MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

LECTURE HANDOUTS

L1

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **I - Electrostatics** **Date of Lecture:**

Topic of Lecture: Review of vector algebra and Coordinate Systems

Introduction :A coordinate system is a way of uniquely specifying the location of any position in space with respect to a reference origin. Any point is defined by the intersection of three mutually perpendicular surfaces. The coordinate axes are then defined by the normals to these surfaces at the point.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra

Introduction to Coordinate Systems:

- Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion
- Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied
- A point or vector can be represented in any curvilinear coordinate system, which may be orthogonal or non-orthogonal
- An orthogonal system is one in which the coordinates arc mutually perpendicular

Orthogonal coordinate systems:

- Cartesian (or rectangular)
- Circular cylindrical
- Spherical
- Elliptic cylindrical
- Parabolic cylindrical
- Conical
- Prolatespheroidal
- Oblate spheroidal
- Ellipsoidal

Cartesian (or Rectangular) Coordinate System:

- The vector **A** is readily written in terms of the cartesian unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$

$$\mathbf{A} = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$$

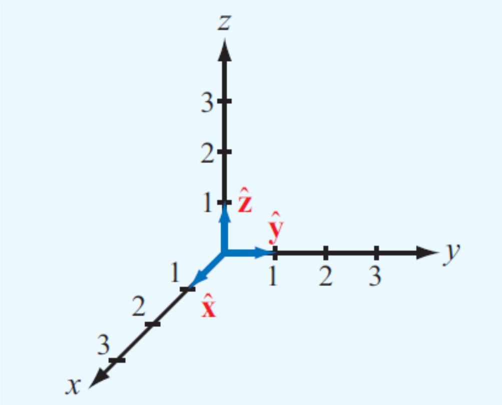
- In linear algebra $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are known as basis vectors, each having unit length, i.e., $|\hat{\mathbf{x}}|$ and mutually orthogonal

- Also, the length of **A** is

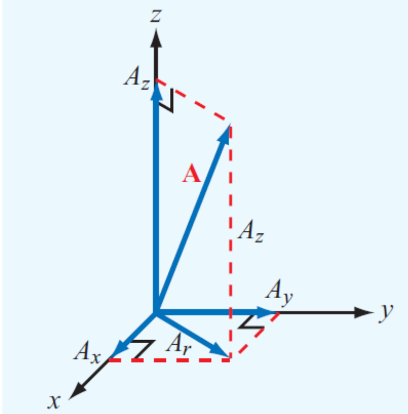
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and the unit vector in the **A** direction is

$$\hat{\mathbf{a}} = \frac{\mathbf{A}}{A} = \frac{\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$



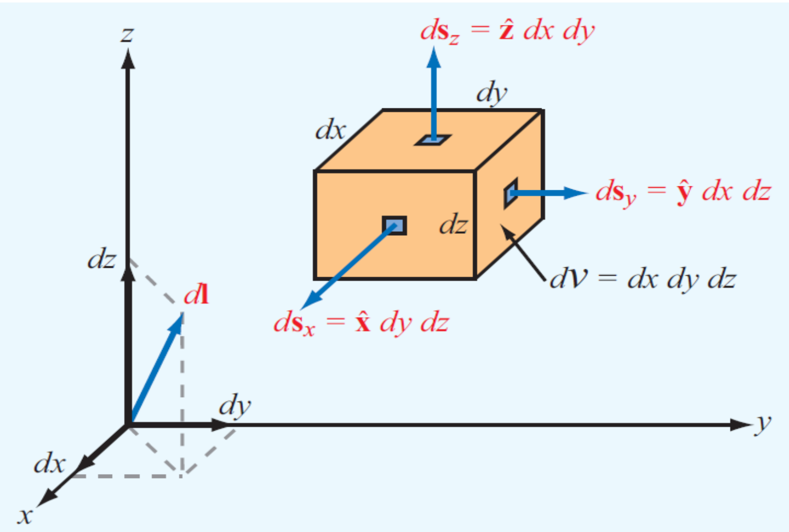
(a) Base vectors



(b) Components of A

- There three orthogonal coordinate systems in common usage in electromagnetics:
  - The Cartesian or rectangular system: $\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$
  - The cylindrical system: $\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$
  - The spherical system: $\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$

Differential Length, Area & Volume in Rectangular System



Differential Length

$$d\mathbf{l} = \hat{\mathbf{x}} dl_x + \hat{\mathbf{y}} dl_y + \hat{\mathbf{z}} dl_z = \hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$$

Differential Area

- A vector, $d\mathbf{s}$, that is normal to the two coordinates describing the scalar area ds

- There are three different differential areas, $d\mathbf{s}$, to consider:

$$d\mathbf{s}_x = \hat{\mathbf{x}}\, dl_y\, dl_z = \hat{\mathbf{x}}\, dy\, dz \quad (y - z\text{-plane})$$
$$d\mathbf{s}_y = \hat{\mathbf{x}}\, dx\, dz \quad (x - z\text{-plane})$$
$$d\mathbf{s}_z = \hat{\mathbf{x}}\, dx\, dy \quad (x - y\text{-plane})$$

Differential Volume

$$d\mathcal{V} = dx\, dy\, dz$$

Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_1_Electromagnetic_Introduction_and_Vector _ Analysis.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics,Tata McGraw-Hill,7th Edition, 2012. P.No: 3

Course Faculty

Verified by HOD

Course Name with Code : 19ECC05 – ELECTROMAGNETIC FIELDS

Course Faculty : Mr.A.Kumaravel

Unit : I - Electrostatics **Date of Lecture:**

Topic of Lecture: Line, surface and volume integrals

Introduction: A coordinate system is a way of uniquely specifying the location of any position in space with respect to a reference origin. Any point is defined by the intersection of three mutually perpendicular surfaces. The coordinate axes are then defined by the normals to these surfaces at the point.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Cartesian Coordinate Systems

Cylindrical Co-ordinate systems:

- The cylindrical system is used for problems involving cylindrical symmetry
- It is composed of: (1) the radial distance $r \in [0, \infty)$, (2) the azimuthal angle, $\phi \in [0, 2\pi)$, and $z \in (-\infty, \infty)$, which can be thought of as height
- As in the case of the Cartesian system, $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\phi}}$, and $\hat{\mathbf{z}}$ are mutually perpendicular or orthogonal to each other, e.g., $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = 0$, etc.
- Likewise the cross product of the unit vectors produces the cyclical result

$$\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}, \quad \hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}, \quad \hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$$

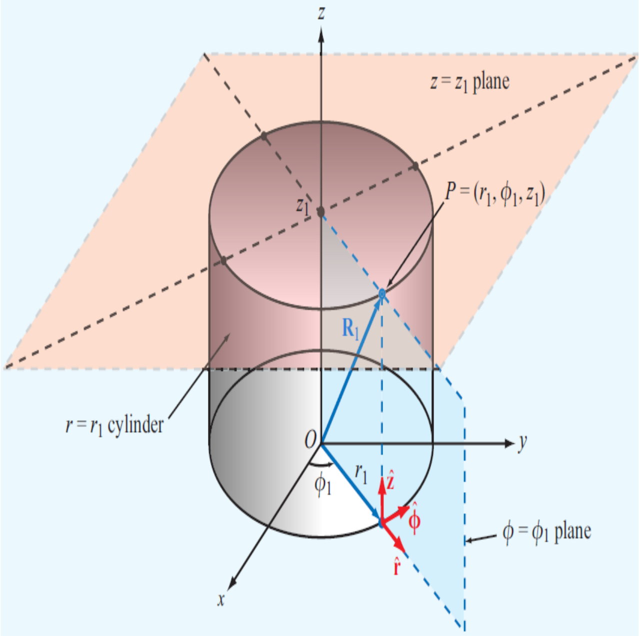
- The general vector expansion

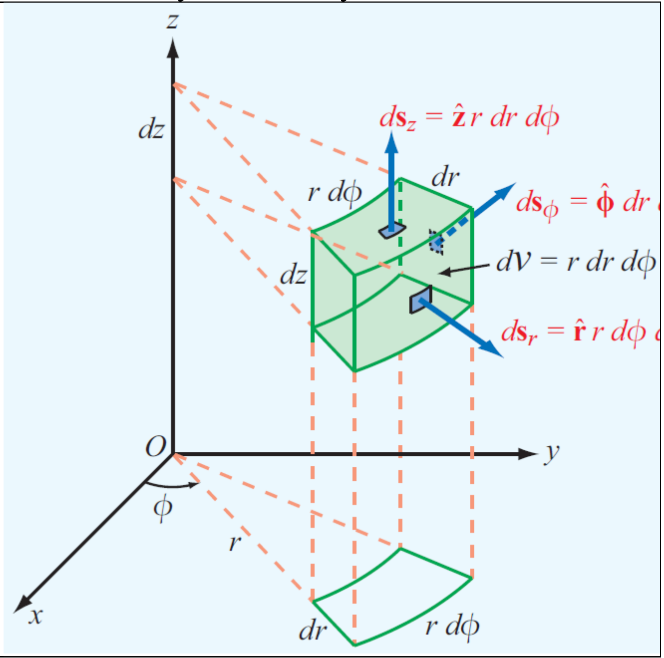
$$\mathbf{A} = \hat{\mathbf{a}}|\mathbf{A}| = \hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$$

is obvious, as is the scalar length

$$|\mathbf{A}| = \sqrt{A_r^2 + A_\phi^2 + A_z^2}$$

A point in the Cylindrical System:Differential Quantities in Cylindrical System:





Differential Quantities

- The differential quantities do not follow from the Cartesian system
- The differential length of the azimuthal component is also a function of the radial component, i.e.,

$$dl_r = dr, \quad dl_\phi = rd\phi, \quad dl_z = dz$$

- In the end

$$d\mathbf{l} = \hat{\mathbf{r}}dr + \hat{\boldsymbol{\phi}}rd\phi + \hat{\mathbf{z}}dz$$

- The differential surface follows likewise

$$d\mathbf{s}r = \hat{\mathbf{r}}\, r\, d\phi\, dz \quad (\phi - z \text{ cylindrical surface})$$
$$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\, dr\, dz \quad (r - z \text{ plane})$$
$$d\mathbf{s}_z = \hat{\mathbf{z}}\, dr\, d\phi \quad (r - \phi \text{ plane})$$

- The differential area is likely the most familiar from calculus

$$d\mathcal{V} = r\, dr\, \phi\, dz$$

Spherical Coordinate Systems:

- In this coordinate system a single range variable R plus two angle variables θ and ϕ are employed

- It is composed of: (1) the radial distance $r \in [0, \infty)$, (2) the azimuthal angle (same as cylindrical), $\phi \in [0, 2\pi)$, and the *zenith angle* $\theta \in [0, \pi]$, which is measured from the positive z-axis

- All coordinates are again mutually orthogonal to span a 3D space

- The cross product of the unit vectors produces the cyclical result

$$\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}, \quad \hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}, \quad \hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$$

- The general vector expansion

$$\mathbf{A} = \hat{\mathbf{a}}|\mathbf{A}| = \hat{\mathbf{R}} A_R + \hat{\boldsymbol{\theta}} A_\theta + \hat{\boldsymbol{\phi}} A_\phi$$

  is obvious, as is the scalar length

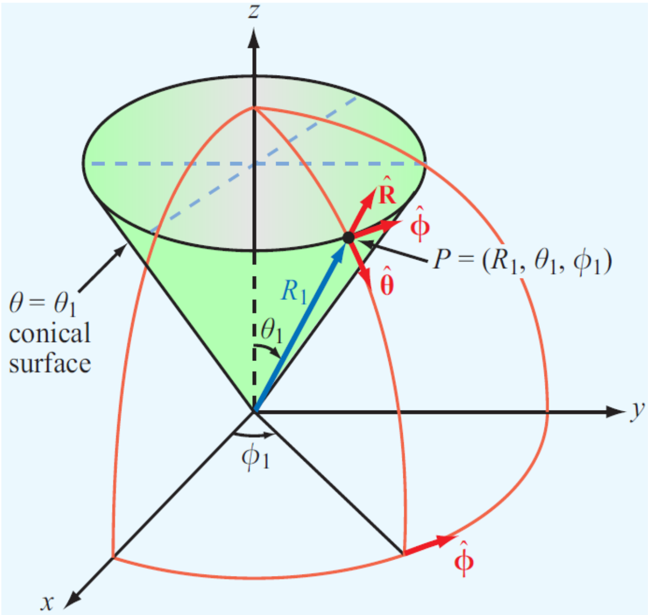
$$|\mathbf{A}| = \sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$$

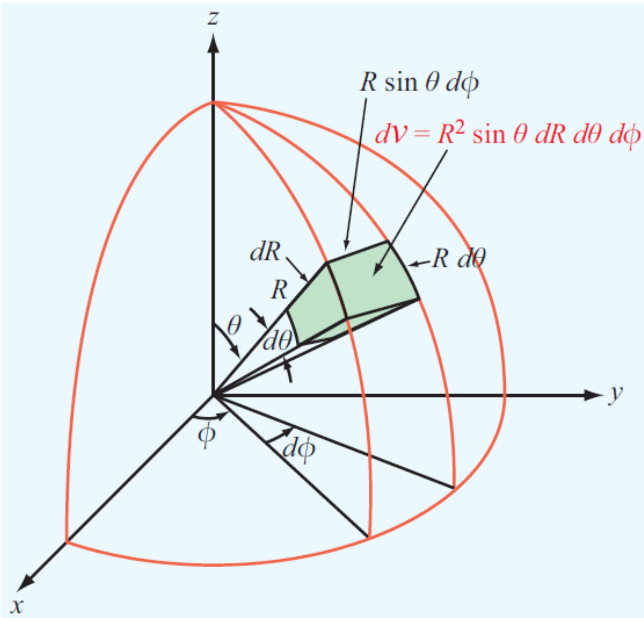
- The position vector $\mathbf{R}_1$ (3.12) is

$$\mathbf{R}_1 = \overrightarrow{OP} = \hat{\mathbf{R}} R_1,$$

  but needs knowledge of θ_1 and ϕ_1 to be complete

<u>A poin on Spherical System: Differential Quantities in Sphercal System:</u>





Differential Quantities

- The differential quantities are different yet again from the Cartestian and the cylindrical systems
- The differential length of the zenith component is like the azimuthal component in the cylindrical system
- The differential length of the azimuthal component is now a function of both the radial component and the zenith component, i.e.,

$$dl_R = dR, \quad dl_\theta = Rd\theta, \quad dl_\phi = R\sin\theta d\phi$$

- In the end

$$d\mathbf{l} = \hat{\mathbf{R}}dr + \hat{\boldsymbol{\theta}}\,Rd\theta + \hat{\boldsymbol{\phi}}\,R\sin\theta dz$$

- The differential surface follows

$$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi \quad (\theta - \phi \text{ spherical surface})$$
$$d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}\;R\sin\theta dR\,d\phi \quad (R - \phi \text{ conical plane})$$
$$d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta \quad (R - \theta \text{ plane})$$

- Again the differential area is likely the most familiar from calculus

$$d\mathcal{V} = R^2\sin\theta\,dR\,d\theta\,d\phi$$

Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_1_Electromagnetic_Introduction_and_Vector_ Analysis.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012. P.No: 4

Course Faculty

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LECTURE HANDOUTS

L3

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **I - Electrostatics** **Date of Lecture:**

Topic of Lecture: Gradient of a scalar field, Divergence of a vector field

Introduction:A coordinate system is a way of uniquely specifying the location of any position in space with respect to a reference origin. Any point is defined by the intersection of three mutually perpendicular surfaces. The coordinate axes are then defined by the normals to these surfaces at the point.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra

Definition of Curl, Divergence and Gradient:

The Gradient of a scalar field:

- Gradient is defined in scalar field and it gives maximum rate of change of field while going from one "equipotential" surface to the nearby "equipotential" surface in particular direction. Gradient of scalar field is vector field.

- Gradient is the multidimensional rate of change of given function. "Gradient vector is a representative of such vectors which give the value of differentiation (means characteristic of curve in terms of increasing & decreasing value in 3 or multi dimensions) in all the 360° direction for the given point on the curve."

If $U(x, y, z)$ is a scalar field, ie a scalar function of position $\mathbf{r} = [x, y, z]$ in 3 dimensions, then its **gradient** at any point is defined in Cartesian co-ordinates by

$$\text{grad}U = \frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\,.$$

It is usual to define the **vector operator**

$$\nabla = \hat{\imath}\,\frac{\partial}{\partial x} + \hat{\jmath}\,\frac{\partial}{\partial y} + \hat{k}\,\frac{\partial}{\partial z}$$

which is called "del" or "nabla". Then

$$\text{grad}U \equiv \nabla U$$

The Divergence of a vector field:

- The divergence computes a scalar quantity from a vector field by differentiation.
- The gradient is a vector operation which operates on a scalar function to produce a vector whose magnitude is the maximum rate of change of the function at the point of

the gradient and which is pointed in the direction of that maximum rate of change.

More precisely, if $\mathbf{a}(x, y, z)$ is a vector function of position in 3 dimensions, that is $\mathbf{a} = a_1\hat{\boldsymbol{\imath}} + a_2\hat{\boldsymbol{\jmath}} + a_3\hat{\boldsymbol{k}}$, then its divergence at any point is defined in Cartesian co-ordinates by

$$\text{div}\mathbf{a} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$$

We can write this in a simplified notation using a scalar product with the $\boldsymbol{\nabla}$ vector differential operator:

$$\text{div}\mathbf{a} = \left(\hat{\boldsymbol{\imath}}\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}\right) \cdot \mathbf{a} = \boldsymbol{\nabla} \cdot \mathbf{a}$$

Notice that the divergence of a vector field is a scalar field.

The Curl of a vector field:

- Curl is defined as the circulation of a vector per unit area. It is the cross product of the Del operator and any vector field. Circulation implies the angular at every point of the vector field.

So far we have seen the operator $\boldsymbol{\nabla}$

- Applied to a scalar field $\boldsymbol{\nabla}U$; and
- Dotted with a vector field $\boldsymbol{\nabla} \cdot \mathbf{a}$.

You are now overwhelmed by that irrestible temptation to

- cross it with a vector field $\boldsymbol{\nabla} \times \mathbf{a}$

This gives the **curl of a vector field**

$$\boldsymbol{\nabla} \times \mathbf{a} \equiv \text{curl}(\mathbf{a})$$

We can follow the pseudo-determinant recipe for vector products, so that

$$\boldsymbol{\nabla} \times \mathbf{a} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

$$= \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial y}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\hat{\boldsymbol{k}}$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=vvzTEbp9lrc

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 51

Course Faculty

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LECTURE HANDOUTS

L4

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **I - Electrostatics** **Date of Lecture:**

Topic of Lecture: Divergence theorem

Introduction: The divergence theorem is a mathematical statement of the physical fact that, in the absence of the creation or destruction of matter, the density within a region of space can change only by having it flow into or away from the region through its boundary.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Gradient, Divergence & Curl

Definition of Divergence Theorem:

- The divergence theorem states that the surface integral of a vector field over a closed surface, which is called the flux through the surface, is equal to the volume integral of the divergence over the region inside the surface. In one dimension, it is equivalent to integration by parts.

