t_0 for which we want to compute $y(t_0)$,

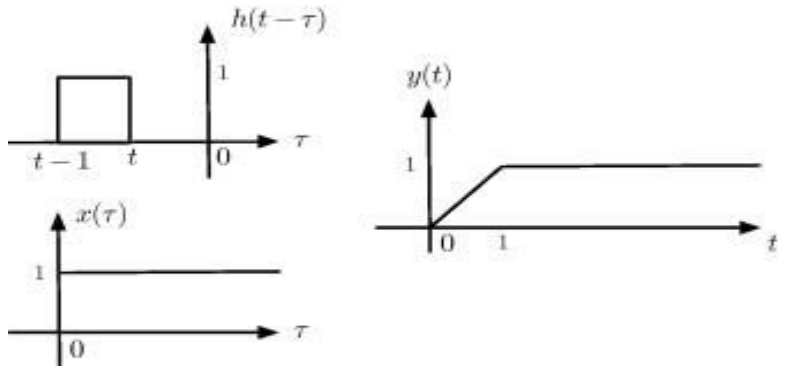
2.Obtaining as functions of τ, the stationary $x(\tau)$ signal and the reflected and delayed (shifted right) t_0 sec $h(t_0 - \tau)$ impulse response,

3.Obtaining the product $x(\tau)h(t_0 - \tau)$ and integrating it from 0 to t_0 to obtain $y(t_0)$.

4.Increasing the time value, moving from $-\infty$ to ∞.

The output is zero for $t < 0$ since the initial conditions are zero and the system is causal. In the above steps we can interchange the input and the impulse response and obtain identical results. It can be seen that the convolution integral is computationally very intensive as the above steps need to be done for each value of $t > 0$.

The input signal $x(\tau) = u(\tau)$, as a function of τ, and the reflected and delayed impulse response $h(t - \tau)$, also as a functions of τ, for some value of $t < 0$



when $t = 0$, $h(-\tau)$ is the reflected version of the impulse response, and for $t > 0$ then $h(t - \tau)$ is $h(-\tau)$ shifted by t to the right. As t goes from $-\infty$ to ∞, $h(t - \tau)$ moves from left to right while $x(\tau)$ remains stationary.

If $t < 0$, then $h(t - \tau)$ and $x(\tau)$ do not overlap and so the convolution integral is zero, or $y(t) = 0$ for $t < 0$. That is, the system for $t < 0$ has not yet been affected by the input.

For $t \geq 0$ and $t - 1 < 0$, or equivalently $0 \leq t < 1$, $h(t - \tau)$, and $x(\tau)$ increasingly overlap and the integral increases linearly from 0 at $t = 0$ to 1 when $t = 1$. So that $y(t) = t$ for $0 \leq t < 1$. That is, for these times the system is reacting slowly to the input.

▪

For $t \geq 1$, the overlap of $h(t - \tau)$ and $x(\tau)$ remains constant, and as such the integral is unity from then on, or $y(t) = 1$ for $t \geq 1$. The response for $t \geq 1$ has attained steady state. Thus the complete response is given as

$y(t)=r(t)-r(t-1)$

where $r(t) = tu(t)$, the ramp function.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=-oNqg6JH0eI
- https://www.sciencedirect.com/topics/computer-science/impulse-response-function

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no :4.16 TO 4.19)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 28

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture: Step Response

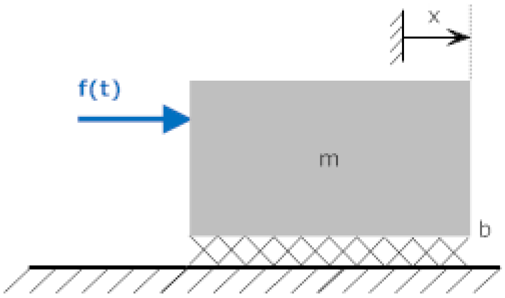
Introduction:

The step response is the time behavior of the outputs of a general system when its inputs change from zero to one in a very short time. The concept can be extended to the abstract mathematical notion of a dynamical system using an evolution parameter.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Step Signal

If the input force of the following system is a unit step, find v(t). Also shown is a free body diagram.



The differential equation describing the system is

$$m\dot{v} + bv = f(t)$$

so the transfer function is determined by taking the Laplace transform (with zero initial conditions) and solving for V(s)/F(s)

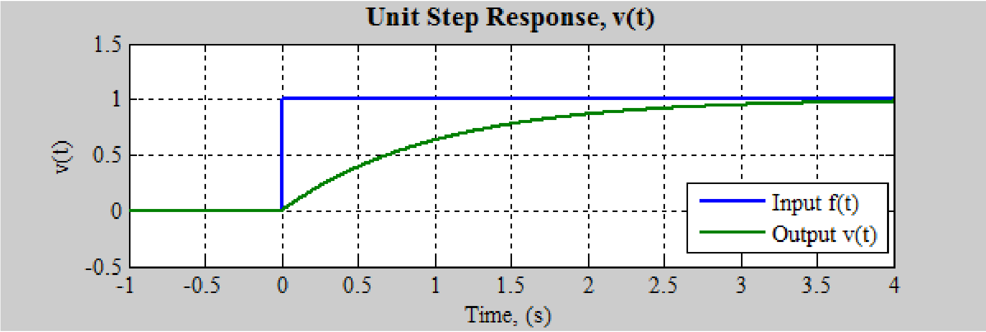
$$msV(s) + bV(s) = F(s)$$

$$\frac{V(s)}{F(s)} = H(s) = \frac{1}{ms+b} = \frac{1/m}{s+b/m}$$

To find the unit step response, multiply the transfer function by the unit step (1/s) and solve by looking up the inverse transform in the Laplace Transform table (Asymptotic exponential)

$$V(s) = F(s)H(s) = \frac{1}{s}\frac{1/m}{s+b/m}$$

$$v(t) = \frac{1}{b}\left(1 - e^{-(b/m)t}\right)$$



Using the general form of the step response of a first order system we get

$$v(t) = v(\infty) + \left(v(0^+) - v(\infty)\right)e^{-\frac{t}{\tau}}$$

$$= H(0) + \left(H(\infty) - H(0)\right)e^{-\frac{t}{\tau}}$$

$$= \frac{1}{b} + \left(0 - \frac{1}{b}\right)e^{-(b/m)t}$$

$$= \frac{1}{b}\left(1 - e^{-(b/m)t}\right)$$

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=-oNqg6JH0eI**
- https://www.sciencedirect.com/topics/computer-science/impulse-response-function

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 4.46 TO 4.51)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 29

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : III– Linear Time-Invariant Continuous Time Systems

Date of Lecture:

Topic of Lecture: Stability

Introduction: Stability, in mathematics, condition in which a slight disturbance in a system does not produce too disrupting an effect on that system. In terms of the solution of a differential equation, a function *f*(*x*) is said to be stable if any other solution of the equation that starts out sufficiently close to it when $x = 0$ remains close to it for succeeding values of x.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Linear time invariant systems

A System is said to be stable if every bounded input produces a bounded output.

The input x(n) is said to bounded if there exists some finite number Mx such that $|x(n)| \leq M_x < \infty$.

The output y(n) is said to bounded if there exists some finite number My such that $|y(n)| \leq M_y < \infty$.

Linear convolution is given by

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Taking the absolute value of both sides

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right|$$

The absolute values of total sum is always less than or equal to sum of the absolute values of individually terms. Hence

$$|y(n)| \leq \left| \sum_{k=-\infty}^{\infty} h(k)\ x(n-k) \right|$$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)|$$

The input x(n) is said to bounded if there exists some finite number Mx such that $|x(n)| \leq Mx < \infty$.

Hence bounded input x(n) produces bounded output y(n) in the LSI system only if

$$\sum_{k=-\infty} |h(k)| < \infty$$

With this condition satisfied, the system will be stable. The above equation states that the LSI system is stable if its unit sample response is absolutely summable. This is necessary and sufficient condition for the stability of LSI system.

Stability theorem

Let dxdt=f(x)dxdt=f(x) be an autonomous differential equation. Suppose x(t)=x∗x(t)=x∗ is an equilibrium, i.e., f(x∗)=0f(x∗)=0.

Then

- if f′(x∗)<0f′(x∗)<0, the equilibrium x(t)=x∗x(t)=x∗ is stable, and
- if f′(x∗)>0f′(x∗)>0, the equilibrium x(t)=x∗x(t)=x∗ is unstable.

Solve $\frac{dy}{dx} + \frac{y}{x} = x^3$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=7q33RFkMMpY
- https://mathinsight.org/stability_equilibria_differential_equation

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 4.45)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 30

HANDOUTS ECTURE

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture: Convolution Integrals

Introduction: convolution is a mathematical operation on two functions (f and g) that produces a third function that expresses how the shape of one is modified by the other. The term convolution refers to both the result function and to the process of computing it.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Convolution Of Two Signals

CONVOLUTION INTEGRAL

• An approach (available tool or operation) to describe the input-output relationship for LTI Systems

– Remember h(t) is T[d(t)]

– Unit impulse function □ the impulse response

• It is possible to use h(t) to solve for any input-output relationship

• Any input can be expressed using the unit impulse function

$$f(t)x(t) = f(t)\dot{y}(t) + f(t)ay(t)$$

$$= \frac{d}{dt}(f(t)y(t))$$

which implies

$$\dot{f}(t)y(t) + f(t)\dot{y}(t) = f(t)\dot{y}(t) + af(t)y(t)$$

By using continuous time convolution integral, obtain the response of the system to unit step input signal. Given the impulse response

$$h(t) = \frac{R}{L} e^{-tR/L} u(t)$$

Solution : The convolution integral is,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Here $x(\tau) = u(\tau) = 1$ for $\tau \geq 0$. Hence above equation will be,.

$$y(t) = \int_{0}^{\infty} 1 \cdot \frac{R}{L} e^{-(t-\tau)R/L}\, u(t-\tau)\, d\tau$$

Here $u(t-\tau) = 1$ for $t \geq \tau$ or $\tau \leq t$.

Hence above equation will be,

$$y(t) = \frac{R}{L} \int_{0}^{t} e^{-(t-\tau)R/L}\, d\tau$$

$$= \frac{R}{L} \int_{0}^{t} e^{-tR/L} \cdot e^{\tau R/L}\, d\tau$$

$$= \frac{R}{L} e^{-tR/L} \int_{0}^{t} e^{\tau R/L}\, d\tau$$

$$= \frac{R}{L} \cdot e^{-tR/L} \cdot \frac{1}{R/L} \left[e^{\tau R/L} \right]_0^t$$

$$= 1 - e^{-tR/L} \qquad \text{for } t \geq 0$$

This is the output of the system.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=3GGT7AFXe1I
- https://www.jhu.edu/bmesignals/Lectures/Convolution.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no :4.12)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture:

Properties of convolution integral

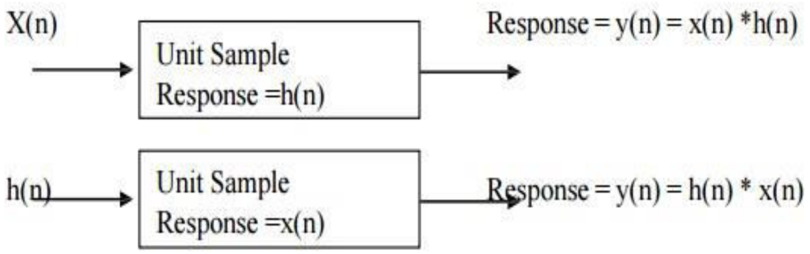
Introduction: convolution is a mathematical operation on two functions (f and g) that produces a third function that expresses how the shape of one is modified by the other. The term convolution refers to both the result function and to the process of computing it.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Convolution

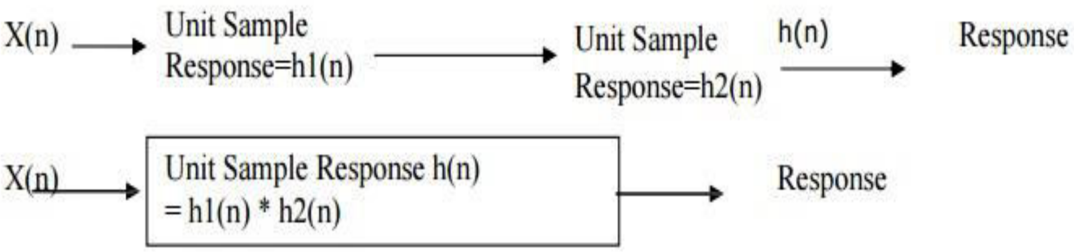
1. Commutative Law: (Commutative Property of Convolution)

x(n) * h(n) = h(n) * x(n)