The flux of a differentiable vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ *across a closed oriented surface* $S \subset \mathbb{R}^3$ *in the direction of the surface outward unit normal vector* $\mathbf{n}$ *satisfies the equation*

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, d\sigma = \iiint_V (\nabla \cdot \mathbf{F}) \, dV,$$

where $V \subset \mathbb{R}^3$ *is the region enclosed by the surface* S.

Remarks:

- The volume integral of the divergence of a field $\mathbf{F}$ in a volume V in space equals the outward flux (normal flow) of $\mathbf{F}$ across the boundary S of V.
- The expansion part of the field $\mathbf{F}$ in V minus the contraction part of the field $\mathbf{F}$ in V equals the net normal flow of $\mathbf{F}$ across S out of the region V.

In the last few lectures we have been studying some results which relate an integral over a domain to another integral over the boundary of that domain. In this lecture we will study a result, called divergence theorem, which relates a triple integral to a surface integral where the surface is the boundary of the solid in which the triple integral is defined.

Let D be a plane region enclosed by a simple smooth closed curve C. Suppose $F(x,y) = M(x,y)i + N(x,y)j$ is such that M and N satisfy the conditions given in Green's theorem. If the curve C is defined by $R(t) = x(t)i + y(t)j$ then the vector $\mathbf{n} = \frac{dy}{ds}i - \frac{dx}{ds}j$ is a unit normal to the curve C because the vector $T = \frac{dx}{ds}i + \frac{dy}{ds}j$ is a unit tangent to the curve C. By Green's theorem

$$\oint_C (F \cdot \mathbf{n})ds = \oint_C Mdy - Ndx = \iint_D \left(\frac{\partial M}{\partial x} - (-\frac{\partial N}{\partial y})\right) dxdy = \iint_D \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dxdy.$$

Since $divF = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$, Green's theorem takes the following form:

$$\iint_D divF dxdy = \oint_C (F \cdot \mathbf{n})ds.$$

We will now generalize this form of Green's theorem to a vector field F defined on a solid.

Theorem: Let D be a solid in $\mathbb{R}^3$ bounded by piecewise smooth (orientable) surface S. Let $F(x,y,z) = P(x,y,z)i + Q(x,y,z)j + R(x,y,z)k$ be a vector field such that P, Q and R are continuous and have continuous first partial derivatives in an open set containing D. Suppose $\mathbf{n}$ is the unit outward normal to the surface S. Then

$$\iiint_D divF \; dV \;=\; \iint_S F \cdot \mathbf{n} \; d\sigma.$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=1qLb0B40YnA

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 42

Course Faculty

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MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

LECTURE HANDOUTS

L5

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **I - Electrostatics** **Date of Lecture:**

Topic of Lecture: Curl of a vector field, Stoke's theorem, Helmholtz's theorem

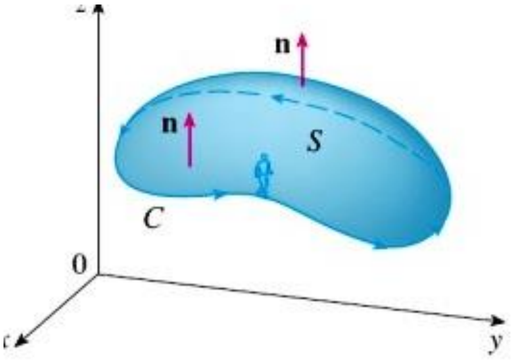
Introduction: The divergence theorem is a mathematical statement of the physical fact that, in the absence of the creation or destruction of matter, the density within a region of space can change only by having it flow into or away from the region through its boundary.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Gradient, Divergence & Curl

Definition of Stokes Theorem:

- It states that the integral of the tangential component of a vector field F around C is equal to the integral of the normal component of curl F over s. Write the expression for force on a test charge q that moves with velocity V in magnetic field of flux density B.
- Stokes' Theorem relates a surface integral over a surface *S* to a line integral around the boundary curve of *S* (a space curve).
- The orientation of *S* induces the positive orientation of the boundary curve *C*.
- If you walk in the positive direction around *C* with your head pointing in the direction of **n**, the surface will always be on your left.



- Let Sbe an oriented piecewise-smooth surface bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation.

- Let **F** be a vector field whose components have continuous partial derivatives on an open region in R3 that contains *S*.
- Then, $\int_C$ F.dr =$\iint_S$ curl F.ds
- The positively oriented boundary curve of the oriented surface *S* is often written as ∂*S*.
- So, the theorem can be expressed as:
- $\iint_S$ curl F.ds = $\int_{\partial S}$ F.dr

Helmholtz's theorem

Theorem 12. Helmholtz' Theorem. *Let* $\mathbf{F}(\mathbf{r})$ *be any continuous vector field with continuous first partial derivatives. Then* $\mathbf{F}(\mathbf{r})$ *can be uniquely expressed in terms of the negative gradient of a scalar potential* $\phi(\mathbf{r})$ *and the curl of a vector potential* $\mathbf{a}(\mathbf{r})$, *as embodied in Eqs. (A.10) and (A.11).*

$$\mathbf{F}(\mathbf{r}) = -\nabla\phi(\mathbf{r}) + \nabla \times \mathbf{a}(\mathbf{r}), \qquad (A.10)$$

where the scalar potential $\phi(\mathbf{r})$ is given by Eq. (A.6) and the vector potential $\mathbf{a}(\mathbf{r})$ is given by Eq. (A9). This expression may also be written as

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_\ell(\mathbf{r}) + \mathbf{F}_t(\mathbf{r}), \qquad (A.11)$$

Any vector field $\mathbf{v}$ satisfying

$$[\nabla \cdot \mathbf{v}]_\infty = 0$$
$$[\nabla \times \mathbf{v}]_\infty = 0$$

may be written as the sum of an irrotational part and a solenoidal part,

$$\mathbf{v} = -\nabla\phi + \nabla \times \mathbf{A},$$

where

$$\phi = \int_V \frac{\nabla \cdot \mathbf{v}}{4\pi|\mathbf{r}' - \mathbf{r}|} d^3\mathbf{r}'$$
$$\mathbf{A} = \int_V \frac{\nabla \times \mathbf{v}}{4\pi|\mathbf{r}' - \mathbf{r}|} d^3\mathbf{r}'.$$

Video Content / Details of website for further learning (if any):

- https://www.youtube.com/watch?v=1qLb0B40YnA

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 42

Course Faculty

Verified by HOD

L6

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**I-Electrostatics** **Date of Lecture:**

Topic of Lecture: Electric field, Coulomb's law, Electric potential

Introduction :

- Electric potential at a point in an electric field is defined as the amount of work to be done to bring a unit positive electric charge from infinity to that point.
- Similarly, the potential difference between two points is defined as the work required to be done for bringing a unit positive charge from one point to other point.
- When a body is charged, it can attract an oppositely charged body and can repulse a similar charged body. That means, the charged body has ability of doing work. The ability of doing work of a charged body is defined as electrical potential of that body.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Electric scalar potential

Electric potential is graded as electrical level, and difference of two such levels, causes current to flow between them. This level must be measured from a reference zero level. The earth potential is taken as zero level. Electric potential above the earth potential is taken as positive potential and the electric potential below the earth potential is negative.

The unit of electric potential is volt. To bring a unit charge from one point to another, if one joule work is done, then the potential difference between the points is said to be one volt. So, we can say,

$$volt = \frac{joules}{coulomb}$$

The curl of a gradient is always zero so that means that the electric field can be represented as the gradient of some function but that function has to be a scalar because gradients act on scalars. This is what is called the scalar potential ϕ:

$\vec{E} = -\nabla\phi$.

Scalar potentials are generally observed under static field conditions where as vector potentials are observed under dynamic conditions.

From electrostatics we know that the electric field can be expressed as a gradient of scalar potential

$\vec{E} = -\vec{\nabla} V$

Where V is the **scalar electric potential**.The above equation is not valid for electrodynamics. The equation is modified to

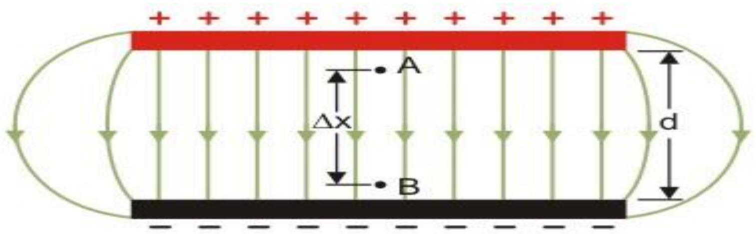
$\vec{E} = -\vec{\nabla} V - \partial\vec{A} / \partial t$

where, $\vec{A}$ is commonly referred as the **magnetic vector potential**, and the curl of it gives the corresponding magnetic field

$\vec{B} = \vec{\nabla} \times \vec{A}$

<u>Relation between potential and electric field</u>

Consider a *uniform* field (as exists near the centre of the space between two parallel, oppositely charged metal plates).



Consider moving a small positive charge, q, from point A to point B.Let the magnitude of the potential difference between points A and B be ΔV.In moving a positive charge from A to B work is done *by the field* so the potential at B is *less than* the potential at A.Therefore represent the potential difference between A and B as -ΔV.

Work done, w moving the charge q is given by

$$w = F\Delta x$$

Therefore, the relation between potential difference and field strength is found by simply dividing the

$$\frac{w}{q} = \frac{F\Delta x}{q}$$

$$-\Delta V = E\Delta x$$

which is usually written as

$$E = -\left(\frac{\Delta V}{\Delta x}\right)$$

and the term in brackets is called the *potential gradient*, as it represents the slope (gradient) of a graph of potential against distance.

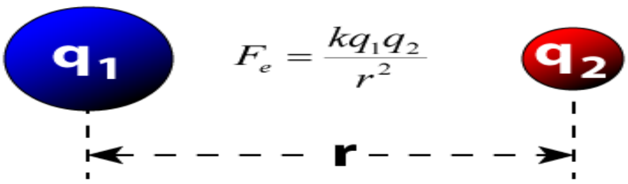
This equation shows that alternative units for measuring field strength are Volts per metre, Vm^{-1}

This means that the magnitude of the field strength between the two parallel plates is simply given by,

$$E = \frac{V}{d}$$

Coulomb's law

- Coulomb's Law gives an idea about the force between two point charges. By the word point charge, we mean that in physics, the size of linear charged bodies is very small as against the distance between them. Therefore, we consider them as point charges as it becomes easy for us to calculate the force of attraction/ repulsion between them.



Let's say that there are two charges q_1 and q_2. The distance between the charges is 'r', and the force of attraction/repulsion between them is 'F'. Then

$$\mathbf{F \propto q_1 q_2 \quad Or \quad F \propto 1/r^2}$$

$$\mathbf{F = k\, q_1 q_2 / r^2}$$

where k is proportionality constant and equals to $1/4\pi\varepsilon_0$. Here, ε_0 is the epsilon naught and it signifies permittivity of a vacuum. The value of k comes $9 \times 10^9\ Nm^2/\ C^2$ when we take the S.I unit of value of ε_0 is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

Video Content / Details of website for further learning (if any): https://www.youtube.com/watch?v=XsroSiKLwAM https://www.youtube.com/watch?v=p8OSoburdt0
Important Books/Journals for further learning including the page nos.: William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(124-129,149-155)

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LECTURE HANDOUTS

L7

ECE **II/III**

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**I- Electrostatics** **Date of Lecture:**

Topic of Lecture: Electric flux density and dielectric constant

Introduction :

- Electric flux is defined as the total number of electric lines of force emanating from a charged body. An electric field is represented by electric flux.
- Like magnetic flux, electric flux lines are not always closed loop. This is because, an isolated magnetic north pole or an isolated magnetic south pole do not exist practically, but an isolated positively charged body and an isolated negatively charged body can exist.
- Electric flux is denoted with Ψ

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

<u>Electric Flux density</u>

Electric flux density is defined as the amount of flux passes through unit surface area in the space imagined at right angle to the direction of electric field. The expression of electric field at a point is given by,

$$E = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}$$

Where, Q is the charge of the body by which the field is created. R is the distance of the point from the center of the charged body.

As, we know, Q = Ψ, The above equation can be rewritten as,

$$E = \frac{\psi}{4\pi\epsilon_0\epsilon_r r^2} \Rightarrow \epsilon_0\epsilon_r E = \frac{\psi}{4\pi r^2}$$

This is the expression of flux per unit area since, 4πr2 is the surface area of the imaginary spare of radius r. This is the flux passing through per unit area at a distance r from the center of the charge. This is called electric flux density at the said point. Generally it is denoted as,

$$D = \epsilon_0\epsilon_r E$$

From, the above expression of D, it is clear that electric field intensity and electric field density are in same phasor.As the number of electric lines of force emanated from a charged body is equal to the quantity of charge of the body measured in coulombs, we can also define the electric flux density at any point in the electric field of the body as the number of lines of force passing through a unit surface area at that point.

Gauss law

Gauss's law states that the net flux of an electric field in a closed surface is directly proportional to the enclosed electric charge. It is one of the four equations of Maxwell's laws of electromagnetism.Another statement of Gauss's law states that the net flux of a given electric field through a given surface, divided by the enclosed charge should be equal to a constant**.** Gauss's law in integral form is given below:

$\int E \cdot dA = Q/\varepsilon_0$ ….. (1)

Where,

- **E** is the electric field vector
- Q is the enclosed electric charge
- ε_0 is the electric permittivity of free space
- **A** is the outward pointing normal area vector

Flux is a measure of the strength of a field passing through a surface. Electric flux is defined as

$$\Phi=\int E\cdot dA \quad \text{.... (2)}$$

Gauss's law implies that the net electric flux through any given closed surface is zero unless the volume bounded by that surface contains a net charge.

Applications of Gauss Law

1. In the case of a charged ring of radius R on its axis at a distance x from the centre of the ring.

2. In case of an infinite line of charge, at a distance 'r'. $E = (1/4 \times \pi r \varepsilon_0)(2\pi/r) = \lambda/2\pi r\varepsilon_0$. Where λ is the linear charge density.

3. The intensity of the electric field near a plane sheet of charge is $E = \sigma/2\varepsilon_0 K$ where σ = surface charge density.

4. The intensity of the electric field near a plane charged conductor $E = \sigma/K\varepsilon_0$ in a medium of dielectric constant K. If the dielectric medium is air, then $E_{air} = \sigma/\varepsilon_0$.

5. The field between two parallel plates of a condenser is $E = \sigma/\varepsilon_0$, where σ is the surface charge density.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=6STe-VIZUcs
https://www.youtube.com/watch?v=UbGd-vTrZlc

Important Books/Journals for further learning including the page nos.:

E.C.Jordan & K.G. Balmain, "Electromagnetics waves and radiating systems", PHI, II edition, 2011,Pg.no.(160-164)

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LECTURE HANDOUTS

L8

ECE

II/III

Course Name with Code :19ECC05-Electromagnetic Fields

Course Faculty :Mr.A.Kumaravel

Unit :I- Electrostatics **Date of Lecture:**

Topic of Lecture: Boundary conditions

Introduction :

- When electric or magnetic fields go across the boundary of material media their values might or might not change. There are 4 possibilities.
- These values depend upon the surface current charge densities and the volume charge densities present on the surface of the media.

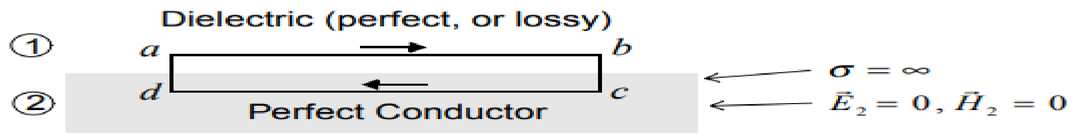
Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Boundary conditions for Electric Field

Fields inside conductor are zero: E2=0 ,H2 =0

Boundary condition for tangential component of electric field:



$$\int_a^b \bar{E}\cdot d\bar{l} + \int_b^c \bar{E}\cdot d\bar{l} + \int_c^d \bar{E}\cdot d\bar{l} + \int_d^a \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int_{abcd} \bar{B}\cdot d\bar{s}$$

Area enclosed is infinitely small. Therefore RHS $\rightarrow 0$.

"ad" and "cb" are infinitely small. Therefore, $\int_a^b$ and $\int_d^a \rightarrow 0$.

$\bar{E}_2$ (inside conductor) is zero. Therefore, $\int_c^d \rightarrow 0$.

This leaves:

$$\int_a^b \bar{E}\cdot d\bar{l} = 0$$

If ab distance is finite, but small, then $\bar{E}$ is constant, and we have:

$E_t \cdot \Delta = 0$ $\quad$ $E_t =$ component of $\vec{E}$ along ab; i.e., tangential component

Thus, $E_t = 0$.

In vector form, this can be written:

$\hat{e}_n \times \vec{E} = 0$, where $\hat{e}_n$ is unit normal.

A very important corollary of this boundary condition is that, since $E_t = 0$ at the surface of a perfect conductor,

$$\int_a^b \vec{E} \cdot d\vec{l} = 0$$

between any point on or inside the conductor. Therefore, **a perfect conductor is an equipotential surface**.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=jUk5nRlxj_8

Important Books/Journals for further learning including the page nos.:

William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.no.(197-206)

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L9

LECTURE HANDOUTS

ECE **II/III**

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**I-Electrostatics** **Date of Lecture:**

Topic of Lecture: Capacitance- Parallel plate capacitors, Electrostatic energy

Introduction :

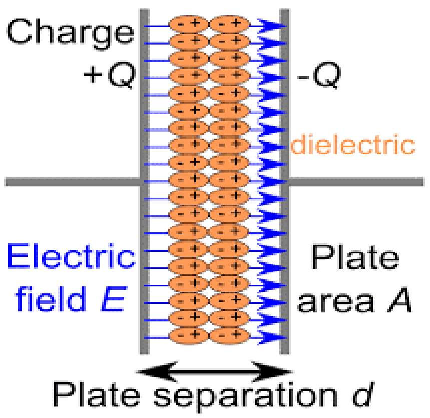
- Parallel Plate Capacitors are the type of capacitors which that have an arrangement of electrodes and insulating material (dielectric).
- The two conducting plates act as electrodes.
- There is a dielectric between them.
- This acts as a separator for the plates.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

<u>Capacitance of parallel plate capacitor</u>

The two plates of parallel plate capacitor are of equal dimensions. They are connected to the power supply. The plate, connected to the positive terminal of the battery, acquires a positive charge. On the other hand, the plate, connected to the negative terminal of battery acquires a negative charge. Due to the attraction charges are in a way trapped within the plates of the capacitor.



Principle of Parallel Plate Capacitor

A certain amount of charge is given to a plate. If we supply more charge, the potential increases and it could lead to a leakage in the charge. If we get another plate and place it next to this positively charged plate, then negative charge flows towards the side of this plate which is closer to the positively charged plate.The capacitance of the parallel plate capacitor determines the amount of charge that it can hold. If you see the above equation, you will see that greater the value of C, greater will be the charge that a capacitor can hold. Therefore we can see that the capacitance depends upon:

- The distance d between two plates.
- The area A of the medium between the plates.

According to the Gauss law, we can write the electric field as:

$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Ed = V = \frac{Qd}{\varepsilon_0 d}$$

Since we know that the capacitance is defined as V = Q/C, so we can write capacitance as:

$$C = \frac{\varepsilon_0 A}{d}$$

When the plates are placed very close and the area of plates are large, we get the maximum capacitance.The effect of dielectric material, inserted between the two plates. Materials have a permeability which is given by the relative permeability k. The capacitance is thus:

$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=sVq5PQYjjkU

Important Books/Journals for further learning including the page nos.:

William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.no.(184-196)

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LECTURE HANDOUTS **L10**

ECE **II/III**

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Lorentz force equation

Introduction :

Lorentz force is defined as the combination of the magnetic and electric force on a point charge due to electromagnetic fields. It is used in electromagnetism and is also known as the electromagnetic force. In the year 1895, Hendrik Lorentz derived the modern formula of Lorentz force.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Lorentz force

Lorentz force explains the mathematical equations along with the physical importance of forces acting on the charged particles that are traveling through the space containing electric as well as the magnetic field. This is the importance of the Lorentz force.

Lorentz force formula for the charged particle is as follows:

$$F=q(E+v*B)$$

Where,

- F is the force acting on the particle
- q is the electric charge of the particle
- v is the velocity
- E is the external electric field
- B is the magnetic field

Lorentz force formula for continuous charge distribution is as follows:

$$dF = dq(E + v * B)$$

Where,

- dF is a force on a small piece of the charge
- dq is the charge of a small piece

When a small piece of charge distribution is divided by the volume dV, the following is the formula:

$$f = \rho (E + v^* B)$$

Where,

- f is the force per unit volume
- ρ is the charge density

With the help of the right-hand rule, it becomes easy to find the direction of the magnetic part of the force.

Applications of Lorentz Force

The following are the applications of Lorentz force:

- Cyclotrons and other particle accelerators use Lorentz force.
- A bubble chamber uses Lorentz force to produce the graph for getting the trajectories of charged particles.
- Cathode ray tube televisions use the concept of Lorentz force to deviate the electrons in a straight line so land on specific spots on the screen.

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=1kydon2HxQA

Important Books/Journals for further learning including the page nos.:
William H Hayt and Jr John A Buck, “Engineering Electromagnetics”, Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(94,137)

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LECTURE HANDOUTS

L11

ECE

II/III

Course Name with Code :19ECC05-Electromagnetic Fields

Course Faculty :Mr.A.Kumaravel

Unit :II-Magnetostatics **Date of Lecture:**

Topic of Lecture: Ampere's law

Introduction :

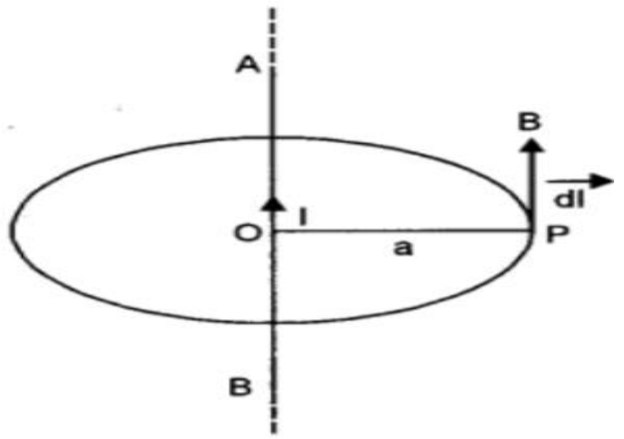
- Ampere's Circuital Law states the relationship between the current and the magnetic field created by it. This law states that the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Ampere's circuital law

Consider a long straight conductor carrying a current I amp. It is desired to find the magnetic flux density at P at a distance a from the conductor. The lines of force will be circular with the center on the conductor. Using-amperes-circuital-law-or-biot-and-savarts-law-show-that-magnetic-flux-density-point.



Ampere's Circuital Law states the relationship between the current and the magnetic field created by it.This law states that the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \, I$$

It alternatively says, the integral of magnetic field intensity (H) along an imaginary closed path is equal to the current enclosed by the path.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \, I$$

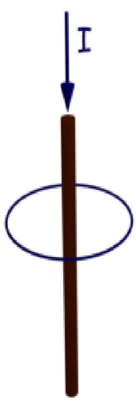
$$\Rightarrow \oint \frac{\overrightarrow{B}}{\mu_0} \cdot \overrightarrow{dl} = I$$

$$\Rightarrow \oint \overrightarrow{H} \cdot \overrightarrow{dl} = I$$

$$\left[\because \overrightarrow{H} = \frac{\overrightarrow{B}}{\mu_0} \right]$$

Let us take an electrical conductor, carrying a current of I ampere, downward as shown in the figure below.

Let us take an imaginary loop around the conductor. We also call this loop as amperian loop.



$$2\pi r\, B = \mu_0\, I$$

$$\Rightarrow \frac{B}{\mu_0} = \frac{I}{2\pi r}$$

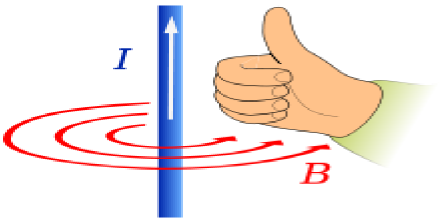
$$\Rightarrow H = \frac{I}{2\pi r}$$

Instead of one current carrying conductor, there are N number of conductors carrying same current I, enclosed by the path, then

$$H = \frac{NI}{2\pi r}$$

Magnetic flux density:

The magnetic flux density or magnetic induction is the number of lines of force passing through a unit area of material, B. The unit of magnetic induction is the tesla (T). ... For paramagnetic materials, χ is a small positive constant, and for diamagnetic materials it is a much smaller negative constant.All moving charged particles produce magnetic fields. Moving point charges, such as electrons, produce complicated but well known magnetic fields that depend on the charge, velocity, and acceleration of the particles. Magnetic field lines form in concentric circles around a cylindrical current-carrying conductor, such as a length of wire. The direction of such a magnetic field can be determined by using the "right-hand grip rule". The strength of the magnetic field decreases with distance from the wire. (For an infinite length wire the strength is inversely proportional to the distance.)



Video Content / Details of website for further learning (if any):
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Important Books/Journals for further learning including the page nos.:
William H Hayt and Jr John A Buck, “Engineering Electromagnetics”, Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(94,137)

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LECTURE HANDOUTS

L 12

ECE

II/III

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II- Magnetostics** **Date of Lecture:**

Topic of Lecture: Vector magnetic potential, Biot-Savart law and applications

Introduction :

- The Biot Savart Law is used to determine the magnetic field intensity H near a current-carrying conductor or we can say, it gives the relation between magnetic field intensity generated by its source current element.
- The law was stated in the year 1820 by Jean Baptisle Biot and Felix Savart. The direction of the magnetic field follows the right-hand rule for the straight wire.
- Biot Savart law is also known as Laplace's law or Ampere's law.

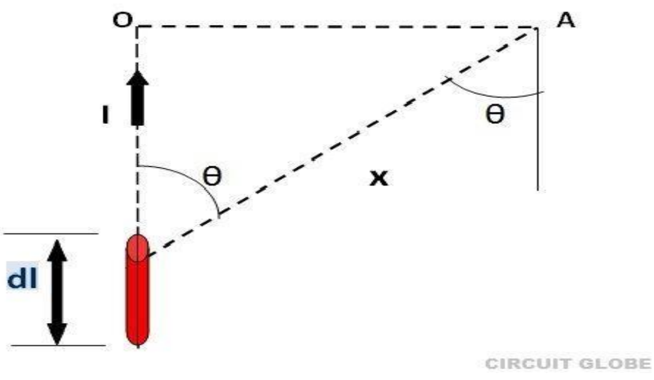
Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

<u>Biot Savart law</u>

The **Biot Savart Law** is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. Biot–Savart law is consistent with both Ampere's circuital law and Gauss's theorem. The Biot Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.

$$Hence,\ dB \propto \frac{Idl\sin\theta}{r^2}\quad or\quad dB = k\frac{Idl\sin\theta}{r^2}$$



The magnetic intensity dH at a point A due to current I flowing through a small element dl is

1. Directly proportional to current (I)
2. Directly proportional to the length of the element (dl)
3. Directly proportional to the sine of angle θ between the direction of current and the line joining the element dl from point A.
4. Inversely proportional to the square of the distance (x) of point A from the element dl.

$$dH = \frac{\mu_0 \mu_r}{4\pi} x \, Idl \, Sin\theta / x^2$$

$$dH = k \, x \, Idl \, Sin\theta / x^2$$

$$dH \propto Idl \, Sin\theta / x^2$$

where k is constant and depends on the magnetic properties of the medium.

$$K = \mu_0 \mu_r / 4\pi$$

μ_0 = absolute permeability of air or vacuum and its value is 4 x 10^{-7} Wb/A-m

μ_r= relative permeability of the medium

Video Content / Details of website for further learning (if any):
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LECTURE HANDOUTS

L13

ECE

II/III

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Magnetic field intensity and idea of relative permeability

Introduction :

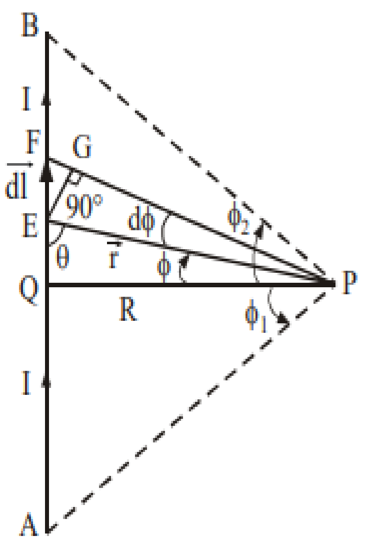
- Magnetic field due to an infinitely long straight current carrying wire - definition. B=(2пr)μ0I where B is the magnitude of magnetic field, r is the distance from the wire where the magnetic field is calculated, and I is the applied current.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Magnetic Field intensity

Suppose AB is a straight conductor carrying a current of I and magnetic field intensity is to be determined at point P. According to Biot-Savart law the magnetic field at P



$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\,\overrightarrow{dl}\times\vec{r}}{r^3}$$

angle between $I\,\overrightarrow{dl}$ and $\vec{r}$ is $(180 - \theta)$ so

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin(180-\theta)}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2} \qquad \ldots(i)$$

Now $EG = EF\sin\theta$

$= dl\sin\theta$

and $EG = EP\sin d\phi = r\sin d\phi$

$= r\,d\phi$

so $dl\sin\theta = r\,d\phi$ ...(ii)

so from (i)

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\phi}{r} \qquad \ldots(iii)$$

from Δ EQP, $r = \frac{R}{\cos\phi}$

so $$dB = \frac{\mu_0}{4\pi}\frac{I\cos\phi\,d\phi}{R} \qquad \ldots(iv)$$

Then the total magnetic field at point P due to the entire conductor is

$$B = \int_{-\phi_1}^{\phi_2}\frac{\mu_0}{4\pi}\frac{I}{R}\cos\phi\,d\phi$$

$$= \frac{\mu_0}{4\pi}\frac{I}{R}[\sin\phi]_{-\phi_1}^{\phi_2}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\phi_1 + \sin\phi_2)} \qquad \ldots(v)$$

for any conductor of infinite length

$$\phi_1 = \phi_2 = 90°$$

so $$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}\ NA^{-1}\ m^{-1}$$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=1kydon2HxQA

Important Books/Journals for further learning including the page nos.:
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LECTURE HANDOUTS **L14**

ECE **II/III**

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Magnetic circuits

Introduction :

- Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force.
- However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current.
- We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Force on a wire carrying a current placed in a magnetic field

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced. If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an ×.

These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an ×. A composite sketch of the magnetic circles is shown in Figure. where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.

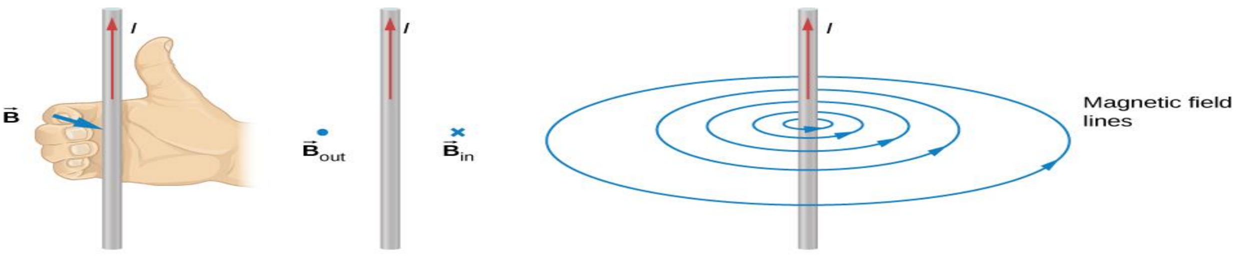

Figure (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in Figure. The length and cross-sectional area of the section are **dl** and **A**, respectively, so its volume is V=A·dl. The wire is formed from material that contains **n** charge carriers per unit volume, so the number of charge carriers in the section is nA·dl. If the charge carriers move with drift velocity $\vec{v}$ dv→d the current **I** in the wire is

$$I=neAv_d$$

The magnetic force on any single charge carrier is ev→d×B→, so the total magnetic force dF→ on the nA·dl charge carriers in the section of wire is

$$d\vec{F}=(nA\cdot dl)e\vec{v}_d\times\vec{B}$$

We can define **dl** to be a vector of length **dl** pointing along $\vec{v}$ dv→d, which allows us to rewrite this equation as

$$d\vec{F}=neAv_d d\vec{l}\times\vec{B}$$

or

$$d\vec{F}=Id\vec{l}\times\vec{B}$$

Video Content / Details of website for further learning (if any):
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LECTURE HANDOUTS

L 15

ECE

II/III

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Behaviour of magnetic materials

Introduction :

The vector potential of a small current loop at the origin (in the xy-plane); a magnetic dipole field. The strength of A is proportional to I_{ab}, which is the current times the area of the loop. This product is called the magnetic dipole moment (or, often, just "magnetic moment") of the loop.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Magnetic Moment

The magnetic moment is the magnetic strength and orientation of a magnet or other object that produces a magnetic field. Examples of objects that have magnetic moments include: loops of electric current (such as electromagnets), permanent magnets, moving elementary particles (such as electrons), various molecules, and many astronomical objects (such as many planets, some moons, stars, etc).

The magnetic moment can be defined as a vector relating the aligning torque on the object from an externally applied magnetic field to the field vector itself. The relationship is given by:

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$$

where τ is the torque acting on the dipole, B is the external magnetic field, and m is the magnetic moment.For a current loop, this definition leads to the magnitude of the magnetic dipole moment equaling the product of the current times the area of the loop. Further, this definition allows the calculation of the expected magnetic moment for any known macroscopic current

distribution. The magnetic dipole moment of a system is the negative gradient of its intrinsic energy, Uint, with respect to external magnetic field:

$$\mathbf{m} = -\hat{\mathbf{x}}\frac{\partial U_{\text{int}}}{\partial B_x} - \hat{\mathbf{y}}\frac{\partial U_{\text{int}}}{\partial B_y} - \hat{\mathbf{z}}\frac{\partial U_{\text{int}}}{\partial B_z}.$$

The magnetic moment is a quantity that describes the magnetic strength of an entire object. Therefore, it is useful to define the magnetization field M as:

$$\mathbf{M} = \frac{\mathbf{m}_{\Delta V}}{V_{\Delta V}},$$

where mΔV and VΔV are the magnetic dipole moment and volume of a sufficiently small portion of the magnet ΔV. This equation is often represented using derivative notation such that

$$\mathbf{M} = \frac{\mathrm{d}\mathbf{m}}{\mathrm{d}V},$$

Instead the parameter that is listed is residual flux density (or remanence), denoted Br. The formula needed in this case to calculate m in (units of A·m2) is:

$$\mathbf{m} = \frac{1}{\mu_0}\mathbf{B}_r V,$$

where:

- Br is the residual flux density, expressed in teslas.
- V is the volume of the magnet (in m3).
- μ0 is the permeability of vacuum (4π×10−7 H/m).

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=1kydon2HxQA

Important Books/Journals for further learning including the page nos.:
William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(94,137)

Course Faculty

Verified by HOD

Course Name with Code :19ECC05-Electromagnetic Fields

Course Faculty :Mr.A.Kumaravel

Unit :II- Magnetostatics **Date of Lecture:**

Topic of Lecture: boundary conditions

Introduction :

Boundary condition is defined such that the velocity on the walls is equal to zero and the initial condition is defined as a prescribed pressure difference between inlet and outlet (along the x-axis).

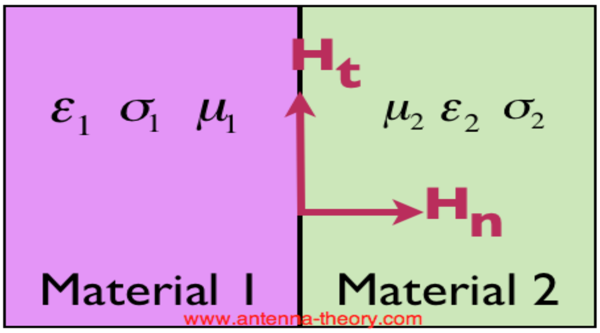
Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Magnetic boundary conditions

The "boundary conditions" on the electric and magnetic fields (i.e. relationships between the electric and magnetic fields on either side of a boundary) from Maxwell's equations. These boundary conditions are important for understanding the behaviour of electromagnetic fields in accelerator components.Just as the electric field obeys certain rules, the magnetic field (H-field) also obeys certain rules along the boundary of two different materials. And again, the rules depend on whether we are discussing the tangential component (Ht) or the normal/perpendicular component (Hn) to a surface. Now, just as there exists the parameter "permittivity" that is associated with electric fields, there exists an analogous parameter for magnetic fields - permeability, written by the symbol μ.

This is a property of a material which basically describes how a material concentrates magnetic fields. The units are measured in Henries/meter [H/m], which is a measure of inductance over a length.



The rules are very similar to the Electric Field boundary condition case. First, since there is no such things as magnetic charge, we will have the normal component of the magnetic flux density (B) continuous across a boundary:

$$B_{n1} = B_{n2}$$
$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

The equation states that the component of the magnetic flux density that is perpendicular to the material change is continuous across the boundary. That is, the vector Bn1 (normal component of B immediately inside region 1) is equal to the vector Bn2 (normal component of B immediately inside region 2). And since B and H are related by the permeability, we know how the normal component of the magnetic field Hn changes across the boundary.

For the tangential magnetic field (Ht) at a material discontinuity. Recall that magnetic fields are created due to electric current flowing. Hence, if no electric current is flowing on the surface (I=0), then the magnetic field will be continuous across a material boundary change:

$$H_{T1} = H_{T2}$$

Video Content / Details of website for further learning (if any):
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Course Faculty

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Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Inductance and inductors

Introduction :

Solenoid is the generic term for a coil of wire used as an electromagnet. It also refers to any device that converts electrical **energy** to mechanical **energy** using a solenoid. The device creates a magnetic field from electric current and uses the magnetic field to create **linear** motion.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

If the current is constant, the magnetic fluxthrough the loop is also constant. However, if the current **I** were to vary with time—say, immediately after switch S is closed—then the magnetic fluxΦm would correspondingly change. Then Faraday's law tells us that an emf ϵϵ would be induced in the circuit, where

$$\epsilon = -\frac{d\Phi_m}{dt}.$$

Since the magnetic field due to a current-carrying wire is directly proportional to the current, the flux due to this field is also proportional to the current; that is,

$$\Phi_m \propto I.$$

This can also be written as

$$\Phi_m = LI$$

where the constant of proportionality **L** is known as the **self-inductance** of the wire loop. If the loop has **N** turns, this equation becomes

$$\boxed{N\Phi_m = LI}$$

By convention, the positive sense of the normal to the loop is related to the current by the right-hand rule. The normal points downward. With this convention, Φm is positive in equation, so L always has a positive value.