2. Associate Law: (Associative Property of Convolution)

[x(n) * h1(n)] * h2(n) = x(n) * [h1(n) * h2(n)]



D.1.3 Distributivity Property

Convolution is also distributive,

$$x(t)*\left[h_1(t)+h_2(t)\right]=x(t)*h_1(t)+x(t)*h_2(t).$$

$$x(t)*\left[h_1(t)+h_2(t)\right]=\int_{-\infty}^{\infty}x(t)\left[h_1(t-\tau)+h_2(t-\tau)\right]d\tau$$

$$x(t)*\left[h_1(t)+h_2(t)\right]=\int_{-\infty}^{\infty}x(t)h_1(t-\tau)d\tau+\int_{-\infty}^{\infty}x(t)h_2(t-\tau)d\tau$$

$$x(t)*\left[h_1(t)+h_2(t)\right]=x(t)*h_1(t)+x(t)*h_2(t)$$

Video Content / Details of website for further learning (if any):

- http://www.brainkart.com/article/Linear-Time-Invariant--Continuous-Time-Systems_13354/
- https://www.youtube.com/watch?v=_HATc2zAhcY

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.(Page no :4.13)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE
(An Autonomous Institution)
(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 32

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : III– Linear Time-Invariant Continuous Time Systems

Date of Lecture:

Topic of Lecture:

Graphical Method Procedure to Perform Convolution

Introduction: Graphical evaluation of convolution (flip n drag) is a very useful, helpful and indispensable method which aids in a very quick visual anticipation of the output, in terms of the input sequences.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Linera time invariant systems

Steps for Graphical Convolution x (t)* h (t)

1. **Re-Write the signals as functions of τ**: $x(\tau)$ and $h(\tau)$

> Note: I use τ for what the book uses λ.... It is not a big deal as they are just dummy variables!!!

2. **Flip** just one of the signals around $t = 0$ to get either $x(-\tau)$ or $h(-\tau)$
   a. It is usually best to flip the signal with shorter duration
   b. For notational purposes here: we'll flip $h(\tau)$ to get $h(-\tau)$

3. **Find Edges** of the flipped signal
   a. Find the left-hand-edge τ-value of $h(-\tau)$: call it $\tau_{L,0}$
   b. Find the right-hand-edge τ-value of $h(-\tau)$: call it $\tau_{R,0}$

4. **Shift** $h(-\tau)$ by an arbitrary value of t to get $h(t - \tau)$ and **get its edges**
   a. Find the left-hand-edge τ-value of $h(t - \tau)$ as a function of t: call it $\tau_{L,t}$
      - **Important**: It will *always* be… $\tau_{L,t} = t + \tau_{L,0}$

   b. Find the right-hand-edge τ-value of $h(t - \tau)$ as a function of t: call it $\tau_{R,t}$
      - **Important**: It will *always* be… $\tau_{R,t} = t + \tau_{R,0}$

   Note: If the signal you flipped is NOT finite duration, one or both of $\tau_{L,t}$ and $\tau_{R,t}$ will be infinite ($\tau_{L,t} = -\infty$ and/or $\tau_{R,t} = \infty$)

5. **Find Regions of τ-Overlap**

a. What you are trying to do here is find intervals of t over which the product $x(\tau)\ h(t - \tau)$ has a single mathematical form in terms of τ

b. In each region find: Interval of t that makes the identified overlap happen

c. Working examples is the best way to learn how this is done

Tips: Regions should be contiguous with no gaps!!!
Don't worry about $<$ vs. $\leq$ etc.

6. For Each Region: **Form the Product $x(\tau)\ h(t - \tau)$ and Integrate**

a. Form product $x(\tau)\ h(t - \tau)$

b. Find the Limits of Integration by finding the interval of τ over which the product is nonzero

   i. Found by seeing where the edges of $x(\tau)$ and $h(t - \tau)$ lie

   ii. Recall that the edges of $h(t - \tau)$ are $\tau_{L,t}$ and $\tau_{R,t}$, which often depend on the value of t

   - So… the limits of integration may depend on t

c. Integrate the product $x(\tau)\ h(t - \tau)$ over the limits found in 6b

   i. The result is generally a function of t, but is only valid for the interval of t found for the current region

   ii. Think of the result as a "time-section" of the output $y(t)$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=zQ7Khy-MifQ
- https://www.youtube.com/watch?v=jvT_wOJd95g

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :4.22)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

L 33

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture:

Fourier Transforms in Analysis of CT Systems

Introduction: The Laplace transformation is used to study the transient evolution of the system´s response from the initial state to the final sinusoid steady state. It Includes not only the transient phenomenon from the initial state of the system but also the final sinusoid steady state.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Fourier transform

Introduction: In analyzing a system, one usually recounts Time – Invariant, Linear Differential Equations of second or high orders. Generally, it is difficult to obtain solutions of these equations in closed form via the solution methods in ordinary differential equations.

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega \qquad (5\text{-}8)$$

where ω is the angular frequency and

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \qquad (5\text{-}9)$$

is called Fourier transform of $f(t)$. In fact, (5-8) can be further written as

$$\begin{aligned} f(t) &= \int_{-\infty}^{\infty}\left(\frac{1}{2\pi}F(j\omega)e^{j\omega t}\right)d\omega \\ &= \int_{-\infty}^{\infty}\left(\frac{1}{2\pi}F(j\omega)(\cos\omega t + j\sin\omega t)\right)d\omega \end{aligned} \qquad (5\text{-}10)$$

which implies that the function $f(t)$ is composed of the sinusoidal functions of all the frequencies from $\omega = -\infty$ to $\omega = \infty$. Moreover, Fourier transform can be represented by the so-called Euler form as

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = |F(j\omega)|e^{j\angle F(j\omega)} \qquad (5\text{-}11)$$

with magnitude $|F(j\omega)|$ and phase $\angle F(j\omega)$.

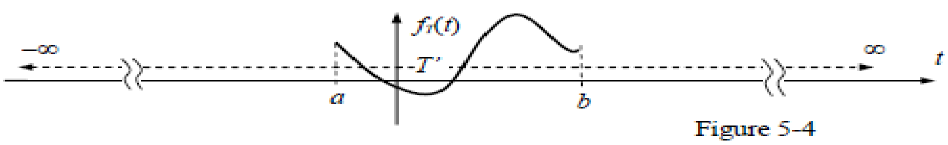


Figure 5-4

The Fourier transform coefficients could be taken as a set of Fourier series whose period T, of the periodic function is near the infinity. In a rectangular waveform, the infinite period means a single pulse. The figure shows the behaviour of f (ω) is shown for negative frequencies.

Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equation. It finds very wide applications in various areas of physics , electrical engineering, control engineering, optics, mathematics , signal processing.

it must satisfy the following Dirichlet conditions:

1) f(t) must be piecewise continuous which means that it must be single valued but can have a finite number of finite isolated discontinuities for t > 0. 2) f(t) must be exponential order which means that f(t) must remain less than $se\ -a0t$ as t approaches ∞ where S is a positive constant and $a0$ is a real positive number.

$$F(s)=\int_0^\infty f(t)e^{-st}\,dt \qquad (5\text{-}13)$$

It is noticed that if $s=j\omega$, along the imaginary axis, then (5-13) becomes

$$F(j\omega)=\int_0^\infty f(t)e^{-j\omega t}\,dt \qquad (5\text{-}14)$$

Viewing from (5-11), it is known that if $f(t)$ is a function staring from $t=0$, i.e., $f(t)=0$ for $t<0$, then

$$F(j\omega)=\int_{-\infty}^\infty f(t)e^{-j\omega t}\,dt=\int_0^\infty f(t)e^{-j\omega t}\,dt \qquad (5\text{-}15)$$

which is the same as Laplace trasform (5-13) with $s=j\omega$. This is the relation between Fourier transform and Laplace transform.

For the detail of Fourier transform and Laplace transform, please refer to textbooks of Engineering Mathematics or System Engineering.

Video Content / Details of website for further learning (if any):

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/Lecture%206%20-%20Systems%20&%20Laplace%20Transform%20(x1).pdf
- https://www.youtube.com/watch?v=S7zGQWX3FZQ

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :7.54)

Course Faculty

Verified by HOD

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture:

Laplace Transforms in Analysis of CT Systems

Introduction: The Laplace transformation is used to study the transient evolution of the system´s response from the initial state to the final sinusoid steady state. It Includes not only the transient phenomenon from the initial state of the system but also the final sinusoid steady state.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Laplace transform

Introduction:

Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equation. It finds very wide applications in various areas of physics , electrical engineering, control engineering, optics, mathematics , signal processing.

it must satisfy the following Dirichlet conditions:

- f(t) must be piecewise continuous which means that it must be single valued but can have a finite number of finite isolated discontinuities for t > 0. 2) f(t) must be exponential order which means that f(t) must remain less than se^{-a0t} as t approaches ∞ where S is a positive constant and $a0$ is a real positive number.
- Laplace transform methods can be employed to study circuits in the *s*-domain.
- Laplace techniques convert circuits with voltage and current signals that change with time to the *s*-domain so you can analyze the circuit's action using only algebraic techniques.

Connection constraints are those physical laws that cause element voltages and currents to behave in certain ways when the devices are interconnected to form a circuit.

$$F(s)=\int_{0}^{\infty} f(t)e^{-st}\,dt \quad (5\text{-}13)$$

It is noticed that if $s=j\omega$, along the imaginary axis, then (5-13) becomes

$$F(j\omega)=\int_{0}^{\infty} f(t)e^{-j\omega t}\,dt \quad (5\text{-}14)$$

Viewing from (5-11), it is known that if $f(t)$ is a function staring from t=0, i.e., $f(t)$=0 for t<0, then

$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt=\int_{0}^{\infty} f(t)e^{-j\omega t}\,dt \quad (5\text{-}15)$$

which is the same as Laplace trasform (5-13) with $s=j\omega$. This is the relation between Fourier transform and Laplace transform.

For the detail of Fourier transform and Laplace transform, please refer to textbooks of Engineering Mathematics or System Engineering.

Video Content / Details of website for further learning (if any):

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/Lecture%206%20-%20Systems%20&%20Laplace%20Transform%20(x1).pdf
- https://www.youtube.com/watch?v=S7zGQWX3FZQ

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :7.54)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 35

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture:

Laplace Transform in Analyzing Electrical Network

Introduction: The Laplace transformation is used to study the transient evolution of the system´s response from the initial state to the final sinusoid steady state. It Includes not only the transient phenomenon from the initial state of the system but also the final sinusoid steady state.

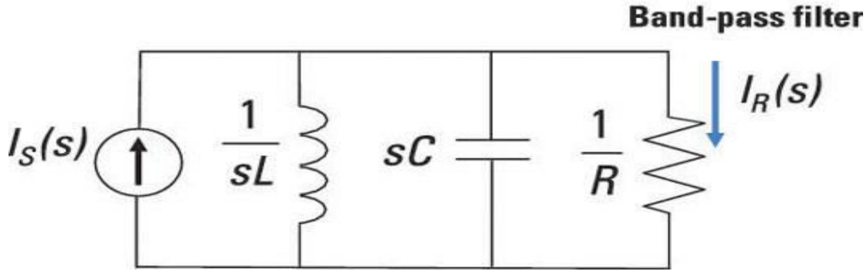
Prerequisite knowledge for Complete understanding and learning of Topic:

- Laplace transform

Introduction: In analyzing a system, one usually recounts Time – Invariant, Linear Differential Equations of second or high orders.

- Generally, it is difficult to obtain solutions of these equations in closed form via the solution methods in ordinary differential equations.
- Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equation.
- It finds very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics , signal processing.
- it must satisfy the following Dirichlet conditions:
- There are many applications for an RLC circuit, including band-pass filters, band-reject filters, and low-/high-pass filters.
- You can use series and parallel RLC circuits to create band-pass and band-reject filters. An RLC circuit has a resistor, inductor, and capacitor connected in series or in parallel.

$$T(j\omega) = \frac{I_R(s)}{I_s(s)} = \left(\frac{1}{RC}\right)\frac{j\omega}{\left[\left(\frac{1}{LC} - \omega^2\right) + \left(\frac{1}{RC}\right)j\omega\right]}$$



$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c1} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

The bandwidth BW and quality factor Q are

$$BW = \omega_{c2} - \omega_{c1} = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{BW} = R\sqrt{\frac{C}{L}}$$

Video Content / Details of website for further learning (if any):

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/Lecture%206%20-%20Systems%20&%20Laplace%20Transform%20(x1).pdf
- https://www.youtube.com/watch?v=S7zGQWX3FZQ

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :7.74)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 36

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **III– Linear Time-Invariant Continuous Time Systems**

Date of Lecture:

Topic of Lecture: Laplace Transform in Analyzing Electrical Network

Introduction: The Laplace transformation is used to study the transient evolution of the system´s response from the initial state to the final sinusoid steady state. It Includes not only the transient phenomenon from the initial state of the system but also the final sinusoid steady state.