For a loop with **N** turns, ϵ=−NdΦm/dt, so the induced emf may be written in terms of the self-inductance as

$$\boxed{\epsilon = -L\frac{dI}{dt}.}$$

When using this equation to determine **L**, it is easiest to ignore the signs of ϵϵ and dI/dt, and calculate **L** as

$$L = \frac{|\epsilon|}{|dI/dt|}.$$

Since self-inductance is associated with the magnetic field produced by a current, any configuration of conductors possesses self-inductance

In accordance with Lenz's law, the negative sign in Equation indicates that the induced emf across an inductor always has a polarity that **opposes** the change in the current. For example, if the current flowing from **A** to **B** were increasing, the induced emf(represented by the imaginary battery) would have the polarity shown in order to oppose the increase. If the current from **A** to **B** were decreasing, then the induced emf would have the opposite polarity, again to oppose the change in current .Finally, if the current through the inductor were constant, no emf would be induced in the coil.

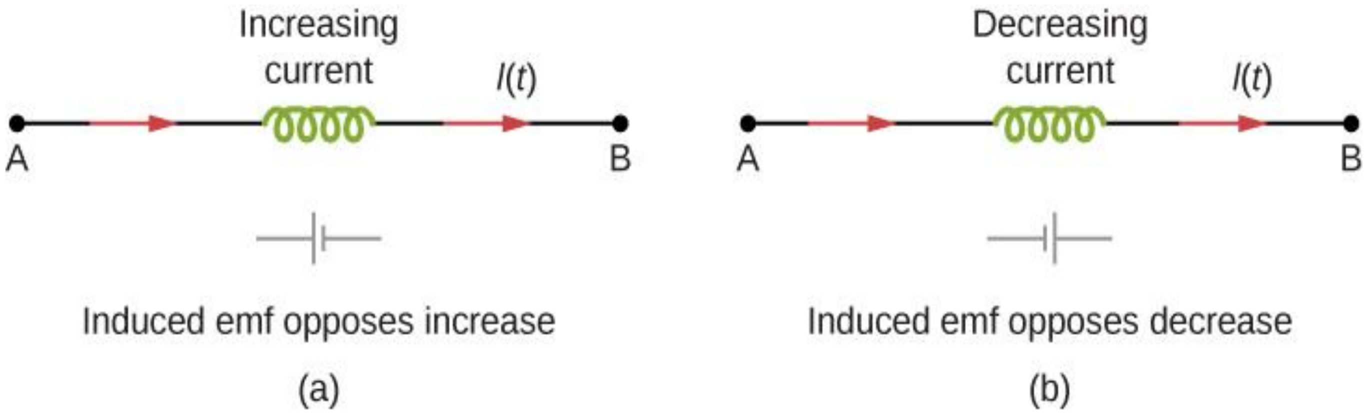

Figure :The induced emf across an inductor always acts to oppose the change in the current. This can be visualized as an imaginary battery causing current to flow to oppose the change in (a) and reinforce the change in (b).

Video Content / Details of website for further learning (if any):
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Important Books/Journals for further learning including the page nos.:
William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(94,137)

Course Faculty

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MUTHAYAMMAL ENGINEERING COLLEGE

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(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

LECTURE HANDOUTS

L 18

ECE

II/III

Course Name with Code :**19ECC05-Electromagnetic Fields**

Course Faculty :**Mr.A.Kumaravel**

Unit :**II-Magnetostatics** **Date of Lecture:**

Topic of Lecture: Magnetic energy, Magnetic forces and torques

Introduction :

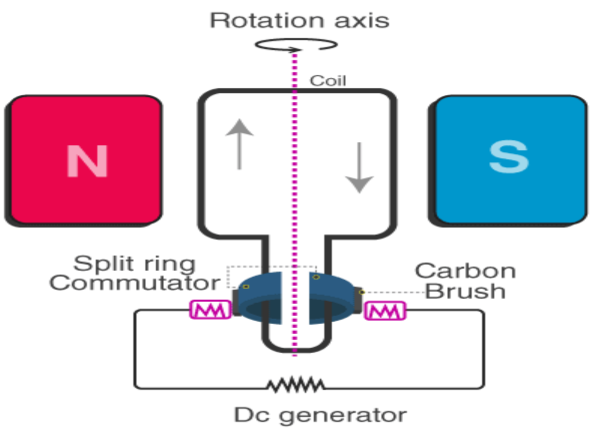
A magnetic dipole is the limit of either a closed loop of electric current or a pair of poles as the dimensions of the source are reduced to zero while keeping the magnetic moment constant. A steady current I passing through a rectangular loop placed in a uniform magnetic field experiences a torque. It does not experience a net force. This behaviour is similar to the of an electric dipole in a uniform electric field.

Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

<u>Torque on a loop carrying a current</u>

Let us consider a rectangular loop such that it carries a current of magnitude I. If we place this loop in a magnetic field, it experiences a torque but no net force, quite similar to what an electric dipole experiences in a uniform electric field.



Let us now consider the case when the magnetic field B is in the plane with the rectangular loop. No force is exerted by the field on the arms of the loop that is parallel to the magnets, but the arms perpendicular to the magnets experience a force given by F_1,

$$F_1 = IbB$$

This force is directed into the plane.

Similarly, we can write the expression for a force F_2 which is exerted on the arm CD,

$$F_2 = IbB = F_1$$

We see that the net force on the loop is zero and the torque on the loop is given by,

$$\tau = F_1 \frac{a}{2} + F_2 \frac{a}{2}$$

$$\tau = IbB\frac{a}{2} + IbB\frac{a}{2} = I(ab)B = IAB$$

Where ab is the area of the rectangle. Here, the torque tends to rotate the loop in the anti-clockwise direction.

Let the angle between the field and the normal to the coil be given by θ. We can see that the forces on the arms BC and DA will always act opposite to each other and will be equal in magnitude. Since these forces are the equal opposite and collinear at all points, they cancel out each other's effect and this results in zero-force or torque. The forces on the arms AB and CD are given by F1 and F2. These forces are equal in magnitude and opposite in direction and can be given by,

$$F_1 = F_2 = IbB$$

These forces are not collinear and thus act as a couple exerting a torque on the coil. The magnitude of the torque can be given by,

$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$$\tau = IabB\sin\theta$$

$$\tau = IAB\sin\theta$$

Video Content / Details of website for further learning (if any):
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Course Faculty

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LECTURE HANDOUTS

L 19

ECE

II / III

Course Name with Code : 19ECC05 – ELECTROMAGNETIC FIELDS

Course Faculty :Mr.A.Kumaravel

Unit : III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS

Date of Lecture:

Topic of Lecture: Faraday's law

Introduction: Faraday's law of electromagnetic induction (referred to as Faraday's law) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction.

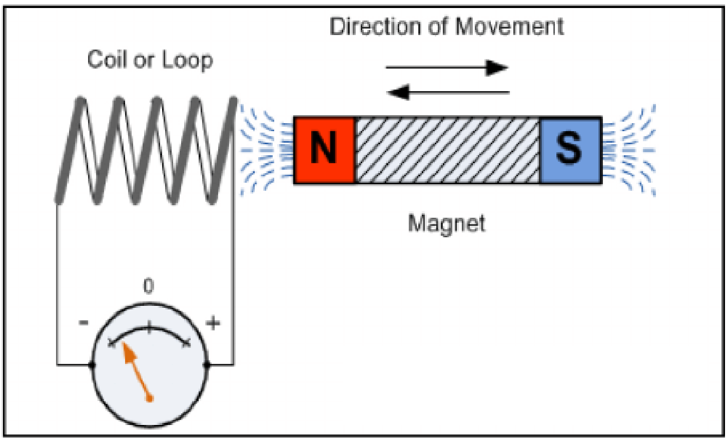
Prerequisite knowledge for Complete understanding and learning of Topic:

- Physics

Faraday's Laws of Electromagnetic Induction:

- Faraday's law states that a current will be induced in a conductor which is exposed to a changing magnetic field.
- Lenz's law of electromagnetic induction states that the direction of this induced current will be such that the magnetic field created by the induced current ***opposes*** the initial changing magnetic field which produced it.
- The direction of this current flow can be determined using Fleming's right-hand rule.
- Faraday's law of induction explains the working principle of transformers, motors, generators, and inductors.

Faraday's Experiment:



- In this experiment, Faraday takes a magnet and a coil and connects a galvanometer across the coil.

- At starting, the magnet is at rest, so there is no deflection in the galvanometer i.e the needle of the galvanometer is at the center or zero position.

- When the magnet is moved towards the coil, the needle of the galvanometer deflects in one direction.

- When the magnet is held stationary at that position, the needle of galvanometer returns to zero position.

- Now when the magnet moves away from the coil, there is some deflection in the needle but opposite direction, and again when the magnet becomes stationary, at that point respect to the coil, the needle of the galvanometer returns to the zero position.

- Similarly, if the magnet is held stationary and the coil moves away, and towards the magnet, the galvanometer similarly shows deflection.

- It is also seen that the faster the change in the magnetic field, the greater will be the induced EMF or voltage in the coil.

- From this experiment, Faraday concluded that whenever there is relative motion between a conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage across a coil.

Faraday's First Law

- Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

Method to change the magnetic field:

- By moving a magnet towards or away from the coil
- By moving the coil into or out of the magnetic field
- By changing the area of a coil placed in the magnetic field
- By rotating the coil relative to the magnet

Faraday's Second Law

- It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of the number of turns in the coil and flux associated with the coil.

The rate of change of flux linkage

$$E = N\frac{d\phi}{dt}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = -N\frac{d\phi}{dt}$$

Applications of Faraday's Law:

- Power transformers function based on Faraday's law
- The basic working principle of the electrical generator is Faraday's law of mutual induction.
- The Induction cooker is the fastest way of cooking. It also works on the principle of mutual

induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that the flow of current always produces heat in it.

- Electromagnetic Flow Meter is used to measure the velocity of certain fluids. When a magnetic field is applied to an electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to the velocity of fluid flowing.
- A form base of Electromagnetic theory, Faraday's idea of lines of force is used in well known Maxwell's equations. According to Faraday's law, change in magnetic field gives rise to change in electric field and the converse of this is used in Maxwell's equations.
- It is also used in musical instruments like an electric guitar, electric violin, etc.
-

Video Content / Details of website for further learning (if any):

- https://www.electrical4u.com/faraday-law-of-electromagnetic-induction/
- https://www.daenotes.com/electronics/basic-electronics/faraday-laws-of-electromagnetic-induction
-

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics,Tata McGraw-Hill,7th Edition, 2012. P.No: 3

Course Faculty

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Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS**

Date of Lecture:

Topic of Lecture: Maxwell's Second Equation in integral form from Faraday's Law

Introduction: The integral forms of Maxwell's equations describe the behaviour of electromagnetic field quantities in all geometric configurations. The differential forms of Maxwell's equations are only valid in regions where the parameters of the media are constant or vary smoothly i.e. in regions. In order for a differential form to exist, the partial derivatives must exist, and this requirement breaks down at the boundaries between different materials.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Differential Equations
- Integral Equations

The Third Maxwell's equation (Faraday's law of electromagnetic induction)

- According to Faraday's law of electromagnetic induction

$$\varepsilon = -N\frac{d\emptyset_m}{dt} \quad \text{................ (5)}$$

Since emf is related to electric field by the relation

$$\varepsilon = \int \vec{E}.\overrightarrow{dA}$$

Also $\emptyset_m = \int \vec{B}.\overrightarrow{dA}$

Put these values in equation (5) we have

$$\int \vec{E}.\overrightarrow{dA} = -N \int \vec{E}.\overrightarrow{dA} \int \vec{B}.\overrightarrow{dA}$$

For N=1 ,we have

$$\int \vec{E}.\overrightarrow{dA} = -\frac{d}{dt}\int \vec{B}.\overrightarrow{dA} \quad \text{............ (6)}$$

It is the integral form of Maxwell's 3rd equation.

Applying Stokes Theorem on L.H.S of equation (6) we have

$$\int(\vec{\nabla} \times \vec{E})d\vec{A} = -\frac{d}{dt}\int \vec{B}.\overrightarrow{dA}$$

$$\int(\vec{\nabla} \times \vec{E})d\vec{A} + \frac{d}{dt}\int \vec{B}.\overrightarrow{dA} = 0$$

$$(\vec{\nabla} \times \vec{E}) + \frac{d\vec{B}}{dt} = 0$$

$$(\vec{\nabla} \times \vec{E}) = -\frac{d\vec{B}}{dt}$$

- It is the differential form of Maxwell's third equation.

The Fourth Maxwell's equation (Ampere's law)

- The magnitude of the magnetic field at any point is directly proportional to the strength of the current and inversely proportional to the distance of the point from the straight conductors is called Ampere's law.

$\int \vec{B}.\overrightarrow{ds} = \mu_0 i$ .... (7)

It is the integral form of Maxwell's 4 th equation.

The value of current density

$i = \int \vec{j}.\overrightarrow{dA}$

Now the equation (7) Become

$\int \vec{B}.\overrightarrow{ds} = \mu_0 \int \vec{j}.\overrightarrow{dA}$

Applying Stoke's theorem on L.H.S of above equation, we have

$\int (\vec{\nabla} \times \vec{B}) d\vec{A} = \mu_0 \int \vec{j}.\overrightarrow{dA}$

$\int [(\vec{\nabla} \times \vec{B}) d\vec{A} - \mu_0 \vec{j}]. d\mathbf{A} = 0$

$(\vec{\nabla} \times \vec{B}) = \mu_0 \boldsymbol{j}$

- It is the differential form of Maxwell's Fourth equation.

Video Content / Details of website for further learning (if any):

- https://physicsabout.com/maxwells-equations/

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 32

Course Faculty

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LECTURE HANDOUTS

L 21

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS**

Date of Lecture:

Topic of Lecture: Displacement current

Introduction: An electric current produces a magnetic field around it. J.C. Maxwell showed that for logical consistency, a changing electric field must also produce a magnetic field. Further, since magnetic fields have always been associated with currents, Maxwell postulated that this current was proportional to the rate of change of the electric field and called it displacement current.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Circuit Theory

Displacement current:

- Apart from conduction current, there is another type of current called displacement current. It does not appear from real movement of electric charge as is the case for conduction current.
- Displacement current is a quantity appearing in Maxwell's equations. Displacement current definition is defined in terms of the rate of change of the electric displacement field (D).
- When a capacitor starts charging there is no conduction of charge between the plates. However, because of change in charge accumulation with time above the plates, the electric field changes causing the displacement current as below:

$$I_D = J_D S = S\frac{\partial D}{\partial t}$$

Where,

- S is the area of the capacitor plate.
- I_D is the displacement current.
- J_D is the displacement current density.
- D is related to electric field E as $D=\varepsilon E$
- ε is the permittivity of the medium in between the plates.

Displacement Current Equation:

- Displacement current has the same unit and effect on the magnetic field as is for conduction current depicted by Maxwell's equation-

$$\nabla \times H = J + J_D$$

Where,

- H is related to magnetic field B as $B=\mu H$
- μ is the permeability of the medium in between the plates.
- J is the conducting current density.
- J_D is the displacement current density.

We know that $\nabla.(\nabla \times H) = 0$ and $\nabla.J = -\frac{\partial \rho}{\partial t} = -\nabla.\frac{\partial D}{\partial t}$ using Gauss's law that is $\nabla.D = \rho$

Here, ρ is the electric charge density.

Thus, $J_D = \frac{\partial D}{\partial t}$ displacement current density is necessary to balance RHS with LHS of the equation.

- According to Faraday's law of electromagnetic induction, a time-varying magnetic field induces an emf, According to Maxwell, an electric field sets up a current and hence a magnetic field. Such a current is called displacement current. It follows that a time-varying electric field produces a magnetic field and vice-versa. Hence the behaviour of the electric and magnetic field is symmetric.
- The total current passing through any surface of which the closed loop is the perimeter is the sum of the conduction current and the displacement current.
- This is also known as – Ampere-Maxwell Law. It is important to remember that the displacement and conduction currents have the same physical effects. Here are some points to remember:
- In cases where the electric field does not change with time, like steady electric fields in a conducting wire, the displacement current may be zero.
- In cases like the one explained above, both currents are present in different regions of the space.
- Since a perfectly conducting or insulating medium does not exist, in most cases both the currents can be present in the same region.
- In cases where there is no conduction current but a time-varying electric field, only displacement current is present. In such a scenario we have a magnetic field even when there is no conduction current source nearby.

Video Content / Details of website for further learning (if any):

- https://www.toppr.com/guides/physics/electromagnetic-waves/displacement-current/
- https://web.sonoma.edu/users/f/farahman/sonoma/courses/es430/lectures/Chap6_S11.pdf

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 31

Course Faculty

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LECTURE HANDOUTS

L 22

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS**

Date of Lecture:

Topic of Lecture: Maxwell's equations in integral form and differential form

Introduction: The integral forms of Maxwell's equations describe the behaviour of electromagnetic field quantities in all geometric configurations. The differential forms of Maxwell's equations are only valid in regions where the parameters of the media are constant or vary smoothly i.e. in regions. In order for a differential form to exist, the partial derivatives must exist, and this requirement breaks down at the boundaries between different materials.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Differential Equations
- Integral Equations

The Fourth Maxwell's equation (Ampere's law)

- The magnitude of the magnetic field at any point is directly proportional to the strength of the current and inversely proportional to the distance of the point from the straight conductors is called Ampere's law.

$\int \vec{B}.\vec{ds} = \mu_0 i$ .... (7)

It is the integral form of Maxwell's 4 th equation.

The value of current density

$i = \int \vec{j}.\vec{dA}$

Now the equation (7) Become

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Applying Stoke's theorem on L.H.S of above equation, we have

$\int (\vec{\nabla} \times \vec{B}) d\vec{A} = \mu_0 \int \vec{j}.\vec{dA}$

$\int [(\vec{\nabla} \times \vec{B}) d\vec{A} - \mu_0 j]. dA = 0$

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The Third Maxwell's equation (Faraday's law of electromagnetic induction)

- According to Faraday's law of electromagnetic induction

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Important Books/Journals for further learning including the page nos.: • M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 32

Course Faculty

Verified by HOD

LECTURE HANDOUTS

L 23

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS**

Date of Lecture:

Topic of Lecture: Equation expressed in point form

Introduction: Maxwell's Equations in Time-Harmonic Form. This is known as phasor form or the time-harmonic form of Maxwell's Equations. It is perfectly legitimate, because this form tells us how the waves behave if they are oscillating at frequency f, and all waves can be decomposed into the sum of simple oscillating waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Maxell's Equations

Maxwell's equation in phasor form:

- Maxwell's Equations are commonly written in a few different ways.

1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

- The above equations are known as "point form" because each equality is true at every point in space. However, if we integrate the point form over a volume, we obtain the integral form. There is also Time-Harmonic Form, and Maxwell's Equations written only with **E** and **H**. And one form uses imaginary magnetic charge, which can be useful for some problem solving.

- If the point forms of Maxwell's Equations are true at every point, then we can integrate them over any volume (*V*) or through any surface and they will still be true.

$$\int_V \nabla \cdot \mathbf{A}\, dV = \oint_S \mathbf{A} \cdot \mathbf{dS} \qquad \text{[Divergence Theorem]}$$

$$\int (\nabla \times \mathbf{A}) \cdot \mathbf{dS} = \oint_L \mathbf{A} \cdot \mathbf{dL} \qquad \text{[Stokes' Theorem]}$$

Note: These Theorems are true for any vector field

- Maxwell's Equations in Integral Form.

1. $\oint\!\!\oint_S \mathbf{D}\cdot\mathbf{dS} = Q_{enc}$ = Amount of Charge Within Surface S
2. $\oint\!\!\oint_S \mathbf{B}\cdot\mathbf{dS} = 0$
3. $\oint_L \mathbf{E}\cdot\mathbf{dL} = -\iint_S \frac{\partial \mathbf{B}}{\partial t}\cdot\mathbf{dS}$
4. $\oint_L \mathbf{H}\cdot\mathbf{dL} = I_{enc} + \iint_S \frac{\partial \mathbf{D}}{\partial t}\cdot\mathbf{dS}$

$$[I_{enc} = \iint_S \mathbf{J}\cdot\mathbf{dS}]$$

www.maxwells-equations.com

Time-Harmonic Form of Maxwell's Equations

- We know from the theory of Fourier Transforms that every signal in time can be rewritten as the sum of sinusoids (sign or cosine). Using a little more complex math and we can specify the time variation in terms of the sum of sinusoids written in complex form:

$$e^{i\omega t} = e^{i2\pi ft} \qquad i = \sqrt{-1}$$

- In the previous Equation, f is the frequency we are interested in, which is equal to $\omega = 2\pi f$. Hence, the time derivative of the function in the above Equation is the same as the original function multiplied by $i\omega$. This means we can replace the time-derivatives in the point-form of Maxwell's Equations in integral form as in the following:

1. $\nabla\cdot\mathbf{D} = \rho_V$
2. $\nabla\cdot\mathbf{B} = 0$
3. $\nabla\times\mathbf{E} = -i\omega\mathbf{B} = -i2\pi f\cdot\mathbf{B}$
4. $\nabla\times\mathbf{H} = i\omega\mathbf{D} + \mathbf{J}$

- This is known as phasor form or the time-harmonic form of Maxwell's Equations. It is perfectly legitimate, because this form tells us how the waves behave if they are oscillating at frequency f, and all waves can be decomposed into the sum of simple oscillating waves.

Video Content / Details of website for further learning (if any):

- http://www.maxwells-equations.com/forms.php

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 33

Course Faculty

Verified by HOD

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS** **Date of Lecture:**

Topic of Lecture: Maxwell's equations in integral form and differential form

Introduction: The integral forms of Maxwell's equations describe the behaviour of electromagnetic field quantities in all geometric configurations. The differential forms of Maxwell's equations are only valid in regions where the parameters of the media are constant or vary smoothly i.e. in regions. In order for a differential form to exist, the partial derivatives must exist, and this requirement breaks down at the boundaries between different materials.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Vector Algebra
- Differential Equations
- Integral Equations

Maxwell's First Equation

- The Gauss's law states that flux passing through any closed surface is equal to 1/ε0 times the total charge enclosed by that surface.

$$\emptyset_E=\frac{q}{\epsilon 0} \text{------------- (1)}$$

$$Also\ \ \emptyset_E = \int \vec{E}.d\vec{A} \text{------ (2)}$$

Comparing equation (1) and (2) we have

$$\int \vec{E}.d\vec{A} = \frac{q}{\epsilon 0} \text{ ---------- (3)}$$

- It is the integral form of Maxwell's 1st equation.

The value of total charge in terms of volume charge density is $q=\int \rho dv$ So equation (3) becomes

$$\int \vec{E}.d\vec{A} = \frac{1}{\epsilon 0} \int \rho dv$$

Applying divergence theorem on left hand side of above equation we have

$$\int (\vec{\nabla}.\text{E})d.\text{V} = \frac{1}{\epsilon 0} \int \rho dv$$

$$\int (\vec{\nabla}.\text{E})d.\text{V} - \frac{1}{\epsilon 0} \int \rho dv = 0$$

$$\int [(\vec{\nabla}.\text{E}) - \frac{\rho}{\epsilon 0}]\ dv = 0$$

$$(\vec{\nabla}.\text{E}) - \frac{\rho}{\epsilon 0} = 0$$

$$(\vec{\nabla}.\text{E}) = \frac{\rho}{\epsilon 0}$$

- The magneto motive force around a closed path is equal to the sum of the conduction current and displacement current enclosed by the path. In other words, magnetic voltage around a closed path is equal to the electric current through the path.

The Second Maxwell's equation (Gauss's law for magnetism)

- The Gauss's law for magnetism states that net flux of the magnetic field through a closed surface is zero because monopoles of a magnet do not exist.

  $\int \vec{B}.\overrightarrow{dA}=0$ .... (4)

  It is the integral form of Maxwell's second equation.

  Applying divergence theorem

  $\int (\vec{\nabla}.\vec{B})dV=0$

  This implies that:

  $\vec{\nabla}.\vec{B}=0$

  It is called differential form of Maxwell's second equation.

Video Content / Details of website for further learning (if any):

- https://physicsabout.com/maxwells-equations/

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 32

Course Faculty

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Course Name with Code :19ECC05-Electromagnetic Fields

Course Faculty :Mr.A.Kumaravel

Unit : III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS **Date of Lecture:**

Topic of Lecture: Electromagnetic boundary conditions

Introduction :

Boundary condition is defined such that the velocity on the walls is equal to zero and the initial condition is defined as a prescribed pressure difference between inlet and outlet (along the x-axis).

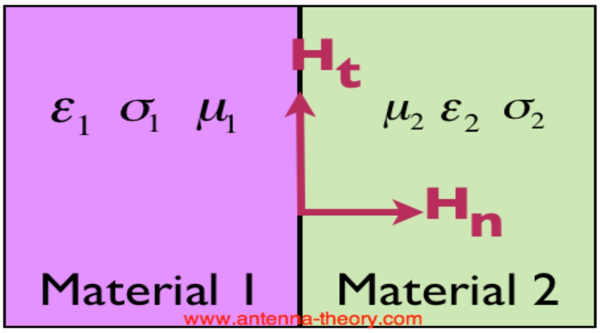
Prerequisite knowledge for Complete understanding and learning of Topic:

- **Physics**

Magnetic boundary conditions

The "boundary conditions" on the electric and magnetic fields (i.e. relationships between the electric and magnetic fields on either side of a boundary) from Maxwell's equations. These boundary conditions are important for understanding the behaviour of electromagnetic fields in accelerator components.Just as the electric field obeys certain rules, the magnetic field (H-field) also obeys certain rules along the boundary of two different materials. And again, the rules depend on whether we are discussing the tangential component (Ht) or the normal/perpendicular component (Hn) to a surface. Now, just as there exists the parameter "permittivity" that is associated with electric fields, there exists an analogous parameter for magnetic fields - permeability, written by the symbol μ.

This is a property of a material which basically describes how a material concentrates magnetic fields. The units are measured in Henries/meter [H/m], which is a measure of inductance over a length.



The rules are very similar to the Electric Field boundary condition case. First, since there is no such things as magnetic charge, we will have the normal component of the magnetic flux density (B) continuous across a boundary:

$$B_{n1} = B_{n2}$$
$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

The equation states that the component of the magnetic flux density that is perpendicular to the material change is continuous across the boundary. That is, the vector Bn1 (normal component of B immediately inside region 1) is equal to the vector Bn2 (normal component of B immediately inside region 2). And since B and H are related by the permeability, we know how the normal component of the magnetic field Hn changes across the boundary.

For the tangential magnetic field (Ht) at a material discontinuity. Recall that magnetic fields are created due to electric current flowing. Hence, if no electric current is flowing on the surface (I=0), then the magnetic field will be continuous across a material boundary change:

$$H_{T1} = H_{T2}$$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=1kydon2HxQA

Important Books/Journals for further learning including the page nos.:
William H Hayt and Jr John A Buck, "Engineering Electromagnetics", Tata Mc Graw-Hill Publishing Company Ltd, New Delhi, 2016,Pg.No.(94,137)

Course Faculty

Verified by HOD

LECTURE HANDOUTS **L 26**

ECE **II / III**

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS** **Date of Lecture:**

Topic of Lecture: Wave equations and solutions

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Propagation of Electromagnetic Waves in a Conducting Medium:

We will consider a plane electromagnetic wave travelling in a linear dielectric medium such as air along the z direction and being incident at a conducting interface. The medium will be taken to be a linear medium. So that one can describe the electrodynamics using only the E and H vectors. We wish to investigate the propagation of the wave in the conducting medium.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

Let us take $\vec{E}, \vec{H}$ and $\vec{k}$ to be respectively in x, y and z direction. We the nhave,

$$(\nabla \times \vec{E})_y = \frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

i.e.,

$$\frac{\partial E_x}{\partial z} + \mu \frac{\partial H_y}{\partial t} = 0 \qquad (1)$$

and

$$(\nabla \times \vec{H})_x = -\frac{\partial H_y}{\partial z} = \sigma E_x + \epsilon \frac{\partial E_x}{\partial t}$$

i.e.

$$\frac{\partial H_y}{\partial z} + \sigma E_x + \epsilon \frac{\partial E_x}{\partial t} = 0 \quad (2)$$

where A, B, C and D are constants to be determined. If the values of the electric field at z=0 is E_0 and that of the magnetic field at z=0 is H_0, we have $A = E_0$ and $C = H_0$.
In order to determine the constants B and D, let us return back to the original first order equations (1) and (2)

$$\frac{\partial E_x}{\partial z} + i\mu\omega H_y = 0$$
$$\frac{\partial H_y}{\partial z} + \sigma E_x + i\omega\epsilon E_x = 0$$

Substituting the solutions for E and H

$$\gamma E_0 \sinh(\gamma z) + B\gamma \cosh(\gamma z) + i\omega\mu(H_0 \cosh(\gamma z) + D\sinh(\gamma z)) = 0$$

This equation must remain valid for all values of z, which is possible if the coefficients of $sinh$ and cosh terms are separately equated to zero,

$$E_0\gamma + i\omega\mu D = 0$$
$$B\gamma + i\omega\mu H_0 = 0$$

The former gives,

where A, B, C and D are constants to be determined. If the values of the electric field at z=0 is E_0 and that of the magnetic field at z=0 is H_0, we have $A = E_0$ and $C = H_0$.
In order to determine the constants B and D, let us return back to the original first order equations (1) and (2)

$$\frac{\partial E_x}{\partial z} + i\mu\omega H_y = 0$$
$$\frac{\partial H_y}{\partial z} + \sigma E_x + i\omega\epsilon E_x = 0$$

Substituting the solutions for E and H

$$\gamma E_0 \sinh(\gamma z) + B\gamma \cosh(\gamma z) + i\omega\mu(H_0 \cosh(\gamma z) + D\sinh(\gamma z)) = 0$$

This equation must remain valid for all values of z, which is possible if the coefficients of $sinh$ and cosh terms are separately equated to zero,

$$E_0\gamma + i\omega\mu D = 0$$
$$B\gamma + i\omega\mu H_0 = 0$$

The former gives,

Video Content / Details of website for further learning (if any):
• http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf
Important Books/Journals for further learning including the page nos.:
• William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

Course Faculty

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MUTHAYAMMAL ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Accredited by NAAC & Affiliated to Anna University)
Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

Estd. 2000

LECTURE HANDOUTS

L 27

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **III - TIME-VARYING FIELDS AND MAXWELL's EQUATIONS**

Date of Lecture:

Topic of Lecture: Time-harmonic fields

Introduction: Transformer emf devices are based on the generation of emf in a stationary circuit, in which the emf is generated by a time-varying magnetic field linking the circuit. Motional emf devices are based on the generation of emf due to a moving conductor within a stationary magnetic field.

Prerequisite knowledge for Complete understanding and learning of Topic:

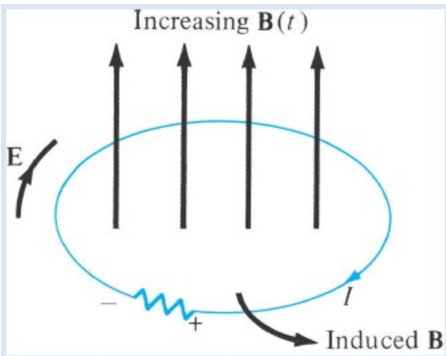
- Physics
- Basic Electrical Engineering

Transformer and motional electromotive forces:

The variation of flux with time may be caused in three ways:

- 1) By having a stationary loop in a time-varying B field. (Transformer induction)
- 2) By having a time-varying loop area in static B field. (motional induction)
- 3) By having a time-varying loop area in a time-varying B field. (general case, transformer and motional induction)

Stationary loop in Time-Varying B field (Transformer EMF):



$$V_{emf} = \oint_L E.dl = -\int_S \frac{\partial B}{\partial t}.dS$$

Stationary loop in Time-Varying B field (Transformer EMF):

$$V_{emf} = \oint_L E.dl = -\int_S \frac{\partial B}{\partial t}.dS$$

By applying Stoke's theorem

$$\int_S (\nabla \times E).dS = -\int_S \frac{\partial B}{\partial t}.dS \quad \Rightarrow \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

- This is one of Maxwell's equations for time-varying fields. Note that (time-varying E field is non-conservative)

Moving loop in static B field (Motional EMF):

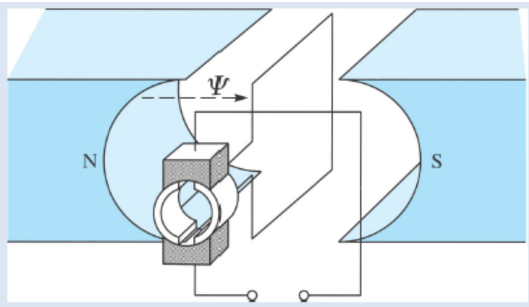
- Consider a conducting loo moving with uniform velocity **u**, the emf induced in the loop is

$$V_{emf} = \oint_L E_m.dl = \oint_L (u \times B).dl$$

- This type of emf is called motional emf or flux-cutting emf (due to motion action).

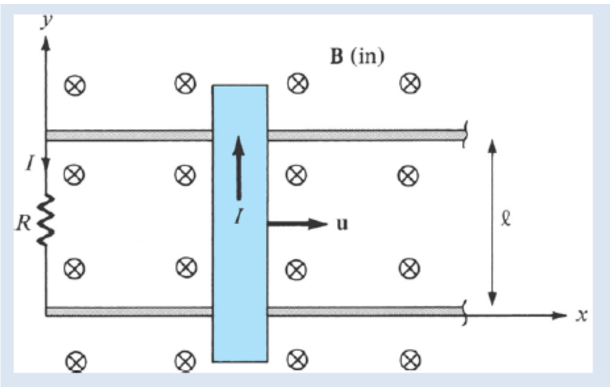
Moving loop in static B field (Motional EMF):

- It is kind of emf found in electrical machines such as motors and generators. Given below is in example of dc machine , where voltage is generated as the coil rotates within the magnetic field.



Moving loop in static B field (Motional EMF):

- Another example of motional emf is illustrated below, where a conducting bar is moving between a pair of rails.



Moving loop in static B field (Motional EMF):

* Recall that the force on a charge moving with uniform velocity u in a a magnetic field B is: $F_m = Q\mathbf{u} \times \mathbf{B}$

* We define the motional electric field E_m as

$$E_m = \frac{F_m}{Q} = u \times B$$

$$V_{emf} = \oint_L E_m . dl = \oint_L (u \times B) . dl$$

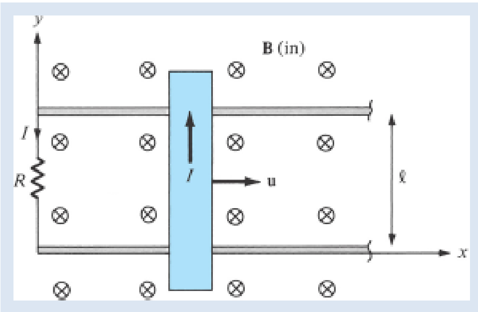
By applying Stoke's theorem,

$$\int_S (\nabla \times E_m) . dS = \int_S \nabla \times (u \times B) . dS$$

$$or \qquad \nabla \times E_m = \nabla \times (u \times B)$$

$$V_{emf} = \oint_L E_m . dl = \oint_L (u \times B) . dl$$

Moving loop in static B field (Motional EMF):



$$V_{emf} = \oint_L E_m . dl = \oint_L (u \times B) . dl$$

- The integral is zero along the portion of the loop where u=0. (e.g. *dl* is taken along the rod in the shown figure.
- The direction of the induced current is the same that of Em or uxB. The limits of integration are selected in the direction opposite to the direction of u x B to satisfy Lenz's law. (e.g. induced current flows in the rod along ay, the integration over L is along -ay).

Moving loop in Time Varying Field:

- Both Transformer emf and motional emf are present.

$$V_{emf} = \oint_L E.d\mathrm{l} = -\int_S \frac{\partial B}{\partial t}.d\mathrm{S} + \oint_L (u \times B).d\mathrm{l}$$

Transformer   Motional

$$or\ \nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t} + \nabla \times (\mathrm{u} \times \mathrm{B})$$

Note that eq. $V_{emf} = -\frac{d\Psi}{dt}$ can always be applied in place of the equations in cases A, B, and C.

Video Content / Details of website for further learning (if any):

- https://web.sonoma.edu/users/f/farahman/sonoma/courses/es430/lectures/Chap6_S11.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012. P.No: 4

Course Faculty

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LECTURE HANDOUTS

L 28

ECE

II/III

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Transmission line Parameters

Introduction :

Energy can be transmitted either by radiation of free electromagnetic waves as in the radio (or) it can be constant to move (or) carried in various conductor element known as transmission line.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Lumped and Distributed Elements

Detailed content of the Lecture:

Transmission line Parameters:

The parameters of a transmission line are:

Resistance(R)
Inductance (L)
Capacitance (C)
Conductance (G)

- Resistance (R) is defined as the loop resistance per unit length of the wire. Its unit is ohm/Km
- Inductance (L) is defined as the loop inductance per unit length of the wire. Its unit is Henry/Km
- Capacitance (C) is defined as the loop capacitance per unit length of the wire. Its unit is Farad/Km
- Conductance (G) is defined as the loop conductance per unit length of the wire. Its unit is mho/Km

4 per-unit-length parameters:
C = capacitance/length [F/m]
L = inductance/length [H/m]
R = resistance/length [W/m]
G = conductance/length [/m or S/m]

Secondary constants:

The secondary constants of a line are: Characteristic Impedance and Propagation Constant. Since the line constants R, L, C, G are distributed through the entire length of the line, they are called as distributed elements. They are also called as primary constants.