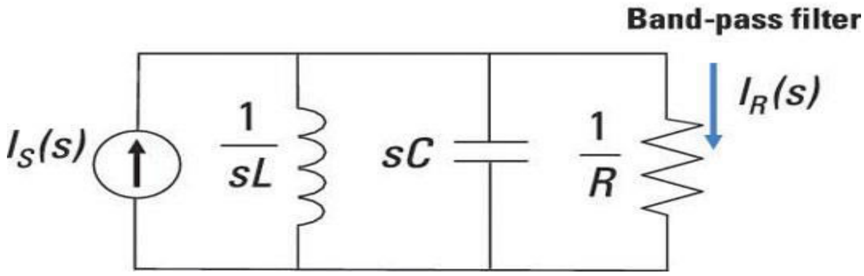
Prerequisite knowledge for Complete understanding and learning of Topic:

- Laplace transform

Introduction: In analyzing a system, one usually recounts Time – Invariant, Linear Differential Equations of second or high orders.

- Generally, it is difficult to obtain solutions of these equations in closed form via the solution methods in ordinary differential equations.
- Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equation.
- It finds very wide applications in various areas of physics , electrical engineering, control engineering, optics, mathematics , signal processing.
- it must satisfy the following Dirichlet conditions:
- There are many applications for an RLC circuit, including band-pass filters, band-reject filters, and low-/high-pass filters.
- You can use series and parallel RLC circuits to create band-pass and band-reject filters. An RLC circuit has a resistor, inductor, and capacitor connected in series or in parallel.

$$T(j\omega) = \frac{I_R(s)}{I_s(s)} = \left(\frac{1}{RC}\right)\frac{j\omega}{\left[\left(\frac{1}{LC} - \omega^2\right) + \left(\frac{1}{RC}\right)j\omega\right]}$$



$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c1} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

The bandwidth *BW* and quality factor *Q* are

$$BW = \omega_{c2} - \omega_{c1} = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{BW} = R\sqrt{\frac{C}{L}}$$

Video Content / Details of website for further learning (if any):

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/Lecture%206%20-%20Systems%20&%20Laplace%20Transform%20(x1).pdf
- https://www.youtube.com/watch?v=S7zGQWX3FZQ

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :7.74)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 37

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Discrete Time Fourier Transform – Definition

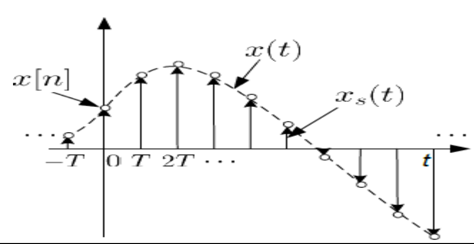
Introduction: The DTFT is often used to analyze samples of a continuous function. The term *discrete-time* refers to the fact that the transform operates on discrete data, often samples whose interval has units of time. From uniformly spaced samples it produces a function of frequency that is a periodic summation of the continuous Fourier transform of the original continuous function.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DFT

The discrete-time Fourier transform of a discrete set of real or complex numbers x[n], for all integers n, is a Fourier series, which produces a periodic function of a frequency variable. When the frequency variable, ω, has normalized units of radians/sample, the periodicity is 2π, and the Fourier series

$$x_s(t) = x(t) \cdot i(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t - kT)$$



$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t-kT) = \sum_{k=-\infty}^{\infty} x[k]\delta(t-kT)$$

Instead of operating on sampled signals of length N (like the DFT), the DTFT operates on sampled signals $x(n)$ defined over all integers $n \in \mathbb{Z}$. As a result, the DTFT frequencies form a *continuum*.

That is, the DTFT is a function of *continuous* frequency $\tilde{\omega} \in [-\pi, \pi)$, while the DFT is a function of discrete frequency ω_k, $k \in [0, N-1]$.

$$X_s(j\Omega) = \int_{-\infty}^{\infty} x_s(t)e^{-j\Omega t}dt = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n]\delta(t-nT)e^{-j\Omega t}dt$$
$$= \sum_{n=-\infty}^{\infty} x[n] \int_{-\infty}^{\infty} \delta(t-nT)e^{-j\Omega t}dt = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega nT}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega+2k\pi)n} = X(e^{j(\omega+2k\pi)})$$

Video Content / Details of website for further learning (if any):

- https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- https://www.youtube.com/watch?v=QLCXSxgxRPY

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.1)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 38

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis ff Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Discrete Time Fourier Transform – Properties

Introduction: The DTFT is often used to analyze samples of a continuous function. The term *discrete-time* refers to the fact that the transform operates on discrete data, often samples whose interval has units of time. From uniformly spaced samples it produces a function of frequency that is a periodic summation of the continuous Fourier transform of the original continuous function.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Linearity:**

  The DTFT is linear.

  If

  $$x_1[n] \xrightarrow{DTFT} X_1(\omega)$$

  and

  $$x_2[n] \xrightarrow{DTFT} X_2(\omega)$$

  then

  $$ax_1[n] + bx_2[n] \xrightarrow{DTFT} aX_1(\omega) + bX_2(\omega)$$

- DFT

Time Shifting and Frequency Shifting:

If,

$$x[n] \xrightarrow{DTFT} X(\omega)$$

then,

$$x[n - n_o] \xrightarrow{DTFT} e^{-j\omega n_o} X(\omega)$$

and,

$$e^{j\omega_0 n} x[n] \xrightarrow{DTFT} X(\omega - \omega_0)$$

Time and Frequency Scaling:

Time reversal

Let us find the DTFT of x[-n]

$$x[n] \xrightarrow{DTFT} X(\omega)$$

$$\therefore x[-n] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} x[-n]e^{-j\omega n}$$

$$= \sum_{m=-\infty}^{\infty} x[m]e^{-j(-\omega)m}$$

$$= X(-\omega)$$

$$\therefore x[-n] \xrightarrow{DTFT} X(-\omega)$$

Time expansion:

It is very difficult for us to define x[an] when a is not an integer. However if a is an integer other than 1 or -1 then the original signal is not just speeded up. Since n can take only integer values, the resulting signal consists of samples of x[n] at **an.**

If k is a positive integer, and we define the signal

$$x_k[n] = x[\frac{n}{k}] \quad \text{if } n \text{ is a multiple of } k;$$

$$= 0 \quad \text{if } n \text{ is not a multiple of } k.$$

then

$$x_k[n] \xrightarrow{DTFT} X(k\omega)$$

Convolution Property :

Let h[n] be the impulse response of a discrete time LSI system. Then the frequency response of the LSI system is

$$h[n] \xrightarrow{DTFT} H(\omega)$$

Now

$$x[n] \xrightarrow{DTFT} X(\omega)$$

and

$$y[n] \xrightarrow{DTFT} Y(\omega)$$

If

$$y[n] = x[n] * h[n]$$

then

$$Y(\omega) = X(\omega)H(\omega)$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-28.pdf
- https://www.comm.utoronto.ca/~dkundur/course_info/455/2013F/Tables2.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 39

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Discrete Time Fourier series – Definition

Introduction: Fourier series: a complicated waveform analyzed into a number of harmonically related sine and cosine functions

Prerequisite knowledge for Complete understanding and learning of Topic:

- Fourier series

A periodic signal with period of T,

$$x(t) = x(t+T) \text{ for all } t, \qquad (3.16)$$

We introduced two basic periodic signals in Chapter 1, the sinusoidal signal

$$x(t) = \cos\omega_0 t, \qquad (3.17)$$

and the periodic complex exponential

$$x(t) = e^{j\omega_0 t}, \qquad (3.18)$$

Both these signals are periodic with fundamental frequency ω_0 and fundamental period $T = 2\pi/\omega_0$. Associated with the signal in Eq. (3.18) is the set of *harmonically related* complex exponentials

$$\phi_k(t) = e^{jk\omega_0 t} = e^{jk(2\pi/T)t}, \qquad k = 0, \pm 1, \pm 2, \ldots\ldots \qquad (3.19)$$

Video Content / Details of website for further learning (if any):

- https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- https://www.youtube.com/watch?v=QLCXSxgxRPY

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.1)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 40

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Discrete Time Fourier series – Properties

Introduction: Fourier series: a complicated waveform analyzed into a number of harmonically related sine and cosine functions

Prerequisite knowledge for Complete understanding and learning of Topic:

- Fourier series

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega},\ a>0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega},\ a>0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a\|t\|} \leftrightarrow \frac{2a}{a^2+\omega^2},\ a>0$	16	$\delta(t-t_o) \leftrightarrow e^{-j\omega t_o}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a\|\omega\|},\ a>0$	17	$e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega-\omega_o)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2},\ a>0$	18	$\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}},\ a>0$	19	$\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
7	$\mathrm{rect}(\frac{t}{\tau}) \leftrightarrow \tau\,\mathrm{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]+\frac{j\omega}{\omega_o^2-\omega^2}$
8	$\mathrm{sinc}(Wt) \leftrightarrow \frac{\pi}{W}\mathrm{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]+\frac{\omega_o}{\omega_o^2-\omega^2}$
9	$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2}\mathrm{sinc}^2(\frac{\omega\tau}{4})$	22	$\mathrm{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\mathrm{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W}\Delta(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$
11	$e^{-at}\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2+\omega_o^2},\ a>0$	24	$\sum_{n=-\infty}^{\infty}\delta(t-nT) \leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\frac{2\pi}{T})$
12	$e^{-at}\cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2+\omega_o^2},\ a>0$	25	$\sum_{n=-\infty}^{\infty}f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty}\frac{1}{T}F(\omega-n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$		

Video Content / Details of website for further learning (if any):
• https://en.wikipedia.org/wiki/Discrete_Fourier_transform • https://www.youtube.com/watch?v=QLCXSxgxRPY
Important Books/Journals for further learning including the page nos.:
Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.4)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Sampling Theorem

Introduction: In signal processing, sampling is the reduction of a continuous-time signal to a discrete-time signal. A common example is the conversion of a sound wave (a continuous signal) to a sequence of samples (a discrete-time signal).

Prerequisite knowledge for Complete understanding and learning of Topic:

- Sampling

A sample is a value or set of values at a point in time and/or space. A sampler is a subsystem or operation that extracts samples from a continuous signal. A theoretical ideal sampler produces samples equivalent to the instantaneous value of the continuous signal at the desired points.

Let us consider an analog signal $x(t)$ whose spectrum is bandlimited to f_m Hz. This means that the signal $x(t)$ has no frequency components beyond f_m Hz.