Characteristic impedance is the impedance measured at the sending end of the line. It is given by

$Z0 = Z/Y$, where
$Z = R + j\omega L$ is the series impedance
$Y = G + j\omega C$ is the shunt admittance

Propagation constant is defined as the natural logarithm of the ratio of the sending end current or voltage to the receiving end current or voltage of the line. It gives the manner in the wave is propagated along a line and specifies the variation of voltage and current in the line as a function of distance. Propagation constant is a complex quantity and is expressed as $\gamma = \alpha + j\beta$. The real part is called the attenuation constant α whereas the imaginary part of propagation constant is called the phase constant β.

$\gamma = ZY$, where
$Z = R + j\omega L$ is the series impedance
$Y = G + j\omega C$ is the shunt admittance

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=_S2K5BT3_Y4
https://www.oreilly.com/library/view/electric-power-transmission/9789332503410/xhtml/chapter002.xhtml

Important Books/Journals for further learning including the page nos.:
John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 195

Course Faculty

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LECTURE HANDOUTS | **L 29**

ECE | **II/III**

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: General solutions of transmission line

Introduction :

A finite line is a line having a finite length on the line. It is a line, which is terminated, in its characteristic impedance (ZR=Z0), so the input impedance of the finite line is equal to the characteristic impedance (Zs=Z0).

Prerequisite knowledge for Complete understanding and learning of Topic:

- Transmission line Parameters

Detailed content of the Lecture:

The transmission line - general solution:

R= series resistance, ohms per unit length of line(includes both wires)

L= series inductance, henrys per unit length of line

C= capacitance between conductors, faradays per unit length of line

G= shunt leakage conductance between conductors, mhos per unit length Of line

ωL = series reactance, ohms per unit length of line

Z = R+jωL

ωL = series susceptance, mhos per unit length of line

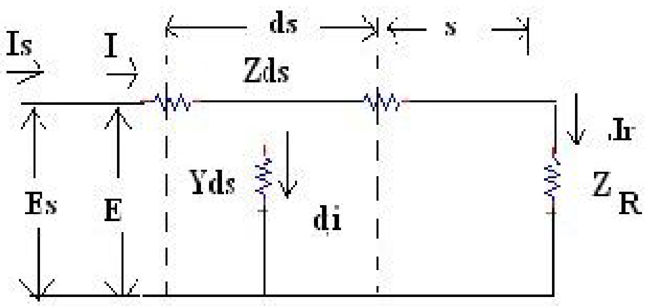
Y = G+jωC

S = distance to the point of observation, measured from the receiving end of the line

I = Current in the line at any point

E= voltage between conductors at any point

l = length of line



This results indicates two solutions, one for the plus sign and the other for the minus sign before the radical. The solution of the differential; equations are

$$E = Ae^{\sqrt{AYs}} + Be^{-\sqrt{ZY}s} \qquad (7)$$

$$I = Ce^{\sqrt{ZYs}} + De^{-\sqrt{ZY}s} \qquad (8)$$

Where A,B,C,D are arbitrary constants of integration.

Since the distance is measured from the receiving end of the line, it is possible to conditions such that at

$$s = 0, I = I_{R,} E = E_R$$

The n equation 7 & 8 becomes

$$E_R = A + B$$

$$I_R = C + D \qquad (9)$$

$$\frac{dE}{ds} = A\sqrt{ZY}\, e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s}$$

$$IZ = A\sqrt{ZY}\, e^{\sqrt{ZY}s} - B\sqrt{ZY}\, e^{-\sqrt{ZY}s} \qquad (11)$$

$$I = A\sqrt{\frac{Y}{Z}} e^{\sqrt{ZY}s} - B\sqrt{\frac{Y}{Z}} e^{-\sqrt{ZY}s}$$

$$\frac{dI}{ds} = C\sqrt{ZY}\, e^{\sqrt{ZY}s} - \sqrt{ZY}\, e^{-\sqrt{ZY}s} \qquad (12)$$

$$E = C\sqrt{\frac{Z}{Y}}e^{\sqrt{ZY}s} - D\sqrt{\frac{Y}{Z}}e^{-\sqrt{ZY}} \qquad (13)$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}}$$

$$E_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}}$$

Simultaneous solution of equation 9 ,12 and 13, along with the fact that $E_R = I_R Z_R$ and that $\sqrt{Z/Y}$ has been identified as the Z_0 of the line,leads to solution for the constants of the above equations as

$$A = \frac{E_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)$$

$$B = \frac{E_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}} = \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)$$

$$C = \frac{I_R}{2} + \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)$$

$$D = \frac{I_R}{2} - \frac{E_R}{2}\sqrt{\frac{Y}{Z}} = \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)$$

The solution of the differential equations of the transmission line may be written

$$E = \frac{E_R}{2}\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}s} + \frac{E_R}{2}\left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}s}$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}s} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}s} \qquad (14)$$

The above equations are very useful form for the voltage and current at any point on a transmission line.

Video Content / Details of website for further learning (if any):
http://www.ee.ic.ac.uk/hp/staff/dmb/courses/ccts1/01700_LinesA.pdf

Important Books/Journals for further learning including the page nos.:
John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 236

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LECTURE HANDOUTS

L 30

ECE

II/III

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Wavelength, velocity of propagation

Introduction :

The distance the wave travels along the line while the phase angle is changing through 2Π radians is called a wavelength.

Wavelength $\lambda = 2\Pi/\beta$

Velocity of propagation $v = \omega / \beta$

Prerequisite knowledge for Complete understanding and learning of Topic:

- Transmission line Parameters

Detailed content of the Lecture:

Wavelength, velocity of propagation:

Wavelength

The distance the wave travels along the line while the phase angle is changed through 2∏ radians is called wavelength.

$$\lambda = 2\pi / \beta$$

The change of 2π in phase angle represents one cycle in time and occurs in a distance of one wavelength,

$$\lambda = v/f$$

Velocity

$$V = f\lambda$$

$$V = \omega / \beta$$

This is the velocity of propagation along the line based on the observation of the change in the phase angle along the line.It is measured in miles/second if ß is in radians per meter.

We know that

$$Z = R + j\,\omega L$$

$$Y = G + j\,\omega C$$

Then

$$\gamma = \alpha + j\,\beta = \sqrt{ZY}$$

$$= \sqrt{RG - \omega^2 LC + j\omega\,(LG + CR)}$$

Squaring on both sides

$$\alpha^2 + 2j\alpha\beta - \beta^2 = RG - \omega^2 LC + j\omega\,(LG + RC)$$

Equating real parts and imaginary parts we get

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + CR)}}{2}}$$

And the equation for ß is

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC) + \omega^2\,(LG + CR)}}{2}}$$

Wavelength $\lambda = 2\Pi/\beta$

Velocity of propagation $v = \omega / \beta$

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=5MfWO_S6M2o

Important Books/Journals for further learning including the page nos.:
John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 245

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LECTURE HANDOUTS | **L 31**

ECE | **II/III**

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: The distortion-less line

Introduction :

A line, which has neither frequency distortion nor phase distortion is called a distortion less line. The condition for a distortion less line is RC=LG. Also,

a) The attenuation constant α should be made independent of frequency.
b) The phase constant β should be made dependent of frequency.
c) The velocity of propagation is independent of frequency.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Waveform distortion

Detailed content of the Lecture:

The distortion-less line:

Then the phase constant be a direct fuction of frequency.

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC) + \omega^2 (LG + CR)}}{2}}$$

The above equation shows that if the the term under the second radical be reduced to equal

$$(RG + \omega^2 LC)^2$$

Then the required condition for ß is obtained. Expanding the term under the internal radical and forcing the equality gives

$$R^2G^2 - 2\omega^2 LCRG + \omega^4 L^2C^2 + \omega^2 L^2G^2 + 2\omega^2 LCRG + \omega^2 C^2 R^2 = (RG + \omega^2 LC)^2$$

This reduces to

$$-2\omega^2 LCRG + \omega^2 L^2G^2 + \omega^2 C^2 R^2 = 0$$

$$(LG - CR)^2 = 0$$

Therefore the condition that will make phase constant a direct form od=f frequency is

$$LG = CR$$

A hypothetical line might be built to fulfill this condition. The line would then have a value of ß obtained by use of the above equation.

Already we know that the formula for the phase constant

$$\beta = \square \sqrt{LC}$$

Then the velocity of propagation will be

$$v = 1/\sqrt{LC}$$

This is the same for the all frequencies, thus eliminating the delay distortion.

We know that the equation for attenuation constant

$$\alpha = \sqrt{\frac{RG - \square^2 LC + \sqrt{(RG - \square^2 LC)^2 + \square^2 \ (LG + CR)}}{2}}$$

May be made independent of frequency if the term under the internal radical is forced to reduce to

$$(RG + \square^2 LC)^2$$

Analysis shows that the condition for the distortion less line $LG = CR$, will produce the desired result, so that it is possible to make attenuation constant and velocity independent of frequency

$$LG = CR$$

$$\frac{L}{C} = \frac{R}{G}$$

Require a very large value of L, since G is small. If G is intentionally increased, α and attenuation are increased, resulting in poor line efficiency.

To reduce R raises the size and cost of the conductors above economic limits, so that the hypothetical results cannot be achieved.

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=xMKW07rnn4c

Important Books/Journals for further learning including the page nos.:

John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.

Page nos.: 250

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LECTURE HANDOUTS

L 32

ECE

II/III

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Reflections on a line not terminated in Z0

Introduction :

If the load impedance is not equal to the source impedance, then all the power that are transmitted from the source will not reach the load end and hence some power is wasted. This is called impedance mismatch condition.

Prerequisite knowledge for Complete understanding and learning of Topic:

- The distortion-less line

Detailed content of the Lecture:

Line not terminated in Z0 :

Reflection occurs because of the following cases:

1) when the load end is open circuited
2) when the load end is short-circuited
3) when the line is not terminated in its characteristic impedance

When the line is either open or short circuited, then there is not resistance at the receiving end to absorb all the power transmitted from the source end. Hence all the power incident on the load gets completely reflected back to the source causing reflections in the line. When the line is terminated in its characteristic impedance, the load will absorb some power and some will be reflected back thus producing reflections.

Assume again we launched a signal $\mathbf{v}_+, \mathbf{i}_+$ at some frequency. Then, $\mathbf{v}_+ / \mathbf{i}_+ = Z_o$. But at the load, $\mathbf{v}_L / \mathbf{i}_L = Z_L$. If $Z_L \neq Z_o$, $\mathbf{v}_L$ cannot be $\mathbf{v}_+$ and $\mathbf{i}_L$ cannot be $\mathbf{i}_+$. The boundary condition at the load can only be satisfied if **there is also a reflected wave** $\mathbf{v}_-, \mathbf{i}_-$. This wave propagates backward in the $-\mathbf{z}$ direction. Then, at any point on the line:

$$\mathbf{v} = V_+ e^{j(\omega t - \beta z)} + V_- e^{j(\omega t + \beta z)} = \mathbf{v}_+ + \mathbf{v}_-$$

$$\mathbf{i} = \frac{V_+}{Z_o} e^{j(\omega t - \beta z)} - \frac{V_-}{Z_o} e^{j(\omega t + \beta z)} = \mathbf{i}_+ + \mathbf{i}_- \qquad \mathbf{i}_- = -\frac{\mathbf{v}_-}{Z_o}$$

Note the $-$ sign on the current of the reflected wave! Now we can satisfy the condition at the load $\mathbf{v}/\mathbf{i} = Z_L$. Only one value of V_- will satisfy.

Let us define:

$$\boxed{\frac{\mathbf{v}_-}{\mathbf{v}_+} = \Gamma, \text{ the reflection coefficient}}$$

Then:

$$\mathbf{v}_+ + \Gamma\,\mathbf{v}_+ = \mathbf{v} \qquad (1)$$

$$\frac{\mathbf{v}_+}{Z_o} - \Gamma\frac{\mathbf{v}_+}{Z_o} = \frac{\mathbf{v}}{Z_L} \qquad (2)$$

We solve for Γ to get: (Problem: Solve to Γ)

$$\boxed{\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}}$$

Note: Γ is **complex, in general**, because Z_L may be complex.

$$-1 \le |\Gamma| \le 1$$

Video Content / Details of website for further learning (if any):

https://www.youtube.com/watch?v=48r450lYX8U

Important Books/Journals for further learning including the page nos.:

John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 256

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LECTURE HANDOUTS

L 33

ECE

II/III

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Reflection coefficient Reflection factor and reflection loss

Introduction :

When the load impedance (ZR) is not equal to characteristic impedance(z0) of the transmission line(i.e ZR≠Z0) reflection takes place.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Line not terminated in Z0

Detailed content of the Lecture:

Reflection factor and reflection loss:

Most practical circuits/devices (whether the device be a transmitting antenna, a TV, a receiver, a microwave amplifier, an oscilloscope, etc.) have complex input impedance. Thus, if such a device is fed by a transmission line, the transmission line sees a complex load. Without something special being done, the line is not matched to the load. There is a reflected wave.

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad \text{(complex, in general)}$$

$$\mathbf{v}_T = V_+ e^{-j\beta z} + \Gamma V_+ e^{+j\beta z}$$

$$\mathbf{i}_T = \frac{V_+}{Z_o} e^{-j\beta z} - \Gamma \frac{V_+}{Z_o} e^{+j\beta z}$$

Assume for simplicity that $\mathbf{v}_+ = V_o \angle 0°$ and that the load is at $Z = 0$. There-fore, on the transmission line, $z = -l$. Then,

$$\mathbf{v}_T = V_o e^{j\beta l} + \Gamma V_o e^{-j\beta l}$$

$$\mathbf{i}_T = \frac{V_o}{Z_o} e^{j\beta l} - \Gamma \frac{V_o}{Z_o} e^{-j\beta l},$$

where ***l* is the distance measured backwards from the load.**

The input impedance at a point l distance back from the load is:

$$Z_{in} = Z_o \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = Z_o \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

Substituting the value of Γ and rearranging, we get the alternate fo

$$Z_{in}(-l) = Z_o \frac{Z_L \cos\beta l + j Z_o \sin\beta l}{Z_o \cos\beta l + j Z_L \sin\beta l} = Z_o \frac{Z_L + j Z_o \tan\beta l}{Z_o + j Z_L \tan\beta l}$$

When $Z_L = Z_o$, we get $Z_{in} = Z_o$.

Now returning to the voltage and current:

$$\mathbf{v} = V_o e^{j\beta l}\left[1 + \Gamma e^{-2j\beta l}\right] \qquad Z_{in} = Z_o \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

$$\mathbf{i} = \frac{V_o}{Z_o} e^{j\beta l}\left[1 - \Gamma e^{-2j\beta l}\right] \qquad \textit{another form}$$

REFLECTION FACTOR

$$K = \frac{|2\sqrt{Z_1 Z_2}|}{|Z_1 + Z_2|}$$

The term K denotes the reflection factor. This ratio indicates the change in current in the load due to reflection at the mismatched junction and is called the reflection factor.

REFLECTION LOSS

Reflection loss is defined as the number of nepers or decibles by which the current in the load under image matched conditions would exceed the current actually flowing in the load. This reflection loss involves the reciprocal of the reflection factor K.

Reflection loss, nepers= $\ln\left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right|$

Reflection loss, db = $20\log\left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right|$

Video Content / Details of website for further learning (if any):
https://www.microwaves101.com/encyclopedias/voltage-standing-wave-ratio-vswr

Important Books/Journals for further learning including the page nos.:
John D Ryder, “Networks Lines and Field” Prentice Hall India, Second Edition 2010.
Page nos.: 265

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LECTURE HANDOUTS

L 34

ECE

II/III

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Standing Waves, Nodes, Standing Wave Ratio

Introduction :

If the transmission is not terminated in its characteristic impedance ,then there will be two waves traveling along the line which gives rise to standing waves having fixed maxima and fixed minima.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Line not terminated in Z0

Detailed content of the Lecture:

STANDING WAVES

When the transmission line is not matched with its load i.e., load impedance is not equal to the characteristic impedance ($Z_R = Z_0$), the energy delivered to the load is reflected back to the source.

The combination of incident and reflected waves give rise to the standing waves.

STANDING-WAVE RATIO

The measurement of standing waves on a transmission line yields information about equipment operating conditions. Maximum power is absorbed by the load when ZL = Z0. If a line has no standing waves, the termination for that line is correct and maximum power transfer takes place.

$$VSWR = \frac{|V_{MAX}|}{|V_{MIN}|}$$

You have probably noticed that the variation of standing waves shows how near the rf line is to being terminated in Z0. A wide variation in voltage along

STANDING WAVE RATIO

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio or voltage standing wave ratio (VSWR)

$$S = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

Voltage equation is

$$V = \frac{V_R(Z_R + Z_0)}{2Z_R}\left[e^{j\beta x} + K e^{-j\beta x}\right]$$

Maxima of voltage occurs at which the incident and reflected waves are in phase

$$V_{max} = \frac{V_R(Z_R + Z_0)}{2Z_R}[1 + K]$$

Minima of voltage occurs at which the incident and reflected waves are out of phase

$$V_{min} = \frac{V_R(Z_R + Z_0)}{2Z_R}[1 - K]$$

$$\frac{V_{max}}{V_{min}} = \frac{1 + |K|}{1 - |K|}$$

$$|K| = \frac{\left|\frac{V_{max}}{V_{min}}\right| - 1}{\left|\frac{V_{max}}{V_{min}}\right| + 1}$$

$$|K| = \frac{|V_{max}| - |V_{min}|}{|V_{min}| + |V_{min}|}$$

Video Content / Details of website for further learning (if any):
https://www.khanacademy.org/science/high-school-physics/waves-and-sound/standing-waves-2/v/standing-waves-on-strings

Important Books/Journals for further learning including the page nos.:
John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 291

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LECTURE HANDOUTS **L 35**

ECE **II/III**

Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Smith Chart and its applications

Introduction : The Smith Chart is a fantastic tool for visualizing the impedance of a transmission line and antenna system as a function of frequency. Smith Charts can be used to increase understanding of transmission lines and how they behave from an impedance viewpoint.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Impedance matching by stub
- Standing Waves, Nodes, Standing Wave Ratio

Detailed content of the Lecture:

Smith Charts were originally developed around 1940 by Phillip Smith as a useful tool for making the equations involved in transmission lines easier to manipulate. See, for instance, the input impedance equation for a load attached to a transmission line of length L and characteristic impedance Z0. With modern computers, the Smith Chart is no longer used to the simplify the calculation of transmission line equatons; however, their value in visualizing the impedance of an antenna or a transmission line has not decreased.

The greatest advantage of the Smith Chart for Transmission Line is that travel along a lossless line corresponds to movement along a correctly drawn constant SWR circle. Close examination of the chart axes shows the chart has been drawn for use with normalized impedance and admittance. This avoids the need to have Smith charts for every imaginable value of line characteristic impedance. (If a particular value of Z0 is employed widely or exclusively, it becomes worthwhile to construct a chart for that, particular value of Z0. For example, the General Radio Company makes a 50-Ω chart for use with its transmission equipment. It may also be used for any other 50-Ω situations and avoids the need for normalization.) Also note that the chart covers a distance of only a half-wavelength, since conditions repeat exactly every half-wavelength on a lossless line. The impedance at 17.716 λ away from a load on a line is exactly the same as the impedance 0.216 λ from that load and can be read from the chart.