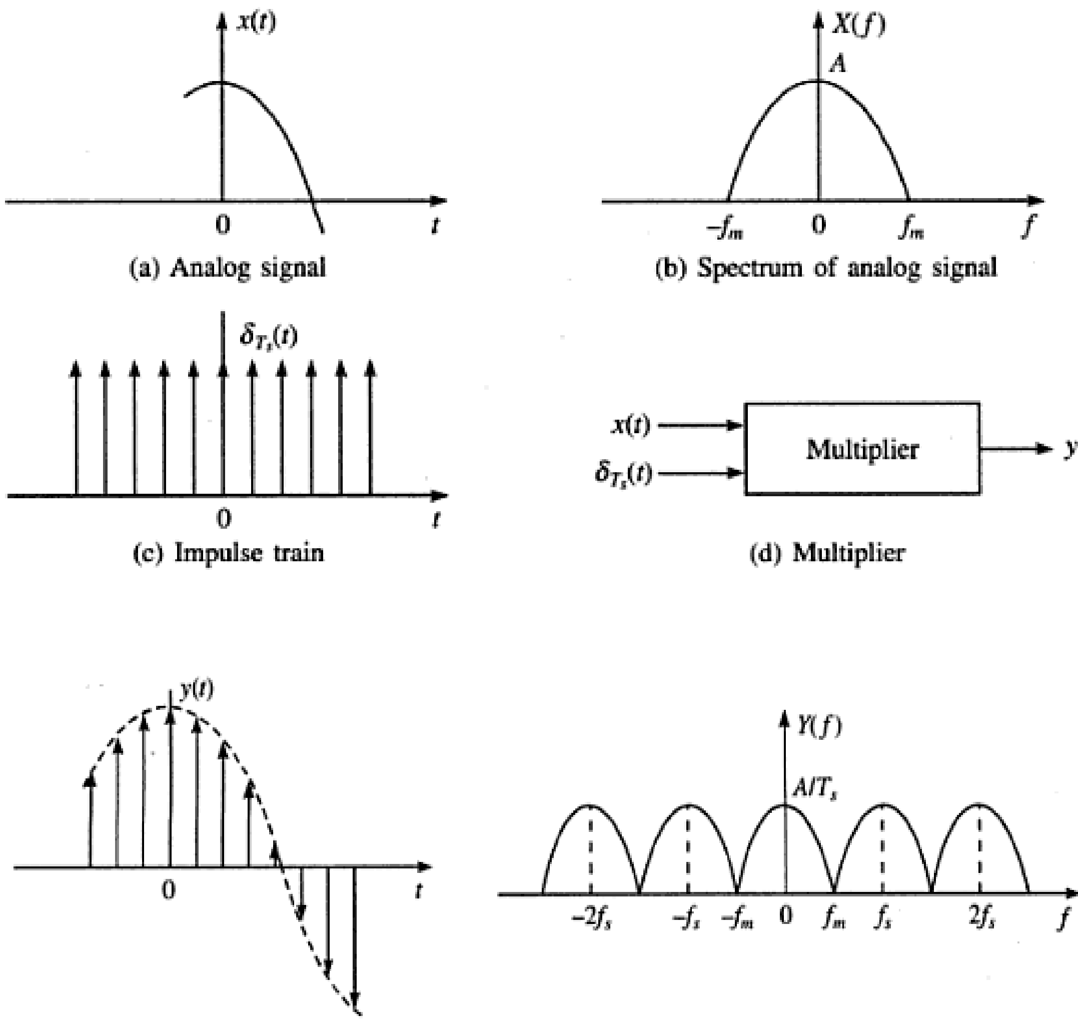
As the impulse train $\delta_{T_s}(t)$ is a periodic signal with period T_s, it may be expressed as a Fourier series as under:

Therefore, $$X(f) = 0 \text{ for } |f| > f_m$$

i.e., $$y(t) = x(t)\ \delta_{T_s}(t)$$

$$\delta_{T_s}(T) = \frac{1}{T_s}[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \cdots]$$

where $$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$



(a) Analog signal (b) Spectrum of analog signal

(c) Impulse train (d) Multiplier

(e) Sampled signal (f) Spectrum of sampled signal

Figure 8.1 Fourier transform series.

Therefore, $$y(t) = \frac{1}{T_s}[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + \cdots]$$

Fourier transform of $x(t)$ is $X(f)$.

Fourier transform of $2x(t) \cos \omega_s t$ is $[X(f - f_s) + X(f + f_s)]$

Fourier transform of $2x(t) \cos \omega_s t$ is $[X(f - 2f_s) + X(f + 2f_s)]$ and so on.

Therefore $Y(f) = \frac{1}{T_s}[X(f) + X(f - f_s) + X(f + f_s) + X(f - 2f_s) + X(f + f_s) + \cdots]$

Therefore,
$$Y(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Video Content / Details of website for further learning (if any):

- https://www.investopedia.com/terms/s/sampling.asp#:~:text=Sampling%20is%20a%20process%20used,random%20sampling%20or%20systematic%20sampling.

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : 9.1- 9.37)

Course Faculty

Verified by HOD

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Z Transform

Introduction: The Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DTFT

Definition

- Given a finite length signal as $x[n]$, the z-transform is defined

$$X(z) = \sum_{k=0}^{N} x[k]z^{-k} = \sum_{k=0}^{N} x[k](z^{-1})^{k}$$

where the sequence support interval is [0, N], and z is any complex number

- This transformation produces a new representation of $x[n]$ denoted $X[z]$
- Returning to the original sequence (*inverse z-transform*) $x[n]$ requires finding the coefficient associated with the nth power of z^{-1}
- The z-transform is particularly useful in the analysis and design of LTI systems

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \sum_{k=0}^{M} h[k]z^{-k}$$

1. **Find the z-transform of $x(n) = a^n \sin \Omega_o n\, u(n)$.** **(Oct./Nov. – 2008, 6 Marks)**

$x(n) = a^n \sin(\Omega_o n)\, u(n)$

Let $x_1(n) = \sin(\Omega_o n)\, u(n)$

$X(z) = Z\{e^{j\Omega on} - e^{-j\Omega on}/2j\}\, u(n)$

$=1/2j\, [Z\{e^{j\Omega on} u(n)\} - Z\{e^{j\Omega on} u(n)\}]$

$=1/2j[1/1-e^{j\Omega o} z^{-1} - 1/1-e^{-j\Omega o} z^{-1}]$

ROC : $|z| > |e^{j}\Omega o|$ and $|z| > |e^{-j\Omega o}|$

$=1/2j[1-e^{-j\Omega on} z^{-1} - 1 + e^{j\Omega on} z^{-1}/(1-e^{j\Omega on} z^{-1})(1-e^{-j\Omega on} z^{-1})$, ROC : $|z|>1$

$= 1/2j\, [2j\sin \Omega_o z^{-1}]/1-z^{-1}.2\cos\Omega o + z^{-2}]$

$=z^{-1} \sin \Omega o /1-2z^{-1} \cos\Omega o + z^{-2}$, ROC : $|z| > 1$

$=X_1(z/a)$, ROC : $|a|\, r_1 < |z| < |a|\, r_2$, By scaling in z-domain

Replacing z by z/a in $X_1(z) = (z/a)^{-1} \sin \Omega_o /1-2(z/a)^{-1} \cos \Omega_o + (z/a)^{-2}$,

ROC : $|z| > 1\, |a|$ i.e. $|z| > |a|$

Video Content / Details of website for further learning (if any):

- **https://learn.lboro.ac.uk/archive/olmp/olmp_resources/pages/workbooks_1_50_jan2008/Workbook21/21_2_bscs_z_trnsfm_thry.pdf**

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :10.1-10.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 43

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: The region of convergence for Z transform

Introduction: The open region where X(z) converges is referred to as the region of convergence (ROC) of X(z). In other discussions, the ROC may mean the region where X(z) converges.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DTFT

Zeros and Poles of z-Transform

- Assume that X(z) is rational, i.e., X(z)=P1 (z)/P2 (z), where P1 (z) and P2 (z) are two polynomials.
- The roots of P1 (z)=0 are called the zeros of X(z), and the roots of P2 (z)=0 are called the poles of X(z). Zeros and poles are indicated with o and × in the complex plane, respectively.
- The algebraic expression of X(z) can be specified by its zeros and poles except for a scale factor.

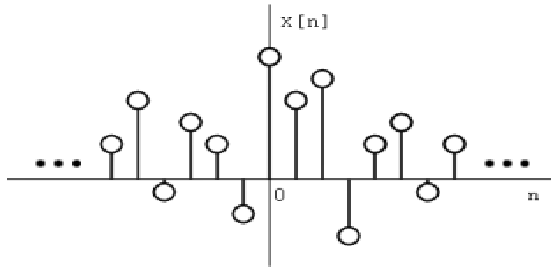
Region of Convergence of z-Transform

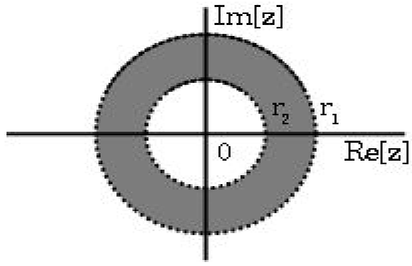
The open region where X(z) converges is referred to as the region of convergence (ROC) of X(z).

In other discussions, the ROC may mean the region where X(z) converges.

It should be noted that both the algebraic expression and the ROC of X(z) are required to specify x(n) uniquely.

Assume that x(n) has the z-transform X(z). For different types of x(n), the ROC of X(z) has different types.





Example: Find the Zeros of

$$h[n] = \delta[n] + \frac{1}{6}\delta[n-1] - \frac{1}{6}\delta[n-2]$$

- The z-transform is

$$\begin{aligned} H(z) &= 1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2} \\ &= \left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right) \\ &= \left(z + \frac{1}{2}\right)\left(z - \frac{1}{3}\right) / z^2 \end{aligned}$$

(1) If x(n) is of finite duration, then the ROC of X(z) is the entire plane, i.e., z∈C. Especially, z=0 needs to be excluded from the ROC if x(n) has nonzero values for n>0.

(2) If x(n) is right-sided, then the ROC of X(z) is the exterior of a circle centered about the origin, i.e., |z|>r0 (3) If x(n) is left-sided, then the ROC of X(z) is the interior of a circle centered about the origin, i.e., |z|0. (4) If x(n) is two-sided, then the ROC of X(z) is a ring centered about the origin, i.e., r1<|z

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=a05UJmw5wak**
- http://fourier.eng.hmc.edu/e102/lectures/Z_Transform/node3.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :10.6-10.9)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture:

The inverse Z transform

Introduction:

In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform. This similarity is explored in the theory of time-scale calculus.

Prerequisite knowledge for Complete understanding and learning of Topic:

Basics of z-transform

1. **Find the inverse z-transform of X(z) = z+4/(z² – 4z+3)**

$X(z) = z+4/(z^2 -4z + 3)$

$X(z) / z = z+4 / z(z^2 -4z + 3) = z+4 / z(z-10 (z-3)$

$= A_1/z + A_2/z-1 + A_3/z-3$

$A_1 = z.X(z)/z \mid_{z=0} = z+4/(z-1) (z-3) \mid_{z=0} = 4/3$

$A_2=(z-1) X(z)/z \mid_{z=1} = z+4/z(z-3) \mid_{z=1} = -5/2$

$A_3=(z-3) X(z)/z_{\ z=3} = z+4/z(z-1) \mid z=3 = 7/6$

$X(z)/z = 4/3/z – 5/2/z-1 + 7/6/z-3$

$X(z) = 4/3 – 5/2 . 1/1-z^{-1}+ 7/6 .1/1-3z^{-1}$

$x(n) = 4/3\, \delta(n) - 5/2\, (1)^n\, u(n) + 7/6 (3)^n\, u(n)$

$= 4/3\, \delta(n) - 5/2\, u(n) + 7/6\, (3)^n\, u(n)$

2. Find the inverse z-transform of $X(z) = 1/1\text{-}1.5z^{-1} + 0.5z^{-2}$ for ROC : $0.5 < |z| < 1$.

$X(z) = z^2/z^2 - 1.5z+0.5$

$X(z)/z = z/z^2 - 1.5z + 0.5 = z/(z-1)(z-0.5)$

$X(z)/z = A_1 /z\text{-}1 + A_2/z\text{-}0.5$

$A_1 = (z-1) . z/(z-1)(z-0.5) \;|_{z=1} = 1/1\text{-}0.5 = 2$

$A_2 = (z-0.5). z/(z-1)(z-0.5) \;|_{z=0.5} = 0.5/0.5\text{-}1 = -1$

$X(z) /z = 2/z\text{-}1\text{-}1/z\text{-}0.5$

$X(z) = 2z/z\text{-}1 - z/z\text{-}0.5$

$=2/1\text{-}z^{-1}\text{-} 1/1\text{-}0.5z^{-1}$

Taking invers z-transform

$x(n) = 2[2\text{-}1^n\, u(-n-1)] - (0.5)^n\, u(n)$

$= -2u(-n-1)-(0.5)^n u(n)$

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=Gr8OwFx3sqA**
- https://web.eecs.umich.edu/~aey/eecs206/lectures/zfer.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :10.33-10.55)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 45

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: Properties of Z Transform

Introduction:

In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform. This similarity is explored in the theory of time-scale calculus.

Prerequisite knowledge for Complete understanding and learning of Topic:

Basics of z-transform

ii.Time shifting : $x(n-k) \overset{z}{\leftrightarrow} z^{-k} X(z)$

* This property says that any shift of 'k' samples in time domain sequence is equivalent to multiplying 'z' transform by z^{-k}.
* This property is useful in converting difference equation to system function.

Linearity

$$a_1 x_1(n) + a_2 x_2(n) \overset{z}{\longleftrightarrow} a_1 x_1(z) + a_2 x_2(z)$$

Proof :

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [a_1 x_1(n) + a_2 x_2(n)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a_1 x_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} a_2 x_2(n) z^{-n}$$

$$= a_1 \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + a_2 \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} \quad \text{Since } a_1 \text{ and } a_2 \text{ are constants.}$$

$$= a_1 X_1(z) + a_2 X_2(z)$$

iii. Scaling in z-domain : $a^n x(n) \overset{z}{\leftrightarrow} X(z^{-1})$, ROC : $1/r_2 < |z| < 1/r_1$

- If the sequence is reversed in time domain, then powers of 'z' are reversed in z- domain.
- The ROC from $r_1 < |z| < r_2$ is changed to $1/r_2 < |z| < 1/r_1$.

iv. Time reversal : $x(-n) \overset{z}{\leftrightarrow} X(z^{-1})$, ROC : $1/r_2 < |z| < 1/r_1$.

- If the sequence is reversed in time domain, then powers sof 'z' are reversed in z-domain.
- The ROC from $r_1 < |z| < r_2$ is changed to $1/r_2 < |z| < 1/r_1$.

v. Convolution : $x_1(n) * x_2(n) \overset{z}{\leftrightarrow} X_1(z). X_2(z)$.

- The convolution in time domain is equivalent to product in z-domain.
- This property is useful in filtering

Video Content / Details of website for further learning (if any):

- **https://www.tutorialspoint.com/digital_signal_processing/dsp_z_transform_properties.htm**
- https://www.youtube.com/watch?v=RYXlHkqqdh8

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no :10.15-10.26)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture: The unilateral Z transform

Introduction:

In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform. This similarity is explored in the theory of time-scale calculus.

Prerequisite knowledge for Complete understanding and learning of Topic:

Basics of z-transform

Unilateral Z-transform

The *unilateral* z-transform of an arbitrary signal x[n] is defined as

$$\mathcal{UZ}[x[n]] = X(z) \stackrel{\triangle}{=} \sum_{n=-\infty}^{\infty} x[n]u[n]z^{-n} = \sum_{n=0}^{\infty} x[n]z^{-n}$$

- When the unilateral z-transform is applied to find the transfer function of an LTI system, it is always assumed to be causal, and the ROC is always the exterior of a circle.
- The unilateral z-transform of any signal is identical to its bilateral Laplace transform.
- However, if, the two z-transforms are different. Some of the properties of the unilateral z-transform different from the bilateral z-transform are listed below.

$$z\left[\sum_{m=0}^{\infty} x[m]z^{-m} - x[0]\right] = zX(z) - zx[0]$$

where we have assumed $m = n + 1$.

Time Delay

$$\begin{aligned}\mathcal{UZ}\left[x[n-1]\right] &= \sum_{n=0}^{\infty} x[n-1]z^{-n} = z^{-1}\sum_{m=-1}^{\infty} x[m]z^{-m} \\ &= z^{-1}\left[\sum_{m=0}^{\infty} x[m]z^{-m} + zx[-1]\right] = z^{-1}X(z) + x[-1]\end{aligned}$$

where $m = n - 1$. Similarly, we have

$$\begin{aligned}\mathcal{UZ}[x[n-2]] &= \sum_{n=0}^{\infty} x[n-2]z^{-n} = z^{-2}\sum_{m=-2}^{\infty} x[m]z^{-m} \\ &= z^{-2}\left[\sum_{m=0}^{\infty} x[m]z^{-m} + zx[-1] + z^2x[-2]\right]\end{aligned}$$

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=PRpXPqrV2ms**
- https://unacademy.com/lesson/unilateral-z-transform-and-solving-of-difference-equations/HKRVR17P

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :10.55)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 47

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture:Geometric evaluation of the Fourier transform from the pole zero plot

Introduction: In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform. This similarity is explored in the theory of time-scale calculus.

Prerequisite knowledge for Complete understanding and learning of Topic:

Z-transform poles and zeroes

Determine z-transform, ROC, pole-zero locations of the following functions :

(i) $a^n \cos(\Omega_0 n)\, u(n)$ *for* $\Omega_0 = 2\pi$ *get pole-zero plot.*

(ii) $0.2^n \{u(n) - u(n-4)\}$ **(Jan./Feb.-2004, 10 Marks)**

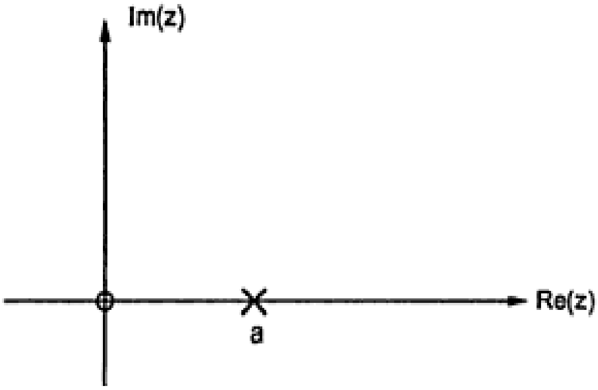
With $\Omega_0 = 2\pi$,

$$X(z) = \frac{1 - \left(\frac{z}{a}\right)^{-1} \cos 2\pi}{1 - 2\left(\frac{z}{a}\right)^{-1} \cos 2\pi + \left(\frac{z}{a}\right)^{-2}}$$

$$= \frac{1 - a z^{-1}}{1 - 2 a z^{-1} + a^2 z^{-2}}$$

$$= \frac{z}{z-a}$$

Pole : z = a



(ii) $x(n) = n\,\alpha^n u(-n)$

$$\alpha^n u(n) \xleftrightarrow{z} \frac{1}{1-\alpha z^{-1}} \quad \text{or} \quad \frac{z}{z-\alpha} \qquad ROC: |z| > |\alpha|$$

$$\therefore \quad \alpha^{-n} u(n) \xleftrightarrow{z} \frac{z}{z-\frac{1}{\alpha}} \qquad ROC: |z| > \frac{1}{|\alpha|}$$

By time reversal property, $x(-n) \xleftrightarrow{z} X\left(z^{-1}\right)$.

$$\therefore \quad \alpha^n u(-n) \xleftrightarrow{z} \frac{z^{-1}}{z^{-1}-\frac{1}{\alpha}} \qquad ROC: \left|z^{-1}\right| > \frac{1}{|\alpha|} \text{ or } |z| < |\alpha|$$

$$\xleftrightarrow{z} \frac{1}{1-\alpha^{-1} z}$$

Now by differentiation property,

$$n\,\alpha^n u(-n) \xleftrightarrow{z} -z\frac{d}{dz}\left[\frac{1}{1-\alpha^{-1}z}\right]$$

$$\xleftrightarrow{z} -\frac{\alpha^{-1}z}{\left(1-\alpha^{-1}z\right)^2}$$

Video Content / Details of website for further learning (if any):

- **https://www.slader.com/discussion/question/use-geometric-evaluation-from-the-pole-zero-plot-to-determine-the-magnitude-of-the-fourier-transform/**

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 10.58)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 48

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **IV- Analysis of Discrete Time Signal**

Date of Lecture:

Topic of Lecture:

The relationship between Z transform and DTFT

Introduction:

In mathematics and signal processing, the Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform. This similarity is explored in the theory of time-scale calculus.

Prerequisite knowledge for Complete understanding and learning of Topic:

Z-transform and DTFT

Obtain the relation between z-transform and DTFT.

X(z) = $\sum_{n=-\infty}^{\infty} x(n) z-n \ By\ definition\ of\ z-transform$

We know that $z = re^{j\Omega}$, where r = |z| and $\Omega = \angle z$.

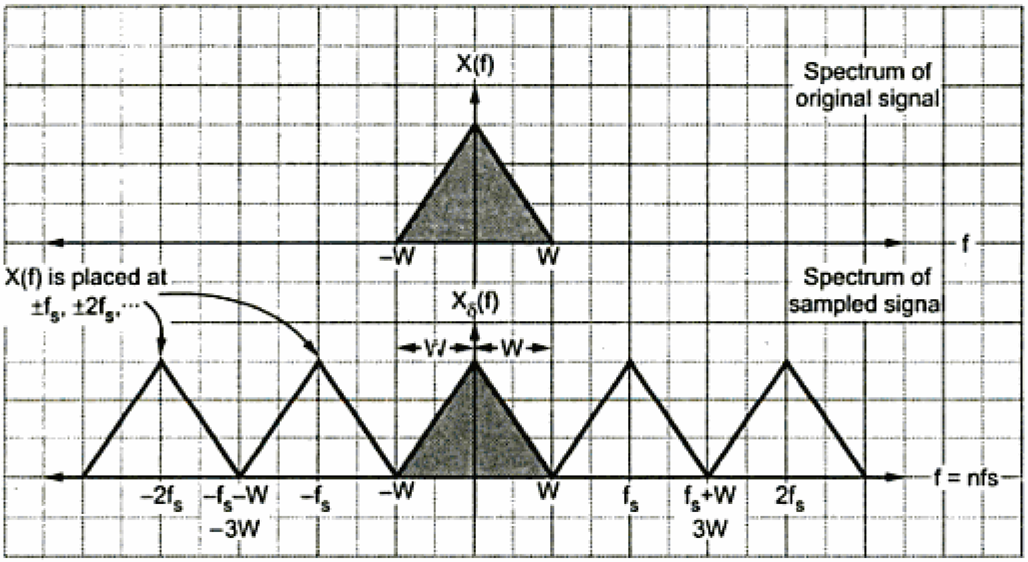
X(z) = $\sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\Omega n}$

DTFT is given as, $X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\ e^{-j\Omega n}$. Above equation is DTFT of $x(n)\ r^{-n}$. If we evaluate X(z) on unit cirucle, then |z| = r=1. Hence above equation will be,

$X(z) = \sum_{n=-\infty}^{\infty} x(n)\ e^{-j\Omega n}$. , r = |z| = 1

= $X(\Omega)$ or DTFT

$X(\Omega) = X(z)\ |_{z=e^{j\Omega}}$



or $$X(f) = \frac{1}{f_s} X_\delta(f)$$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

DTFT is, $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$\therefore$ $$X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi fn}$$

Video Content / Details of website for further learning (if any):

- http://www.commsp.ee.ic.ac.uk/~tania/teaching/dsp/Lectures%2034%20DTFT%20DFT%20 and%20z-%20Transforms.pdf

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008.
(Page no : 10.62-10.64)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 49

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : V- Linear Time Invariant-Discrete Time Systems

Date of Lecture:

Topic of Lecture: Difference Equation

Introduction: A differential equation is an equation that relates one or more functions and their derivatives.[1] In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DFT

The difference equation of the system is given as

$$y(n) = \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

Find out (i) System function (ii) Unit sample response (iii) Pole zero plot (iv) Transfer function and its magnitude phase plot (v) Magnitude/phase response using geometric interpretation.