Applications of Smith Chart

1. Admittance calculations. This application is based on the fact that the impedance measured at Q is equal to the admittance at P, if P and Q are λ/4 apart and lie on the same SWR circle. This is shown in Figure 7-12. The impedance at Q is 1 – j1, and a very simple calculation shows that if the impedance is 0.5 + j0.5, as it was at P, then the corresponding admittance is indeed 1 – j1, as read off at Q.Since the complete circle of the Smith Chart for Transmission Line, represents a half-wavelength along the line, a quarter-wavelength corresponds to a semicircle. It is not necessary to measure λ/4 around the circle from P, but merely to project the line through P and the center of the chart until it intersects the drawn circle at Q on the other side. (Although such an application is not very important in itself, it has been found of great value in familiarizing students with the chart and with the method of converting it for use as an admittance chart, this being essential for stub calculations.)

2. Calculation of the impedance or admittance at any point, on any transmission line, with any load, and simultaneous calculation of the SWR on the liner This may be done for lossless or lossy lines, but is much easier for lossless lines.

3. Calculation of the length of a short-circuited piece of transmission line to give a required capacitive or inductive reactance. This is done by starting at the point 0, j0 on the left-hand side rim of the chart, and traveling toward the generator until the correct value of reactance is reached. Alternatively, if a susceptance of known value is required, start at the right-hand rim of the chart at the point ∞,j∞ and work toward the generator again. This calculation is always performed in connection with short-circuited stubs.

Before plotting on a Smith Chart we need to study a few terms such as transmission line, characteristic impedance, standing wave, etc.

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=rUDMo7hwihs

Important Books/Journals for further learning including the page nos.:
John D Ryder, "Networks Lines and Field" Prentice Hall India, Second Edition 2010.
Page nos.: 327

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Course Name with Code : **19ECC05- ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel, AP/ECE**

Unit : **IV - Transmission Lines at radio frequency** **Date of Lecture:**

Topic of Lecture: Single stub matching using Smith chart

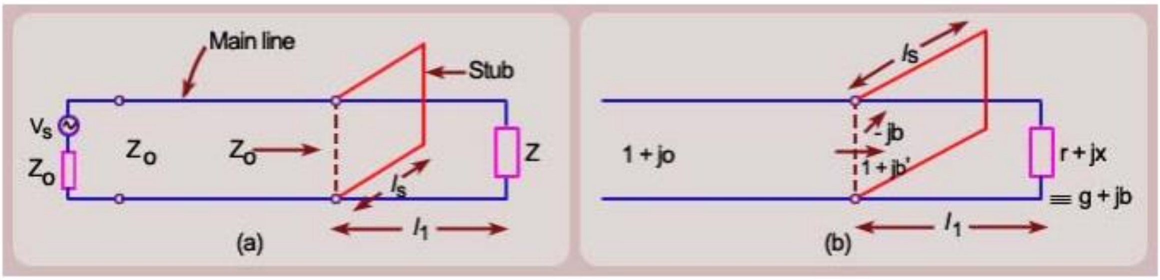
Introduction : For the maximum transfer of power, the sending end impedance and the receiving end impedance of a transmission line should be matched perfect. In practical cases, this impedance matching is not perfect. So a stub is placed on the transmission line and its length position is adjusted for maximum power transfer.

Prerequisite knowledge for Complete understanding and learning of Topic:

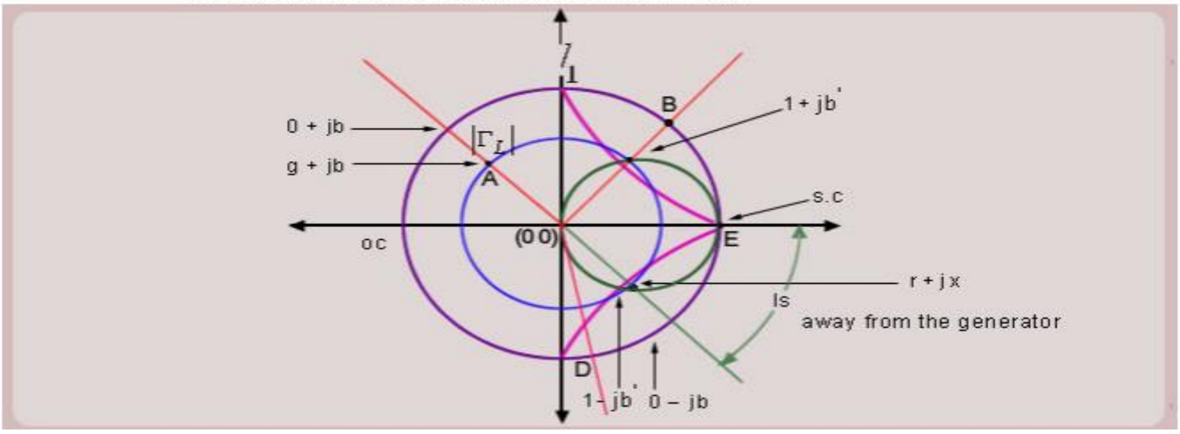
- Impedance matching by stub
- Standing Waves, Nodes, Standing Wave Ratio

Detailed content of the Lecture:

Single-Stub Matching Technique A stub is a short-circuited section of a transmission line connected in parallel to the main transmission line. A stub of appropriate length is placed at some distance from the load such that the impedance seen beyond the stub is equal to the characteristic impedance. Suppose we have a load impedance connected to a transmission line with characteristic impedance (Figure a). The objective here is that no reflection should be seen by the generator. In other words, even if there are standing waves in the vicinity of the load , the standing waves must vanish beyond certain distance from the load. Conceptually this can be achieved by adding a stub to the main line such that the reflected wave from the short-circuit end of the stub and the reflected wave from the load on the main line completely cancel each other at point B to give no net reflected wave beyond point B towards the generator. We make use of Smith chart for this purpose



Since we have a parallel connection of transmission lines, it is more convenient to solve the problem using admittances rather than impedances. To convert the impedance into admittance also we make use of the Smith chart and avoid any analytical calculation. Now onwards treat the Smith chart as the admittance chart.



Matching Procedure

- First mark the load admittance on the admittance smith chart (A).
- Plot the constant circle on the smith chart .Move on the constant circle till you intersect the constant circle this point of intersection corresponds to point (B). The distance traversed on the constant circle is . This is the location of placing the stub on the transmission line from the load end .
- Find constant suseptance circle.
- Find mirror image of the circle to get circle.
- Mark on the outer most circle (D).
- From (D) move circular clockwise upto s.c point (E) to get the stub length .

Advantage

The single-stub matching technique is superior to the quarter wavelength transformer as it makes use of only one type of transmission line for the main line as well as the stub. This technique also in principle is capable of matching any complex load to the characteristic impedance/admittance. The single stub matching technique is quite popular in matching fixed impedances at microwave frequencies.

Drawback

The single stub matching technique although has overcome the drawback of the quarter wavelength transformer technique, it still is not suitable for matching variable impedances. A change in load impedance results in a change in the length as well as the location of the stub. Even if changing length of a stub is a simpler task, changing the location of a stub may not be easy in certain transmission line configurations. For example, if the transmission line is a co-axial cable, the connection of a stub would need drilling of a hole in the outer conductor.

Video Content / Details of website for further learning (if any):
https://www.youtube.com/watch?v=r78seP5upKA

Important Books/Journals for further learning including the page nos.:
John D Ryder, “Networks Lines and Field” Prentice Hall India, Second Edition 2010.
Page nos.: 331

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II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **V – PLANE ELECTROMAGNETIC WAVE** **Date of Lecture:**

Topic of Lecture: Uniform Plane Waves

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Waves and wave equation.

Uniform Plane waves:

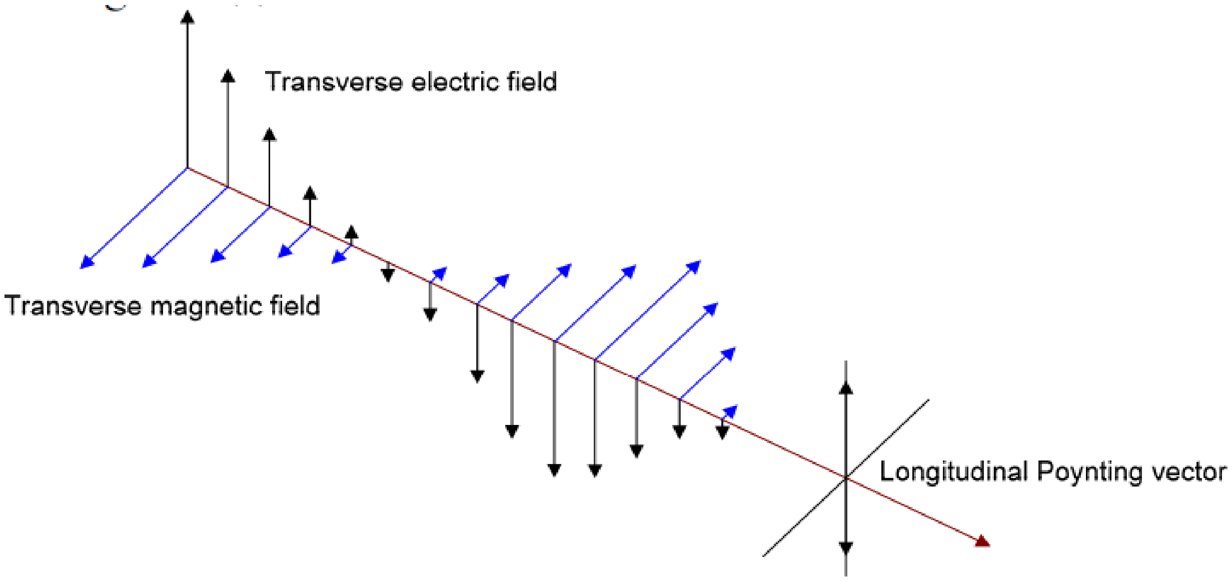
- defines a plane surface and hence the name plane waves
- Since the field strength is uniform everywhere it is also known as uniform plane waves
- A plane wave is a constant-frequency wave whose wave fronts (surfaces of constant phase) are infinite parallel of constant amplitude normal to the phase velocity vector

  Properties of a uniform plane wave:
  - Electric and magnetic field are perpendicular to each other
  - No electric or magnetic field in the direction of propagation (Transverse electromagnetic wave: TEM wave)
  - The value of the magnetic field is equal to the magnitude of the electric field divided by η 0 (~377 Ohm) at every instant

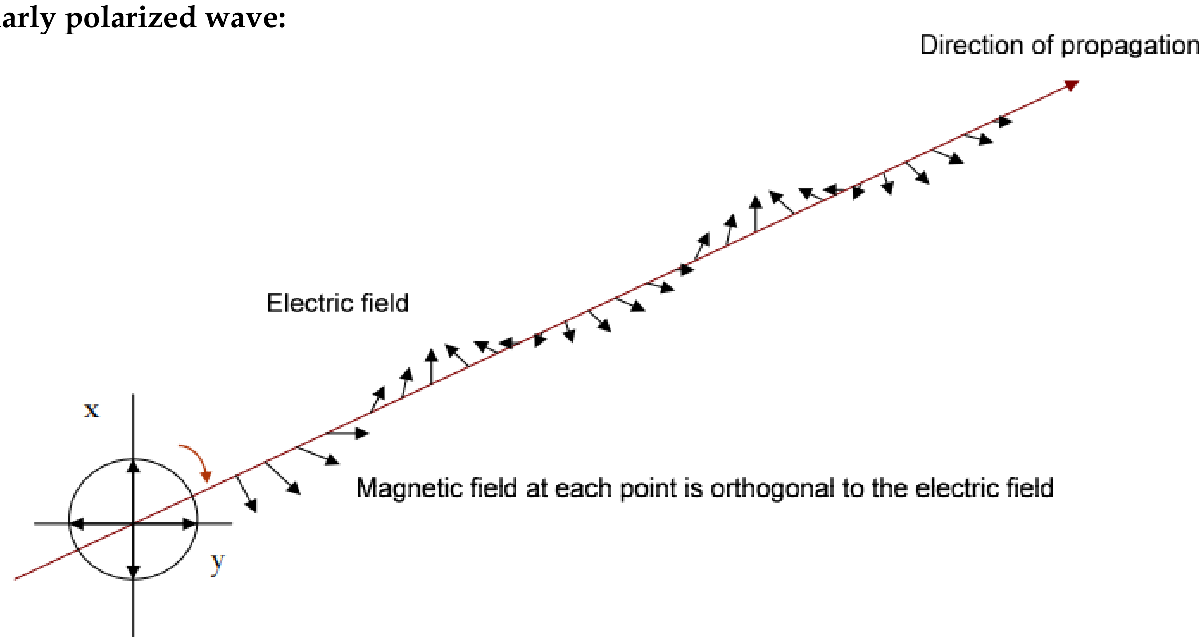
Wave polarization

- Polarization of plane wave refers to the orientation of electric field vector,
- Which may be in fixed direction or may change with time.
- Polarization is the curve traced out by the tip of the arrow representing the instantaneous electric field
- The electric field must be observed along the direction of propagation

Plane wave:



Circularly polarized wave:



Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012. P. No: 3

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ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **V – PLANE ELECTROMAGNETIC WAVE** **Date of Lecture:**

Topic of Lecture: Maxwell's equation in Phasor form
Introduction: Maxwell's Equations in Time-Harmonic Form. This is known as phasor form or the time-harmonic form of Maxwell's Equations. It is perfectly legitimate, because this form tells us how the waves behave if they are oscillating at frequency f, and all waves can be decomposed into the sum of simple oscillating waves.
Prerequisite knowledge for Complete understanding and learning of Topic: • Vector Algebra • Maxell's Equations

Maxwell's equation in phasor form:

- Maxwell's Equations are commonly written in a few different ways.

$$1.\quad \nabla \cdot \mathbf{D} = \rho_V$$

$$2.\quad \nabla \cdot \mathbf{B} = 0$$

$$3.\quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$4.\quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

- The above equations are known as "point form" because each equality is true at every point in space. However, if we integrate the point form over a volume, we obtain the integral form. There is also Time-Harmonic Form, and Maxwell's Equations written only with **E** and **H**. And one form uses imaginary magnetic charge, which can be useful for some problem solving.

- If the point forms of Maxwell's Equations are true at every point, then we can integrate them over any volume (*V*) or through any surface and they will still be true.

$$\int_V \nabla \cdot \mathbf{A}\, dV = \oint_S \mathbf{A} \cdot \mathbf{dS} \qquad \text{[Divergence Theorem]}$$

$$\int (\nabla \times \mathbf{A}) \cdot \mathbf{dS} = \oint_L \mathbf{A} \cdot \mathbf{dL} \qquad \text{[Stokes' Theorem]}$$

Note: These Theorems are true for any vector field

- Maxwell's Equations in Integral Form.

1. $\oint\!\!\oint_S \mathbf{D}\cdot d\mathbf{S} = Q_{enc}$ = Amount of Charge Within Surface S

2. $\oint\!\!\oint_S \mathbf{B}\cdot d\mathbf{S} = 0$

3. $\oint_L \mathbf{E}\cdot d\mathbf{L} = -\iint_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$

4. $\oint_L \mathbf{H}\cdot d\mathbf{L} = I_{enc} + \iint_S \frac{\partial \mathbf{D}}{\partial t}\cdot d\mathbf{S}$

$$[I_{enc} = \iint_S \mathbf{J}\cdot d\mathbf{S}]$$

www.maxwells-equations.com

Time-Harmonic Form of Maxwell's Equations

- We know from the theory of Fourier Transforms that every signal in time can be rewritten as the sum of sinusoids (sign or cosine). Using a little more complex math and we can specify the time variation in terms of the sum of sinusoids written in complex form:

$$e^{i\omega t} = e^{i2\pi ft} \qquad i = \sqrt{-1}$$

- In the previous Equation, f is the frequency we are interested in, which is equal to $\omega = 2\pi f$. Hence, the time derivative of the function in the above Equation is the same as the original function multiplied by $i\omega$. This means we can replace the time-derivatives in the point-form of Maxwell's Equations in integral form as in the following:

1. $\nabla\cdot\mathbf{D} = \rho_V$
2. $\nabla\cdot\mathbf{B} = 0$
3. $\nabla\times\mathbf{E} = -i\omega\mathbf{B} = -i2\pi f\cdot\mathbf{B}$
4. $\nabla\times\mathbf{H} = i\omega\mathbf{D} + \mathbf{J}$

- This is known as phasor form or the time-harmonic form of Maxwell's Equations. It is perfectly legitimate, because this form tells us how the waves behave if they are oscillating at frequency f, and all waves can be decomposed into the sum of simple oscillating waves.