Solution: (i) To find system function :

Taking z-transform of given difference equation,

$$Z\{y(n)\} = Z\left\{\frac{1}{2}x(n) + \frac{1}{2}x(n-1)\right\}$$

Applying the linearity and time shift properties,

$$Y(z) = \frac{1}{2}X(z) + \frac{1}{2}z^{-1}X(z) = \left(\frac{1}{2} + \frac{1}{2}z^{-1}\right)X(z)$$

$$\therefore \quad \frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{2}z^{-1}$$

(ii) To find unit sample response :

A unit sample response $h(n)$ is obtained by taking inverse z-transform of $H(z)$. i.e.,

$$h(n) = IZT\ \{H(z)\} = IZT\ \left\{\frac{1}{2} + \frac{1}{2} z^{-1}\right\}$$

Applying the linearity and time shifting properties and from z-transform pair $\delta(n) \leftrightarrow 1$, we can write,

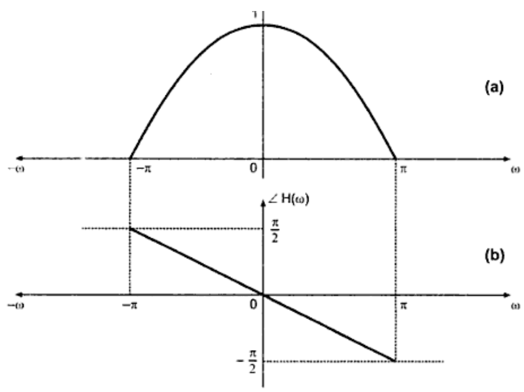
$$h(n) = \frac{1}{2}\,\delta(n) + \frac{1}{2}\,\delta(n-1)$$

(iii) To determine pole zero plot :

Consider the system function $H(z)$,

$$H(z) = \frac{1}{2} + \frac{1}{2} z^{-1}$$

Let us convert the powers of 'z' to positive by rearranging the equation as follows ;



). i.e.,
r δ(n) ↔ 1,
lows ;

Plots of magnitude and phase of transfer function
(a) : Plot of magnitude transfer function |H (ω)| versus ω
(b) : Plot of phase of the transfer function ∠ H (ω) versus ω

Video Content / Details of website for further learning (if any):

- **https://byjus.com/maths/differential-equation/**
- https://www.math24.net/linear-differential-equations-first-order/

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no 11.1-11.1:)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 50

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time Invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Problems in Difference Equation

Introduction: A differential equation is an equation that relates one or more functions and their derivatives.[1] In applications, the functions generally represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DFT

1. Determine the complete response of the system

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\,y(t) = \frac{dx(t)}{dt}$$

$$with\quad y(0)=0,\ \left.\frac{dy(t)}{dt}\right|_{t=0} = 1\ and\ x(t) = e^{-2t}\,u(t)$$

Solution

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4\,y(t) = \frac{dx(t)}{dt}$$

The complete response will be given as,

$$y(t) = y^{(n)}(t) + y^{(p)}(t)$$

To determine $y^{(n)}(t)$

The given differential equation has order $N = 2$.

$$r^2 + 5r + 4 = 0$$

Roots of this equation will be,

$$r_1 = -4 \text{ and } r_2 = -1$$

$$y^{(n)}(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$$= c_1 e^{-4t} + c_2 e^{-t}$$

To determine $y^{(p)}(t)$

The input is $x(t) = e^{-2t} u(t)$.

$$y^{(p)}(t) = k e^{-2t}$$

Hence putting $y(t) = y^{(p)}(t) = k e^{-2t}$

$$x(t) = e^{-2t} u(t)$$

$$\frac{d^2}{dt^2}\left(k e^{-2t}\right) + 5 \frac{d}{dt}\left(k e^{-2t}\right) + 4 \times k e^{-2t} = \frac{d}{dt}\left(e^{-2t}\right)$$

$$4 k e^{-2t} - 10 k e^{-2t} + 4 k e^{-2t} = -2 e^{-2t}$$

$$k = 1$$

$$y^{(p)}(t) = e^{-2t}$$

To determine $y(t)$

Putting $y^{(p)(}$t) from above equation and $y^{(p)}$(t)

$$y(t) = c_1 e^{-4t} + c_2 e^{-t} + e^{-2t}$$

Now let us use initial conditions to determine the values of c1 and c2. Putting y(0)=o

$$0 = c_1 + c_2 + 1 \Rightarrow c_1 + c_2 = -1$$

$$\frac{dy(t)}{dt} = -4 c_1 e^{-4t} - c_2 e^{-t} - 2 e^{-2t}$$

Putting $\left.\frac{dy(t)}{dt}\right|_{t=0} = 1$ in above equation,

$$1 = -4 c_1 - c_2 - 2 \Rightarrow 4 c_1 + c_2 = -3$$

Solving for c1 and c2 , we get

$$c_1 = -\frac{2}{3} \text{ and } c_2 = -\frac{1}{3}$$

Hence the complete response becomes

$$y(t) = -\frac{2}{3} e^{-4t} - \frac{1}{3} e^{-t} + e^{-2t}$$

This is the required response of the system considering input as well as initial conditions.

Video Content / Details of website for further learning (if any): • **https://www.math24.net/linear-differential-equations-first-order/**
Importan Books/Journals for further learning including the page nos.: Ramesh Babu “Signals And System”, Scitec, Fourth Edition, 2008. (Page no : 11.3-11.4)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS & 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time Invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Block Diagram Representation

Introduction: A mathematical block diagram gives a graphically representation of a mathematical model. The block diagram in itself gives good information of the structure of the model

Prerequisite knowledge for Complete understanding and learning of Topic:

- Differential equation

The output of first system is $y_1(t)$. It can be given as,

$$y_1(\tau) = x(\tau) * h_1(\tau)$$

$$= \int_{-\infty}^{\infty} x(m)\ h_1(\tau - m)\, dm \qquad \dots (3.2.21)$$

Here separate variables τ and m are used. Putting above equation for $y_1(\tau)$ in equation 3.2.20.

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(\tau - m) h_2(t-\tau)\, dm\, d\tau$$

Here put $\tau - m = n$, then we get

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(n)\cdot h_2(t-m-n)\, dm\, dn$$

Fig. 3.2.21 Cascade connection of two LTI systems

The output $y(t)$ of the second system can be given as,

$$y(t) = y_1(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} y_1(\tau)\, h_2(t-\tau)\, d\tau \qquad \dots (3.2.20)$$

The output of first system is $y_1(t)$. It can be given as,

$$y_1(\tau) = x(\tau) * h_1(\tau)$$

$$= \int_{-\infty}^{\infty} x(m)\ h_1(\tau - m)\, dm \qquad \text{... (3.2.21)}$$

Here separate variables τ and m are used. Putting above equation for $y_1(\tau)$ in equation 3.2.20.

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(\tau - m) h_2(t-\tau) dm\, d\tau$$

Here put $\tau - m = n$, then we get

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(m)\, h_1(n) \cdot h_2(t - m - n)\, dm\, dn$$

$$= \int_{-\infty}^{\infty} x(m) \left[\int_{-\infty}^{\infty} h_1(n) \cdot h_2((t-m)-n)\, dn \right] dm \qquad \text{... (3.2.22)}$$

The integration in square brackets indicate convolution of $h_1(t)$ and $h_2(t)$ evaluated at t – m. i.e.,

$$\int_{-\infty}^{\infty} h_1(n)\, h_2\ ((t-m)-n)\ dn = h(t-m)$$

Putting this value in equation 3.2.22 we get,

$$y(t) = \int_{-\infty}^{\infty} x(m)\, h(t-m)\, dm$$

$$= x(t) * h(t)$$

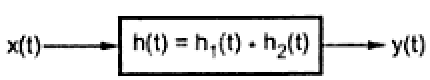


Fig. 3.2.22 Equivalent of cascade connection of Fig. 3.2.21

Thus if the two systems are connected in cascade, the overall impulse response is equal to convolution of two impulse responses. This is shown in Fig. 3.2.22 .

We know that,

$$y_1(t) = x(t) * h_1(t)$$

and $$y(t) = y_1(t) * h_2(t)$$

Putting for $y_1(t)$ in above equation,

$$y(t) = [x(t) * h_1(t)] * h_2(t) \quad ... (3.2.23)$$

And from Fig. 3.2.22 we can write,

$$y(t) = x(t) * h(t)$$

$$= x(t) * [h_1(t) * h_2(t)] \quad ... (3.2.24)$$

Thus equation 3.2.23 and above equation prove associative property. i.e.,

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)] \quad ... (3.2.25)$$

3. Distributive property of convolution :

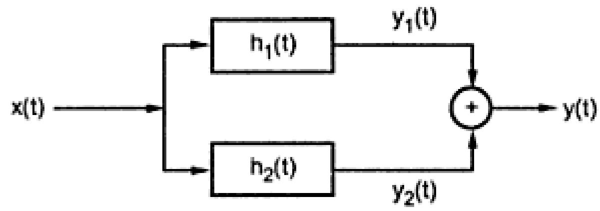


Fig. 3.2.23 Parallel connection of the systems

Consider the two systems connected in parallel as shown in Fig. 3.2.23.

The overall output is,

$$y(t) = y_1(t) + y_2(t)$$

$$= x(t) * h_1(t) + x(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h_1(t-\tau)\, d\tau + \int_{-\infty}^{\infty} x(\tau)\, h_2(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) \{h_1(t-\tau) + h_2(t-\tau)\}\, d\tau = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= x(t) * h(t)$$

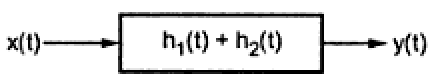


Fig. 3.2.24 Equivalent system of Fig. 3.2.23

Here $h(t) = h_1(t) + h_2(t)$. Thus impulse responses of the parallel connected systems are added. i.e.

This proves the distributive property which can be stated as,

$$x(t) * h_1(t) + x(t) * h_2(t) = x(t) * \{h_1(t) + h_2(t)\} \quad ... (3.2.26)$$

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=ekAK9VizaNs**
- **http://celinagrace.com/lm-q720/block-diagram-reduction-problems-and-solutions-pdf.html**

Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : 11.4-11.5)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 52

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : V- Linear Time Invariant-Discrete Time Systems

Date of Lecture:

Topic of Lecture: Problems in Block Diagram Representation

Introduction: A mathematical block diagram gives a graphically representation of a mathematical model. The block diagram in itself gives good information of the structure of the model

Prerequisite knowledge for Complete understanding and learning of Topic:

- Differential equation

For the interconnection system of the system shown in fig, obtain the overall impulse response in terms of impulse response of individual subsystems.

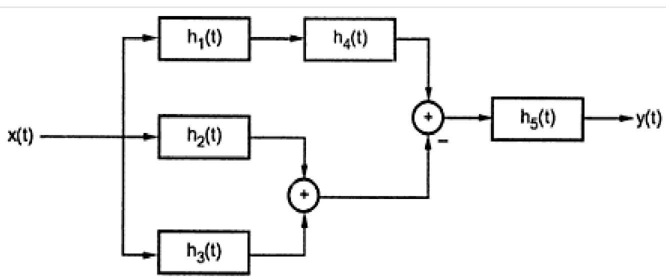


Fig. 3.2.25 Interconnection of the systems

Solution : Let us combine cascade connection of $h_1(t)$ and $h_4(t)$. Similarly combine parallel connection of $h_2(t)$ and $h_3(t)$. This is shown below.

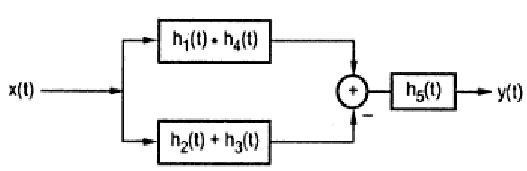


Fig. 3.2.26 Simplified version of Fig. 3.2.25

Now let us combine parallel combination of the two systems. This is shown below.

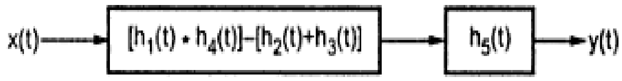


Fig. 3.2.27 Further simplified version of Fig. 3.2.26

From above figure we can write the overall impulse response as,

$$h(t) = \left\{ h_1(t) * h_4(t) - \left[h_2(t) + h_3(t)\right]\right\} * h_5(t)$$

$$= \int_{-\infty}^{\infty} x(m) \left[\int_{-\infty}^{\infty} h_1(n) \cdot h_2((t-m)-n)\, dn \right] dm \qquad \text{... (3.2.22)}$$

The integration in square brackets indicate convolution of $h_1(t)$ and $h_2(t)$ evaluated at t – m. i.e.,

$$\int_{-\infty}^{\infty} h_1(n)\, h_2\, ((t-m)-n)\, dn = h(t-m)$$

Putting this value in equation 3.2.22 we get,

$$y(t) = \int_{-\infty}^{\infty} x(m)\, h(t-m)\, dm$$

$$= x(t) * h(t)$$

Video Content / Details of website for further learning (if any):

- **http://celinagrace.com/lm-q720/block-diagram-reduction-problems-and-solutions-pdf.html**

Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : 11.6-11.6)

Course Faculty

Verified by HOD

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : V- Linear Time Invariant-Discrete Time Systems

Date of Lecture:

Topic of Lecture: Impulse Response

Introduction: A mathematical block diagram gives a graphically representation of a mathematical model. The block diagram in itself gives good information of the structure of the model

Prerequisite knowledge for Complete understanding and learning of Topic:

- Differential equation

Example 5.3.1 : *Determine the range of values 'a' and 'b', for which the LTI system with impulse response*

$$h(n) = \begin{cases} a^n, & n \geq 0 \\ b^n, & n < 0 \end{cases} \quad \text{... (5.3.9)}$$

is stable.

Solution : The condition for stability is given by equation 5.3.8 as,

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Splitting the summation according to equation 5.3.9 and putting for $h(n)$ in above equation we get,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} |a^n| + \sum_{n=-\infty}^{-1} |b^n| \quad \text{... (5.3.10)}$$

$$= \frac{1}{|b|} + \frac{1}{|b^2|} + \frac{1}{|b^3|} + \frac{1}{|b^4|} + \ldots\ldots$$

$$= \frac{1}{|b|}\left[1 + \frac{1}{|b|} + \frac{1}{|b^2|} + \ldots\right]$$

Let us consider the summation terms in above equation separately. The first summation can be written as,

$$\sum_{n=0}^{\infty} \left|a^n\right| = 1+|a|+\left|a^2\right|+\left|a^3\right|+.......$$

This is a standard geometric series and it converges to $\frac{1}{1-|a|}$ if $|a|<1$. If $|a|>1$, the series does not converge and it becomes unstable. Thus,

$$\sum_{n=0}^{\infty} \left|a^n\right| = \frac{1}{1-|a|}, \text{ if } |a|<1 \qquad \text{... (5.3.11)}$$

Now let us consider the second summation in equation 5.3.10. It can be rearranged as follows :

$$\sum_{n=-\infty}^{-1} \left|b^n\right| = \sum_{n=1}^{\infty} \frac{1}{\left|b^n\right|}$$

The part inside square brackets in above equation is the geometric series and it converges to, $\frac{1}{1-\frac{1}{|b|}}$, if $\frac{1}{|b|}<1$ i.e. $|b|>1$. Hence equation 5.3.12 becomes,

$$\sum_{n=-\infty}^{-1} \left|b^n\right| = \frac{1}{|b|}\cdot\frac{1}{1-\frac{1}{|b|}}, \text{ if } |b|>1$$

$$= \frac{1}{|b|-1}, \text{ if } |b|>1 \qquad \text{... (5.3.13)}$$

Putting the values of summations obtained in the above equation and equation 5.3.11 in equation 5.3.10 we get,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \frac{1}{1-|a|}+\frac{1}{|b|-1}, \text{ if } |a|<1<|b|$$

Thus the geometric series converges if $|a|<1<|b|$. Thus the system will be stable if $|a|<1<|b|$.

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=plor6ywAXXg
- https://courses.engr.illinois.edu/ece401/sp2017/lecture8impulse.pdf

Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :11.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 54

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time Invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Convolution Sum

Introduction: Convolution is a mathematical way of combining two signals to form a third signal. It is the single most important technique in Digital Signal Processing. Convolution is important because it relates the three signals of interest: the input signal, the output signal, and the impulse response.

Prerequisite knowledge for Complete understanding and learning of Topic:

- convolution

The convolution sum is a fast way to find the coefficients of the polynomial resulting from the multiplication of two polynomials.

(a)Suppose x[n]=u[n]−u[n−3] find its Z-transform X(z), a second-order polynomial in z−1.

(b)Multiply X(z) by itself to get a new polynomial Y(z)=X(z)X(z)=X2(z). Find Y(z).

(c)Do graphically the convolution of x[n] with itself and verify that the result coincides with the coefficients of Y(z).

(d)Use the *conv* function to find the coefficients of Y(z).

Example 5.3.8 : *Determine the inverse z-transform of* $Y(z)=\frac{z}{(z-1)^3}$ *using convolution*

[Madras University April - 95, 99]

Solution : Let us write the given function as,

$$Y(z) = \frac{1}{z-1}\frac{z}{(z-1)^2}$$

$$= X(z)\ H(z) \qquad \text{... (5.3.27)}$$

Here $X(z)=\frac{1}{z-1}$ and $H(z)=\frac{z}{(z-1)^2}$. Here we have selected $X(z)$ and $H(z)$ as a product of two functions to give $Y(z)$.

We can write $X(z)$ as,

$$X(z) = \frac{1}{z-1}$$

$$= z^{-1}\frac{1}{1-z^{-1}}$$

The inverse z-transform of above function becomes,

$x(n) = u(n-1)$ with the help of time shifting property.

Similarly consider the function $H(z)$. i.e.,

$$H(z) = \frac{z}{(z-1)^2}$$

The above function can be written as,

$$H(z) = \frac{z^{-1}}{\left(1-z^{-1}\right)^2}$$

The inverse z-transform of this function is,

$$h(n) = n\,u(n)$$

Equation 5.3.27 can be written in time domain as,

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

We know that $x(n)=u(n-1)$. In the above equation we can write

$$y(n) = \sum_{k=-\infty}^{\infty} u(k-1)\, h(n-k)$$

We know that $u(k-1)=1$ for $k \geq 1$.

Hence above equation becomes,

$$y(n) = \sum_{k=1}^{\infty} 1\, h(n-k)$$

Putting for $h(n)=n\,u(n)$ in above equation,

$$y(n) = \sum_{k=1}^{\infty} (n-k)\, u(n-k)$$

We know that $u(n-k)=1$ for $n \geq k$ or

$u(n-k)=1$ for $k \leq n$

Hence $y(n)$ can be written as,

$$y(n) = \sum_{k=1}^{n} (n-k)\, 1$$

For $k=n$, $n-k=0$, hence above equation becomes,

For $k=n$, $n-k=0$, hence above equation becomes,

$$y(n) = \sum_{k=1}^{n-1} (n-k)$$

By rearranging the above equation,

$$y(n) = \sum_{k=1}^{n-1} k \qquad \text{... (5.3.30)}$$

Here let us use the finite series formula : $\sum_{k=1}^{N} n = \frac{N(N+1)}{2}$. Hence above equation becomes,

$$y(n) = \frac{(n-1)\,(n-1+1)}{2}\, u(n)$$

$$= \frac{n\,(n-1)}{2}\, u(n)$$

Video Content / Details of website for further learning (if any):

- **https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-5.pdf**

Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : 11.8-11.9)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 55

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time Invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Discrete Fourier Series
Introduction: Fourier series coefficients represent various frequencies present in the signal. It is nothing but spectrum of the signal.
Prerequisite knowledge for Complete understanding and learning of Topic: • Discrete Fourier Series

Find the Fourier series for the periodic signal $x(t) = t$, $0 \le t \le 1$ and repeats every 1 sec.

(April/May-2003, 10 Marks)

Sol. : The signal $x(t)$ can be represented over one period as,

$$x(t) = t \quad \text{for } 0 < t < 1$$

Period $T_0 = 1$

From equation 2.3.7 C_n is given as,

$$C_n = \frac{1}{T_0} \int_{t}^{t+T_0} x(t)\, e^{-j2\pi n t / T_0}\, dt$$

$$= \frac{1}{1} \int_0^1 t \cdot e^{-j2\pi n t} \; dt \qquad \text{since } T_0 = 1$$

Here we will use $\int x e^{ax} \, dx = e^{ax} \left[\frac{x}{a} - \frac{1}{a^2} \right]$ from appendix C – 30.

Let $a = -j\,2\pi n$, then C_n will be,

$$C_n = \frac{-j}{2\pi n} \quad \text{[for } n \neq 0 \text{ on simplification]} \qquad \text{... (2.3.72)}$$

With $n = 0$ in equation 2.3.7, C_0 will be,

$$C_0 = \frac{1}{T_0} \int_t^{t+T_0} x(t) \; dt \quad \text{i.e. } a_0 = \int_0^1 t \; dt$$

$$= \frac{1}{2} \qquad \text{... (2.3.73)}$$

From equation 2.3.72 magnitude of C_n will be,

$$|C_n| = \sqrt{0 + \frac{1}{(2\pi n)^2}} = \frac{1}{2\pi n} \qquad \text{... (2.3.74)}$$

$$\phi_n = arg\; (C_n) = -tan^{-1} \left(\frac{-1/2\pi n}{0} \right)$$

$$= \pm 90°$$

Since phase spectrum is odd function,

$$arg\; (C_n) = \left. \begin{matrix} -90° \text{ for } n > 0 \\ +90° \text{ for } n < 0 \end{matrix} \right\} \qquad \text{... (2.3.75)}$$

$$x(t) = \frac{1}{j} e^{-j3} e^{j\omega_0 t} - \frac{1}{j} e^{j3} e^{-j\omega_0 t} + \frac{1}{2j} e^{j3\omega_0 t} - \frac{1}{2j} e^{-j3\omega_0 t} \qquad \text{...(1)}$$

Step 3 : Now consider the synthesis equation of Fourier series,

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

Putting for k = – 3 to + 3 in above equation,

$$x(t) = \sum_{k=-3}^{3} X(k) e^{jk\omega_0 t}$$

$$= X(-3) e^{-j3\omega_0 t} + X(-2) e^{-j2\omega_0 t} + X(-1) e^{-j\omega_0 t} + X(0)$$
$$+ X(1) e^{j\omega_0 t} + X(2) e^{j2\omega_0 t} + X(3) e^{j3\omega_0 t}$$

Comparing above equation with equation (1),

$$X(-3) = -\frac{1}{2j},\; X(-2) = 0,\; X(-1) = -\frac{1}{j} e^{j3},\; X(0) = 0$$

$$X(1) = \frac{1}{j} e^{-j3},\; X(2) = 0,\; X(3) = \frac{1}{2j}$$

Video Content / Details of website for further learning (if any): • **https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-5.pdf**
Importan Books/Journals for further learning including the page nos.: Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.1-8.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 56

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : SIGNALS AND SYSTEMS / 19ECC02

Course Faculty : Mrs.S.Punitha

Unit : V- Linear Time Invariant-Discrete Time Systems

Date of Lecture:

Topic of Lecture: Problems in Discrete Fourier Series

Introduction: Fourier series coefficients represent various frequencies present in the signal. It is nothing but spectrum of the signal.

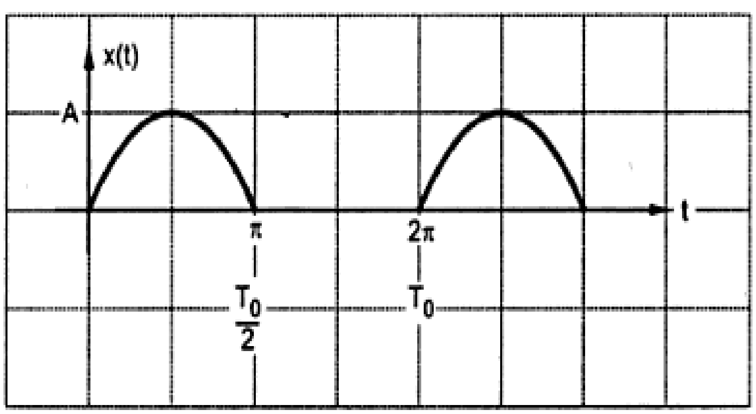
Prerequisite knowledge for Complete understanding and learning of Topic:

- Discrete Fourier Series

3. *Determine the trigonometric Fourier series representation of the half wave rectifier output.*

(April/May-2004, 10 Marks)

i. : Fig. 1 shows the half wave rectifier output.



$$= \frac{A}{\pi}\int_0^{\pi} \sin t \sin kt\, dt \qquad since\ \omega_0 = 1$$

$$= \frac{A}{\pi}\int_0^{\pi} \frac{\cos(t-kt)-\cos(t+kt)}{2}\, dt$$

$$= \frac{A}{2\pi}\left[\int_0^{\pi} \cos(1-k)t\, dt - \int_0^{\pi} \cos(1+k)t\, dt\right]$$

$$= \frac{A}{2\pi}\left\{\left[\frac{\sin(1-k)t}{1-k}\right]_0^{\pi} - \left[\frac{\sin(1+k)t}{1+k}\right]_0^{\pi}\right\}$$

$$= 0$$

Step 5 : To express Fourier series

$$x(t) = a(0) + \sum_{k=1}^{\infty} a(k)\cos k\,\omega_0\, t + \sum_{k=1}^{\infty} b(k)\sin k\,\omega_0\, t$$

Putting values in above equation,

$$x(t) = \frac{A}{\pi} + \sum_{k=2,4,6,\dots}^{\infty} \frac{2A}{\pi(1-k^2)}\cos k\,\omega_0 t + 0$$

$$= \frac{A}{\pi}\left\{1 + \sum_{k=2,4,6,\dots}^{\infty} \frac{2}{1-k^2}\cos kt\right\} \qquad since\ \omega_0 = 1$$

Video Content / Details of website for further learning (if any):

- https://nptel.ac.in/content/storage2/courses/117101055/downloads/Lec-5.pdf

Importan Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :8.1-8.7)

Course Faculty

Verified by HOD

LECTURE HANDOUTS

ECE | **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time Invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Z Transform