Video Content / Details of website for further learning (if any):

- http://www.maxwells-equations.com/forms.php

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 33

Course Faculty

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LECTURE HANDOUTS

L 39

ECE

II / III

Course Name with Code : 19ECC05 – ELECTROMAGNETIC FIELDS

Course Faculty : Mr.A.Kumaravel

Unit : V – PLANE ELECTROMAGNETIC WAVE **Date of Lecture:**

Topic of Lecture: Wave equations in phasor form

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Wave equations in phasor form :

Maxwell's Equations M1-3 are as for insulators:

$$\nabla.\mathbf{D} = \rho_C \qquad \nabla.\mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

but M4 in conductors includes a free current density $\mathbf{J}_C = \sigma\mathbf{E}$:

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J_C}$$

$$\nabla \times \mathbf{B} = \mu_0\epsilon_r\epsilon_0\frac{\partial \mathbf{E}}{\partial t} + \mu_0\sigma\mathbf{E}$$

where we have again assumed that $\mu_r = 1$

Taking the curl of M3:

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = \nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E}$$

using M1 with the assumption that the free charge density is zero:

$$\nabla \rho_c = \epsilon_r \epsilon_0 \nabla(\nabla.\mathbf{E}) = 0$$

Subtituting for **B** from M4 leads to a modified wave equation:

$$\nabla^2\mathbf{E} = \frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = \mu_0 \epsilon_r \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \sigma \frac{\partial \mathbf{E}}{\partial t}$$

The first order time derivative is proportional to the conductivity σ

This acts as a damping term for waves in conductors

Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

Course Faculty

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LECTURE HANDOUTS | **L 40**

ECE | **II / III**

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **V – PLANE ELECTROMAGNETIC WAVES** **Date of Lecture:**

Topic of Lecture: Plane waves in free space and in a homogenous material

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Reflection of plane waves by a perfect dielectric at Normal incidence:

Reflection at a dielectric boundary

An electromagnetic wave of real (positive) frequency ω can be written

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0\,\mathrm{e}^{\,i\,(\mathbf{k}\cdot\mathbf{r}-\omega\,t)},$$

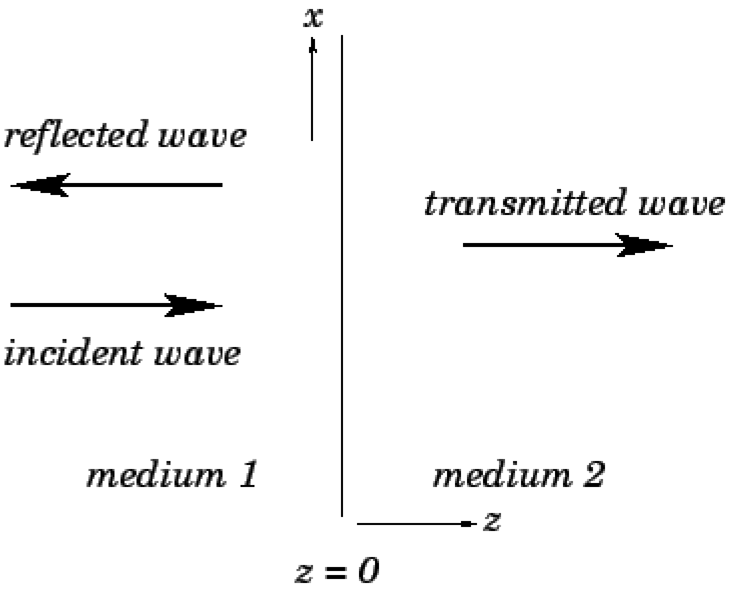
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0\,\mathrm{e}^{\,i\,(\mathbf{k}\cdot\mathbf{r}-\omega\,t)}.$$

The wave-vector, $\mathbf{k}$, indicates the direction of propagation of the wave, and also its phase-velocity, v, via

$$v = \frac{\omega}{k}.$$

Since the wave is transverse in nature, we must have $\mathbf{E}_0\cdot\mathbf{k} = \mathbf{B}_0\cdot\mathbf{k} = 0$. Finally, the familiar Maxwell equation

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$



Without loss of generality, we can assume that the incident wave is polarized in the x direction. Hence, the incident wave can be written

$$\mathbf{E}(z,t) = E_i\, \mathrm{e}^{\,i\,(k_1\, z - \omega\, t)}\, \hat{\mathbf{x}},$$

$$\mathbf{B}(z,t) = \frac{E_i}{v_1}\, \mathrm{e}^{\,i\,(k_1\, z - \omega\, t)}\, \hat{\mathbf{y}},$$

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Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

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LECTURE HANDOUTS

L 41

ECE

II / III

Course Name with Code : 16ECD03 – ELECTROMAGNETIC FIELDS

Course Faculty : Mr.A.Kumaravel

Unit : V – PLANE ELECTROMAGNETIC WAVE **Date of Lecture:**

Topic of Lecture: Wave equation for a conducting medium

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Propagation of Electromagnetic Waves in a Conducting Medium:

We will consider a plane electromagnetic wave travelling in a linear dielectric medium such as air along the z direction and being incident at a conducting interface. The medium will be taken to be a linear medium. So that one can describe the electrodynamics using only the E and H vectors. We wish to investigate the propagation of the wave in the conducting medium.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu\frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon\frac{\partial \vec{E}}{\partial t} = \sigma\vec{E} + \epsilon\frac{\partial \vec{E}}{\partial t}$$

Let us take $\vec{E}, \vec{H}$ and $\vec{k}$ to be respectively in x, y and z direction. We the nhave,

$$(\nabla \times \vec{E})_y = \frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t}$$

i.e.,

$$\frac{\partial E_x}{\partial z} + \mu\frac{\partial H_y}{\partial t} = 0 \qquad (1)$$

and

$$(\nabla \times \vec{H})_x = -\frac{\partial H_y}{\partial z} = \sigma E_x + \epsilon\frac{\partial E_x}{\partial t}$$

i.e.

$$\frac{\partial H_y}{\partial z} + \sigma E_x + \epsilon\frac{\partial E_x}{\partial t} = 0 \quad (2)$$

where A, B, C and D are constants to be determined. If the values of the electric field at z=0 is E_0 and that of the magnetic field at z=0 is H_0, we have $A = E_0$ and $C = H_0$.
In order to determine the constants B and D, let us return back to the original first order equations (1) and (2)

$$\frac{\partial E_x}{\partial z} + i\mu\omega H_y = 0$$
$$\frac{\partial H_y}{\partial z} + \sigma E_x + i\omega\epsilon E_x = 0$$

Substituting the solutions for E and H

$$\gamma E_0 \sinh(\gamma z) + B\gamma \cosh(\gamma z) + i\omega\mu(H_0 \cosh(\gamma z) + D \sinh(\gamma z)) = 0$$

This equation must remain valid for all values of z, which is possible if the coefficients of *sinh* and cosh terms are separately equated to zero,

$$E_0\gamma + i\omega\mu D = 0$$
$$B\gamma + i\omega\mu H_0 = 0$$

The former gives,

where A, B, C and D are constants to be determined. If the values of the electric field at z=0 is E_0 and that of the magnetic field at z=0 is H_0, we have $A = E_0$ and $C = H_0$.
In order to determine the constants B and D, let us return back to the original first order equations (1) and (2)

$$\frac{\partial E_x}{\partial z} + i\mu\omega H_y = 0$$
$$\frac{\partial H_y}{\partial z} + \sigma E_x + i\omega\epsilon E_x = 0$$

Substituting the solutions for E and H

$$\gamma E_0 \sinh(\gamma z) + B\gamma \cosh(\gamma z) + i\omega\mu(H_0 \cosh(\gamma z) + D \sinh(\gamma z)) = 0$$

This equation must remain valid for all values of z, which is possible if the coefficients of *sinh* and cosh terms are separately equated to zero,

$$E_0\gamma + i\omega\mu D = 0$$
$$B\gamma + i\omega\mu H_0 = 0$$

The former gives,

Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf

Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

Course Faculty

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LECTURE HANDOUTS

L 42

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **V – PLANE ELECTROMAGNETIC WAVES** **Date of Lecture:**

Topic of Lecture: Propagation in good conductors

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Plane waves in good conductors:

in a good conductor
(a) higher the frequency, lower is the skin depth
(b) higher is the conductivity, lower is the skin depth and
(c) higher is the permeability, lower is the skin depth

The solution can be written as an attenuated plane wave:

$$E_x = E_0 e^{i(\omega t - \beta z)} e^{-\alpha z}$$

Substituting this back into the modified wave equation:

$$(-i\beta - \alpha)^2 E_0 = \mu_0 \epsilon_r \epsilon_0 (i\omega)^2 E_0 + \mu_0 \sigma (i\omega) E_0$$

Equating the real and imaginary parts:

$$-\beta^2 + \alpha^2 = -\mu_0 \epsilon_r \epsilon_0 \omega^2 \qquad 2\beta\alpha = \mu_0 \sigma \omega$$

For a **good conductor** with $\sigma \gg \epsilon_r \epsilon_0 \omega$:

$$\alpha = \beta = \sqrt{\frac{\mu_0 \sigma \omega}{2}}$$

For a perfect conductor $\sigma \to \infty$ *there are no waves and* $\mathbf{E} = 0$

- Taking the real part, we have,

$$E_x(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

- Substituting the values of α and β for good conductors, we have,

$$E_x(z,t) = E_0 e^{-\sqrt{\pi f \mu \sigma} z} \cos\left(\omega t - \sqrt{\pi f \mu \sigma} z\right)$$

- Now using the point form of Ohm's law for conductors, we can write

$$J_x = \sigma E_x(z,t) = \sigma E_0 e^{-\sqrt{\pi f \mu \sigma} z} \cos\left(\omega t - \sqrt{\pi f \mu \sigma} z\right)$$

the phase velocity and wavelength inside a good conductor is

$$v_p = \frac{\omega}{\beta} = \omega\delta; \lambda = \frac{2\pi}{\beta} = 2\pi\delta$$

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Important Books/Journals for further learning including the page nos.:

- William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

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LECTURE HANDOUTS

L 43

ECE

II / III

Course Name with Code : 19ECC05 – ELECTROMAGNETIC FIELDS

Course Faculty : Mr.A.Kumaravel

Unit : V – PLANE ELECTROMAGNETIC WAVES **Date of Lecture:**

Topic of Lecture: Skin effect. Group velocity

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.
The short answer is that conductivity means that there are free electrons, and free electrons will always react to electromagnetic waves. According to Lenz's Law, these reactions will always be such that they diminish or cancel the electromagnetic waves. So even before we do the math, we know that conductivity will cause electromagnetic waves to sinusoidally decay.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Plane waves in lossy dielectrics:

A lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to the imperfect dielectric. A lossy dielectric is a partially conducting medium.

- **Vector wave equation or vector Helmholtz's equation**

(i) For field: $\nabla^2 \overline{E}_S - \gamma^2 \overline{E}_S = 0$

(ii) For field: $\nabla^2 \overline{H}_S - \gamma^2 \overline{H}_S = 0$

- Where **γ** is Propagation constant of medium and $\mathbf{E_S}$, $\mathbf{H_S}$ is Source electric and magnetic fields respectively.

$\gamma = \alpha + j\beta \quad and \quad \gamma^2 = j\omega\mu(\sigma + j\omega \in)$

- Where α is Attenuation constant (Neper/m) and β is Phase constant (rad/m).

$$\alpha = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\in}\right]^2}-1\right]}\ ;\ \beta = \omega\sqrt{\frac{\mu\in}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\in}\right]^2}+1\right]}$$

- **Field equation of EM wave in s-domain**

$$E_S(z) = E_0 e^{-\alpha z} e^{j(\omega t-\beta z)}\hat{i}_x$$
$$H_S(z) = H_0 e^{-\alpha z} e^{j(\omega t-\beta z)}\hat{i}_y$$

- **Field equation of EM wave in time domain**

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t-\beta z)\hat{i}_x$$
$$H(z,t) = H_0 e^{-\alpha z}\cos(\omega t-\beta z)\hat{i}_y$$

Skin Effect:

Skin effect is the tendency of an alternating electric current (AC) to become distributed within a conductor such that the current density is largest near the surface of the conductor and decreases exponentially with greater depths in the conductor.

The electric current flows mainly at the "skin" of the conductor, between the outer surface and a level called the **skin depth**. Skin depth depends on the frequency of the alternating current; as frequency increases, current flow moves to the surface, resulting in less skin depth. Skin effect reduces the effective cross-section of the conductor and thus increases its effective resistance.
The skin depth is thus defined as the depth below the surface of the conductor at which the current density has fallen to 1/e (about 0.37) of J_S.

The general formula for the skin depth is:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}\ \sqrt{\sqrt{1+(\rho\omega\varepsilon)^2}+\rho\omega\varepsilon}$$

where

ρ = resistivity of the conductor
ω = angular frequency of current = $2\pi f$, where f is the frequency.
μ = permeability of the conductor, $\mu_r\mu_0$
μ_r = relative magnetic permeability of the conductor
μ_0 = the permeability of free space
ε = permittivity of the conductor, $\varepsilon_r\varepsilon_0$
ε_r = relative permittivity of the conductor
ε_0 = the permittivity of free space

At frequencies much below $1/\rho\epsilon$ the quantity inside the large radical is close to unity and the formula is more usually given as:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}.$$

Video Content / Details of website for further learning (if any):
• http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf
Important Books/Journals for further learning including the page nos.:
• William H. Hayt, J A Buck, Engineering Electromagnetics, Tata McGraw-Hill, 7th Edition, 2012.

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Rasipuram - 637 408, Namakkal Dist., Tamil Nadu

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LECTURE HANDOUTS

L 44

ECE

II / III

Course Name with Code : **19ECC05 – ELECTROMAGNETIC FIELDS**

Course Faculty : **Mr.A.Kumaravel**

Unit : **V – PLANE ELECTROMAGNETIC WAVE** **Date of Lecture:**

Topic of Lecture: Electromagnetic power flow and Poynting vector

Introduction: Poynting vector, a quantity describing the magnitude and direction of the flow of energy in electromagnetic waves. It is named after English physicist John Henry Poynting, who introduced it in 1884.The Poynting vector *S* is defined as to be equal to the cross product (1/μ)E × B, where μ is the permeability of the medium through which the radiation passes (*see* magnetic permeability), E is the amplitude of the electric field, and B is the amplitude of the magnetic field.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Electrostatics
- Magnetostatics

Poynting Vector and the flow of power:

Poynting Vector and Power Flow in Electromagnetic Fields: Electromagnetic waves can transport energy as a result of their travelling or propagating characteristics. Starting from Maxwell's Equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Together with the vector identity

$$\nabla . \left(\vec{E} \times \vec{H}\right) = \vec{H} . \nabla \times \vec{E} - \vec{E} . \nabla \times \vec{H}$$

One can write

$$\nabla . \left(\vec{E} \times \vec{H}\right) = -\vec{H} . \frac{\partial \vec{B}}{\partial t} - \vec{E} . \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$\nabla.\left(\vec{E}\times\vec{H}\right) = -\vec{H}.\frac{\partial\vec{B}}{\partial t} - \vec{E}.\vec{J} - \vec{E}.\frac{\partial\vec{D}}{\partial t}$$

In simple medium where $\in, \mu$ and σ are constant,

$$\vec{H}.\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$$

$$\vec{E}.\frac{\partial\vec{D}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\mu E^2\right) \quad \text{and} \quad \vec{E}.\vec{J} = \sigma E^2$$

$$\nabla.\left(\vec{E}\times\vec{H}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2$$

Divergence theorem states,

$$\oint_S \left(\vec{E}\times\vec{H}\right).d\vec{S} = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right)dV - \int_V \sigma E^2 dV$$

This equation is referred to as Poynting theorem and it states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume minus the conduction losses.

In the equation, the following term represents the rate of change of the stored energy in the electric and magnetic fields

$$\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\in E^2 + \frac{1}{2}\mu H^2\right)dV$$

On the other hand, the power dissipation within the volume appears in the following form

$$\int_V \sigma E^2 dV$$

Hence the total decrease in power within the volume under consideration:

$$\oint_S \left(\vec{E}\times\vec{H}\right).d\vec{S} = \oint_S \vec{P}.d\vec{S}$$

Here $\vec{P} = \vec{E}\times\vec{H}$ (W/mt^2) is called the Poynting vector and it represents the power density vector associated with the electromagnetic field. The integration of the Poynting vector over any closed surface gives the net power flowing out of the surface.

Video Content / Details of website for further learning (if any):

- https://acikders.ankara.edu.tr/mod/resource/view.php?id=49914

Important Books/Journals for further learning including the page nos.:

- M.N.O.Sadiku, Elements of Engineering Electromagnetics; Oxford University Press;4th Edition, 2006. P.No: 51

Course Faculty

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Course Name with Code : 16ECD03 – ELECTROMAGNETIC FIELDS

Course Faculty : Mr.A.Kumaravel

Unit : V – PLANE ELECTROMAGNETIC WAVES **Date of Lecture:**

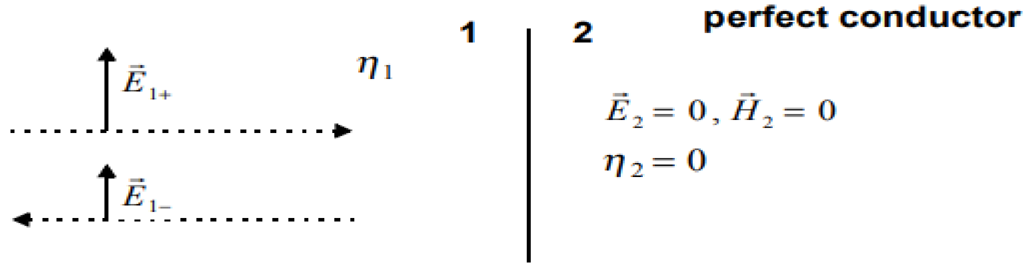
Topic of Lecture: Normal incidence at a plane conducting boundary

Introduction: Waves with constant amplitude is called plane waves. plane waves with uniform phase is called uniform plane waves.
The short answer is that conductivity means that there are free electrons, and free electrons will always react to electromagnetic waves. According to Lenz's Law, these reactions will always be such that they diminish or cancel the electromagnetic waves. So even before we do the math, we know that conductivity will cause electromagnetic waves to sinusoidally decay.

Prerequisite knowledge for Complete understanding and learning of Topic:

- Plane Waves and wave equation.

Normal Incidence Plane Wave Reflection at Perfect Conductor



At the boundary, since $\vec{E}_2$ and $\vec{H}_2$ are both 0, then:

$$\vec{E}_{tot} = \vec{E}_{1+} + \vec{E}_{1-} = 0 \qquad E \text{ tangential} = 0 \text{ (no charge)}$$

Solution exists for $\vec{E}_{1+} = -\vec{E}_{1-} = E$

Then, $\vec{E}_{tot}(z) = \vec{E}_{1+}\left(e^{-j\beta_1 z} - e^{+j\beta_2 z}\right)\hat{e}_x = -2\,j\,E\sin\beta_1 z\,\hat{e}_x$

This is our old friend, the standing wave! Let's check ρ and τ.

Recall that $\rho = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$, since $\eta_2 = 0$ and $\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1} = 0$.

The total field is: $\vec{E}_{tot}(z,t) = \mathrm{Re}\left[e^{j\omega}\vec{E}_{tot}(z)\right] = 2\sin(\beta_1 z)\sin\omega t\,\hat{e}_x$.

$$\vec{H}_{tot}(z) = \left(\frac{E_+}{\eta_1} e^{-j\beta_1 z} + \frac{E_-}{-\eta_1} e^{+j\beta_2 z} \right) \hat{e}_y$$

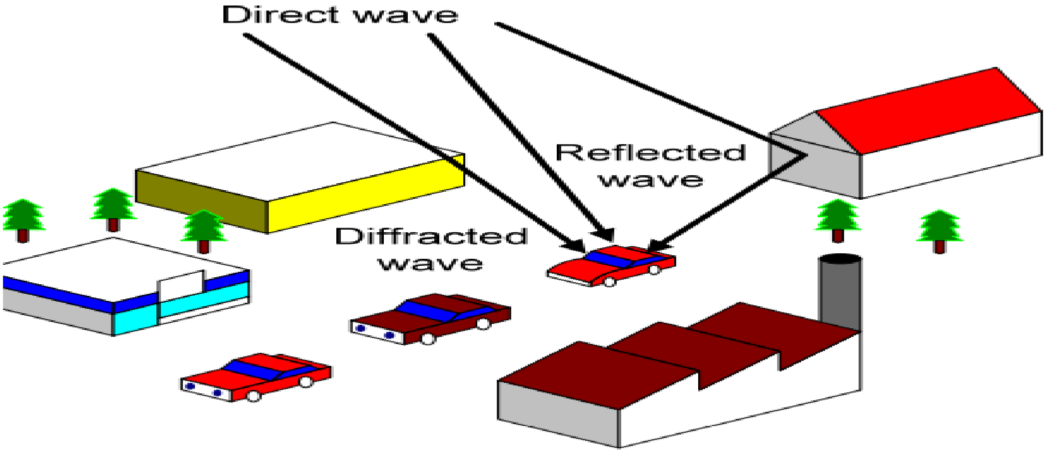
But the boundary conditions on E have that $E_+ = -E_-$.

Then:

$$\vec{H}_{tot}(z) = \frac{E_+}{\eta_1}\left(e^{-j\beta_1 z} + e^{+j\beta_2 z}\right)\hat{e}_y = 2\frac{E_+}{\eta_1}\cos\beta z\,\hat{e}_y$$

or, in the time domain representation:

$$\vec{H}_{tot}(z,t) = 2\frac{E_+}{\eta_1}\cos\beta z\cos\omega t\,\hat{e}_y$$



Video Content / Details of website for further learning (if any):

- http://ocw.utm.my/file.php/213/CHAPTER_5_Electromagnetic_Introduction_and_plane waves.pdf

Important Books/Journals for further learning including the page nos.:

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