Introduction: The Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DTFT

Definition

- Given a finite length signal as $x[n]$, the *z*-transform is defined

$$X(z) = \sum_{k=0}^{N} x[k]z^{-k} = \sum_{k=0}^{N} x[k](z^{-1})^{k}$$

where the sequence support interval is [0, *N*], and *z* is any complex number

- This transformation produces a new representation of $x[n]$ denoted $X[z]$
- Returning to the original sequence (*inverse z-transform*) $x[n]$ requires finding the coefficient associated with the *n*th power of z^{-1}
- The *z*-transform is particularly useful in the analysis and design of LTI systems

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \sum_{k=0}^{M} h[k]z^{-k}$$

1. Find the z-transform of $x(n) = a^n \sin \Omega_o n\, u(n)$. **(Oct./Nov. – 2008, 6 Marks)**

$x(n) = a^n \sin (\Omega_o n)\, u(n)$

Let $x_1(n) = \sin (\Omega_o n)\, u(n)$

$X(z) = Z\{e^{j\Omega on} - e^{-j\Omega on}/2j\}\, u(n)$

$=1/2j\, [Z\{e^{j\Omega on}u(n)\} - Z\{e^{j\Omega on}u(n)\}]$

$=1/2j[1/1-e^{j\Omega o}\, z^{-1} - 1/1-e^{-j\Omega o}\, z^{-1}]$

ROC : $|z| > |e^{j}\Omega o|$ and $|z| > |e^{-j\Omega o}|$

$=1/2j[\, 1-e^{-j\Omega on}\, z^{-1}-1+e^{j\Omega on}\, z^{-1}/(1-e^{j\Omega on}\, z^{-1})(\, 1-e^{-j\Omega on}\, z^{-1})$, ROC : $|z|>1$

$= 1/2j\, [\, 2j\sin \Omega_o\, z^{-1}]/1-z^{-1}.2 \cos\Omega o + z^{-2}]$

$=z^{-1} \sin \Omega o\, /1-2z^{-1} \cos\Omega o + z^{-2}$, ROC : $|z| >1$

$=X_1(z/a)$, ROC : $|a|\, r_1 < |z| < |a|\, r_2$, By scaling in z-domain

Replacing z by z/a in $X_1(z) = (z/a)^{-1} \sin \Omega_o\, /1-2(z/a)^{-1} \cos \Omega_o + (z/a)^{-2}$,

ROC : $|z| > 1\, |a|$ i.e. $|z| > |a|$

Video Content / Details of website for further learning (if any):

- **https://www.youtube.com/watch?v=ftdkaJjqyAs**
- https://lpsa.swarthmore.edu/ZXform/FwdZXform/FwdZXform.html

Important Books/Journals for further learning including the page nos.:

Ramesh Babu “Signals And System”, Scitec, Fourth Edition, 2008. (Page no :10.1-10.7)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 58

LECTURE HANDOUTS

ECE

II/III

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Problems in Z Transform

Introduction: The Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace transform.

Prerequisite knowledge for Complete understanding and learning of Topic:

- DTFT

$$i)\ X(z) = \frac{1}{1-az^{-1}}\ ,\ ROC : |z|>|a|$$

$$ii)\ X(z) = \frac{1}{1-az^{-1}}\ ,\ ROC : |z|<|a|$$

1-

$1-az^{-1})\overline{1}$

1

Solution :

(i) $$X(z) = \frac{1}{1-az^{-1}}\ ,\ ROC : |z|>|a|$$

$a^2 z^{-2}$

$a^2 z^{-2} - a^3 z^{-3}$

$a^3 z^{-3}$

$a^3 z^{-3} - a^4 z^{-4}$

$a^4 z^{-4}$ ...

Thus we have, $X(z) = \frac{1}{1-az^{-1}} = 1 + az^{-1} + a^2 z^{-2} + a^3 z^{-3} + \cdots$

Taking inverse z-transform, $x(n) = \{1, a, a^2, a^3, \cdots\}$

$$= a^n u(n)$$

$$= \frac{1}{-az^{-1}+1}$$ Equation rearranged to get positive powers of 'z'.

$$-a^{-1}z - a^{-2}z^2 - a^{-3}z^3 - a^{-4}z^4 \quad \leftarrow \textit{Positive powers of 'z'}$$

$$-az^{-1}+1 \,\big)\, 1$$

$$1 - a^{-1}z$$

$$a^{-1}z$$

$$a^{-1}z - a^{-2}z^2$$

$$a^{-2}z^2$$

$$a^{-2}z^2 - a^{-3}z^3$$

$$a^{-3}z^3$$

$$a^{-3}z^3 - a^{-4}z^4$$

$$a^{-4}z^4 \ \ldots$$

Thus we have, $X(z) = \frac{1}{1-az^{-1}} = -a^{-1}z - a^{-2}z^2 - a^{-3}z^3 - a^{-4}z^4 \ldots$

Rearranging above equation, $= \ldots\ldots - a^{-4}z^4 - a^{-3}z^3 - a^{-2}z^2 - a^{-1}z$

Taking inverse z-transform, $x(n) = \{\cdots -a^{-4}, -a^{-3}, -a^{-2}, -a^{-1}\}$
$\uparrow$

$$= -a^n u(-n-1)$$

Video Content / Details of website for further learning (if any):

- https://dsp.stackexchange.com/questions/53064/recursive-form-representation-of-iir-filter

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no : 3.58)

Course Faculty

Verified by HOD

MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, accredited by NAAC & Affiliated to Anna University)

Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

L 59

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Analysis of Recursive System

Introduction: A recursive system is a system in which current output depends on previous output(s) and input(s) but in non-recursive system current output does not depend on previous output(s).

Prerequisite knowledge for Complete understanding and learning of Topic:

- Filters

A recursive system is a system in which current output depends on previous output(s) and input(s) but in non-recursive system current output does not depend on previous output(s).

The system with memory is not necessarily a recursive system. For example, in FIR systems for input x[n] and output y[n] if we have

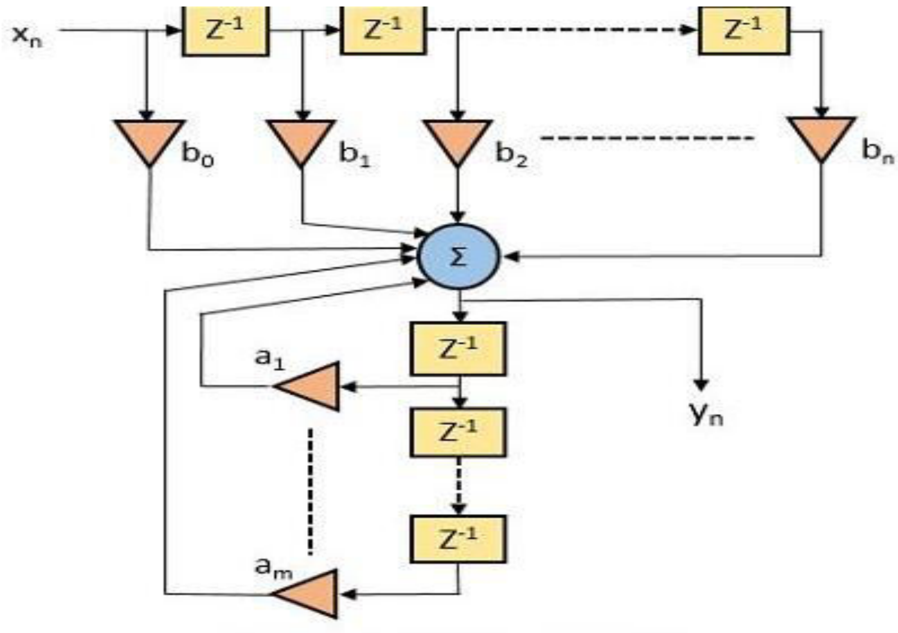
y[n] = 0.5(x[n]+x[n-1]) then the current output does not depend on previous output but depends on current input and previous input. But for IIR case like an accumulator

y[n] = y[n-1] +x[n], current output is depended on previous output as well as on current input (generally current and previous inputs). So accumulator is a recursive system.

Fast computation of the convolution of a signal or image using finite impulse response (FIR) filters is one of the basic problems in the theory of signal and image processing. At present, there are several approaches to solving this problem.

$$y(n) = \sum_{k=0}^{N-1} h(k)\, x(n-k)$$

Hence the output is the summation of a finite quantity of finite samples of input values. So, these are highly stable. The generalized equation for FIR filter is given as:



- FIR filter generates an output of a dynamic system using the samples of present and past input values. While an IIR filter uses present and past input values along with the past output value to generate the present output.
- FIR filters do not use feedback circuitry, while IIR filters make use of feedback loop in order to provide previous output in conjunction with current input. FIR filters support linear phase filtering, however, IIR filters do not support linear phase filtering.
- As FIR filters have more strength in comparison to IIR filters thus these offer more flexible operation than the IIR filters.
- The transfer function of the system having an FIR filter contains only zeros. While in the case of IIR filters the transfer function contains both poles and zeros.
- FIR filters show non-recursive behavior. But IIR filters show recursive behavior. Due to the absence of the feedback loop, the implementation of FIR filters in a system is quite easy in comparison to IIR filters in which a feedback loop is present.
- The delay in providing a response in case of FIR filters is more than that of IIR filter. FIR filters are more stable as here the present output does not hold any relationship with the previous output. But IIR filters are less stable as it makes use of previous output samples as well.FIR filter. need greater processing power in comparison to IIR filters.

Video Content / Details of website for further learning (if any):

- https://dsp.stackexchange.com/questions/53064/recursive-form-representation-of-fir-filter

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :3.58)

Course Faculty

Verified by HOD

Course Name with Code : **SIGNALS AND SYSTEMS / 19ECC02**

Course Faculty : **Mrs.S.Punitha**

Unit : **V- Linear Time invariant-Discrete Time Systems**

Date of Lecture:

Topic of Lecture: Analysis of Non-Recursive System

Introduction: In contrast, if y(n) depends only on the present and past inputs, then, Such a system is called non-recursive system. Finite Impulse Response systems (FIR) have the above form.

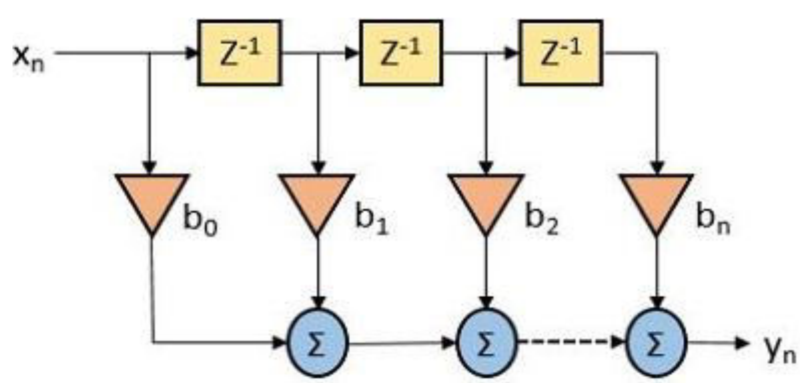
Prerequisite knowledge for Complete understanding and learning of Topic:

- Filter

Non Recursive Filter

A digital filter that lacks feedback; that is, its output depends on present and past input values only and not on previous output values in non recursive filter

In non-recursive filters , the output y at the moment t is a function of only input values x(t-z),z>1 corresponding to the time moments t-z. A non-recursive filter is also known as an FIR (or Finite Impulse Response) filter.



$$y(n) = \sum_{k=0}^{N-1} h(k)\, x(n-k)$$

A non recursive filter is that in which the current output (yn) is calculated solely from the current and previous input values (xn, xn-1, xn-2, …). A recursive filter is one which in addition to input values also uses previous output values. These, like the previous input values, are stored in the processor's memory.

A type of digital filter that generates a finite impulse response of a dynamic system is known as FIR filters. More simply, we can say, here the impulse response provided by the filter is of finite duration. In FIR filters the response gets fixed to zero in a finite period of time thus it is named so. In the case of FIR filters, the n_{th} *order filter generates n+1 samples* before getting fixed to 0.

Video Content / Details of website for further learning (if any):

- https://dsp.stackexchange.com/questions/53064/recursive-form-representation-of-fir-filter

Important Books/Journals for further learning including the page nos.:

Ramesh Babu "Signals And System", Scitec, Fourth Edition, 2008. (Page no :3.58)

Course Faculty

Verified by HOD